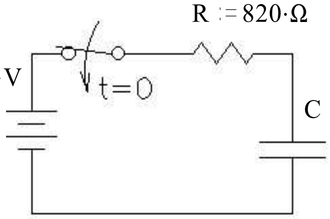
1.5 First-order transient examples

1. a) What is the value of the voltage across C at $t := 25 \cdot \mu s$ if the switch is closed at time t = 0?

$R := 820 \cdot \Omega$ $\quad V_{in} := 9 \cdot V$ $\quad C := 0.1 \cdot \mu F$



$\tau := R \cdot C \qquad \tau = 0.082 \cdot ms$

$v_C(0) = 0 \cdot V \qquad v_C(\infty) = 9 \cdot V$

$$v_C(t) = v_C(\infty) + \left(v_C(0) - v_C(\infty)\right) \cdot e^{-\frac{t}{\tau}} = 9 \cdot V + (0 \cdot V - 9 \cdot V) \cdot e^{-\frac{t}{\tau}} = 2.365 \cdot V$$

b) When will the current throught the resistor be $i_R := 5 \cdot mA$?

$i_R(0) = \frac{V_{in}}{R} = 10.976 \cdot mA \qquad i_R(\infty) = 0 \cdot mA$

$$i_R(t) = i_R(\infty) + \left(i_R(0) - i_R(\infty)\right) \cdot e^{-\frac{t}{\tau}} = 0 \cdot mA + (10.976 \cdot mA - 0 \cdot mA) \cdot e^{-\frac{t}{\tau}} = 10.976 \cdot mA \cdot e^{-\frac{t}{\tau}} = 5 \cdot mA$$

Rearrange the equation: $\quad t = -\tau \cdot \ln\left(\frac{5 \cdot mA}{10.976 \cdot mA}\right) = 64.5 \cdot \mu s$

c) When will the current throught the resistor be $i_R := 20 \cdot mA$?
Since the initial condition is 11mA and the final condition is 0mA, i_R will never be 20mA.

2. A 1000 µF capacitor has an initial charge of 12 volts. A 20 Ω resistor is connected across the capacitor at time t = 0. Find the energy dissipated by the resistor in the first 5 time constants.

After 5 time constants nearly all of the energy initially stored in the capacitor will be dissipated by the resistor. $\quad C := 1000 \cdot \mu F \qquad V_C := 12 \cdot V \qquad W_C := \frac{1}{2} \cdot C \cdot V_C^{\,2} \qquad W_C = 0.072 \cdot joule$

You can get to this answer just by knowing a little about the exponential curve, but what if you want a more accurate answer? Then you'll have to find the remaining voltage across the capacitor at t = 5τ and subtract off the energy left in the capacitor at that time.

$$v_C(0) = 12 \cdot V \quad v_C(\infty) = 0 \cdot V \quad v_C(t) = v_C(\infty) + \left(v_C(0) - v_C(\infty)\right) \cdot e^{-\frac{t}{\tau}} = 0 \cdot V + (12 \cdot V - 0 \cdot V) \cdot e^{-\frac{t}{\tau}} = 12 \cdot V \cdot e^{-\frac{t}{\tau}}$$

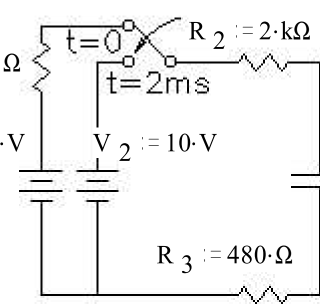
at t = 5τ: $\quad v_C(5 \cdot \tau) = 12 \cdot V \cdot e^{-5} = 81 \cdot mV \qquad \frac{1}{2} \cdot C \cdot (81 \cdot mV)^2 = 3.281 \cdot 10^{-6} \cdot joule$

Not surprisingly, this makes no significant difference: $\quad W_R = W_C - \frac{1}{2} \cdot C \cdot (81 \cdot mV)^2 = 0.072 \cdot joule$

3. The capacitor is initially uncharged. The switch is in the upper position from 0 to 2ms and is switched down at time t = 2ms.

What is the capacitor voltage $V_C(t)$

$R_3 := 480 \cdot \Omega \quad R_1 := 220 \cdot \Omega \quad R_2 := 2 \cdot k\Omega \quad V_1 := 24 \cdot V \quad V_2 := 10 \cdot V \quad C := 0.4 \cdot \mu F$



First interval $\quad v_C(0) = 0 \cdot V \qquad v_C(\infty) = 24 \cdot V$

$\tau := \left(R_1 + R_2 + R_3\right) \cdot C \qquad \tau = 1.08 \cdot ms$

$$v_C(t) = v_C(\infty) + \left(v_C(0) - v_C(\infty)\right) \cdot e^{-\frac{t}{\tau}} = 24 \cdot V + (0 \cdot V - 24 \cdot V) \cdot e^{-\frac{t}{1.08 \cdot ms}}$$

at 2ms $\quad 24 \cdot V - 24 \cdot V \cdot e^{-\frac{2 \cdot ms}{1.08 \cdot ms}} = 20.233 \cdot V$

