Introduction to AC Power, RMS

**DC Power**

$$P = VI = \frac{V^2}{R} = I^2R$$

**AC Power**

$$v(t) = V_P \cos(\omega t)$$

**RMS (Root Mean Square)**

$$P = \frac{V_P^2}{R}$$

$$v(t)^2 = \frac{V_P^2}{2}$$

Could we define an "effective" voltage that would allow us to use the same relationships for AC power as used for DC power?

$$P_{\text{ave}} = \left( \frac{V_P^2}{R} \right) \frac{1}{2} = \left( \frac{V_P^2}{2} \right)$$

$$V_{\text{eff}} = \left( \frac{V_P}{\sqrt{2}} \right) = V_{\text{rms}} = \sqrt{\int_0^T (v(t))^2 dt}$$

**RMS** Root of the Mean of the Square

Use RMS in power calculations

Sinusoids

$$v(t) = V_P \cos(\omega t)$$

$$v(t)^2 = \frac{V_P^2}{2}$$

$$P = \frac{1}{2} V_P^2$$

$$P = \frac{1}{2} \frac{V_P^2}{\sqrt{2}}$$

$$P = \frac{1}{2} \left[ \frac{1}{2} + \frac{1}{2} \cos(2\omega t) \right] dt$$

$$= \frac{1}{2} \frac{V_P^2}{\sqrt{2}}$$
Common household power

\[
\begin{align*}
  f &= 60 \text{ Hz} \\
  \omega &= 377 \text{ rad/ sec} \\
  T &= 16.67 \text{ ms}
\end{align*}
\]

Neutral, N
white
(Also ground)
Line, L
black, 120V
120V
go 

What about other wave shapes??

\[v(t)\]

\[v(t)^2\]

average = \[\frac{V_p^2}{3}\]

\[V_{rms} = \frac{V_p}{\sqrt{3}}\]

Works for all types of triangular and sawtooth waveforms

\[v(t)\]

\[v(t)^2\]

\[V_{rms} = V_p\]

Same for DC

How about AC + DC ?

\[
V_{rms} = \sqrt{\frac{1}{T} \int_0^T (v(t))^2 \, dt}
\]

\[
= \sqrt{\frac{1}{T} \int_0^T \left( V_p \cos(\omega t) + V_{DC} \right)^2 \, dt}
\]

\[
= \sqrt{\frac{1}{T} \int_0^T \left( \left( V_p \cos(\omega t) \right)^2 + 2 \cdot V_p \cos(\omega t) \cdot V_{DC} + V_{DC}^2 \right) \, dt}
\]

\[
= \sqrt{\frac{1}{T} \int_0^T \left( V_p \cos(\omega t) \right)^2 \, dt + \frac{1}{T} \int_0^T 2 \cdot V_p \cos(\omega t) \cdot V_{DC} \, dt + \frac{1}{T} \int_0^T V_{DC}^2 \, dt}
\]

\[
= \sqrt{V_{rmsAC}^2 + 0 + V_{DC}^2}
\]

\[V_{rms} = \sqrt{V_{rmsAC}^2 + V_{DC}^2}\]
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sinusoid: \( V_{\text{rms}} = \frac{V_p}{\sqrt{2}} \quad I_{\text{rms}} = \frac{I_p}{\sqrt{2}} \)

triangular: \( V_{\text{rms}} = \frac{V_p}{\sqrt{3}} \quad I_{\text{rms}} = \frac{I_p}{\sqrt{3}} \)

square: \( V_{\text{rms}} = V_p \quad I_{\text{rms}} = I_p \)

waveform + DC \( V_{\text{rms}} = \sqrt{V_{\text{rmsAC}}^2 + V_{\text{DC}}^2} \)

\[ W_{\text{L}} = P_{\text{L}} \cdot 6 \text{ sec} \quad \text{All converted to heat} \]

\[ P_{\text{L}} = 0.22 \text{ W} \]

rectified average \( V_{\text{ra}} = \frac{1}{T} \int_{0}^{T} |v(t)| \, dt \)

\( V_{\text{ra}} = \frac{2}{\pi} V_p \quad I_{\text{ra}} = \frac{2}{\pi} I_p \)

\( V_{\text{ra}} = \frac{1}{2} V_p \quad I_{\text{ra}} = \frac{1}{2} I_p \)

Most AC meters don't measure true RMS. Instead, they measure \( V_{\text{ra}} \), display \( 1.11V_{\text{ra}} \), and call it RMS. That works for sine waves but not for any other waveform.

