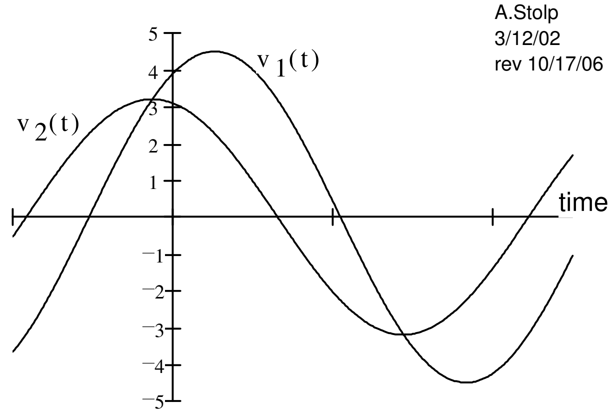
A.Stolp
3/12/02
rev 10/17/06



1. Add the sinusoidal voltages $v_1(t) = 4.5\cdot V\cdot\cos(\omega\cdot t - 30\cdot deg)$
and $v_2(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 15\cdot deg)$

using phasor notation, draw a phasor diagram of the three phasors, then convert back to time domain form.

$v_1(t) = 4.5\cdot V\cdot\cos(\omega\cdot t - 30\cdot deg)$

$\mathbf{V}_1(\omega) = 4.5V\ \underline{/-30^o}$ or: $\mathbf{V}_1(\omega) = 4.5\cdot V\cdot e^{-j\cdot 30\cdot deg}$

and

$v_2(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 15\cdot deg)$

$\mathbf{V}_2(\omega) = 3.2V\ \underline{/15^o}$ or: $\mathbf{V}_2(\omega) = 3.2\cdot V\cdot e^{j\cdot 15\cdot deg}$

I'm going to drop the (ω) notation from the phasor notation, it gets cumbersome, but remember that phasors are in the frequency domain..

$\mathbf{V}_1 = 4.5V\ \underline{/-30^o}$ or: $\mathbf{V}_1 := 4.5\cdot V\cdot e^{-j\cdot 30\cdot deg}$ $\mathbf{V}_2 = 3.2V\ \underline{/15^o}$ or: $\mathbf{V}_2 := 3.2\cdot V\cdot e^{j\cdot 15\cdot deg}$

$4.5\cdot V\cdot\cos(-30\cdot deg) = 3.897\cdot V$ $4.5\cdot V\cdot\sin(-30\cdot deg) = -2.25\cdot V$ $\mathbf{V}_1 = 3.897 - 2.25j\ \cdot V$ \

$3.2\cdot V\cdot\cos(15\cdot deg) = 3.091\cdot V$ $3.2\cdot V\cdot\sin(15\cdot deg) = 0.828\cdot V$ $\mathbf{V}_2 = 3.091 + 0.828j\ \cdot V$ } add /

Add real parts: $3.897 + 3.091 = 6.988$ $\mathbf{V}_3 := \mathbf{V}_1 + \mathbf{V}_2$

Add imaginary parts: $-2.25 + 0.828 = -1.422$ $\mathbf{V}_3 = 6.988 - 1.422j\ \cdot V$ sum

Change $\mathbf{V}_3$ back to polar coordinates:

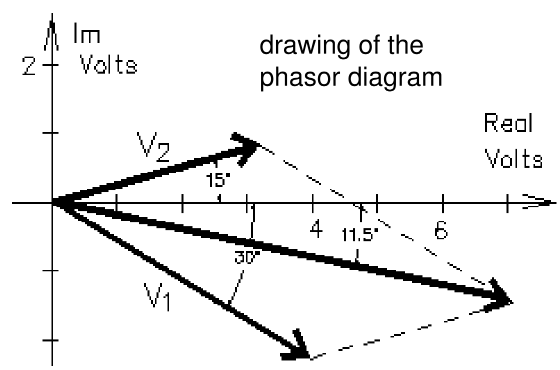
$\sqrt{6.988^2 + 1.422^2} = 7.131$ $\operatorname{atan}\left(\frac{-1.422}{6.988}\right) = -11.502\cdot deg$

OR, in Mathcad notation (you'll see these in future solutions):

$|\mathbf{V}_3| = 7.131\cdot V$ $\arg(\mathbf{V}_3) = -11.5\cdot deg$

Change $\mathbf{V}_3$ back to the time domain:

