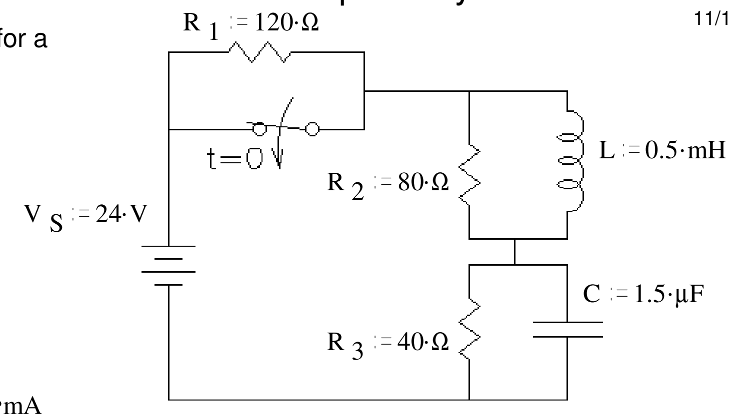
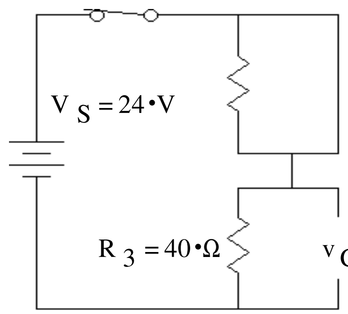
A. Stolp
11/1/06

Ex 1. The switch at right has been in the open position for a long time and is closed (as shown) at time $t = 0$.



a) What are the final conditions of i_L and the v_C?



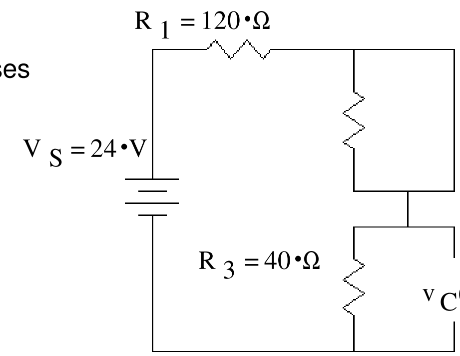
$R_3 := 40\cdot\Omega$

$$i_L(\infty) = \frac{V_S}{R_3} = 600\cdot mA$$

$$v_C(\infty) = V_S = 24\cdot V$$

b) Find the initial condition and intial slope of i_L so that you could find all the constants in $i_L(t)$.
Don't find $i_L(t)$ or it's constants, just the initial conditions.

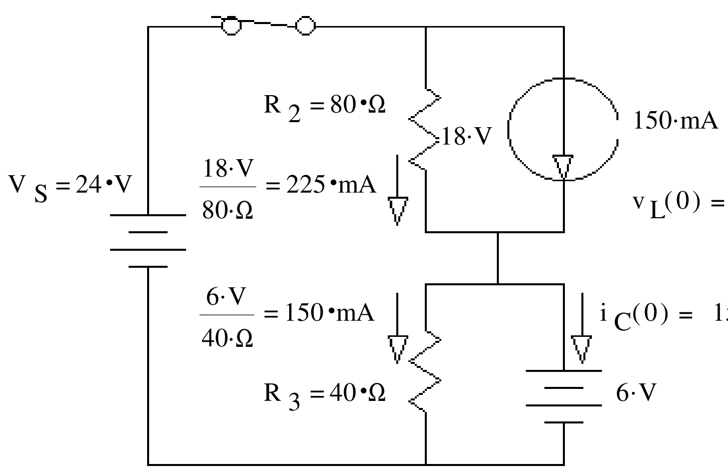
Before the switch closes



$$i_L(0) = \frac{V_S}{R_1 + R_3} = 150\cdot mA$$

$$v_C(0) = V_S\cdot\frac{R_3}{R_1 + R_3} = 6\cdot V$$

Just after the switch closes:



$$v_L(0) = 24\cdot V - 6\cdot V = 18\cdot V$$

$$\frac{d}{di} i_L(0) = \frac{18\cdot V}{L} = 36000\cdot\frac{A}{sec}$$

$$i_C(0) = 150\cdot mA + \frac{18\cdot V}{R_2} - \frac{6\cdot V}{R_3} = 225\cdot mA$$