Some waveforms don't fall into these forms, then you have to perform the math from scratch

For waveform shown

The average DC \( (V_{\text{DC}}) \) value

\[ \frac{2 \cdot V \cdot (4 \text{ ms}) + (-5 \cdot V) \cdot (2 \text{ ms})}{6 \text{ ms}} = -0.333 \cdot V \]

The RMS (effective) value

Graphical way

\[ \frac{4 \cdot V^2 \cdot (4 \text{ ms}) + 25 \cdot V^2 \cdot (2 \text{ ms})}{6 \text{ ms}} = 11 \cdot V^2 \]

\( V_{\text{rms}} := \sqrt{11 \cdot V^2} \quad V_{\text{rms}} = 3.32 \cdot V \)

OR...

\[ V_{\text{rms}} = \sqrt{\frac{1}{T} \int_{0}^{T} (v(t))^2 \, dt} \]

\[ = \sqrt{\frac{1}{6 \text{ ms}} \left[ \int_{0 \text{ ms}}^{4 \text{ ms}} (2 \cdot V)^2 \, dt + \int_{4 \text{ ms}}^{6 \text{ ms}} (-5 \cdot V)^2 \, dt \right]} = \sqrt{\frac{1}{6 \text{ ms}} \left[ 4 \text{ ms} (2 \cdot V)^2 + 2 \text{ ms} (-5 \cdot V)^2 \right]} = 3.32 \cdot V \]

The voltage is hooked to a resistor, as shown, for 6 seconds.

The energy is transferred to the resistor during that 6 seconds:

\[ P_{\text{L}} := \frac{V_{\text{rms}}^2}{R_{\text{L}}} \quad P_{\text{L}} = 0.22 \text{ W} \]

\[ W_{\text{L}} := P_{\text{L}} \cdot 6 \text{ sec} \quad W_{\text{L}} = 1.32 \text{ joule} \]
Use RMS in power calculations

\[ P = I_{\text{Rms}}^2 \cdot R = \frac{V_{\text{Rms}}^2}{R} \]

for Resistors ONLY!!

### Capacitors and Inductors

- Capacitors and Inductors DO NOT dissipate (real) average power.

- Reactive power is negative
  \[ Q_C = -I_{\text{Crms}} \cdot V_{\text{Crms}} \]
  \[ = -I_{\text{Crms}} \cdot \frac{1}{\omega \cdot C} = -V_{\text{Crms}} \cdot \omega \cdot C \]

- Reactive power is positive
  \[ Q_L = I_{\text{Lrms}} \cdot V_{\text{Lrms}} \]
  \[ = I_{\text{Lrms}}^2 \cdot \omega \cdot L = \frac{V_{\text{Lrms}}^2}{\omega \cdot L} \]

If current and voltage are not in phase, only the in-phase part of the current matters for the power-- DOT PRODUCT

- "Lagging" power
- Inductor dominates

- "Leading" Power
- Capacitor dominates
Real Power

\[ P = V \cdot I \cdot \cos(\theta) = I^2 \cdot |Z| \cdot \cos(\theta) = \frac{V^2}{|Z|} \cdot \cos(\theta) \]

\[ P = \text{"Real" Power (average)} = V \cdot I \cdot \text{pf} = I^2 \cdot |Z| \cdot \text{pf} = \frac{V^2}{|Z|} \cdot \text{pf} \]

otherwise....