$v_3(t) = v_1(t) + v_2(t) = 7.13\cdot\cos(\omega\cdot t - 11.5\cdot deg)\cdot V$



drawing of the phasor diagram

2. Two sinusoidal voltages: $v_1(t) = 5\cdot V\cdot\cos(\omega\cdot t + 36.87\cdot deg)$ and $v_2(t) = 3.162\cdot V\cdot\cos(\omega\cdot t - 18.44\cdot deg)$

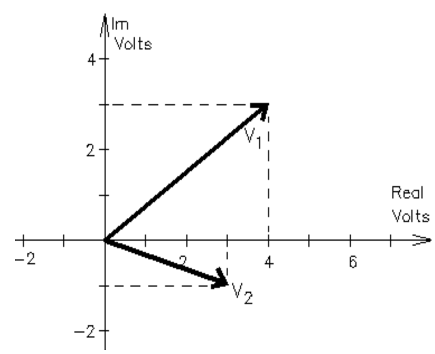
a) using phasor notation, find $v_3 = v_1 - v_2$

$\mathbf{V}_1 := 5\cdot V\cdot e^{j\cdot(36.87\cdot deg)}$ $5\cdot V\cdot\cos(36.87\cdot deg) = 4\cdot V$

$5\cdot V\cdot\sin(36.87\cdot deg) = 3\cdot V$ $\mathbf{V}_1 = 4 + 3j\ \cdot V$

$\mathbf{V}_2 := 3.162\cdot V\cdot e^{j\cdot(-18.44\cdot deg)}$ $3.162\cdot V\cdot\cos(-18.44\cdot deg) = 3\cdot V$

$3.162\cdot V\cdot\sin(-18.44\cdot deg) = -1\cdot V$ $\mathbf{V}_2 = 3 - j\ \cdot V$



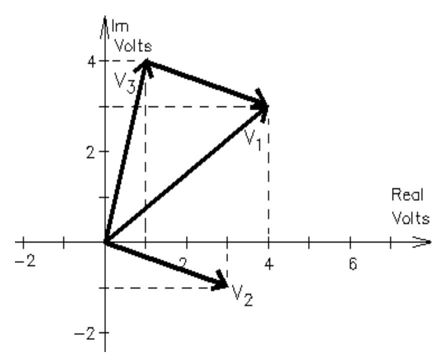
Subtract real parts: $4\cdot V - 3\cdot V = 1\cdot V$ $\mathbf{V}_3 := \mathbf{V}_1 - \mathbf{V}_2$

Subtract imaginary parts: $3\cdot V - -1\cdot V = 4\cdot V$ $\mathbf{V}_3 = 1 + 4j\ \cdot V$

$v_1(t) - v_2(t) = (1 + 4\cdot j)\cdot V$

Magnitude: $\sqrt{(1\cdot V)^2 + (4\cdot V)^2} = 4.123\cdot V$ OR: $|\mathbf{V}_3| = 4.123\cdot V$

Angle: $\operatorname{atan}\left(\frac{4\cdot V}{1\cdot V}\right) = 75.96\cdot deg$ $\arg(\mathbf{V}_3) = 75.96\cdot deg$



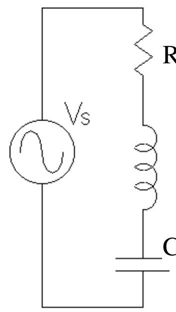
So: $v_3(t) = v_1(t) - v_2(t) = 4.123\cdot V\cdot\cos(\omega\cdot t + 75.96\cdot deg)\cdot V$

3 Find $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$ in polar phasor form. $f := 2\cdot kHz$

$\mathbf{V}(j\omega)$

$6\cdot V\cdot e^{j0}$

$f := 2\cdot kHz$



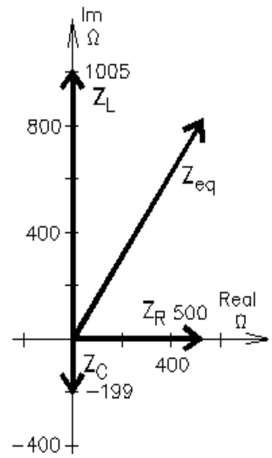
$R := 500\cdot\Omega$ $\quad \omega := 2\cdot\pi\cdot f \quad \omega = 1.257\cdot 10^4 \cdot\frac{rad}{sec}$

$L := 80\cdot mH$ $\quad \mathbf{Z_L} := j\cdot\omega\cdot L \quad \mathbf{Z_L} = 1.005j\cdot k\Omega$

