ECE 2210 AC Power Examples

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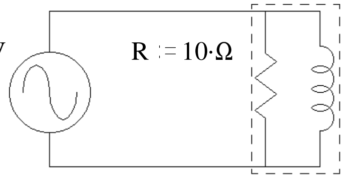
Ex. 1 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

$\mathbf{V}_{in} := 110 \cdot V$ $\quad R := 10 \cdot \Omega \quad L := 25 \cdot mH$

$\omega := 377 \cdot \frac{rad}{sec}$



$$\mathbf{Z} := \frac{1}{\left(\frac{1}{R} + \frac{1}{j \cdot \omega \cdot L}\right)} = \frac{1}{0.1458 \cdot \frac{1}{\Omega} \cdot e^{-j \cdot 46.7 \cdot deg}}$$

$\mathbf{Z} = 4.704 + 4.991j \ \bullet\Omega \qquad |\mathbf{Z}| = 6.859 \bullet \Omega \qquad \theta := \arg(\mathbf{Z}) \qquad \theta = 46.7 \bullet deg \qquad pf := \cos(\theta) \qquad pf = 0.686$

$\mathbf{I} := \frac{\mathbf{V}_{in}}{\mathbf{Z}} \qquad \mathbf{I} = 11 - 11.671j \ \bullet A \qquad |\mathbf{I}| = 16.038 \bullet A \qquad \arg(\mathbf{I}) = -46.7 \bullet deg$

$P := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot pf \qquad P = 1.21 \bullet kW$

$Q := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot \sin(\theta) \qquad Q = 1.284 \bullet kVAR$ OR... $Q := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot \sqrt{1^2 - pf^2} \qquad Q = 1.284 \bullet kVAR$

$\mathbf{S} := \mathbf{V}_{in} \cdot \overline{\mathbf{I}}$ OR.. $\mathbf{S} := P + j \cdot Q \qquad \mathbf{S} = 1.21 + 1.284j \ \bullet kVA \qquad S := \sqrt{Re(\mathbf{S})^2 + Im(\mathbf{S})^2} = |\mathbf{S}| = 1.764 \bullet kVA$

$\operatorname{atan}\left(\frac{Im(\mathbf{S})}{Re(\mathbf{S})}\right) = 46.696 \bullet deg$

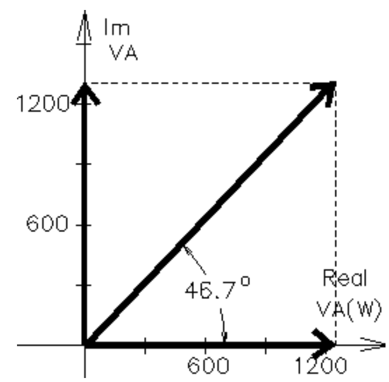
$\mathbf{S} = 1.764kVA \ \underline{/46.7^o}$

OR, since we know that the voltage across each element of the load is V_{in} ...
Real power is dissipated only by resistors

$P := \frac{(|\mathbf{V}_{in}|)^2}{R} \qquad P = 1.21 \bullet kW \qquad Q := \frac{(|\mathbf{V}_{in}|)^2}{\omega \cdot L} \qquad Q = 1.284 \bullet kVAR$

$\mathbf{S} := P + j \cdot Q$

$S = |\mathbf{S}| = \sqrt{P^2 + Q^2} = 1.764 \bullet kVA \qquad pf = \frac{P}{|\mathbf{S}|} = 0.686$



What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

$Im(\mathbf{I}) = -11.671 \bullet A \qquad X_C := \frac{\mathbf{V}_{in}}{Im(\mathbf{I})} \qquad X_C = -9.425 \bullet \Omega = \frac{-1}{\omega \cdot C}$

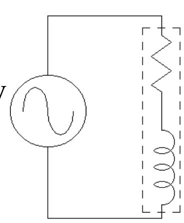
$\frac{1}{|X_C| \cdot \omega} = 281 \bullet \mu F$ OR.. $\omega = \frac{1}{\sqrt{L \cdot C}} \qquad C := \frac{1}{L \cdot \omega^2} \qquad C = 281 \bullet \mu F$

Ex. 2 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

Series R & L

$\mathbf{V}_{in} := 110 \cdot V$

$\omega := 377 \cdot \frac{rad}{sec}$



$R := 10 \cdot \Omega$ load $L := 25 \cdot mH$

$\mathbf{Z} := R + j \cdot \omega \cdot L$

$\mathbf{Z} = 10 + 9.425j \ \bullet \Omega \qquad |\mathbf{Z}| = 13.742 \bullet \Omega$

$\theta := \arg(\mathbf{Z}) \qquad \theta = 43.304 \bullet deg \qquad pf := \cos(\theta) \qquad pf = 0.728$