\[ I_R \rightarrow \text{\Phi} \quad V_R \]

for resistors only part that uses real average power \[ P = I_R^2 \cdot R = \frac{V_R^2}{R} \]

Reactive Power

\[ Q = \text{Reactive "power" } = V \cdot I \cdot \sin(\theta) \]

otherwise....

\[ I_C \rightarrow \text{\Phi} \quad V_C \quad \text{capacitors } \rightarrow -Q \]

\[ Q_C = I_C^2 \cdot X_C = \frac{V_C^2}{X_C} \]

\[ X_C = \frac{1}{\omega \cdot C} \text{ and is a negative number} \]

\[ I_L \rightarrow \text{\Phi} \quad V_L \quad \text{inductors } \rightarrow +Q \]

\[ Q_L = I_L^2 \cdot X_L = \frac{V_L^2}{X_L} \]

\[ X_L = \omega \cdot L \text{ and is a positive number} \]

Complex and Apparent Power

\[ S = \text{Complex "power" } = P + jQ = V \cdot I \cdot j\theta = V \cdot I \cdot j = I^2 \cdot Z \]

\[ \text{NOT} \quad V \cdot I \quad \text{NOR} \quad \frac{V^2}{Z} \]

\[ S = \text{Apparent "power" } = |S| = \sqrt{P^2 + Q^2} = V \cdot I \]

units: VA, kVA, etc. "volt-amp"

Power factor

\[ \text{pf} = \cos(\theta) = \text{power factor (sometimes expressed in \%)} \quad 0 \leq \text{pf} \leq 1 \]

\[ \theta \text{ is the phase angle between the voltage and the current or the phase angle of the impedance. } \theta = \theta_Z \]

\[ \theta < 0 \text{ Load is "Capacitive", power factor is "leading". This condition is very rare} \]

\[ \theta > 0 \text{ Load is "Inductive", power factor is "lagging". This condition is so common you can assume any power factor given is lagging unless specified otherwise. Transformers and motors make most loads inductive.} \]

Industrial users are charged for the reactive power that they use, so power factor < 1 is a bad thing.

Power factor < 1 is also bad for the power company. To deliver the same power to the load, they have more line current (and thus more line losses).

Power factors are "corrected" by adding capacitors (or capacitive loads) in parallel with the inductive loads which cause the problems. (In the rare case that the load is capacitive, the pf would be corrected by an inductor.)
Transformer basics and ratings

A Transformer is two coils of wire that are magnetically coupled.

Transformers are only useful for AC, which is one of the big reasons electrical power is generated and distributed as AC.

Transformer turns and turns ratios are rarely given, \( V_p/V_s \) is much more common where \( V_p/V_s \) is the rated primary over rated secondary voltages. You may take this to be the same as \( N_1/N_2 \) although in reality \( N_2 \) is usually a little bit bigger to make up for losses. Also common: \( V_p : V_s \).

Both RMS

Transformers are rated in VA Transformer Rating (VA) = (rated V) x (rated I) , on either side.

Don't allow voltages over the rated V, regardless of the actual current.
Don't allow currents over the rated I, regardless of the actual voltage.

Ideal Transformers

Ideal: \( P_1 = P_2 \)

power in = power out

Transformation of voltage and current

\[
\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}
\]

Turns ratio

Turns ratio as defined in most books: \( N = \frac{N_1}{N_2} \) Note: some other texts define the turns ratio as: \( N_2 \frac{N_1}{N_2} \) Be careful how you and others use this term.

Transformation of impedance

You can replace the entire transformer and load with \( (Z_{eq}) \). This "impedance transformation" can be very handy.

Transformers can be used for "impedance matching"

This also works the opposite way, to move an impedance from the primary to the secondary, multiply by:

\[
Z_{eq} = N_2 \cdot Z_2 = \left( \frac{N_1}{N_2} \right)^2 Z_2
\]

\[
N_2^2 \frac{N_1}{N_2}
\]
Other Transformers

Multi-tap transformers: Many transformers have more than two connections to primary and/or the secondary. The extra connections are called "taps" and may allow you to select from several different voltages or get more than one voltage at the same time.