$C := 0.4\cdot\mu F$ $\quad \mathbf{Z_C} := \frac{1}{j\cdot\omega\cdot C} \quad \mathbf{Z_C} = -0.199j\cdot k\Omega$

$\mathbf{Z_{eq}} := R + j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C} \quad \mathbf{Z_{eq}} = 500 + 806.366j \cdot\Omega$

$\sqrt{500^2 + 806^2} = 948.491 \quad \operatorname{atan}\left(\frac{806}{500}\right) = 58.187\cdot deg \quad \mathbf{Z_{eq}} = 948.5\Omega\ \underline{/\ 58.2}°$



find the current: $\mathbf{I} := \frac{6\cdot V\cdot e^{j\cdot 0}}{\mathbf{Z_{eq}}}$ magnitude: $\frac{6\cdot V}{948.5\cdot\Omega} = 6.326\cdot mA$ angle: $0\cdot deg - 58.2\cdot deg = -58.2\cdot deg$

$\mathbf{I} = 6.326mA\ \underline{/-58.2}°$

	find the magnitude	find the angle	
$\mathbf{V_R} := \mathbf{I}\cdot R$	$6.326\cdot mA\cdot 500\cdot\Omega = 3.163\cdot V$	$-58.2\cdot deg + 0\cdot deg = -58.2\cdot deg$	$\mathbf{V_R} = 3.163V\ \underline{/-58.2}°$
$\mathbf{V_L} := \mathbf{I}\cdot\mathbf{Z_L}$	$6.326\cdot mA\cdot 1005\cdot\Omega = 6.358\cdot V$	$-58.2\cdot deg + 90\cdot deg = 31.8\cdot deg$	$\mathbf{V_L} = 6.358V\ \underline{/\ 31.8}°$
$\mathbf{V_C} := \mathbf{I}\cdot\mathbf{Z_C}$	$6.326\cdot mA\cdot(-199)\cdot\Omega = -1.259\cdot V$	$-58.2\cdot deg + (90)\cdot deg = 31.8\cdot deg$	$\mathbf{V_C} = -1.259V\ \underline{/\ 31.8}°$
OR:	$6.326\cdot mA\cdot(199)\cdot\Omega = 1.259\cdot V$	$-58.2\cdot deg + (-90)\cdot deg = -148.2\cdot deg$	$\mathbf{V_C} = 1.259V\ \underline{/-148.2}°$

OR, you can also find these voltages directly, using a voltage divider. I.E. to find $\mathbf{V_C}$ directly:

$$\mathbf{V_C} := \frac{\frac{1}{j\cdot\omega\cdot C}}{R + j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C}}\cdot 6\cdot V = \frac{1}{R\cdot(j\cdot\omega\cdot C) + j\cdot\omega\cdot L\cdot(j\cdot\omega\cdot C) + 1}\cdot 6\cdot V = \frac{1}{R\cdot(j\cdot\omega\cdot C) - \omega^2\cdot L\cdot C + 1}\cdot 6\cdot V$$

$$= \frac{1}{\left(1 - \omega^2\cdot L\cdot C\right) + j\cdot\omega\cdot R\cdot C}\cdot 6\cdot V \qquad \left(1 - \omega^2\cdot L\cdot C\right) = -4.053 \qquad j\cdot\omega\cdot R\cdot C = 2.513j$$

$$= \frac{6\cdot V}{-4.053 + 2.513\cdot j}\cdot\frac{(-4.053 - 2.513\cdot j)}{(-4.053 - 2.513\cdot j)} = \frac{6\cdot V\cdot(-4.053 - 2.513\cdot j)}{(-4.053)^2 + 2.513^2}$$

$6\cdot V\cdot(-4.053 - 2.513\cdot j) = -24.318 - 15.078j \cdot V$

$(-4.053)^2 + 2.513^2 = 22.742$

$$= \left(\frac{-24.318}{22.742} - \frac{15.078\cdot j}{22.742}\right)\cdot V = -1.069 - 0.663j \cdot V$$

