ECE 2210    First-Order Transient Examples

Ex1  a) Find the expression for $v_c(t)$ if the switch is closed at time $t = 0$ and $v_c(0) = 0$.

$$v_c(t) = v_c(\infty) + \left(v_c(0) - v_c(\infty)\right) e^{-\frac{t}{\tau}}$$

redraw to find $v_c(\infty)$

$$v_c(\infty) = 9\text{ V}$$

b) What is the voltage across the capacitor, $C$, at $t = 0.1\text{ ms}$?

$$v_c(25\text{ \mu s}) = 9\text{ V} - 9\text{ V} \cdot e^{-\frac{100\text{ \mu s}}{60\text{ \mu s}}} = 7.3\text{ V}$$

c) When will the current through the resistor be $i_R = 5\text{ mA}$?

$$i_R(\infty) = 0\text{ mA} \quad i_R(0) = \frac{9.\text{ V}}{R} = 15\text{ mA} \quad \text{found from drawing}$$

$$i_R(t) = i_R(\infty) + \left(i_R(0) - i_R(\infty)\right) e^{-\frac{t}{\tau}}$$

$$= 0\text{ mA} + (15\text{ mA} - 0\text{ mA}) e^{-\frac{t}{60\text{ \mu s}}}$$

$$= 10.976\text{ mA} \cdot e^{-\frac{t}{60\text{ \mu s}}} = 5\text{ mA} \quad \text{at some time, t}$$

Solve for $t = -\tau \ln \left(\frac{5\text{ mA}}{15\text{ mA}}\right) = 65.92\text{ \mu s}$

d) When will the current through the resistor be $i_R = 20\text{ mA}$?

Since the initial condition is about $15\text{ mA}$ and the final condition is $0\text{ mA}$, $i_R$ will never be $20\text{ mA}$.

Ex2  A 1000 \mu F capacitor has an initial charge of 12 volts. A 20-\Omega resistor is connected across the capacitor at time $t = 0$. Find the energy dissipated by the resistor in the first 5 time constants.

After 5 time constants nearly all of the energy initially stored in the capacitor will be dissipated by the resistor.

$$C := 1000\text{ \mu F} \quad V_C := 12\text{ V} \quad W_C := \frac{1}{2}C \cdot V^2 \quad W_C = 0.072\text{ J}$$

You can get to this answer just by knowing a little about the exponential curve, but what if you want a more accurate answer? Then you'll have to find the remaining voltage across the capacitor at $t = 5t$ and subtract the energy left in the capacitor at that time.

$$v_C(0) = 12\text{ V} \quad v_C(\infty) = 0\text{ V} \quad v_C(t) = v_C(\infty) + \left(v_C(0) - v_C(\infty)\right) e^{-\frac{t}{\tau}}$$

$$= 0\text{ V} + (12\text{ V} - 0\text{ V}) e^{-\frac{t}{\tau}} = 12\text{ V} e^{-\frac{t}{\tau}}$$

at $t = 5t$: $v_C(5t) = 12\text{ V} \cdot e^{-5} = 81\text{ mV}$

$$\frac{1}{2}C \cdot (81\text{ mV})^2 = 3.281 \cdot 10^{-6}\text{ J}$$

Not surprisingly, this makes no significant difference:

$$W_R = W_C - \frac{1}{2}C \cdot (81\text{ mV})^2 = 0.072\text{ J}$$
Ex3 The capacitor is initially uncharged. The switch is in the upper position from 0 to 2\,\text{ms} and is switched down at time \( t = 2\,\text{ms} \).

a) What is the capacitor voltage, \( v_C(t) \)

First interval

\[ v_C(0) = 0 \, \text{V} \]
\[ v_C(\infty) = 24 \, \text{V} \]

\[ v_C(t) = v_C(\infty) + \left( v_C(0) - v_C(\infty) \right) e^{-\frac{t}{\tau}} = 24 \, \text{V} + (0 \, \text{V} - 24 \, \text{V}) e^{-\frac{t}{1.08 \, \text{ms}}} \]

at \( t = 2 \, \text{ms} \):
\[ 24 \, \text{V} - 24 \, \text{V} e^{-\frac{2 \, \text{ms}}{1.08 \, \text{ms}}} = 20.23 \, \text{V} \]