Isolation Transformers: Almost all transformers isolate the primary from the secondary. An Isolation transformer has a 1:1 turns ratio and is just for isolation.

Auto Transformers: Auto transformers have only one winding with taps for various voltages. The primary and secondary are simply parts of the same winding. These parts may overlap. Any regular transformer can be wired as an auto transformer. Auto transformers DO NOT provide isolation.

Vari-AC: A special form of auto transformer with an adjustable tap for an adjustable output voltage.

LVDT: A Linear-Variable-Differential-Transformer has moveable core which couples the primary winding to the secondary winding(s) in such a way the the secondary voltage is proportional to the position of the core. LVDTs are used as position sensors.

Home power

Standard 120 V outlet connections are shown at right. The 3 lines coming into your house are NOT 3-phase. They are +120 V, Gnd, -120 V. (The two 120s are 180° out-of-phase, allowing for 240 V connections)

3-Phase Power (FYI ONLY)

Single phase power pulses at 120 Hz. This is not good for motors or generators over 5 hp. Three phase power is constant as long as the three loads are balanced. Three lines are needed to transmit 3-phase power. If loads are balanced, ground return current will be zero.

Wye connection:
Connect each load or generator phase between a line and ground.

\[
V_{LN} = \frac{V_{LL}}{\sqrt{3}} \quad I_L = \sqrt{3}I_{LL}
\]

Wye

Delta connection:
Connect each load or generator phase between two lines.

\[
V_{LL} = \sqrt{3}V_{LN} \quad I_{LL} = \frac{I_L}{\sqrt{3}}
\]

Delta
3-Phase Power (FYI ONLY)

Common 3-phase voltages:

\[ V_{120} \]
\[ V_{277} \]
\[ V_{120} \]
\[ V_{277} \]

Apparent Power:

\[ S_3\phi = \sqrt{3} V_{LN} I_L = \sqrt{3} V_{LL} I_L \]

Power:

\[ P_3\phi = \sqrt{3} V_{LN} I_L \cos(\theta) = \sqrt{3} V_{LL} I_L \cos(\theta) = S_3\phi \cos(\theta) \]

Reactive power:

\[ Q_3\phi = \sqrt{3} V_{LN} I_L \sin(\theta) \]

\[ = \sqrt{3} \left( \frac{S_3\phi}{P_3\phi} \right) - I_3 \phi \]

\[ I_a = I_L / -\theta \]

\[ V_{an} = V_{LN} \]

\[ I_a = I_L / -\theta - 120^\circ \]

\[ V_{bn} = V_{LN} / -120^\circ \]

\[ I_b = I_L / -\theta - 120^\circ \]

\[ V_{cn} = V_{LN} / -240^\circ = V_{LN} / -120^\circ \]

\[ I_c = I_L / -\theta + 120^\circ \]

\[ V_{CA} = V_{LL} / -210^\circ = V_{LL} / -150^\circ \]

\[ V_{BA} = V_{LL} / -90^\circ \]

neutral (ground at some point)

\[ V_{AN} = V_{BN} = V_{CN} = V_{LN} = V_{LL} \]

\[ I_A = I_B = I_C = I_L = \sqrt{3} I_L \]

\[ V_{AB} = V_{LL} / -30^\circ \]

\[ V_{BC} = V_{LL} / -90^\circ \]

\[ V_{CA} = V_{LL} / -210^\circ = V_{LL} / -150^\circ \]

neutral is not connected at the load

\[ V_{AN} = V_{BN} = V_{CN} = V_{LN} = V_{LL} \]

\[ I_A = I_B = I_C = I_L = \frac{I_L}{\sqrt{3}} \]

To get equivalent line currents with equivalent voltages

\[ Z_Y = \frac{Z_\Delta}{3} \]

\[ Z_\Delta = 3 \cdot Z_Y \]