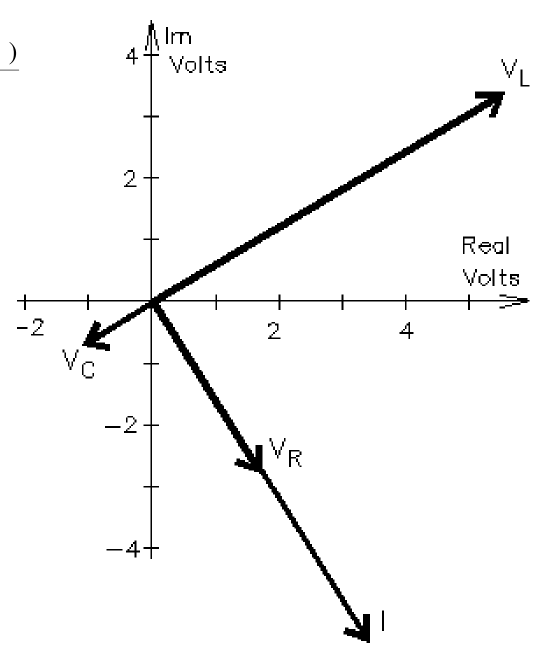
magnitude: $\sqrt{1.069^2 + 0.663^2} = 1.258$

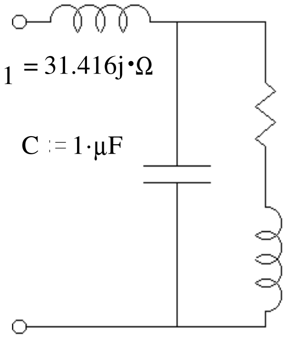
angle: $\operatorname{atan}\left(\frac{-0.663}{-1.069}\right) = 31.81\cdot deg$

but this is actually in the third quadrant, so modify your calculator's results:

$31.81\cdot deg - 180\cdot deg = -148.19\cdot deg$

$= 1.258V\ \underline{/-148.2}°$





4. a) Find $\mathbf{Z_{eq}}$. $f := 2.5 \cdot kHz$ $\quad \omega := 2 \cdot \pi \cdot f$ $\quad \omega = 1.571 \cdot 10^4 \cdot \frac{rad}{sec}$

Left branch

$$\mathbf{Z_l} := \frac{1}{j \cdot \omega \cdot C} \qquad \mathbf{Z_l} = -63.662j \cdot \Omega$$

Right branch

$$\mathbf{Z_r} := j \cdot \omega \cdot L_2 + R \qquad \mathbf{Z_r} = 200 + 125.664j \cdot \Omega$$

$$\mathbf{Z_{eq}} := j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + \frac{1}{\frac{1}{j \cdot \omega \cdot C}}} = j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C} = j \cdot \omega \cdot L_1 + \frac{R + j \cdot \omega \cdot L_2}{1 + j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2)}$$

$$= j \cdot \omega \cdot L_1 + \frac{R + j \cdot \omega \cdot L_2}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}$$

Sometimes it's worth simplifying a little before puttiing in numbers.

$$\mathbf{Z_{eq}} = 31.416 \cdot j \cdot \Omega + \frac{(200 + 125.664 \cdot j) \cdot \Omega}{-0.974 + 3.142 \cdot j} \cdot \left(\frac{-0.974 - 3.142 \cdot j}{-0.974 - 3.142 \cdot j} \right) = 31.416 \cdot j \cdot \Omega + \frac{(200 + 125.664 \cdot j) \cdot (-0.974 - 3.142 \cdot j)}{0.974^2 + 3.142^2}$$

$$= 31.416 \cdot j \cdot \Omega + \frac{((200 \cdot (-0.974) - 125.664 \cdot (-3.142)) + (125.664 \cdot (-0.974) - 200 \cdot 3.142) \cdot j) \cdot \Omega}{0.974^2 + 3.142^2}$$

$$= 31.416 \cdot j \cdot \Omega + \frac{(200.036288 - 750.796736 \cdot j) \cdot \Omega}{10.82084} = 31.416 \cdot j \cdot \Omega + 18.486 \cdot \Omega - 69.384 \cdot j \cdot \Omega = 18.486 - 37.968j \cdot \Omega$$

$$\sqrt{18.49^2 + 37.97^2} = 42.233 \qquad \text{atan}\left(\frac{-37.97}{18.49} \right) = -64.036 \cdot deg \qquad \mathbf{Z_{eq}} = 42.24\Omega \ \underline{/-64.04^o}$$

b) $\mathbf{V_{in}} := 12 \cdot V \cdot e^{j \cdot 20 \cdot deg}$ Find $\mathbf{I_{L1}}$, $\mathbf{V_C}$ $\quad \mathbf{I_{L1}} := \frac{\mathbf{V_{in}}}{\mathbf{Z_{eq}}}$ $\quad \frac{12 \cdot V}{42.24 \cdot \Omega} = 284.091 \cdot mA$ $\quad 20 \cdot deg - (-64.04) \cdot deg = 84.04 \cdot deg$