$\mathbf{I} := \frac{\mathbf{V}_{in}}{\mathbf{Z}} \qquad \mathbf{I} = 5.825 - 5.49j \ \bullet A$

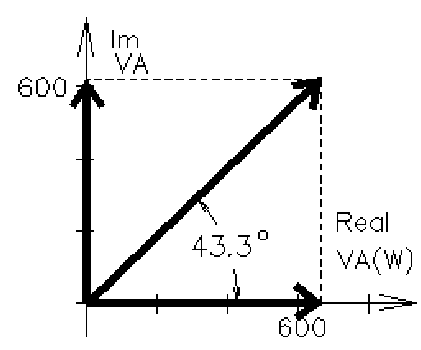
$|\mathbf{I}| = 8.005 \bullet A \qquad \arg(\mathbf{I}) = -43.304 \bullet deg$

$P := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot pf \qquad P = 0.641 \bullet kW$

$Q := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot \sin(\theta) \qquad Q = 0.604 \bullet kVAR$

$\mathbf{S} := \mathbf{V}_{in} \cdot \overline{\mathbf{I}} \qquad \mathbf{S} = 0.641 + 0.604j \ \bullet kVA$

$|\mathbf{S}| = 0.881 \bullet kVA \qquad \arg(\mathbf{S}) = 43.304 \bullet deg \qquad \mathbf{S} = 881VA\underline{/43.3^o}$



OR, if we first find the magnitude of the current which flows through each element of the load...

$$|\mathbf{I}| = \frac{V_{in}}{\sqrt{R^2 + (\omega \cdot L)^2}} = 8.005 \cdot A$$

$P := (|\mathbf{I}|)^2 \cdot R$ $\quad$ $P = 0.641 \cdot kW$ $\quad$ $Q := (|\mathbf{I}|)^2 \cdot (\omega \cdot L)$ $\quad$ $Q = 0.604 \cdot kVAR$

$\mathbf{S} := P + j \cdot Q$ $\quad$ $|\mathbf{S}| = \sqrt{P^2 + Q^2} = 0.881 \cdot kVA$ $\quad$ $pf = \frac{P}{|\mathbf{S}|} = 0.728$

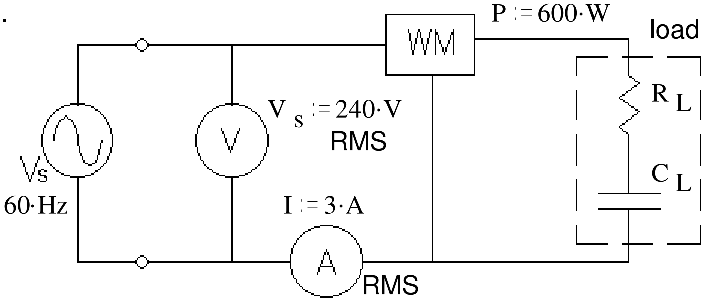
What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

$Q = 603.9 \cdot VAR$ $\quad$ so we need: $Q_C := -Q$ $\quad$ $Q_C = -603.9 \cdot VAR = \frac{V_{in}^2}{X_C}$

$X_C := \frac{V_{in}^2}{Q_C}$ $\quad$ $X_C = -20.035 \cdot \Omega = \frac{-1}{\omega \cdot C}$ $\quad$ $C := \frac{1}{|X_C| \cdot \omega}$ $\quad$ $C = 132 \cdot \mu F$

Check: $\dfrac{1}{\dfrac{1}{R + j \cdot \omega \cdot L} + j \cdot \omega \cdot C} = 18.883 \cdot \Omega$ $\quad$ No j term, so $\theta = 0^o$

Ex. 3 R, & C together are the load in the circuit shown. The RMS voltmeter measures 240 V, the RMS ammeter measures 3 A, and the wattmeter measures 600 W. Find the following: Be sure to show the correct units for each value.



a) The value of the load resistor. R_L = ?

$P = I^2 \cdot R_L$

$R_L := \frac{P}{I^2}$ $\quad$ $R_L = 66.7 \cdot \Omega$

b) The apparent power. |**S**| = ? $\quad$ $S := V_s \cdot I$ $\quad$ $S = 720 \cdot VA$

c) The reactive power. Q = ? $\quad$ $Q := -\sqrt{S^2 - P^2}$ $\quad$ $Q = -398 \cdot VAR$

d) The complex power. **S** = ? $\quad$ $\mathbf{S} := P + j \cdot Q$ $\quad$ $\mathbf{S} = 600 - 398i \cdot VA$

e) The power factor. pf = ? $\quad$ $pf := \frac{P}{V_s \cdot I}$ $\quad$ $pf = 0.833$

f) The power factor is leading or lagging? $\quad$ leading (load is capacitive, Q is negative)

g) The two components of the load are in a box which cannot be opened. Add (draw it) another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and <u>find its value</u>. This component should not affect the real power consumption of the load.

