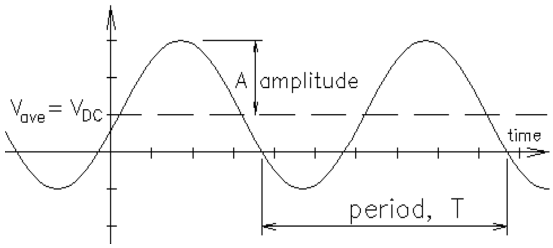
AC Power Notes

A.Stolp 4/7/02, rev 11/3/05



T = Period = repeat time $\quad T = 16.67 \cdot ms$

f = frequency, cycles/sec $\quad f = \dfrac{1}{T} = \dfrac{\omega}{2 \cdot \pi} = 60 \cdot Hz$

ω = radian frequency $\quad \omega = 2 \cdot \pi \cdot f = 377 \cdot \dfrac{rad}{sec}$

A = amplitude = V_p $\quad V_p = 115 \cdot V_{rms} \cdot \sqrt{2} = 163 \cdot V$

DC = average

RMS **R**oot of the **M**ean of the **S**quare

$$V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T (v(t))^2 \, dt}$$

Square, **R**oot, **M**ean (average)

<u>rectified average</u> $\quad V_{ra} = \dfrac{1}{T} \cdot \displaystyle\int_0^T |v(t)| \, dt$

sinusoid: $\quad V_{rms} = \dfrac{V_p}{\sqrt{2}} \quad I_{rms} = \dfrac{I_p}{\sqrt{2}} \quad V_{ra} = \dfrac{2}{\pi} \cdot V_p \quad I_{ra} = \dfrac{2}{\pi} \cdot I_p$

triangular: $\quad V_{rms} = \dfrac{V_p}{\sqrt{3}} \quad I_{rms} = \dfrac{I_p}{\sqrt{3}} \quad V_{ra} = \dfrac{1}{2} \cdot V_p \quad I_{ra} = \dfrac{1}{2} \cdot I_p$

square: $\quad V_{rms} = V_p \quad I_{rms} = I_p \quad V_{ra} = V_{rms} = V_p \quad I_{ra} = I_{rms} = I_p$

waveform + DC $\quad V_{rms} = \sqrt{V_{rmsAC}^2 + V_{DC}^2}$

Most AC meters don't measure true RMS. Instead, they measure V_{ra} , display $1.11 V_{ra}$, and call it RMS. That works for sine waves but not for any other waveform.

Use RMS in power calculations

$$P = I_{Rrms}^2 \cdot R = \frac{V_{Rrms}^2}{R} = V_{rms} \cdot I_{rms} \cdot \cos(\theta) = I_{rms}^2 \cdot |\mathbf{Z}| \cdot \cos(\theta) = \frac{V_{rms}^2}{|\mathbf{Z}|} \cdot \cos(\theta)$$

units: watts, kW, MW, etc.

P = "Real" Power (average) $= V_{rms} \cdot I_{rms} \cdot pf = I_{rms}^2 \cdot |\mathbf{Z}| \cdot pf = \dfrac{V_{rms}^2}{|\mathbf{Z}|} \cdot pf$

Power factor

$pf = \cos(\theta)$ = power factor (sometimes expressed in %) $\quad 0 \le pf \le 1$

θ is the **phase angle** between the voltage and the current or the phase angle of the impedance. $\quad \theta = \theta_Z$

$\theta < 0$ Load is "Capacitive", power factor is "leading". This condition is very rare

$\theta > 0$ Load is "Inductive", power factor is "lagging". This condition is so common you can assume any power factor given is lagging unless specified otherwise. Transformers and motors make most loads inductive.

No average power dissipated in capacitors or inductors.

Q = Reactive "power" $= V_{rms} \cdot I_{rms} \cdot \sin(\theta)$ $\quad$ units: VAR, kVAR, etc. "volt-amp-reactive"

capacitors -> neg Q, inductors -> positive Q

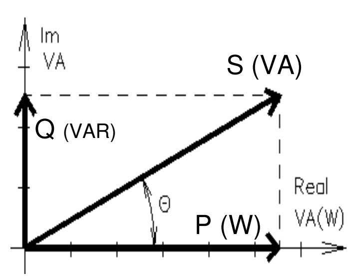
$\mathbf{S}$ = Complex "power" $= \mathbf{V_{rms}} \cdot \overline{\mathbf{I_{rms}}}$ (complex congugate) $= P + jQ = V_{rms} I_{rms} \underline{/\theta}$ $\quad$ units: VA, kVA, etc. "volt-amp"

S = Apparent "power" $= |\mathbf{S}| = V_{rms} \cdot I_{rms} = \sqrt{P^2 + Q^2}$ $\quad$ units: VA, kVA, etc. "volt-amp"

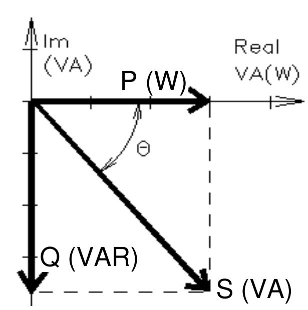
Industrial users are charged for the apparent power that they use, so power factor < 1 is a bad thing.

Power factor < 1 is also bad for the power company. To deliver the same power to the load, they have more line current (and thus more line losses).

