

ECE 2210 / 00 Phasor Examples

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1. Add the sinusoidal voltages $v_1(t) = 4.5 \cdot V \cdot \cos(\omega \cdot t - 30 \cdot \text{deg})$
and $v_2(t) = 3.2 \cdot V \cdot \cos(\omega \cdot t + 15 \cdot \text{deg})$

using phasor notation, draw a phasor diagram of the three phasors, then convert back to time domain form.

$$v_1(t) = 4.5 \cdot V \cdot \cos(\omega \cdot t - 30 \cdot \text{deg})$$

$$\mathbf{V}_1(\omega) = 4.5V \angle -30^\circ \quad \text{or:} \quad \mathbf{V}_1(\omega) = 4.5 \cdot V \cdot e^{-j30 \text{ deg}}$$

and

$$v_2(t) = 3.2 \cdot V \cdot \cos(\omega \cdot t + 15 \cdot \text{deg})$$

$$\mathbf{V}_2(\omega) = 3.2V \angle 15^\circ \quad \text{or:} \quad \mathbf{V}_2(\omega) = 3.2 \cdot V \cdot e^{j15 \text{ deg}}$$

I'm going to drop the (ω) notation from the phasor notation, it gets cumbersome, but remember that phasors are in the frequency domain..

$$\mathbf{V}_1 = 4.5V \angle -30^\circ \quad \text{or:} \quad \mathbf{V}_1 := 4.5 \cdot V \cdot e^{-j30 \text{ deg}}$$

$$\mathbf{V}_2 = 3.2V \angle 15^\circ \quad \text{or:} \quad \mathbf{V}_2 := 3.2 \cdot V \cdot e^{j15 \text{ deg}}$$

$$4.5 \cdot V \cdot \cos(-30 \cdot \text{deg}) = 3.897 \cdot V \quad 4.5 \cdot V \cdot \sin(-30 \cdot \text{deg}) = -2.25 \cdot V$$

$$3.2 \cdot V \cdot \cos(15 \cdot \text{deg}) = 3.091 \cdot V \quad 3.2 \cdot V \cdot \sin(15 \cdot \text{deg}) = 0.828 \cdot V$$

$$\text{Add real parts:} \quad 3.897 + 3.091 = 6.988$$

$$\text{Add imaginary parts:} \quad -2.25 + 0.828 = -1.422$$

$$\mathbf{V}_1 = 3.897 - 2.25j \cdot V \quad \backslash$$

$$\mathbf{V}_2 = 3.091 + 0.828j \cdot V \quad / \quad \text{add}$$

$$\mathbf{V}_3 := \mathbf{V}_1 + \mathbf{V}_2$$

$$\mathbf{V}_3 = 6.988 - 1.422j \cdot V \quad \text{sum}$$

Change $\mathbf{V}_3$ back to polar coordinates:

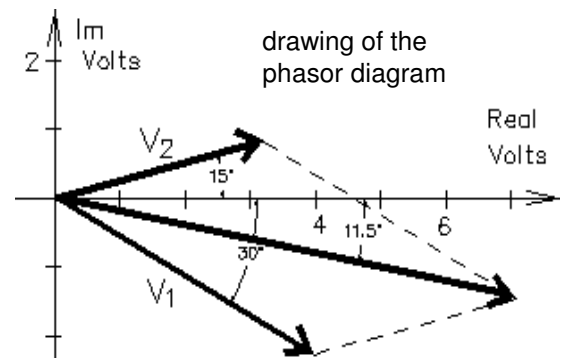
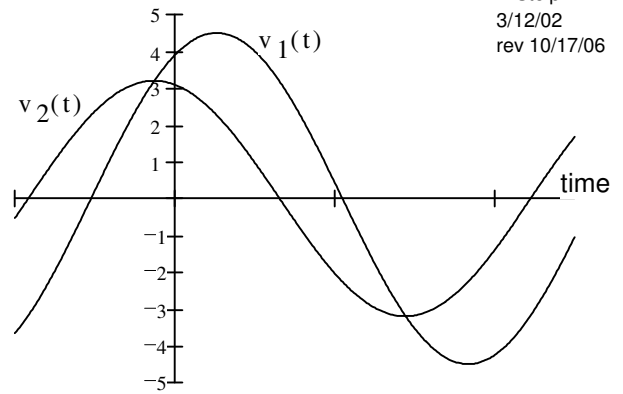
$$\sqrt{6.988^2 + 1.422^2} = 7.131 \quad \text{atan}\left(\frac{-1.422}{6.988}\right) = -11.502 \cdot \text{deg}$$

