ECE 2210 First-Order Transient Examples

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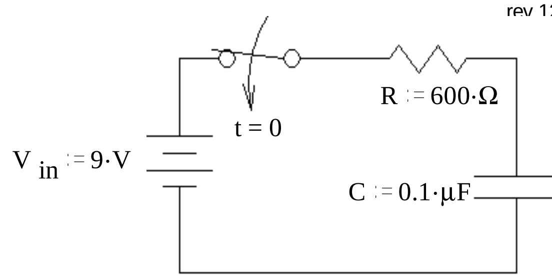
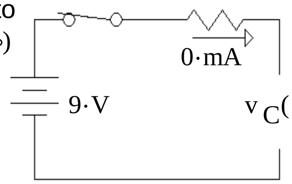
Ex1 a) Find the expression for $v_c(t)$ if the switch is closed at time $t = 0$ and $v_c(0) = 0$.



$$v_C(t) = v_C(\infty) + \left(v_C(0) - v_C(\infty)\right)\cdot e^{-\frac{t}{\tau}}$$

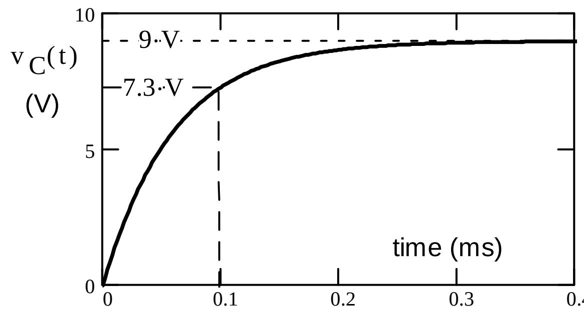
$\tau := R\cdot C \qquad \tau = 60\cdot\mu s$

redraw to find $v_c(\infty)$



$v_C(\infty) = 9\cdot V$

$$v_C(t) = 9\cdot V + (0\cdot V - 9\cdot V)\cdot e^{-\frac{t}{60\cdot\mu s}}$$



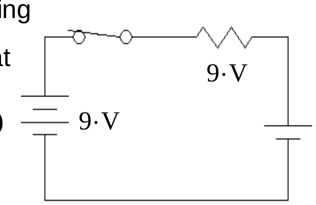
b) What is the voltage across the capacitor, C, at $t = 0.1ms$?

$$v_C(25\cdot\mu s) = 9\cdot V - 9\cdot V\cdot e^{-\frac{100\cdot\mu s}{60\cdot\mu s}} = 7.3\cdot V$$

c) When will the current through the resistor be $i_R := 5\cdot mA$?

$i_R(\infty) = 0\cdot mA \qquad i_R(0) = \frac{9\cdot V}{R} = 15\cdot mA$ found from drawing

redraw at $t = 0^+$ to find $i_R(0)$



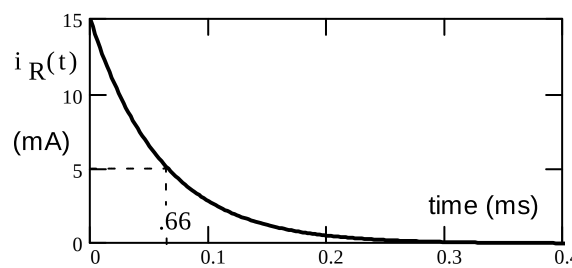
capacitor voltage can't change instantly

$$i_R(t) = i_R(\infty) + \left(i_R(0) - i_R(\infty)\right)\cdot e^{-\frac{t}{\tau}}$$

$$= 0\cdot mA + (15\cdot mA - 0\cdot mA)\cdot e^{-\frac{t}{\tau}}$$

$$= 15\cdot mA\cdot e^{-\frac{t}{60\cdot\mu s}} = 5\cdot mA$$ at some time, t

Solve for $t = -\tau\cdot\ln\left(\frac{5\cdot mA}{15\cdot mA}\right) = 65.92\cdot\mu s$



d) When will the current through the resistor be $i_R := 20\cdot mA$?

Since the initial condition is about $15mA$ and the final condition is $0mA$, i_R will never be $20mA$.

Ex2 A $1000\ \mu F$ capacitor has an initial charge of 12 volts. A $20\text{-}\Omega$ resistor is connected across the capacitor at time $t = 0$. Find the energy dissipated by the resistor in the first 5 time constants.

After 5 time constants nearly all of the energy initially stored in the capacitor will be dissipated by the resistor.