Second interval, define a new time, tt = t - 2ms

$v_C(0) = 20.233\cdot V \qquad v_C(\infty) = 10\cdot V$

$\tau := (R_2 + R_3)\cdot C \qquad \tau = 0.992\cdot ms$

$$v_C(tt) = v_C(\infty) + \left(v_C(0) - v_C(\infty)\right)\cdot e^{-\frac{tt}{\tau}} = 10\cdot V + (20.233\cdot V - 10\cdot V)\cdot e^{-\frac{tt}{0.96\cdot ms}} = 10\cdot V + 10.233\cdot V\cdot e^{-\frac{t-2\cdot ms}{0.96\cdot ms}}$$

$t := 0, .02 .. 6$ ms

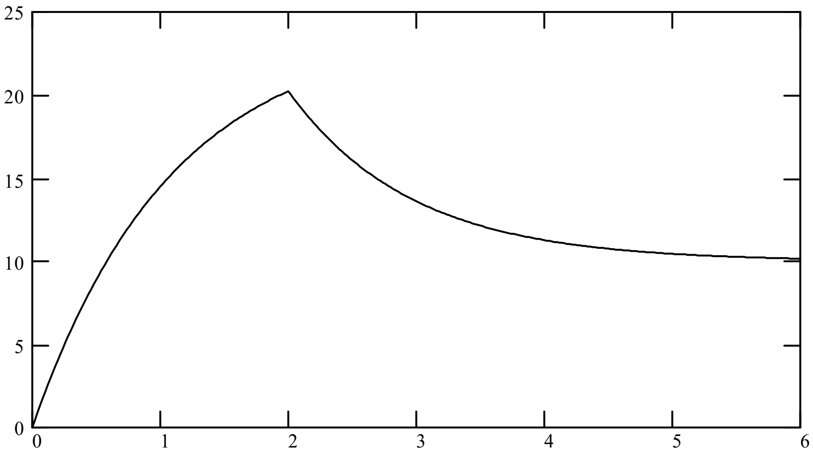
0 < t < 2ms

$V_C(t) = 24\cdot V - 24\cdot V\cdot e^{-\frac{2\cdot ms}{1.08\cdot ms}}$

t > 2ms

$V_C(t) = 10\cdot V + 10.233\cdot V\cdot e^{-\frac{t-2\cdot ms}{0.96\cdot ms}}$

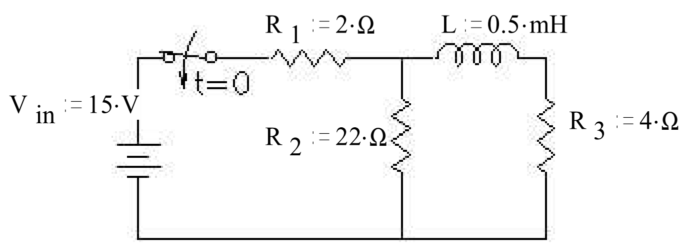
$V_C(t)$



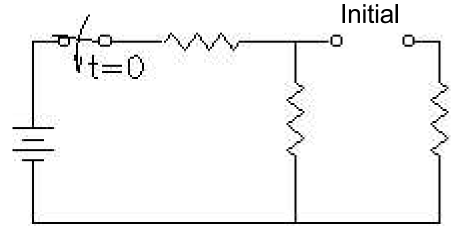
time (ms)

4. For the circuit at right, when is the voltage across R_2 = 10V?

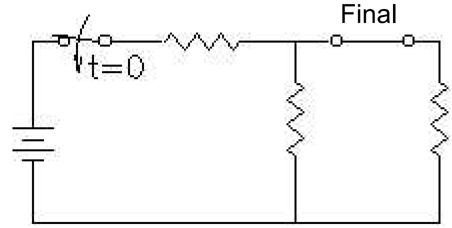
Taking L as the "load" and finding the Thevenin equivalent of the rest:



Initial



Final



$R_{Th} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} + R_3 \qquad R_{Th} = 5.833\cdot\Omega$

$\tau := \frac{L}{R_{Th}} \qquad \tau = 85.714\cdot\mu s$

$v_{R2}(0) = \frac{R_2}{R_1 + R_2}\cdot V_{in} = 13.75\cdot V$

$R_{23} := \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}$

$v_{R2}(\infty) = \frac{R_{23}}{R_1 + R_{23}}\cdot V_{in} = 9.429\cdot V$

$$v_{R2}(t) = v_{R2}(\infty) + \left(v_{R2}(0) - v_{R2}(\infty)\right)\cdot e^{-\frac{t}{\tau}}$$

$$= 9.429\cdot V + (13.75\cdot V - 9.429\cdot V)\cdot e^{-\frac{t}{85.714\cdot\mu s}} = 10V$$

$$t = -\tau\cdot\ln\left(\frac{10\cdot V - 9.429\cdot V}{13.75\cdot V - 9.429\cdot V}\right) = 173\cdot\mu s$$