ECE 2210 AC Power Examples

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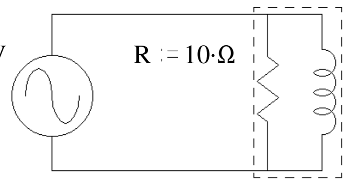
Ex. 1 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.



$V_{in} := 110 \cdot V$ $\quad \omega := 377 \cdot \frac{rad}{sec}$ $\quad R := 10 \cdot \Omega$ $\quad L := 25 \cdot mH$ load

$$\mathbf{Z} := \frac{1}{\left(\frac{1}{R} + \frac{1}{j \cdot \omega \cdot L}\right)} = \frac{1}{0.1458 \cdot \frac{1}{\Omega} \cdot e^{-j \cdot 46.7 \cdot deg}}$$

$\mathbf{Z} = 4.704 + 4.991j \cdot \Omega$ $\quad |\mathbf{Z}| = 6.859 \cdot \Omega$ $\quad \theta := \arg(\mathbf{Z})$ $\quad \theta = 46.7 \cdot deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.686$

$\mathbf{I} := \frac{V_{in}}{\mathbf{Z}}$ $\quad \mathbf{I} = 11 - 11.671j \cdot A$ $\quad |\mathbf{I}| = 16.038 \cdot A$ $\quad \arg(\mathbf{I}) = -46.7 \cdot deg$

$P := |V_{in}| \cdot |\mathbf{I}| \cdot pf$ $\quad P = 1.21 \cdot kW$

$Q := |V_{in}| \cdot |\mathbf{I}| \cdot \sin(\theta)$ $\quad Q = 1.284 \cdot kVAR$ OR... $Q := |V_{in}| \cdot |\mathbf{I}| \cdot \sqrt{1^2 - pf^2}$ $\quad Q = 1.284 \cdot kVAR$

$\mathbf{S} := V_{in} \cdot \overline{I}$ OR.. $\mathbf{S} := P + j \cdot Q$ $\quad \mathbf{S} = 1.21 + 1.284j \cdot kVA$ $\quad S := \sqrt{\mathrm{Re}(\mathbf{S})^2 + \mathrm{Im}(\mathbf{S})^2} = |\mathbf{S}| = 1.764 \cdot kVA$

$\mathrm{atan}\left(\frac{\mathrm{Im}(\mathbf{S})}{\mathrm{Re}(\mathbf{S})}\right) = 46.696 \cdot deg$

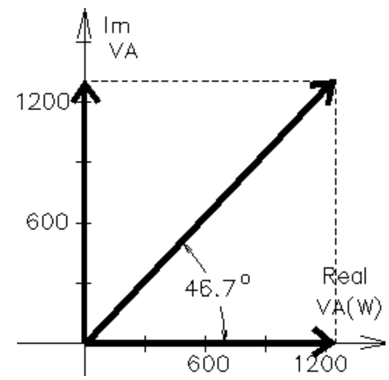
$\mathbf{S} = 1.764kVA \underline{/46.7^o}$

OR, since we know that the voltage across each element of the load is V_{in} ...
Real power is dissipated only by resistors

$P := \frac{(|V_{in}|)^2}{R}$ $\quad P = 1.21 \cdot kW$ $\quad Q := \frac{(|V_{in}|)^2}{\omega \cdot L}$ $\quad Q = 1.284 \cdot kVAR$

$\mathbf{S} := P + j \cdot Q$

$S = |\mathbf{S}| = \sqrt{P^2 + Q^2} = 1.764 \cdot kVA$ $\quad pf = \frac{P}{|\mathbf{S}|} = 0.686$



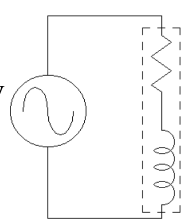
What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

$\mathrm{Im}(\mathbf{I}) = -11.671 \cdot A$ $\quad X_C := \frac{V_{in}}{\mathrm{Im}(\mathbf{I})}$ $\quad X_C = -9.425 \cdot \Omega = \frac{-1}{\omega \cdot C}$

$\frac{1}{|X_C| \cdot \omega} = 281 \cdot \mu F$ OR.. $\omega = \frac{1}{\sqrt{L \cdot C}}$ $\quad C := \frac{1}{L \cdot \omega^2}$ $\quad C = 281 \cdot \mu F$

Ex. 2 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

Series R & L



$V_{in} := 110 \cdot V$ $\quad \omega := 377 \cdot \frac{rad}{sec}$ $\quad R := 10 \cdot \Omega$ load $\quad L := 25 \cdot mH$

$\mathbf{Z} := R + j \cdot \omega \cdot L$

$\mathbf{Z} = 10 + 9.425j \cdot \Omega$ $\quad |\mathbf{Z}| = 13.742 \cdot \Omega$

$\theta := \arg(Z)$ $\quad \theta = 46.696 \cdot deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.686$

$\mathbf{I} := \frac{V_{in}}{\mathbf{Z}}$ $\quad I = 11 - 11.671j \cdot sec^{-1} \cdot coul$

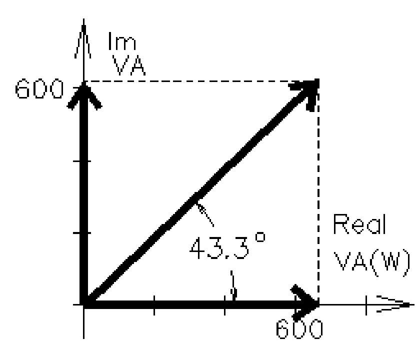
$|\mathbf{I}| = 8.005 \cdot A$ $\quad \arg(I) = -46.696 \cdot deg$