$C := 1000\cdot\mu F \qquad V_C := 12\cdot V \qquad W_C := \frac{1}{2}\cdot C\cdot V_C^{\,2} \qquad W_C = 0.072\cdot joule$

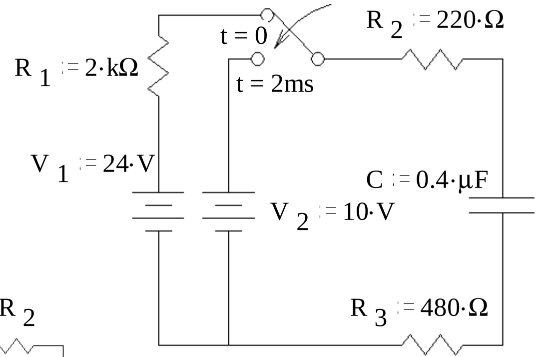
You can get to this answer just by knowing a little about the exponential curve, but what if you want a more accurate answer? Then you'll have to find the remaining voltage across the capacitor at $t = 5t$ and subtract the energy left in the capacitor at that time.

$$v_C(0) = 12\cdot V \quad v_C(\infty) = 0\cdot V \quad v_C(t) = v_C(\infty) + \left(v_C(0) - v_C(\infty)\right)\cdot e^{-\frac{t}{\tau}} = 0\cdot V + (12\cdot V - 0\cdot V)\cdot e^{-\frac{t}{\tau}} = 12\cdot V\cdot e^{-\frac{t}{\tau}}$$

at $t = 5\tau$: $\quad v_C(5\cdot\tau) = 12\cdot V\cdot e^{-5} = 81\cdot mV \qquad \frac{1}{2}\cdot C\cdot(81\cdot mV)^2 = 3.281\cdot 10^{-6}\ \cdot joule$

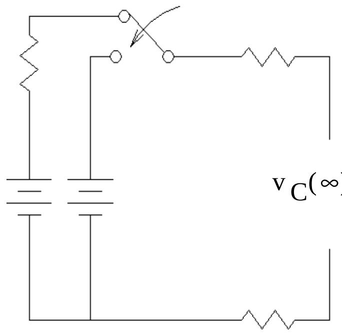
Not surprisingly, this makes no significant difference: $\quad W_R = W_C - \frac{1}{2}\cdot C\cdot(81\cdot mV)^2 = 0.072\cdot joule$

Ex3 The capacitor is initially uncharged. The switch is in the upper position from 0 to 2ms and is switched down at time $t = 2\text{ms}$.

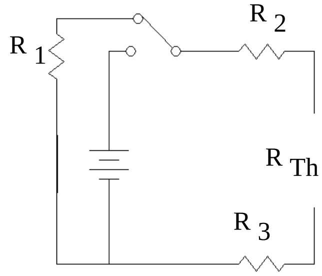


a) What is the capacitor voltage, $v_C(t)$

First interval $v_C(0) = 0\cdot V$



$v_C(\infty) = 24\cdot V$



$R_{Th} := R_1 + R_2 + R_3$

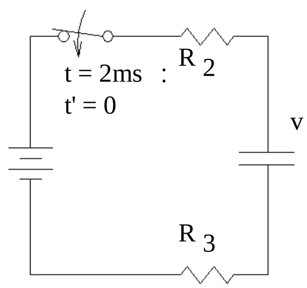
$\tau := R_{Th}\cdot C$

$\tau = 1.08\cdot\text{ms}$

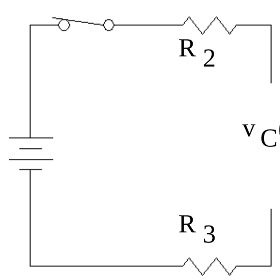
$$v_C(t) = v_C(\infty) + \left(v_C(0) - v_C(\infty)\right)\cdot e^{-\frac{t}{\tau}} = 24\cdot V + (0\cdot V - 24\cdot V)\cdot e^{-\frac{t}{1.08\cdot ms}}$$

at 2ms: $24\cdot V - 24\cdot V\cdot e^{-\frac{2\cdot ms}{1.08\cdot ms}} = 20.23\cdot V$