Power factors are "corrected" by adding large capacitors (or capacitve loads) in parallel with the inductive loads which cause the problems



"Lagging" power



"Leading" Power

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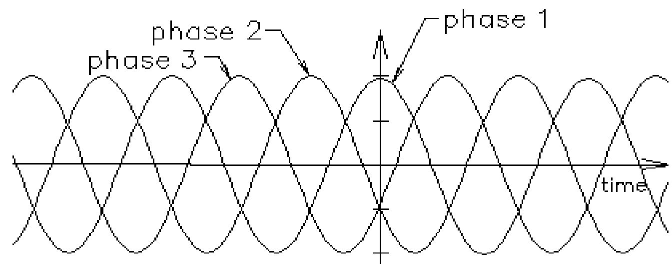


3-Phase Power

Single phase power pulses at 120 Hz. This is not good for motors or generators over 5 hp.

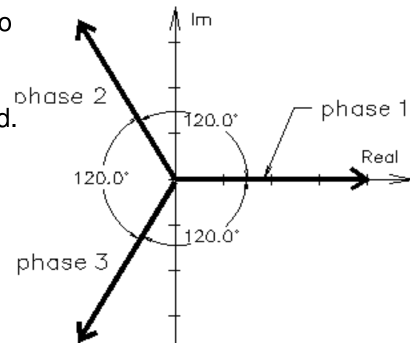
Three phase power is constant as long as the three loads are balanced.

Three lines are needed to transmit 3-phase power.
If loads are balanced, ground return current will be zero

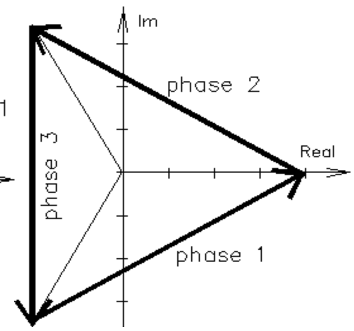
Wye connection:
Connect each of your loads between a line and ground.

Delta connection:
Connect each of your loads between two lines.

$$V_{\Delta} = \sqrt{3}\cdot V_Y \qquad I_{\Delta} = \frac{I_Y}{\sqrt{3}}$$



Wye



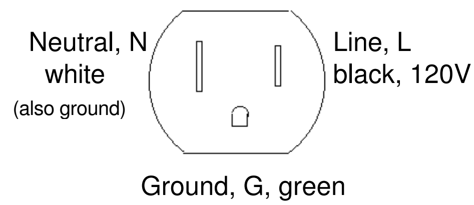
Delta

Home power

Standard 120 V outlet connections are shown at right.

The 3 lines coming into your house are **NOT** 3-phase.
They are +120 V, Gnd, -120 V

(The two 120s are 180º out-of-phase, allowing for 240 V connections)



Transformers (ideal)

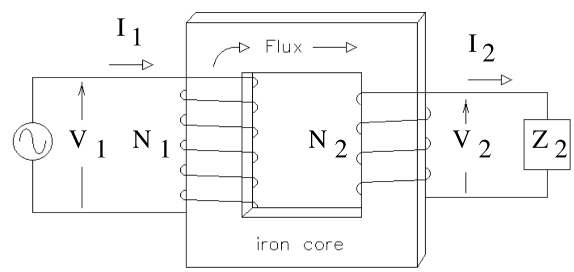
Two coils of wire that are magnetically coupled.

Transformers are only useful for AC, which is one of the big reasons electrical power is generated and distributed as AC.

The higher the voltage of a power transmission line, the lower the current needed (and hence the I^2R losses in the wire).

Transformers are used to increase/decrease voltages/currents for power transmission and power supplies.

Iron-core transformer
primary secondary



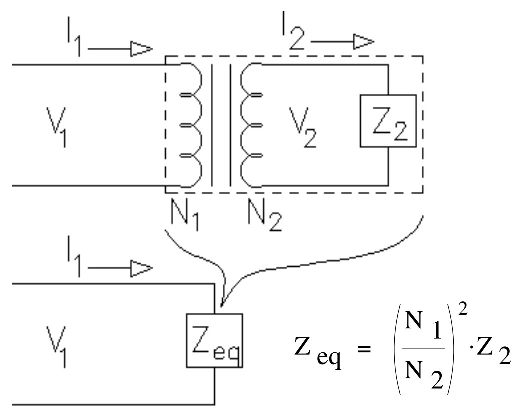
Ideal: $P_1 = P_2$

power in = power out

$$\text{Turns ratio} = N = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

Note: some books define the turns ratio as N_2/N_1

Equivalent impedance in primary: $\mathbf{Z}_{eq} = N^2\cdot\mathbf{Z}_2 = \left(\frac{N_1}{N_2}\right)^2\cdot\mathbf{Z}_2$



$$Z_{eq} = \left(\frac{N_1}{N_2}\right)^2\cdot Z_2$$

You can replace the entire transformer and load with ($\mathbf{Z}_{eq}$).
This "impedance transformation" can be very handy.
Transformers can be used for "impedance matching"

Transformer Rating (VA) = (rated V_{rms}) x (rated I_{rms}) , on either side.

Don't allow voltages over the rated V_{rms}.
Don't allow currents over the rated I_{rms}.