$f = 60 \cdot Hz$ $\quad$ $\omega := 377 \cdot \frac{rad}{sec}$

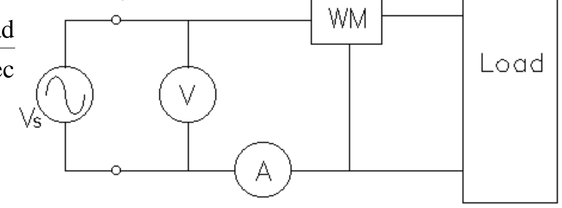
Add an inductor in parallel with load

$Q = -398 \cdot VAR$ $\quad$ so we need: $Q_L := -Q$ $\quad$ $Q_L = 398 \cdot VAR = \frac{V_s^2}{X_L}$

$X_L := \frac{V_s^2}{Q_L}$ $\quad$ $X_L = 144.725 \cdot \Omega = \omega \cdot L$ $\quad$ $L := \frac{|X_L|}{\omega}$ $\quad$ $L = 384 \cdot mH$

Ex. 4 For the 60 Hz load shown in the figure, the RMS voltmeter measures 120 V. The phasor diagram for the power is also shown. Find the following:

$V_s := 120\cdot V$ $\omega := 377\cdot\frac{rad}{sec}$

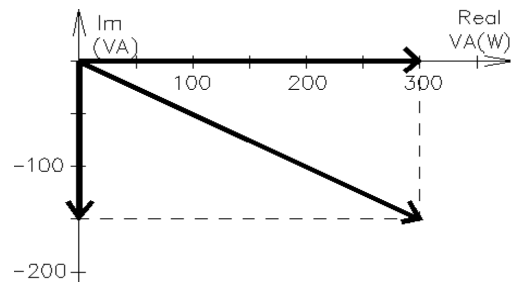


a) The complex power. **S** = ?

$P := 300\cdot W$ $Q := -150\cdot VA$

$\mathbf{S} := P + j\cdot Q$ $\mathbf{S} = 300 - 150j \cdot VA$

b) The apparent power. |**S**| = ? $|\mathbf{S}| = \sqrt{P^2 + Q^2} = 335.4\cdot VA$



c) The power factor. pf = ? $pf := \frac{P}{|\mathbf{S}|}$ $pf = 0.894$

d) The item marked "WM" in the figure is a wattmeter, what does it read? (give a number) $P = 300\cdot W$

e) The item marked "A" in the figure is an RMS ammeter, what does it read? (give a number)

$I := \frac{|\mathbf{S}|}{V_s}$ $I = 2.795\cdot A$ $I = 2.8\cdot A$

f) The power factor is leading or lagging? leading (Q is negative)

g) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and <u>find its value</u>. This component should not affect the real power consumption of the load.

Add an inductor in parallel with load

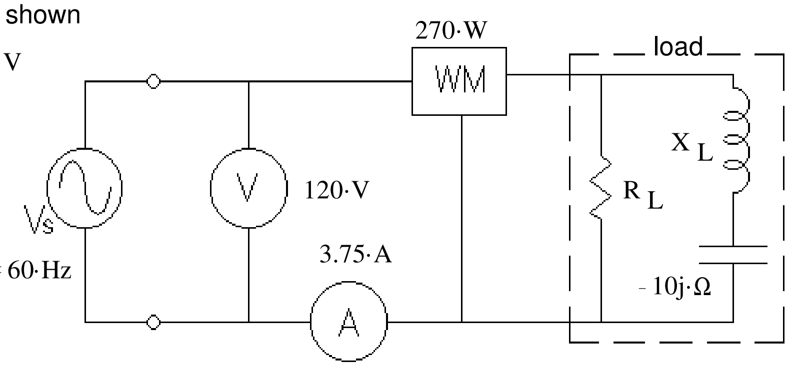
$Q = -150\cdot VAR$ need: $Q_L := -Q$ $Q_L = 150\cdot VAR$ $= \frac{{V_s}^2}{\omega\cdot L}$ $L := \frac{{V_s}^2}{\omega\cdot Q_L}$ $L = 255\cdot mH$

Ex. 5 R, L, & C together are the load in the circuit shown

The RMS voltmeter measures 120 V. $V_s := 120\cdot V$

The wattmeter measures 270 W. $P := 270\cdot W$

The RMS ammeter measures 3.75 A. $I := 3.75\cdot A$



Find the following: Be sure to show the correct units for each value.

a) The value of the load resistor. R_L = ?

$P = \frac{{V_s}^2}{R_L}$ $R_L := \frac{{V_s}^2}{P}$ $R_L = 53.3\cdot\Omega$

b) The magnitude of the impedance of the load inductor (reactance) . $|\mathbf{Z}_L| = X_L$ = ?