Second interval, define a new time, $t' = t - 2\text{ms}$



$v_C(0) = 20.23\cdot V$
$t' = 0$



$v_C(\infty) = 10\cdot V$
$t' = \infty$

R_{Th}

$\tau' := \left(R_2 + R_3\right)\cdot C$

$\tau' = 0.28\cdot\text{ms}$

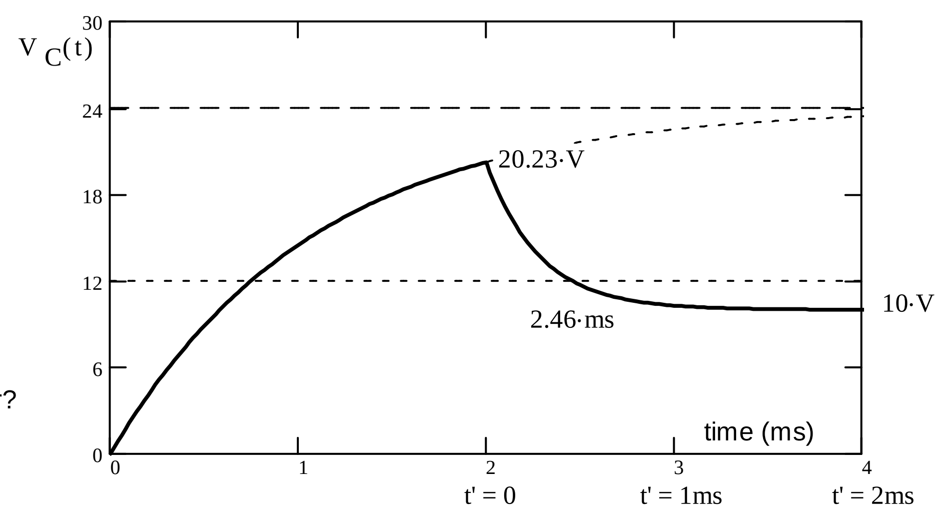
$$v_C(t') = v_C(\infty) + \left(v_C(0) - v_C(\infty)\right)\cdot e^{-\frac{t'}{\tau'}} = 10\cdot V + (20.23\cdot V - 10\cdot V)\cdot e^{-\frac{t'}{0.28\cdot ms}} = 10\cdot V + 10.23\cdot V\cdot e^{-\frac{t-2\cdot ms}{0.28\cdot ms}}$$

$0 < t < 2\text{ms}$

$V_C(t) = 24\cdot V - 24\cdot V\cdot e^{-\frac{2\cdot ms}{1.08\cdot ms}}$

$t > 2\text{ms}$

$V_C(t) = 10\cdot V + 10.23\cdot V\cdot e^{-\frac{t-2\cdot ms}{0.28\cdot ms}}$



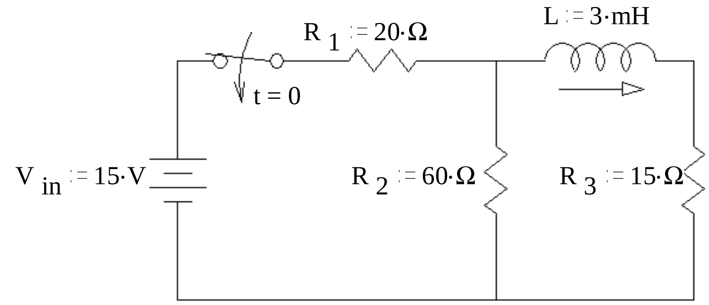
b) When is voltage across the capacitor 12V AND getting smaller?

$$12\cdot V = 10\cdot V + 10.233\cdot V\cdot e^{-\frac{t_{12}}{0.28\cdot ms}}$$

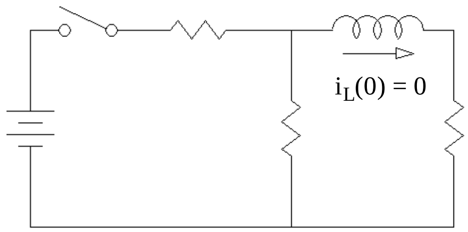
$$\frac{12\cdot V - 10\cdot V}{10.23\cdot V} = e^{-\frac{t_{12}}{0.28\cdot ms}} \qquad \ln\left(\frac{12\cdot V - 10\cdot V}{10.23\cdot V}\right) = -\left(\frac{t_{12}}{0.28\cdot ms}\right) \qquad t_{12} = -0.28\cdot ms\cdot\ln\left(\frac{12\cdot V - 10\cdot V}{10.23\cdot V}\right) = 0.46\cdot\text{ms}$$

$2\cdot ms + 0.46\cdot ms = 2.46\cdot\text{ms}$

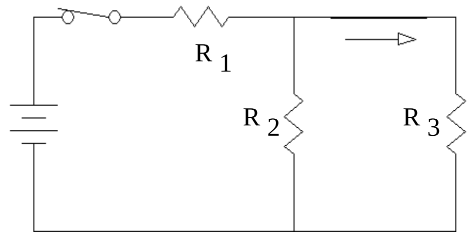
Ex4 a) Find the complete expression for $i_L(t)$.