OR, in Mathcad notation (you'll see these in future solutions):

$$|\mathbf{V}_3| = 7.131 \cdot V \quad \arg(\mathbf{V}_3) = -11.5 \cdot \text{deg}$$

Change $\mathbf{V}_3$ back to the time domain:

$$v_3(t) = v_1(t) + v_2(t) = 7.13 \cdot \cos(\omega \cdot t - 11.5 \cdot \text{deg}) \cdot V$$



2. Two sinusoidal voltages: $v_1(t) = 5 \cdot V \cdot \cos(\omega \cdot t + 36.87 \cdot \text{deg})$ and $v_2(t) = 3.162 \cdot V \cdot \cos(\omega \cdot t - 18.44 \cdot \text{deg})$

a) using phasor notation, find $v_3 = v_1 - v_2$

$$\mathbf{V}_1 := 5 \cdot V \cdot e^{j(36.87 \cdot \text{deg})} \quad 5 \cdot V \cdot \cos(36.87 \cdot \text{deg}) = 4 \cdot V$$

$$5 \cdot V \cdot \sin(36.87 \cdot \text{deg}) = 3 \cdot V$$

$$\mathbf{V}_1 = 4 + 3j \cdot V$$

$$\mathbf{V}_2 := 3.162 \cdot V \cdot e^{j(-18.44 \cdot \text{deg})} \quad 3.162 \cdot V \cdot \cos(-18.44 \cdot \text{deg}) = 3 \cdot V$$

$$3.162 \cdot V \cdot \sin(-18.44 \cdot \text{deg}) = -1 \cdot V$$

$$\mathbf{V}_2 = 3 - j \cdot V$$

$$\text{Subtract real parts:} \quad 4 \cdot V - 3 \cdot V = 1 \cdot V$$

$$\text{Subtract imaginary parts:} \quad 3 \cdot V - -1 \cdot V = 4 \cdot V$$

$$\mathbf{V}_3 := \mathbf{V}_1 - \mathbf{V}_2$$

$$\mathbf{V}_3 = 1 + 4j \cdot V$$

$$v_1(t) - v_2(t) = (1 + 4j) \cdot V$$

$$\text{Magnitude:} \quad \sqrt{(1 \cdot V)^2 + (4 \cdot V)^2} = 4.123 \cdot V$$

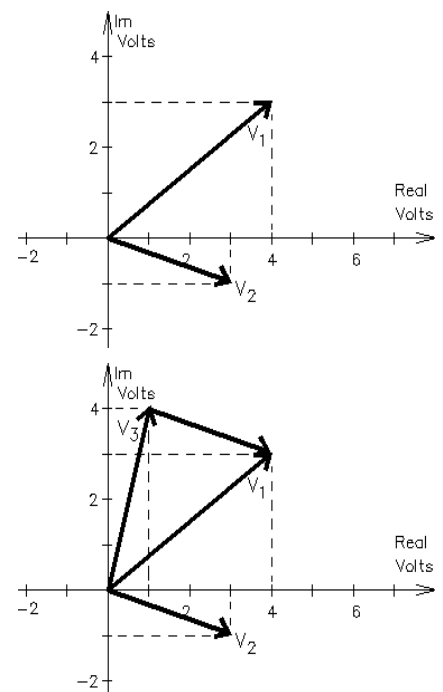
OR:

$$|\mathbf{V}_3| = 4.123 \cdot V$$

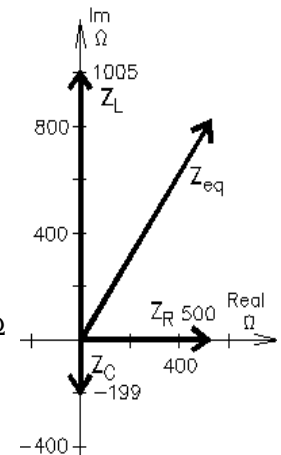
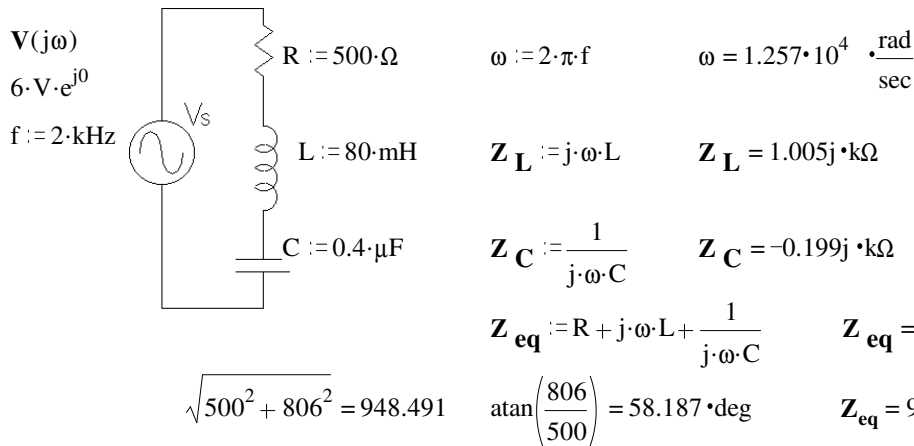
$$\text{Angle:} \quad \text{atan}\left(\frac{4 \cdot V}{1 \cdot V}\right) = 75.96 \cdot \text{deg}$$

$$\arg(\mathbf{V}_3) = 75.96 \cdot \text{deg}$$

$$\text{So:} \quad v_3(t) = v_1(t) - v_2(t) = 4.123 \cdot V \cdot \cos(\omega \cdot t + 75.96 \cdot \text{deg}) \cdot V$$



3 Find V_R , V_L , and V_C in polar phasor form. $f := 2 \cdot \text{kHz}$



find the current: $I := \frac{6 \cdot V \cdot e^{j0}}{Z_{eq}}$ magnitude: $\frac{6 \cdot V}{948.5 \cdot \Omega} = 6.326 \cdot \text{mA}$ angle: $0 \cdot \text{deg} - 58.2 \cdot \text{deg} = -58.2 \cdot \text{deg}$

$I = 6.326 \text{mA} \angle -58.2^\circ$

find the magnitude

find the angle

$V_R := I \cdot R$ $6.326 \cdot \text{mA} \cdot 500 \cdot \Omega = 3.163 \cdot V$ $-58.2 \cdot \text{deg} + 0 \cdot \text{deg} = -58.2 \cdot \text{deg}$ $V_R = 3.163V \angle -58.2^\circ$

$V_L := I \cdot Z_L$ $6.326 \cdot \text{mA} \cdot 1005 \cdot \Omega = 6.358 \cdot V$ $-58.2 \cdot \text{deg} + 90 \cdot \text{deg} = 31.8 \cdot \text{deg}$ $V_L = 6.358V \angle 31.8^\circ$

$V_C := I \cdot Z_C$ $6.326 \cdot \text{mA} \cdot (-199) \cdot \Omega = -1.259 \cdot V$ $-58.2 \cdot \text{deg} + (90) \cdot \text{deg} = 31.8 \cdot \text{deg}$ $V_C = -1.259V \angle 31.8^\circ$

OR: $6.326 \cdot \text{mA} \cdot (199) \cdot \Omega = 1.259 \cdot V$ $-58.2 \cdot \text{deg} + (-90) \cdot \text{deg} = -148.2 \cdot \text{deg}$ $V_C = 1.259V \angle -148.2^\circ$

OR, you can also find these voltages directly, using a voltage divider. I.E. to find V_C directly:

$$V_C := \frac{\frac{1}{j \cdot \omega \cdot C}}{R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C}} \cdot 6 \cdot V = \frac{1}{R \cdot (j \cdot \omega \cdot C) + j \cdot \omega \cdot L \cdot (j \cdot \omega \cdot C) + 1} \cdot 6 \cdot V = \frac{1}{R \cdot (j \cdot \omega \cdot C) - \omega^2 \cdot L \cdot C + 1} \cdot 6 \cdot V$$

$$= \frac{1}{(1 - \omega^2 \cdot L \cdot C) + j \cdot \omega \cdot R \cdot C} \cdot 6 \cdot V \quad (1 - \omega^2 \cdot L \cdot C) = -4.053 \quad j \cdot \omega \cdot R \cdot C = 2.513j$$

$$= \frac{6 \cdot V}{-4.053 + 2.513j} \cdot \frac{(-4.053 - 2.513j)}{(-4.053 - 2.513j)} = \frac{6 \cdot V \cdot (-4.053 - 2.513j)}{(-4.053)^2 + 2.513^2}$$

$$6 \cdot V \cdot (-4.053 - 2.513j) = -24.318 - 15.078j \cdot V$$

$$(-4.053)^2 + 2.513^2 = 22.742$$

$$= \left(\frac{-24.318}{22.742} - \frac{15.078j}{22.742} \right) \cdot V = -1.069 - 0.663j \cdot V$$