$$\mathbf{I_{L1}} = 284mA \ \underline{/84.04^o} = 284 \cdot mA \cdot e^{j \cdot 84.04 \cdot deg} \qquad \mathbf{I_{L1}} = 29.485 + 282.569j \cdot mA$$

$$\mathbf{V_C} := \mathbf{I_{L1}} \cdot (18.486 - 69.384 \cdot j) \cdot \Omega \qquad 284 \cdot mA \cdot \sqrt{18.486^2 + 69.384^2} \cdot \Omega = 20.392 \cdot V$$

$$84.04 \cdot deg + \text{atan}\left(\frac{-69.384}{18.486} \right) = 8.959 \cdot deg \qquad \mathbf{V_C} = 20.4V \ \underline{/8.96^o}$$

To find V_C directly:

You could then use another voltage divider to find $\mathbf{V_R}$ or $\mathbf{V_{L2}}$.

$$\mathbf{V_C} := \frac{\frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C}}{j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C}} \cdot \mathbf{V_{in}} = \frac{1}{j \cdot \omega \cdot L_1 \cdot \left(\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C \right) + 1} \cdot \mathbf{V_{in}} = \frac{1}{\frac{j \cdot \omega \cdot L_1}{R + j \cdot \omega \cdot L_2} - \omega^2 \cdot L_1 \cdot C + 1}$$

$$= \frac{1}{\frac{j \cdot \omega \cdot L_1}{R + j \cdot \omega \cdot L_2} - \omega^2 \cdot L_1 \cdot C + 1} \cdot \mathbf{V_{in}} = \frac{1}{\frac{j \cdot \omega \cdot L_1 \cdot (R - j \cdot \omega \cdot L_2)}{R^2 + (\omega \cdot L_2)^2} - \omega^2 \cdot L_1 \cdot C + 1} \cdot \mathbf{V_{in}}$$

$$= \frac{1}{\left[\frac{\omega^2 \cdot L_1 \cdot L_2}{R^2 + (\omega \cdot L_2)^2} - \omega^2 \cdot L_1 \cdot C + 1 \right] + j \cdot \frac{\omega \cdot L_1 \cdot R}{R^2 + (\omega \cdot L_2)^2}} \cdot \mathbf{V_{in}} = \frac{12 \cdot V \cdot e^{j \cdot 20 \, deg}}{0.58816 \cdot e^{j \cdot 11.039 \, deg}} = \frac{12 \cdot V}{0.58816} \ \underline{/20 - 11.039^o}$$

$= 20.4V \ \underline{/8.96^o}$ Same

$$\mathbf{V_C} = 20.153 + 3.178j \cdot V$$

4. Continued Let's find $\mathbf{I_{L2}}$.

$$\mathbf{Z_r} := j\cdot\omega\cdot L_2 + R$$
$$\mathbf{Z_r} = 200 + 125.664j \ \cdot\Omega \qquad \sqrt{200^2 + 125.664^2} = 236.202 \qquad \operatorname{atan}\left(\frac{125.664}{200}\right) = 32.142\cdot\deg$$

$$\mathbf{I_{L2}} = \frac{20.4\cdot V\cdot e^{j\cdot 8.96\cdot\deg}}{236.202\cdot\Omega\cdot e^{j\cdot 32.142\cdot\deg}} = \frac{20.4\cdot V}{236.202\cdot\Omega}\ \underline{/\ 8.96 - 32.142^o} = 86.4\text{mA}\ \underline{/-23.18}$$

Or, directly by Current divider:
$$\mathbf{I_{L2}} := \frac{\dfrac{1}{R + j\cdot\omega\cdot L_2}}{j\cdot\omega\cdot C + \dfrac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I_{L1}} = \frac{1}{j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right) + 1}\cdot\mathbf{I_{L1}} = \frac{\mathbf{I_{L1}}}{1 - \omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}$$

denominator:
$$\sqrt{\left(1 - \omega^2\cdot C\cdot L_2\right)^2 + (\omega\cdot C\cdot R)^2} = 3.289 \qquad \operatorname{atan}\left(\frac{\omega\cdot C\cdot R}{1 - \omega^2\cdot C\cdot L_2}\right) + 180\cdot\deg = 107.224\cdot\deg$$