Before the switch closes, $t = 0$-



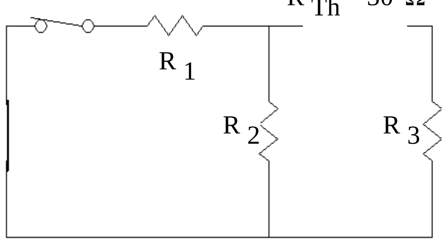
Final time, $t = \infty$



$$R_{23} := \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}$$

$$R_{23} = 12 \cdot \Omega$$

$$R_{Th} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} + R_3 \qquad R_{Th} = 30 \cdot \Omega$$



$$v_{R3}(\infty) = \frac{R_{23}}{R_1 + R_{23}} \cdot V_{in} = 5.625 \cdot V$$

$$i_L(\infty) = \frac{v_{R3}(\infty)}{R_3} = \frac{5.625 \cdot V}{15 \cdot \Omega} = 375 \cdot mA$$

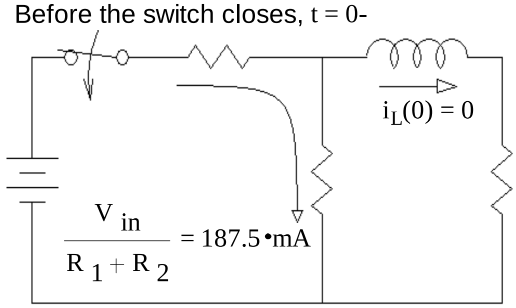
$$\tau := \frac{L}{R_{Th}}$$

$$\tau = 100 \cdot \mu s$$

$$i_L(t) = i_L(\infty) + \left(i_L(0) - i_L(\infty)\right) \cdot e^{-\frac{t}{\tau}} = 375 \cdot mA + (0 \cdot mA - 375 \cdot mA) \cdot e^{\frac{-t}{100 \cdot \mu s}} = 375 \cdot mA - 375 \cdot mA \cdot e^{\frac{-t}{100 \cdot \mu s}}$$

b) When is the voltage across $R_2 = 10V$?

Before the switch closes, $t = 0$-



$$v_{R2}(0) = \frac{R_2}{R_1 + R_2} \cdot V_{in} = 11.25 \cdot V$$

From drawing above at $t = \infty$

$$v_{R2}(\infty) = v_{R3}(\infty) = \frac{R_{23}}{R_1 + R_{23}} \cdot V_{in} = 5.625 \cdot V$$

$$v_{R2}(t) = v_{R2}(\infty) + \left(v_{R2}(0) - v_{R2}(\infty)\right) \cdot e^{-\frac{t}{\tau}}$$

$$= 5.625 \cdot V + (11.25 \cdot V - 5.625 \cdot V) \cdot e^{-\frac{t}{100 \cdot \mu s}}$$

$= 10 \cdot V$ at some time, solving for that time...

$$t = -\tau \cdot \ln\left(\frac{10 \cdot V - 5.625 \cdot V}{11.25 \cdot V - 5.625 \cdot V}\right) = 25 \cdot \mu s$$

Alternatively, when $v_{R2}(t) = 10V$, then $v_{R1}(t) = 5V$ and $i_L(t) = \frac{5 \cdot V}{R_1} - \frac{10 \cdot V}{R_2} = 83.333 \cdot mA$

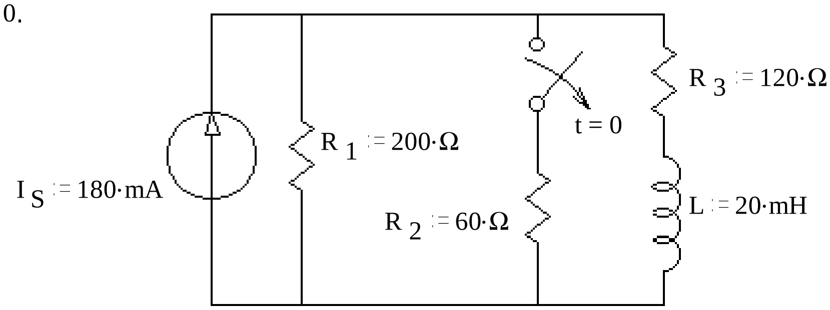
$$t = -\tau \cdot \ln\left(\frac{83.333 \cdot mA - 375 \cdot mA}{-375 \cdot mA}\right) = 25 \cdot \mu s$$

c) What is the $v_L(t)$ expression?