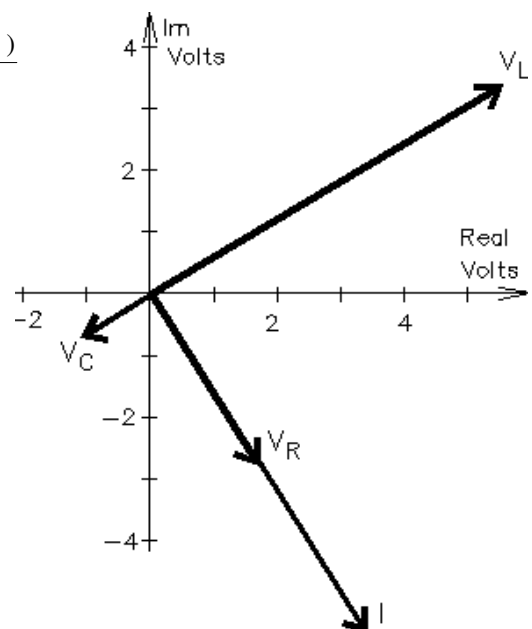
magnitude: $\sqrt{1.069^2 + 0.663^2} = 1.258$

angle: $\text{atan}\left(\frac{-0.663}{-1.069}\right) = 31.81 \cdot \text{deg}$

but this is actually in the third quadrant,
so modify your calculator's results:

$31.81 \cdot \text{deg} - 180 \cdot \text{deg} = -148.19 \cdot \text{deg}$

$= 1.258V \angle -148.2^\circ$



ECE 2210 / 00 Phasor Examples p3

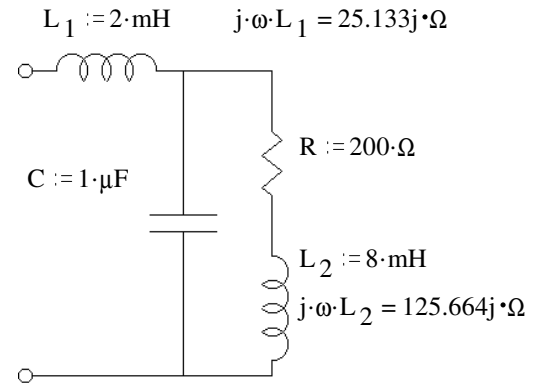
4. a) Find Z_{eq} . $f := 2.5 \cdot \text{kHz}$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 1.571 \cdot 10^4 \cdot \frac{\text{rad}}{\text{sec}}$

Left branch

$$Z_L := \frac{1}{j \cdot \omega \cdot C} \quad Z_L = -63.662j \cdot \Omega$$

Right branch

$$Z_R := j \cdot \omega \cdot L_2 + R \quad Z_R = 200 + 125.664j \cdot \Omega$$



$$Z_{eq} := j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + \frac{1}{\frac{1}{j \cdot \omega \cdot C}}} = j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C} = j \cdot \omega \cdot L_1 + \frac{R + j \cdot \omega \cdot L_2}{1 + j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2)}$$

$$= j \cdot \omega \cdot L_1 + \frac{R + j \cdot \omega \cdot L_2}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}$$

Sometimes it's worth simplifying a little before putting in numbers.