$$\mathbf{I_{L2}} = \frac{284\cdot mA\cdot e^{j\cdot 84.04\cdot\deg}}{3.289\cdot e^{j\cdot 107.224\cdot\deg}} = \frac{284\cdot mA}{3.289}\ \underline{/\ 84.04 - 107.224^o} = 86.4\text{mA}\ \underline{/-23.18}$$

$$\mathbf{I_{L2}} = 79.404 - 34.001j \ \cdot mA$$

How about $\mathbf{I_C}$?
$$\mathbf{I_C} := \frac{\mathbf{V_C}}{\left(\dfrac{1}{j\cdot\omega\cdot C}\right)} = \mathbf{V_C}\cdot j\cdot\omega\cdot C = 20.4\text{V}\ \underline{/\ 8.96^o}\ \ 0.015708\ \underline{/\ 90^o}\ \frac{1}{\Omega} = 320\text{mA}\ \underline{/\ 98.96^o}$$

Or, directly by Current divider:
$$\mathbf{I_C} := \frac{j\cdot\omega\cdot C}{j\cdot\omega\cdot C + \dfrac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I_{L1}} = \frac{j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right)}{j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right) + 1}\cdot\mathbf{I_{L1}} = \frac{-\omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}{1 - \omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}\cdot\mathbf{I_{L1}}$$

numerator:
$$\sqrt{\left(\omega^2\cdot C\cdot L_2\right)^2 + (\omega\cdot C\cdot R)^2} = 3.71 \qquad \operatorname{atan}\left(\frac{\omega\cdot C\cdot R}{-\omega^2\cdot C\cdot L_2}\right) + 180\cdot\deg = 122.142\cdot\deg$$

Second quadrant

denominator is the same as above.

$$\mathbf{I_C} = \frac{3.71\cdot e^{j\cdot 122.14\cdot\deg}}{3.289\cdot e^{j\cdot 107.224\cdot\deg}}\cdot 284\cdot mA\cdot e^{j\cdot 84.04\cdot\deg} = \frac{3.71}{3.289}\cdot 284\cdot mA\ \underline{/\ 122.14 - 107.224 + 84.04^o} = 320\text{mA}\ \underline{/\ 98.96^o}$$

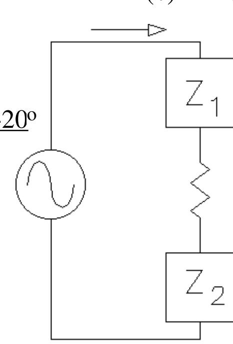
This current is greater than the input current. What's going on?

The angle between $\mathbf{I_C}$ & $\mathbf{I_{L2}}$ is big enough that they somewhat cancel each other out.

Check Kirchoff's Current Law: $\mathbf{I_C} + \mathbf{I_{L2}} = 29.485 + 282.569j \ \cdot mA \ = \ \mathbf{I_{L1}} = 29.485 + 282.569j \ \cdot mA$

5. a) Find $\mathbf{Z_2}$. For partial credit, you must show work and/or intermediate results.

$$i(t) = 25\cdot mA\cdot\cos\left(377\cdot\frac{rad}{sec}\cdot t + 10\cdot\deg\right)$$



$$\mathbf{Z_1} := (120 - 60\cdot j)\cdot\Omega$$
$$\mathbf{V_{in}} = 10\text{V}\ \underline{/-20^o}$$
$$R := 50\cdot\Omega$$
$$\mathbf{Z_2} = ?$$

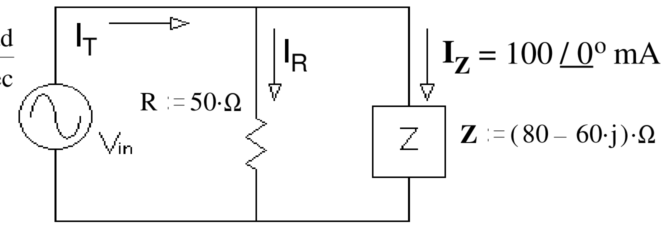
$$\mathbf{I} := 25\cdot mA\cdot e^{j\cdot 10\cdot\deg}$$
$$\mathbf{V_{in}} := 10\cdot V\cdot e^{-j\cdot 20\cdot\deg}$$
$$\mathbf{Z_T} := \frac{\mathbf{V_{in}}}{\mathbf{I}} = \frac{10\cdot V}{25\cdot mA}\ \underline{/\ -20 - 10^o} = 400\Omega\ \underline{/-30^o}$$
$$\mathbf{Z_T} = 346.41 - 200j \ \cdot\Omega$$

$$\mathbf{Z_2} := \mathbf{Z_T} - R - \mathbf{Z_1} = (346.41 - 200\cdot j)\cdot\Omega - 50\cdot\Omega - (120 - 60\cdot j)\cdot\Omega = 176.41 - 140j \ \cdot\Omega$$

b) Circle 1: i) The source current leads the source voltage <--- answer, because $10^o > -20^o$.