Second interval, define a new time, \( t' = t - 2 \, \text{ms} \)

\[ v_C(0) = 20.23 \, \text{V} \]
\[ v_C(\infty) = 10 \, \text{V} \]

\[ v_C(t') = v_C(\infty) + \left( v_C(0) - v_C(\infty) \right) e^{-\frac{t'}{\tau'}} = 10 \, \text{V} + (20.23 \, \text{V} - 10 \, \text{V}) e^{-\frac{t'}{0.96 \, \text{ms}}} = 10 \, \text{V} + 10.23 \, \text{V} e^{-\frac{t'}{0.96 \, \text{ms}}} \]

\[ 0 < t < 2 \, \text{ms} \]
\[ V_C(t) = 24 \, \text{V} - 24 \, \text{V} e^{-\frac{2 \, \text{ms}}{1.08 \, \text{ms}}} \]

\[ t > 2 \, \text{ms} \]
\[ V_C(t) = 10 \, \text{V} + 10.23 \, \text{V} e^{-\frac{t - 2 \, \text{ms}}{0.96 \, \text{ms}}} \]

b) When is voltage across the capacitor 12\,\text{V} AND getting smaller?

\[ 12 \, \text{V} = 10 \, \text{V} + 10.23 \, \text{V} e^{-\frac{t}{0.96 \, \text{ms}}} \]

\[ \frac{12 \, \text{V} - 10 \, \text{V}}{10.23 \, \text{V}} = e^{-\frac{t}{0.96 \, \text{ms}}} \ln \left( \frac{12 \, \text{V} - 10 \, \text{V}}{10.23 \, \text{V}} \right) = -\left( \frac{t}{0.96 \, \text{ms}} \right) \ln \left( \frac{12 \, \text{V} - 10 \, \text{V}}{10.23 \, \text{V}} \right) = 1.57 \, \text{ms} \]

\[ 2 \, \text{ms} + 1.57 \, \text{ms} = 3.57 \, \text{ms} \]
**Ex4**  

a) Find the complete expression for $i_L(t)$.

Before the switch closes, $t = 0^-$

![Circuit Diagram](image)

$\tau := \frac{L}{R_{Th}}$

$\tau = 100 \mu s$

$$i_L(t) = i_L(\infty) + \left( i_L(0) - i_L(\infty) \right) e^{-\frac{t}{\tau}}$$

$$= 375 \text{mA} + (0 \text{mA} - 375 \text{mA}) e^{-\frac{t}{100 \mu s}}$$

$$= 375 \text{mA} - 375 \text{mA} e^{-\frac{t}{100 \mu s}}$$

b) When is the voltage across $R_2 = 10 \text{V}$?

Before the switch closes, $t = 0^-$

![Circuit Diagram](image)

$$V_{in} = 11.25 \text{V}$$

$$v_{R2}(0) = \frac{R_2}{R_1 + R_2} V_{in} = 11.25 \text{V}$$

Alternatively, when $V_{R2}(t) = 10 \text{V}$, then $V_{R1}(t) = 5 \text{V}$ and

$$i_L(t) = \frac{5 \text{V}}{R_1} - \frac{10 \text{V}}{R_2} = 83.333 \text{mA}$$

$$t = -\tau \ln \left( \frac{10 \text{V} - 5.625 \text{V}}{5.625 \text{V} - 5.625 \text{V}} \right) = 25 \mu s$$

From drawing above at $t = \infty$

$$v_{R2}(\infty) = v_{R3}(\infty) = \frac{R_{23}}{R_1 + R_{23}} V_{in} = 5.625 \text{V}$$

$$v_{R2}(t) = v_{R2}(\infty) + \left( v_{R2}(0) - v_{R2}(\infty) \right) e^{-\frac{t}{\tau}}$$

$$= 5.625 \text{V} + (11.25 \text{V} - 5.625 \text{V}) e^{-\frac{t}{100 \mu s}}$$

$$= 10 \text{V}$$

at some time, solving for that time...

$$t = -\tau \ln \left( \frac{10 \text{V} - 5.625 \text{V}}{5.625 \text{V} - 5.625 \text{V}} \right) = 25 \mu s$$

$$c)$$ What is the $v_L(t)$ expression?

$$v_L(t) = v_L(\infty) + \left( v_L(0) - v_L(\infty) \right) e^{-\frac{t}{\tau}}$$

$$= 0 \text{V} + (11.25 \text{V} - 0 \text{V}) e^{-\frac{t}{100 \mu s}}$$

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The switch has been closed for a long time and is opened (as shown) at time $t = 0$.

a) Find the complete expression for $i_L(t)$.

Before the switch opens, $t = 0$.

Final time, $t = \infty$

b) Find $i_L$ at time $t = 1.4\tau$. $i_L(1.4\tau) = 112.5\text{ mA} - 62.5\text{ mA}e^{-1.4\tau} = 112.5\text{ mA} - 62.5\text{ mA}e^{-1.4\tau} = 97.088\text{ mA}$

c) At time $t = 1.4\tau$ the switch is closed again. Find the complete expression for $i_L(t')$, where $t'$ starts at $t = 1.4\tau$.

Be sure to clearly show the time constant.