$$Z_{eq} = 31.416j \cdot \Omega + \frac{(200 + 125.664j) \cdot \Omega \cdot (-0.974 - 3.142j)}{-0.974 + 3.142j} = 31.416j \cdot \Omega + \frac{(200 + 125.664j) \cdot (-0.974 - 3.142j)}{0.974^2 + 3.142^2}$$

$$= 31.416j \cdot \Omega + \frac{((200 \cdot (-0.974) - 125.664 \cdot (-3.142)) + (125.664 \cdot (-0.974) - 200 \cdot 3.142) \cdot j) \cdot \Omega}{0.974^2 + 3.142^2}$$

$$= 31.416j \cdot \Omega + \frac{(200.036288 - 750.796736j) \cdot \Omega}{10.82084} = 31.416j \cdot \Omega + 18.486 \cdot \Omega - 69.384j \cdot \Omega = 18.486 - 37.968j \cdot \Omega$$

$$\sqrt{18.49^2 + 37.97^2} = 42.233 \quad \text{atan}\left(\frac{-37.97}{18.49}\right) = -64.036 \cdot \text{deg} \quad Z_{eq} = 42.24 \Omega \angle -64.04^\circ$$

b) $V_{in} := 12 \cdot V \cdot e^{j20 \cdot \text{deg}}$ Find I_{L1} , V_C $I_{L1} := \frac{V_{in}}{Z_{eq}} = \frac{12 \cdot V}{42.24 \cdot \Omega} = 284.091 \cdot \text{mA}$ $20 \cdot \text{deg} - (-64.04) \cdot \text{deg} = 84.04 \cdot \text{deg}$

$$I_{L1} = 284 \text{mA} \angle 84.04^\circ = 284 \cdot \text{mA} \cdot e^{j84.04 \cdot \text{deg}} \quad I_{L1} = 29.485 + 282.569j \cdot \text{mA}$$

$$V_C := I_{L1} \cdot (18.486 - 69.384j) \cdot \Omega \quad 284 \cdot \text{mA} \cdot \sqrt{18.486^2 + 69.384^2} \cdot \Omega = 20.392 \cdot V$$

$$84.04 \cdot \text{deg} + \text{atan}\left(\frac{-69.384}{18.486}\right) = 8.959 \cdot \text{deg} \quad V_C = 20.4 \text{V} \angle 8.96^\circ$$

To find V_C

directly:

$$V_C := \frac{\frac{1}{R + j \cdot \omega \cdot L_2}}{j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C}} \cdot V_{in} = \frac{1}{j \cdot \omega \cdot L_1 \cdot \left(\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C\right) + 1} \cdot V_{in} = \frac{1}{\frac{j \cdot \omega \cdot L_1}{R + j \cdot \omega \cdot L_2} - \omega^2 \cdot L_1 \cdot C + 1}$$

$$= \frac{1}{\frac{j \cdot \omega \cdot L_1}{R + j \cdot \omega \cdot L_2} - \omega^2 \cdot L_1 \cdot C + 1} \cdot V_{in} = \frac{1}{\frac{j \cdot \omega \cdot L_1 \cdot (R - j \cdot \omega \cdot L_2)}{R^2 + (\omega \cdot L_2)^2} - \omega^2 \cdot L_1 \cdot C + 1} \cdot V_{in}$$

$$= \frac{1}{\left[\frac{\omega^2 \cdot L_1 \cdot L_2}{R^2 + (\omega \cdot L_2)^2} - \omega^2 \cdot L_1 \cdot C + 1\right] + j \cdot \frac{\omega \cdot L_1 \cdot R}{R^2 + (\omega \cdot L_2)^2}} \cdot V_{in} = \frac{12 \cdot V \cdot e^{j20 \cdot \text{deg}}}{0.58816 \cdot e^{j11.039 \cdot \text{deg}}} = \frac{12 \cdot V}{0.58816} \angle 20 - 11.039^\circ$$

$$= 20.4 \text{V} \angle 8.96^\circ \quad \text{Same}$$

$$V_C = 20.153 + 3.178j \cdot V$$

ECE 2210 / 00 Phasor Examples p4

4. Continued Let's find I_{L2} .

$$\begin{aligned} Z_R &:= j \cdot \omega \cdot L_2 + R \\ Z_R &= 200 + 125.664j \cdot \Omega \quad \sqrt{200^2 + 125.664^2} = 236.202 \quad \text{atan}\left(\frac{125.664}{200}\right) = 32.142^\circ \text{deg} \\ I_{L2} &= \frac{20.4 \cdot V \cdot e^{j8.96^\circ}}{236.202 \cdot \Omega \cdot e^{j32.142^\circ}} = \frac{20.4 \cdot V}{236.202 \cdot \Omega} \angle 8.96 - 32.142^\circ = 86.4 \text{mA} \angle -23.18^\circ \end{aligned}$$