ii) The source voltage leads the source current

6. a) Find $\mathbf{V_{in}}$ in polar form.



$\mathbf{I_Z} := 100\cdot mA$ $\mathbf{Z} := (80 - 60\cdot j)\cdot\Omega$ $\omega := 1000\cdot\frac{rad}{sec}$

$\mathbf{V_{in}} := \mathbf{I_Z}\cdot\mathbf{Z}$ $\mathbf{V_{in}} = 8 - 6j \;\bullet V$

$\sqrt{8^2 + 6^2} = 10$ $\text{atan}\left(\frac{-6}{8}\right) = -36.87\;\bullet deg$

$\mathbf{V_{in}}$ = 10V $\underline{/-36.9^o}$

b) Find $\mathbf{I_T}$ in polar form. $\mathbf{I_R} := \frac{\mathbf{V_{in}}}{R} = \frac{10\cdot V}{50\cdot\Omega}\;\underline{/-36.9^o} = \frac{10\cdot V}{50\cdot\Omega}\cdot\cos(-36.9\cdot deg) + j\cdot\frac{10\cdot V}{50\cdot\Omega}\cdot\sin(-36.9\cdot deg) = 160 - 120i \;\bullet mA$

$\mathbf{I_T} := \mathbf{I_R} + \mathbf{I_Z} = (160 - 120\cdot j)\cdot mA + 100\cdot mA = 260 - 120j \;\bullet mA$

$\sqrt{260^2 + 120^2} = 286.356$ $\text{atan}\left(\frac{-120}{260}\right) = -24.78\;\bullet deg$ $\mathbf{I_T}$ = 286mA $\underline{/-24.8^o}$

c) Circle 1: i) The source current leads the source voltage ii) The source voltage leads the source current

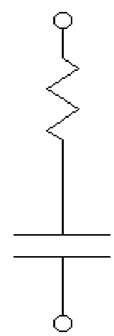
answer i), -24.8° > -36.9°

7. d) The impedance $\mathbf{Z}$ is made of two components in series. What are they and what are their values?

$\mathbf{Z} = 80 - 60j \;\bullet\Omega$

Must have a resistor because there is a real part.

$R := \text{Re}(\mathbf{Z})$ $R = 80\bullet\Omega$



Must have a capacitor because the imaginary part is negative.

$\text{Im}(\mathbf{Z}) = -60\bullet\Omega = \frac{-1}{\omega\cdot C}$ $C := \frac{-1}{\omega\cdot\text{Im}(\mathbf{Z})}$ $C = 16.667\bullet\mu F$

e) The impedance $\mathbf{Z}$ is made of two components in parallel. What are they and what are their values?

$\mathbf{Z} = 80 - 60j \;\bullet\Omega$

Must have a resistor because there is a real part.

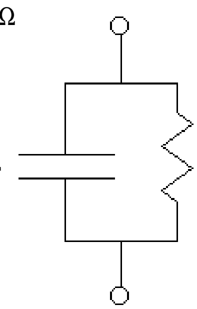
Must have a capacitor because the imaginary part is negative.

$\mathbf{Z} = \frac{1}{\frac{1}{R} + j\cdot\omega\cdot C}$ $\frac{1}{\mathbf{Z}} = \frac{1}{(80 - 60\cdot j)\cdot\Omega}\cdot\left(\frac{80 + 60\cdot j}{80 + 60\cdot j}\right) = \frac{80 + 60\cdot j}{80^2 + 60^2} = \frac{80 + 60\cdot j}{10,000}\cdot\frac{1}{\Omega}$

$\frac{1}{\mathbf{Z}} = 8\bullet 10^{-3} + 6\bullet 10^{-3}j \;\bullet\Omega^{-1} = \frac{1}{R} + j\cdot\omega\cdot C$