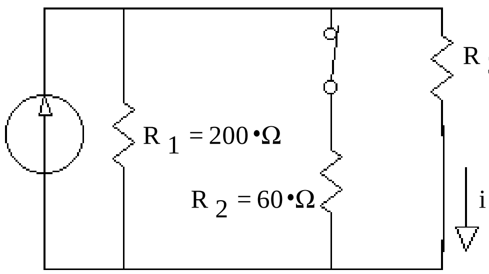
$$v_L(t) = v_L(\infty) + \left(v_L(0) - v_L(\infty)\right) \cdot e^{-\frac{t}{\tau}} = 0 \cdot V + (11.25 \cdot V - 0 \cdot V) \cdot e^{-\frac{t}{100 \cdot \mu s}}$$

Ex5 The switch has been closed for a long time and is opened (as shown) at time t = 0.

a) Find the complete expression for $i_L(t)$.

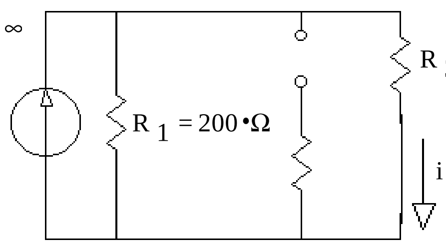


Before the switch opens, t = 0-

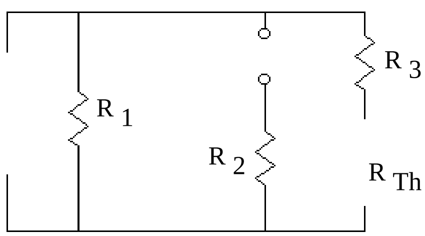


$$i_L(0) = I_S\cdot\frac{\frac{1}{R_3}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = 50\,mA$$

Final time, t = ∞



$$i_L(\infty) = I_S\cdot\frac{\frac{1}{R_3}}{\frac{1}{R_1}+\frac{1}{R_3}} = 112.5\,mA$$

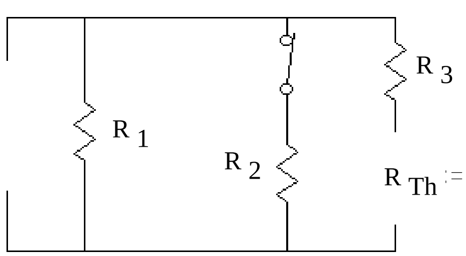


$R_{Th} := R_1 + R_3$ $\quad R_{Th} = 320\,\Omega$ $\quad \tau := \frac{L}{R_{Th}}$ $\quad \tau = 62.5\,\mu s$

$$i_L(t) = i_L(\infty) + \left(i_L(0) - i_L(\infty)\right)\cdot e^{-\frac{t}{\tau}} = 112.5\cdot mA + (50\cdot mA - 112.5\cdot mA)\cdot e^{\frac{-t}{62.5\cdot\mu s}} = 112.5\cdot mA - 62.5\cdot mA\cdot e^{\frac{-t}{62.5\cdot\mu s}}$$

b) Find i_L at time $t = 1.4\tau$. $\quad i_L(1.4\cdot\tau) = 112.5\cdot mA - 62.5\cdot mA\cdot e^{-\frac{1.4\cdot\tau}{\tau}} = 112.5\cdot mA - 62.5\cdot mA\cdot e^{-1.4} = 97.088\,mA$

c) At time $t = 1.4\tau$ the switch is closed again. Find the complete expression for $i_L(t')$, where t' starts at $t = 1.4\tau$. Be sure to clearly show the time constant.



$R_{Th} := \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} + R_3$ $\quad R_{Th} = 166.2\,\Omega$

$\tau := \frac{L}{R_{Th}}$ $\quad \tau = 120.4\,\mu s$

$i_L(0) = 97.1\cdot mA$ from part b)

$i_L(\infty) = 50\cdot mA$ initial value from part a)

$$i_L(t) = i_L(\infty) + \left(i_L(0) - i_L(\infty)\right)\cdot e^{-\frac{t}{\tau}} = 50\cdot mA + (97.1\cdot mA - 50\cdot mA)\cdot e^{\frac{-t'}{120.4\cdot\mu s}} = 50\cdot mA + 47.1\cdot mA\cdot e^{\frac{-t'}{120.4\cdot\mu s}}$$