Or, directly by
Current divider: $I_{L2} := \frac{1}{R + j \cdot \omega \cdot L_2} \cdot I_{L1} = \frac{1}{j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2) + 1} \cdot I_{L1} = \frac{I_{L1}}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}$

denominator: $\sqrt{(1 - \omega^2 \cdot C \cdot L_2)^2 + (\omega \cdot C \cdot R)^2} = 3.289 \quad \text{atan}\left(\frac{\omega \cdot C \cdot R}{1 - \omega^2 \cdot C \cdot L_2}\right) + 180^\circ = 107.224^\circ \text{deg}$

$$I_{L2} = \frac{284 \cdot \text{mA} \cdot e^{j84.04^\circ}}{3.289 \cdot e^{j107.224^\circ}} = \frac{284 \cdot \text{mA}}{3.289} \angle 84.04 - 107.224^\circ = 86.4 \text{mA} \angle -23.18^\circ$$

$$I_{L2} = 79.404 - 34.001j \cdot \text{mA}$$

How about I_C ? $I_C := \frac{V_C}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} = V_C \cdot j \cdot \omega \cdot C = 20.4 \text{V} \angle 8.96^\circ \cdot 0.015708 \angle 90^\circ \cdot \frac{1}{\Omega} = 320 \text{mA} \angle 98.96^\circ$

Or, directly by
Current divider: $I_C := \frac{j \cdot \omega \cdot C}{j \cdot \omega \cdot C + \frac{1}{R + j \cdot \omega \cdot L_2}} \cdot I_{L1} = \frac{j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2)}{j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2) + 1} \cdot I_{L1} = \frac{-\omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R} \cdot I_{L1}$

numerator: $\sqrt{(\omega^2 \cdot C \cdot L_2)^2 + (\omega \cdot C \cdot R)^2} = 3.71 \quad \text{atan}\left(\frac{\omega \cdot C \cdot R}{-\omega^2 \cdot C \cdot L_2}\right) + 180^\circ = 122.142^\circ \text{deg}$
denominator is the same as above. Second quadrant

$$I_C = \frac{3.71 \cdot e^{j122.14^\circ}}{3.289 \cdot e^{j107.224^\circ}} \cdot 284 \cdot \text{mA} \cdot e^{j84.04^\circ} = \frac{3.71}{3.289} \cdot 284 \cdot \text{mA} \angle 122.14 - 107.224 + 84.04^\circ = 320 \text{mA} \angle 98.96^\circ$$

This current is greater than the input current. What's going on?

The angle between I_C & I_{L2} is big enough that they somewhat cancel each other out.

Check Kirchoff's Current Law: $I_C + I_{L2} = 29.485 + 282.569j \cdot \text{mA} = I_{L1} = 29.485 + 282.569j \cdot \text{mA}$

5. a) Find Z_2 . For partial credit, you must show work and/or intermediate results.

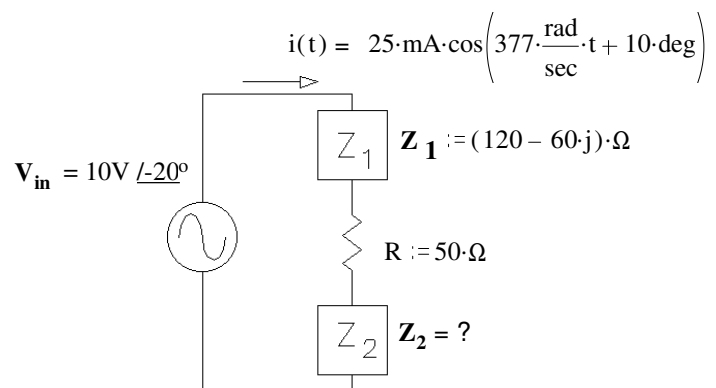
$$I := 25 \cdot \text{mA} \cdot e^{j10^\circ}$$

$$V_{in} := 10 \cdot V \cdot e^{-j20^\circ}$$

$$Z_T := \frac{V_{in}}{I} = \frac{10 \cdot V}{25 \cdot \text{mA}} \angle -20 - 10^\circ = 400 \Omega \angle -30^\circ$$

$$Z_T = 346.41 - 200j \cdot \Omega$$

$$Z_2 := Z_T - R - Z_1 = (346.41 - 200j) \cdot \Omega - 50 \cdot \Omega - (120 - 60j) \cdot \Omega = 176.41 - 140j \cdot \Omega$$