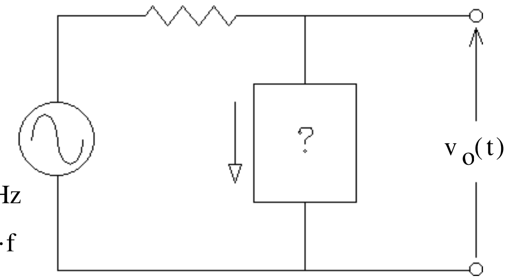
$\frac{1}{R} = .008\cdot\frac{1}{\Omega}$ $R := \frac{1}{.008\cdot\frac{1}{\Omega}}$ $R = 125\bullet\Omega$

$\omega\cdot C = .006\cdot\frac{1}{\Omega}$ $C := \frac{.006\cdot\frac{1}{\Omega}}{\omega}$ $C = 6\bullet\mu F$



$R = 125\bullet\Omega$

8. You need to design a circuit in which the the "output" voltage leads the input voltage ($v_S(t)$) by 40^o of phase.



$v_S(t)$

$f := 1 \cdot kHz$

$\omega := 2 \cdot \pi \cdot f$

$\omega = 6.283 \cdot 10^3 \cdot \frac{rad}{sec}$

a) What should go in the box: R, L, C?

$$\mathbf{V}_o = \frac{\mathbf{Z}_{box}}{R + \mathbf{Z}_{box}} \cdot \mathbf{V}_S$$

angle of $\frac{\mathbf{Z}_{box}}{R + \mathbf{Z}_{box}}$ is 40^o.

This can only happen if the angle of $\mathbf{Z}_{box}$ is positive, so $\mathbf{Z}_{box}$ is a inductor

b) Find its value. $\mathbf{V}_o =$ $\mathbf{V}_o = \frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L} \cdot \mathbf{V}_S$ angle $\frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L}$ is $90 - atan\left(\frac{\omega \cdot L}{R}\right) = 40^o$.

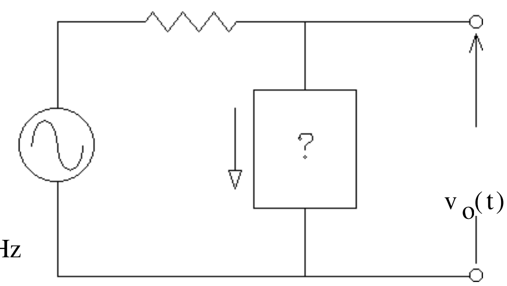
$$atan\left(\frac{\omega \cdot L}{R}\right) = 60^o.$$

$$\frac{\omega \cdot L}{R} = tan(60 \cdot deg) = 1.732$$

$$L := \frac{R \cdot 1.732}{\omega} \qquad L = 34.5 \cdot mH$$

9. You need to design a circuit in which the the "output" voltage lags the input voltage ($v_S(t)$) by 40^o of phase.

$R := 180 \cdot \Omega$



$v_S(t)$

$f := 1 \cdot kHz$

$\omega := 2 \cdot \pi \cdot f$

$\omega = 6.283 \cdot 10^3 \cdot \frac{rad}{sec}$

a) What should go in the box: R, L, C?

$$\mathbf{V}_o = \frac{\mathbf{Z}_{box}}{R + \mathbf{Z}_{box}} \cdot \mathbf{V}_S$$

angle of $\frac{\mathbf{Z}_{box}}{R + \mathbf{Z}_{box}}$ is -40^o.

This can only happen if the angle of $\mathbf{Z}_{box}$ is negative, so $\mathbf{Z}_{box}$ is a capacitor

b) Find its value. $\mathbf{V}_o = \frac{\frac{1}{j \cdot \omega \cdot C}}{R + \frac{1}{j \cdot \omega \cdot C}} \cdot \mathbf{V}_S$ angle $\frac{\frac{1}{j \cdot \omega \cdot C}}{R + \frac{1}{j \cdot \omega \cdot C}}$ is $-90 - atan\left(\frac{-\frac{1}{\omega \cdot C}}{R}\right) = -90 - atan\left(-\frac{1}{\omega \cdot C \cdot R}\right)$

$$atan\left(-\frac{1}{\omega \cdot C \cdot R}\right) = -60^o.$$

$$-\frac{1}{\omega \cdot C \cdot R} = tan(-60 \cdot deg) = -1.732$$

$$C := \frac{1}{\omega \cdot R \cdot 1.732} \qquad C = 0.511 \cdot \mu F$$