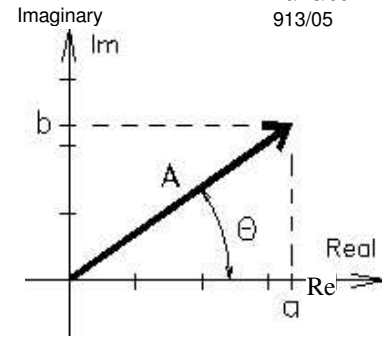


Complex Numbers

ECE 2210 / 00

A.Stolp
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$j = \sqrt{-1}$ the imaginary number



Rectangular Form $A = a + b \cdot j$

$$\text{Re}(A) = a \quad \text{Im}(A) = b$$

Polar Form

$$A = A \cdot e^{j\theta}$$

$$\text{Re}(A) = A \cdot \cos(\theta) \quad \text{Im}(A) = A \cdot \sin(\theta)$$

Conversions

$$A = |A| = \sqrt{a^2 + b^2} \quad \theta = \arg(A) = \text{atan}\left(\frac{b}{a}\right)$$

$$a = A \cdot \cos(\theta) \quad b = A \cdot \sin(\theta)$$

$$A = A \cdot e^{j\theta} = A \cdot \cos(\theta) + A \cdot \sin(\theta) \cdot j \quad A = a + b \cdot j = \left(\sqrt{a^2 + b^2}\right) \cdot e^{j \cdot \text{atan}\left(\frac{b}{a}\right)}$$

Special Cases

$$j := \sqrt{-1} = e^{j \cdot 90\text{-deg}} \quad \frac{1}{j} = -j = e^{-j \cdot 90\text{-deg}} \quad e^{j \cdot 0\text{-deg}} = 1 \quad e^{-j \cdot 180\text{-deg}} = e^{-j \cdot 180\text{-deg}} = -1$$

$$j \cdot e^{j\theta} = e^{j(\theta + 90\text{-deg})}$$

Define a 2nd number: rect: $D = c + d \cdot j$ polar: $D = D \cdot e^{j\phi}$

Equality

$A = D$ if and only if $a = c$ and $b = d$ OR $A = D$ and $\theta = \phi$

Addition and Subtraction

$$A + D = (a + b \cdot j) + (c + d \cdot j) = (a + c) + (b + d) \cdot j$$

$$A - D = (a + b \cdot j) - (c + d \cdot j) = (a - c) + (b - d) \cdot j$$

Convert polars to rectangular form first

Multiplication and Division

$$A \cdot D = (a + b \cdot j) \cdot (c + d \cdot j) = (a \cdot c - b \cdot d) + (b \cdot c + a \cdot d) \cdot j$$

$$\text{Rectangular: } \frac{A}{D} = \frac{a + b \cdot j}{c + d \cdot j} = \frac{a + b \cdot j \cdot c - d \cdot j}{c + d \cdot j \cdot c - d \cdot j} = \frac{a \cdot c + b \cdot d}{c^2 + d^2} + \frac{b \cdot c - a \cdot d}{c^2 + d^2} \cdot j$$

$$\text{Polar: } A \cdot D = A \cdot e^{j\theta} \cdot D \cdot e^{j\phi} = A \cdot D \cdot e^{j(\theta + \phi)}$$

$$\frac{A}{D} = \frac{A \cdot e^{j\theta}}{D \cdot e^{j\phi}} = \frac{A}{D} \cdot e^{j(\theta - \phi)}$$

Powers

$$A^n = A^n \cdot e^{j \cdot n \cdot \theta} = A^n \cdot \cos(n \cdot \theta) + A^n \cdot \sin(n \cdot \theta) \cdot j \quad \text{Convert rectangulars first, usually}$$

Conjugates

complex number

Conjugate

$$A = a + b \cdot j$$

$$\overline{A} = a - b \cdot j$$

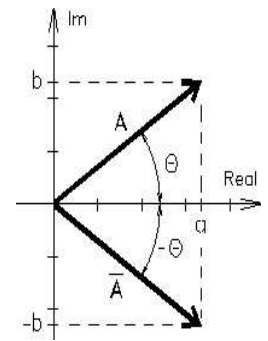
$$\overline{\overline{A}} = A$$

$$A = A \cdot e^{j\theta}$$

$$\overline{A} = A \cdot e^{-j\theta}$$

$$F = \frac{3 + 5 \cdot j}{(2 - 6 \cdot j) \cdot e^{j \cdot 40\text{-deg}}}$$

$$\overline{F} = \frac{3 - 5 \cdot j}{(2 + 6 \cdot j) \cdot e^{-j \cdot 40\text{-deg}}}$$



Euler's equation

$$e^{j\alpha} = \cos(\alpha) + j \cdot \sin(\alpha)$$

$$\text{OR: } \cos(\alpha) = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$\sin(\theta) = \frac{e^{j\alpha} - e^{-j\alpha}}{2 \cdot j}$$

$$e^{j(\omega \cdot t + \theta)} = \cos(\omega \cdot t + \theta) + j \cdot \sin(\omega \cdot t + \theta)$$

$$\text{Re}\left[e^{j(\omega \cdot t + \theta)}\right] = \cos(\omega \cdot t + \theta)$$

If we freeze this at time $t=0$, then we can represent $\cos(\omega \cdot t + \theta)$ by $e^{j\theta}$

Calculus

Remember, when we write $e^{j\theta}$, we really mean $e^{j(\omega \cdot t + \theta)}$

$$\frac{d}{dt} A = \frac{d}{dt} (A \cdot e^{j\theta}) = j \cdot \omega \cdot A \cdot e^{j\theta} = \omega \cdot A \cdot e^{j(\theta + 90\text{-deg})}$$

$$\int A dt = \int A \cdot e^{j\theta} dt = \frac{1}{j \cdot \omega} \cdot A \cdot e^{j\theta} = \frac{1}{\omega} \cdot A \cdot e^{j(\theta - 90\text{-deg})}$$