- b) Circle 1: i) The source current leads the source voltage <--- answer, because $10^\circ > -20^\circ$.
ii) The source voltage leads the source current

ECE 2210 / 00 Phasor Examples p5

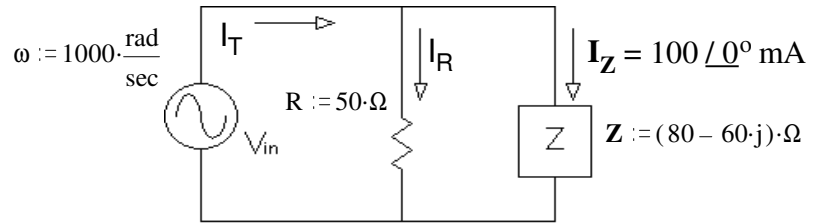
6. a) Find V_{in} in polar form.

$$I_Z := 100 \cdot \text{mA} \quad Z := (80 - 60j) \cdot \Omega$$

$$V_{in} := I_Z \cdot Z \quad V_{in} = 8 - 6j \cdot V$$

$$\sqrt{8^2 + 6^2} = 10 \quad \text{atan}\left(\frac{-6}{8}\right) = -36.87 \cdot \text{deg}$$

$$V_{in} = 10V \angle -36.9^\circ$$



b) Find I_T in polar form. $I_R := \frac{V_{in}}{R} = \frac{10 \cdot V}{50 \cdot \Omega} \angle -36.9^\circ = \frac{10 \cdot V}{50 \cdot \Omega} \cdot \cos(-36.9 \cdot \text{deg}) + j \cdot \frac{10 \cdot V}{50 \cdot \Omega} \cdot \sin(-36.9 \cdot \text{deg}) = 160 - 120i \cdot \text{mA}$

$$I_T := I_R + I_Z = (160 - 120j) \cdot \text{mA} + 100 \cdot \text{mA} = 260 - 120j \cdot \text{mA}$$

$$\sqrt{260^2 + 120^2} = 286.356 \quad \text{atan}\left(\frac{-120}{260}\right) = -24.78 \cdot \text{deg} \quad I_T = 286 \text{mA} \angle -24.8^\circ$$

c) Circle 1: i) The source current leads the source voltage ii) The source voltage leads the source current
answer i), $-24.8^\circ > -36.9^\circ$

7. d) The impedance Z is made of two components in series. What are they and what are their values?

$$Z = 80 - 60j \cdot \Omega$$

Must have a resistor because there is a real part.

$$R := \text{Re}(Z)$$

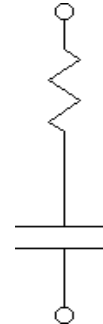
$$R = 80 \cdot \Omega$$

Must have a capacitor because the imaginary part is negative.

$$\text{Im}(Z) = -60 \cdot \Omega = \frac{-1}{\omega \cdot C}$$

$$C := \frac{-1}{\omega \cdot \text{Im}(Z)}$$

$$C = 16.667 \cdot \mu\text{F}$$



e) The impedance Z is made of two components in parallel. What are they and what are their values?

$$Z = 80 - 60j \cdot \Omega$$

Must have a resistor because there is a real part.

Must have a capacitor because the imaginary part is negative.

$$Z = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C}$$

$$\frac{1}{Z} = \frac{1}{(80 - 60j) \cdot \Omega} \cdot \left(\frac{80 + 60j}{80 + 60j} \right) = \frac{80 + 60j}{80^2 + 60^2} = \frac{80 + 60j}{10,000} \cdot \frac{1}{\Omega}$$

$$\frac{1}{Z} = 8 \cdot 10^{-3} + 6 \cdot 10^{-3}j \cdot \Omega^{-1} = \frac{1}{R} + j \cdot \omega \cdot C$$

$$\frac{1}{R} = .008 \cdot \frac{1}{\Omega}$$

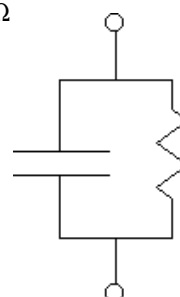
$$R := \frac{1}{.008 \cdot \frac{1}{\Omega}}$$