$P := |V_{in}| \cdot |\mathbf{I}| \cdot pf$ $\quad P = 0.604 \cdot kW$

$Q := |V_{in}| \cdot |\mathbf{I}| \cdot \sin(\theta)$ $\quad Q = 0.641 \cdot kVAR$

$\mathbf{S} := V_{in} \cdot \overline{\mathbf{I}}$ $\quad \mathbf{S} = 0.641 + 0.604j \cdot kVA$

$|\mathbf{S}| = 0.881 \cdot kVA$ $\quad \arg(\mathbf{S}) = 43.304 \cdot deg$ $\quad \mathbf{S} = 881VA\underline{/43.3^o}$



OR, if we first find the magnitude of the current which flows through each element of the load...

$$|\mathbf{I}| = \frac{V_{in}}{\sqrt{R^2 + (\omega \cdot L)^2}} = 8.005 \cdot A$$

$P := (|\mathbf{I}|)^2 \cdot R$ $\quad P = 0.641 \cdot kW$ $\quad Q := (|\mathbf{I}|)^2 \cdot (\omega \cdot L)$ $\quad Q = 0.604 \cdot kVAR$

$\mathbf{S} := P + j \cdot Q$ $\quad |\mathbf{S}| = \sqrt{P^2 + Q^2} = 0.881 \cdot kVA$ $\quad pf = \frac{P}{|\mathbf{S}|} = 0.728$

What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

$Q = 603.9 \cdot VAR$ so we need: $Q_C := -Q$ $\quad Q_C = -603.9 \cdot VAR = \frac{V_{in}^2}{X_C}$

$X_C := \frac{V_{in}^2}{Q_C}$ $\quad X_C = -20.035 \cdot \Omega = \frac{-1}{\omega \cdot C}$ $\quad C := \frac{1}{|X_C| \cdot \omega}$ $\quad C = 132 \cdot \mu F$

Check: $\frac{1}{\frac{1}{R + j \cdot \omega \cdot L} + j \cdot \omega \cdot C} = 18.883 \cdot \Omega$ No j term, so $\theta = 0^o$

Ex. 3

A step-down transformer has an output voltage of 220 V (rms) when the primary is connected across a 560 V (rms) source.

a) If there are 280 turns on the primary winding, how many turns are required on the secondary? $\quad 280 \cdot \frac{220 \cdot volt}{560 \cdot volt} = 110$ turns

b) If the current in the primary is 2.4 A, what current flows in the load connected to the secondary? $\quad 2.4 \cdot amp \cdot \frac{280}{110} = 6.11 \cdot A$

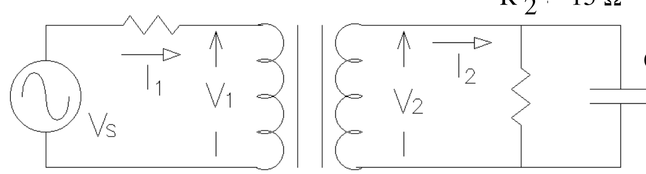
c) If the transformer is rated at 700/275 V, 2.1 kVA, what are the rated primary and secondary currents? pri: $\frac{2.1 \cdot kVA}{700 \cdot V} = 3 \cdot A$ sec: $\frac{2.1 \cdot kVA}{275 \cdot V} = 7.636 \cdot A$

Ex. 4 The transformer shown in the circuit below is ideal. Find the following:

a) $\mathbf{I_1}$ = ?

$R_1 := 20 \cdot \Omega$ $\quad R_2 := 15 \cdot \Omega$

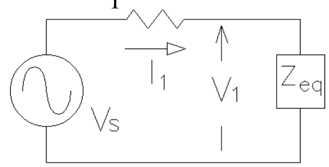
$V_s := 120 \cdot V$ $\quad \omega := 377 \cdot \frac{rad}{sec}$ $\quad C := 40 \cdot \mu F$



$N_1 := 150 \cdot turns$ $\quad N_2 := 50 \cdot turns$ $\quad \mathbf{Z_L} := \frac{1}{\frac{1}{R_2} + j \cdot \omega \cdot C}$

$\mathbf{Z_L} = 14.27 - 3.228j \cdot \Omega$

Make an equivalent circuit: $R_1 := 20 \cdot \Omega$



$\mathbf{Z_{eq}} := \left(\frac{N_1}{N_2}\right)^2 \cdot \mathbf{Z_L}$ $\quad \mathbf{Z_{eq}} = 128.429 - 29.051j \cdot \Omega$

$\mathbf{I_1} := \frac{V_s}{R_1 + \mathbf{Z_{eq}}}$ $\quad \mathbf{I_1} = 0.779 + 0.152j \cdot A$

b) $\mathbf{I_2}$ = ? $\quad \mathbf{I_2} := \left(\frac{N_1}{N_2}\right) \cdot \mathbf{I_1}$ $\quad \mathbf{I_2} = 2.336 + 0.457j \cdot A$

c) $\mathbf{V_2}$ = ? $\quad \mathbf{V_1} := V_s \cdot \frac{\mathbf{Z_{eq}}}{R_1 + \mathbf{Z_{eq}}}$ $\quad \mathbf{V_2} = \left(\frac{N_2}{N_1}\right) \cdot \mathbf{V_1} = 34.809 - 1.016j \cdot V$

OR.. $\mathbf{V_2} = \mathbf{I_2} \cdot \mathbf{Z_L} = 34.809 - 1.016j \cdot V$