$I_R := \frac{V_s}{R_L}$ $I_R = 2.25\cdot A$ $I_L := \sqrt{I^2 - {I_R}^2}$ $I_L = 3\cdot A$ $X := \frac{V_s}{I_L}$ $X = 40\cdot\Omega$

$X_C := -10\cdot\Omega$ $X_L := X - X_C$ $X_L = 50\cdot\Omega$

c) The reactive power. Q = ? $Q := \sqrt{(V_s\cdot I)^2 - P^2}$ $Q = 360\cdot VAR$ positive, because the load is primarily inductive

d) The power factor is leading or lagging? lagging (load is inductive, Q is positive)

e) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and <u>find its value</u>. This component should not affect the real power consumption of the load.

Add a capacitor in parallel with load

$f = 60 \cdot Hz \qquad \omega := 377 \cdot \frac{rad}{sec}$

$Q = 360 \cdot VAR$ so we need: $Q_C := -Q \qquad Q_C = -360 \cdot VAR = -\frac{V_s^2}{\frac{1}{\omega \cdot C}} = -\omega \cdot C \cdot V_s^2$

$C := \frac{Q_C}{-\omega \cdot V_s^2} \qquad C = 66.3 \cdot \mu F$

Ex. 6 A step-down transformer has an output voltage of 220 V (rms) when the primary is connected across a 560 V (rms) source.

a) If there are 280 turns on the primary winding, how many turns are required on the secondary?

$280 \cdot \frac{220 \cdot volt}{560 \cdot volt} = 110$ turns

b) If the current in the primary is 2.4 A, what current flows in the load connected to the secondary?

$2.4 \cdot amp \cdot \frac{280}{110} = 6.11 \cdot A$

c) If the transformer is rated at 700/275 V, 2.1 kVA, what are the rated primary and secondary currents?

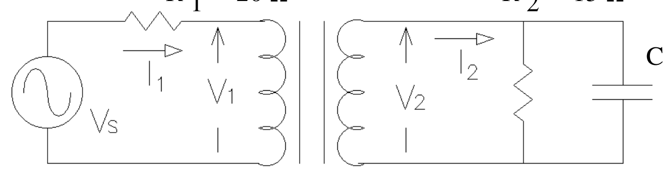
pri: $\frac{2.1 \cdot kVA}{700 \cdot V} = 3 \cdot A$ sec: $\frac{2.1 \cdot kVA}{275 \cdot V} = 7.636 \cdot A$

Ex. 7 The transformer shown in the circuit below is ideal. Find the following:

a) $|\mathbf{I_1}|$ = ?

$V_s := 120 \cdot V \qquad \omega := 377 \cdot \frac{rad}{sec}$

$R_1 := 20 \cdot \Omega \qquad R_2 := 15 \cdot \Omega \qquad C := 40 \cdot \mu F$



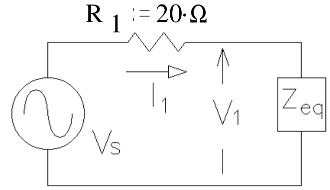
$N_1 := 150 \cdot turns \qquad N_2 := 50 \cdot turns$

$\mathbf{Z_L} := \frac{1}{\frac{1}{R_2} + j \cdot \omega \cdot C}$

$\mathbf{Z_L} = 14.27 - 3.228j \cdot \Omega$

Make an equivalent circuit:

$R_1 := 20 \cdot \Omega$



$\mathbf{Z_{eq}} := \left(\frac{N_1}{N_2}\right)^2 \cdot \mathbf{Z_L} \qquad \mathbf{Z_{eq}} = 128.429 - 29.051j \cdot \Omega$

$R_1 + \mathbf{Z_{eq}} = 148.429 - 29.051j \cdot \Omega \qquad \sqrt{148.429^2 + 29.051^2} = 151.245$

$|\mathbf{I_1}| = \frac{V_s}{|R_1 + \mathbf{Z_{eq}}|} = \frac{V_s}{151.245 \cdot \Omega} = 0.793 \cdot A$

b) $|\mathbf{I_2}|$ = ? $= \left(\frac{N_1}{N_2}\right) \cdot |\mathbf{I_1}| = \frac{150}{50} \cdot .793 \cdot A = 2.379 \cdot A$

c) $|\mathbf{V_1}|$ = ? $= V_s \cdot \left|\frac{\mathbf{Z_{eq}}}{R_1 + \mathbf{Z_{eq}}}\right|$ OR.. $|\mathbf{V_1}| = |\mathbf{I_1}| \cdot |\mathbf{Z_{eq}}| = .793 \cdot A \cdot \sqrt{128.429^2 + 29.051^2} \cdot \Omega = 104.417 \cdot V$