$$R = 125 \cdot \Omega$$

$$\omega \cdot C = .006 \cdot \frac{1}{\Omega}$$

$$C := \frac{.006 \cdot \frac{1}{\Omega}}{\omega}$$

$$C = 6 \cdot \mu\text{F} \quad R = 125 \cdot \Omega$$



ECE 2210 / 00 Phasor Examples p6

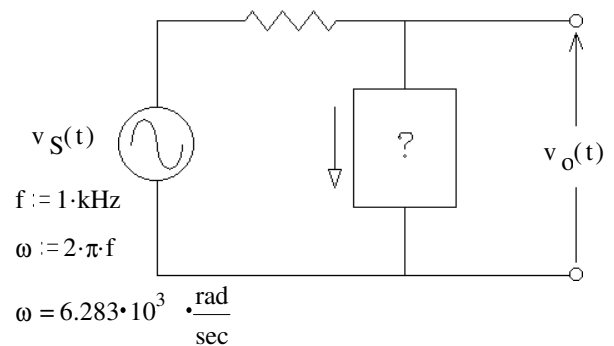
8. You need to design a circuit in which the "output" voltage leads the input voltage ($v_S(t)$) by 40° of phase.

a) What should go in the box: R, L, C?

$$\mathbf{V}_o = \frac{\mathbf{Z}_{\text{box}}}{R + \mathbf{Z}_{\text{box}}} \cdot \mathbf{V}_S$$

$$\text{angle of } \frac{\mathbf{Z}_{\text{box}}}{R + \mathbf{Z}_{\text{box}}} \text{ is } 40^\circ.$$

This can only happen if the angle of $\mathbf{Z}_{\text{box}}$ is positive,
so $\mathbf{Z}_{\text{box}}$ is an inductor



b) Find its value. $\mathbf{V}_o = \frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L} \cdot \mathbf{V}_S$ angle $\frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L}$ is $90 - \text{atan}\left(\frac{\omega \cdot L}{R}\right) = 40^\circ$.

$$\text{atan}\left(\frac{\omega \cdot L}{R}\right) = 60^\circ.$$

$$\frac{\omega \cdot L}{R} = \tan(60^\circ) = 1.732$$

$$L := \frac{R \cdot 1.732}{\omega} \quad L = 34.5 \cdot \text{mH}$$

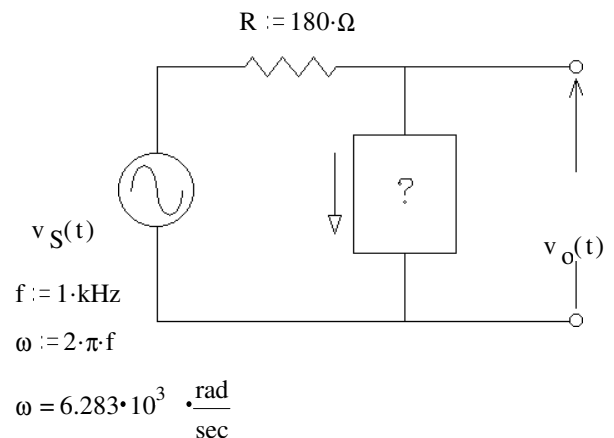
9. You need to design a circuit in which the "output" voltage lags the input voltage ($v_S(t)$) by 40° of phase.

a) What should go in the box: R, L, C?

$$\mathbf{V}_o = \frac{\mathbf{Z}_{\text{box}}}{R + \mathbf{Z}_{\text{box}}} \cdot \mathbf{V}_S$$

$$\text{angle of } \frac{\mathbf{Z}_{\text{box}}}{R + \mathbf{Z}_{\text{box}}} \text{ is } -40^\circ.$$

This can only happen if the angle of $\mathbf{Z}_{\text{box}}$ is negative,
so $\mathbf{Z}_{\text{box}}$ is a capacitor



b) Find its value. $\mathbf{V}_o = \frac{1}{R + \frac{1}{j \cdot \omega \cdot C}} \cdot \mathbf{V}_S$ angle $\frac{1}{R + \frac{1}{j \cdot \omega \cdot C}}$ is $-90 - \text{atan}\left(\frac{1}{\omega \cdot C \cdot R}\right) = -40^\circ$.

$$\text{atan}\left(-\frac{1}{\omega \cdot C \cdot R}\right) = -60^\circ.$$

$$-\frac{1}{\omega \cdot C \cdot R} = \tan(-60^\circ) = -1.732$$

$$C := \frac{1}{\omega \cdot R \cdot 1.732} \quad C = 0.511 \cdot \mu\text{F}$$