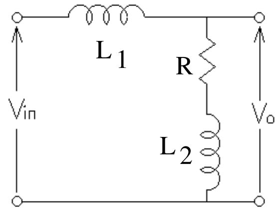
c) Find the initial condition and intial slope of v_C so that you could find all the constants in $v_C(t)$.
Don't find $v_C(t)$ or it's constants, just the initial conditions.

$$v_C(0) = V_S\cdot\frac{R_3}{R_1 + R_3} = 6\cdot V \qquad \frac{d}{di} v_C(0) = \frac{225\cdot mA}{C} = 150000\cdot\frac{V}{sec}$$

Systems

Now that we' ve developed the concept of the transfer function, we can now develop system block diagrams using blocks which contain transfer functions.

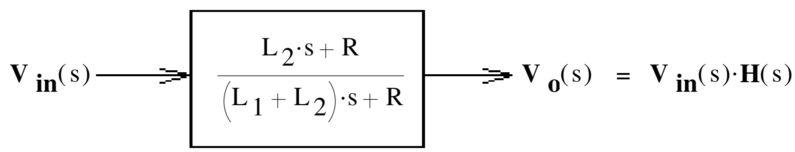
Consider a circuit:



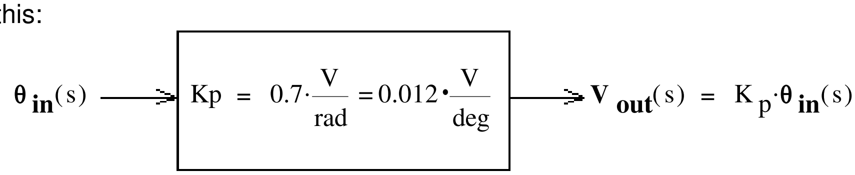
$$\mathbf{H}(s) = \frac{\mathbf{V_o}(s)}{\mathbf{V_{in}}(s)} = \frac{R + L_2 \cdot s}{R + L_1 \cdot s + L_2 \cdot s} = \frac{R + L_2 \cdot s}{R + \left(L_1 + L_2\right) \cdot s}$$

$$= \frac{L_2 \cdot s + R}{\left(L_1 + L_2\right) \cdot s + R}$$

This could be represented in as a block operator:

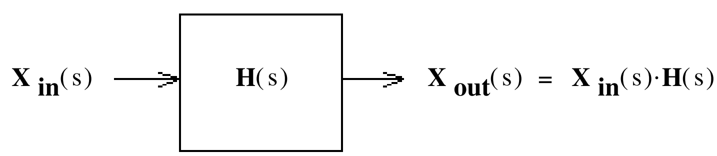


Transfer functions can be written for all kinds of devices and systems, not just electric circuits and the input and output do not have to be similar. For instance, the potentiometers used to measure angular position in the lab servo can be represented like this:



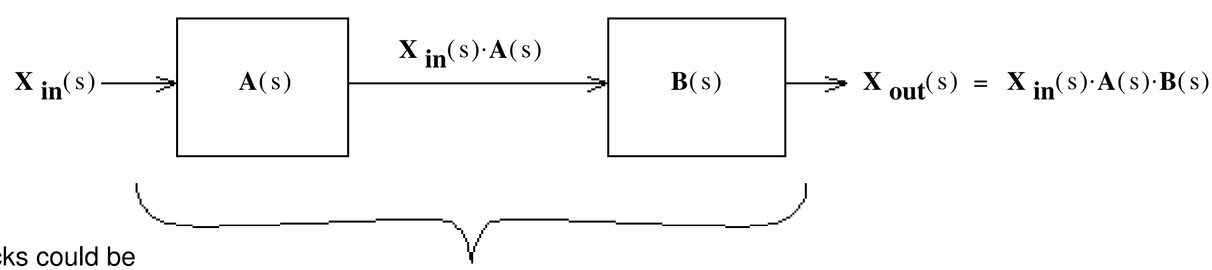
In general:

$$\mathbf{H}(s) = \frac{\mathbf{X_{out}}(s)}{\mathbf{X_{in}}(s)}$$

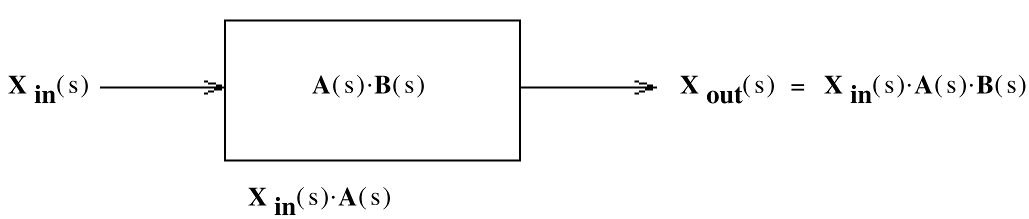


$\mathbf{X_{in}}$ and $\mathbf{X_{out}}$ could be anything from small electrical signals to powerful mechanical motions or forces.

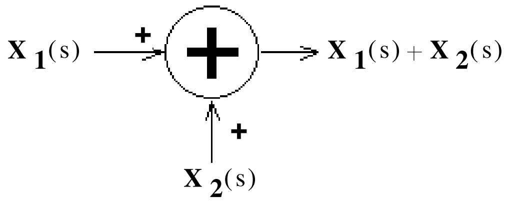
Two blocks with transfer functions $\mathbf{A}(s)$ and $\mathbf{B}(s)$ in a row would look like this:



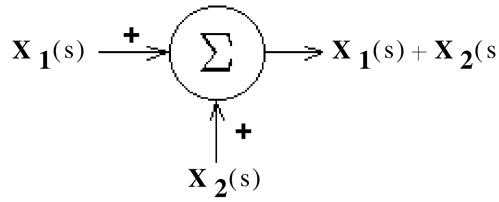
The two blocks could be replaced by a single equivalent block:



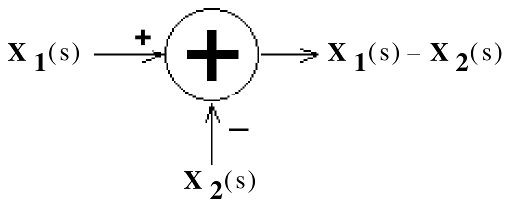
Summer blocks can be used to add signals:



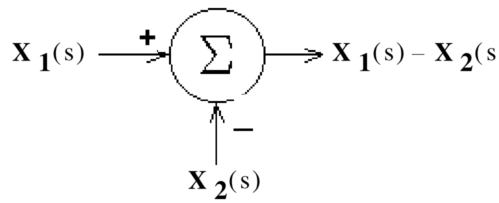
OR



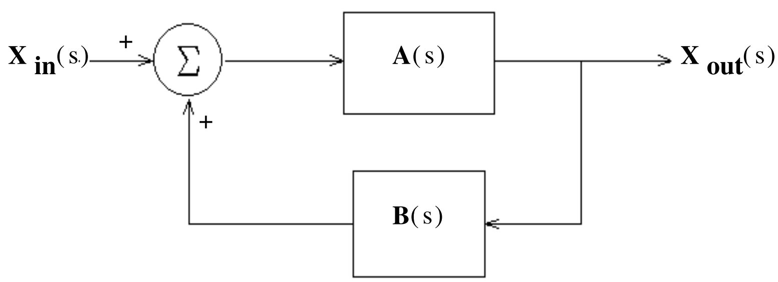
or subtract signals:



OR

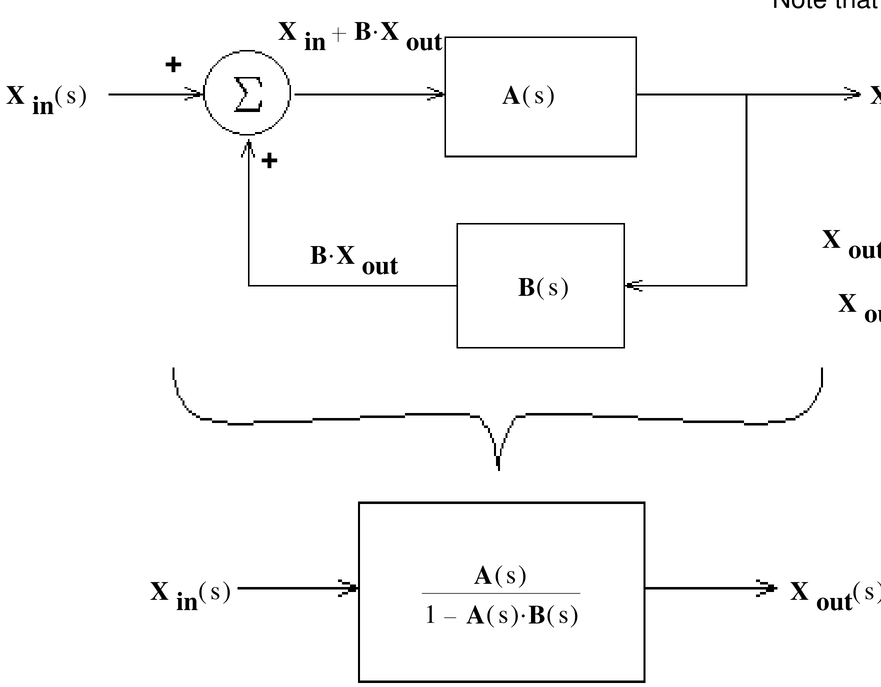


A feedback loop system is particularly interesting and useful:



The entire loop can be replaced by a single equivalent block:

Note that I' ve begun to drop the (s)



$$X_{out}(s) = A\cdot\left(X_{in} + B\cdot X_{out}\right)$$

$$= A\cdot X_{in} + A\cdot B\cdot X_{out}$$

$$X_{out} - A\cdot B\cdot X_{out} = A\cdot X_{in}$$

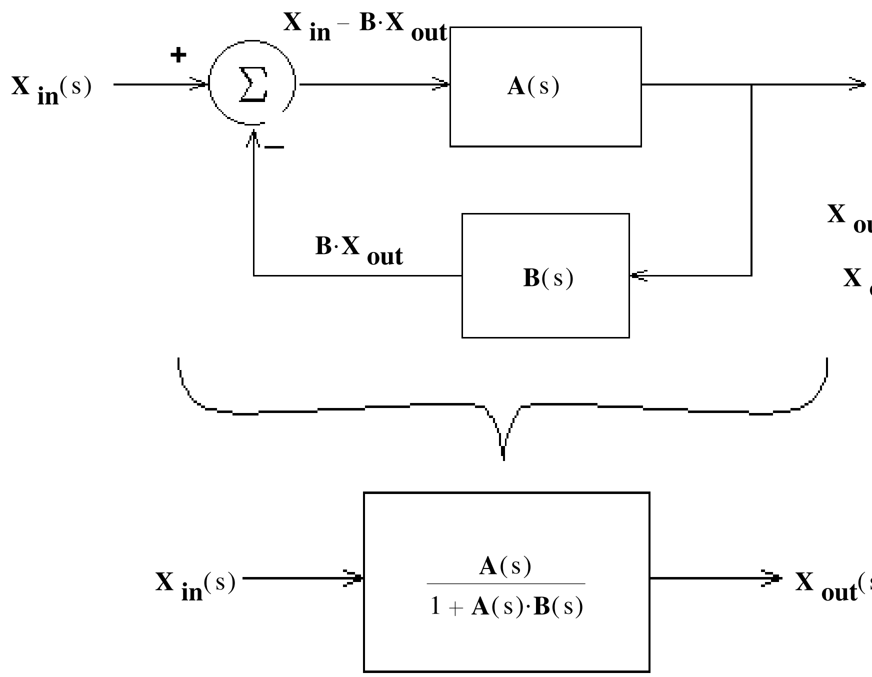
$$X_{out}\cdot(1 - A\cdot B) = A\cdot X_{in}$$

$$\frac{X_{out}}{X_{in}} = \frac{A}{1 - A\cdot B} = H(s)$$

The equivalent transfer function

$A(s)\cdot B(s)$ is called the "loop gain" or "open loop gain"

Negative feedback is more common and is used as a control system:



$$X_{out}(s) = A \cdot \left(X_{in} - B \cdot X_{out}\right)$$

$$= A \cdot X_{in} - A \cdot B \cdot X_{out}$$

$$X_{out} + A \cdot B \cdot X_{out} = A \cdot X_{in}$$

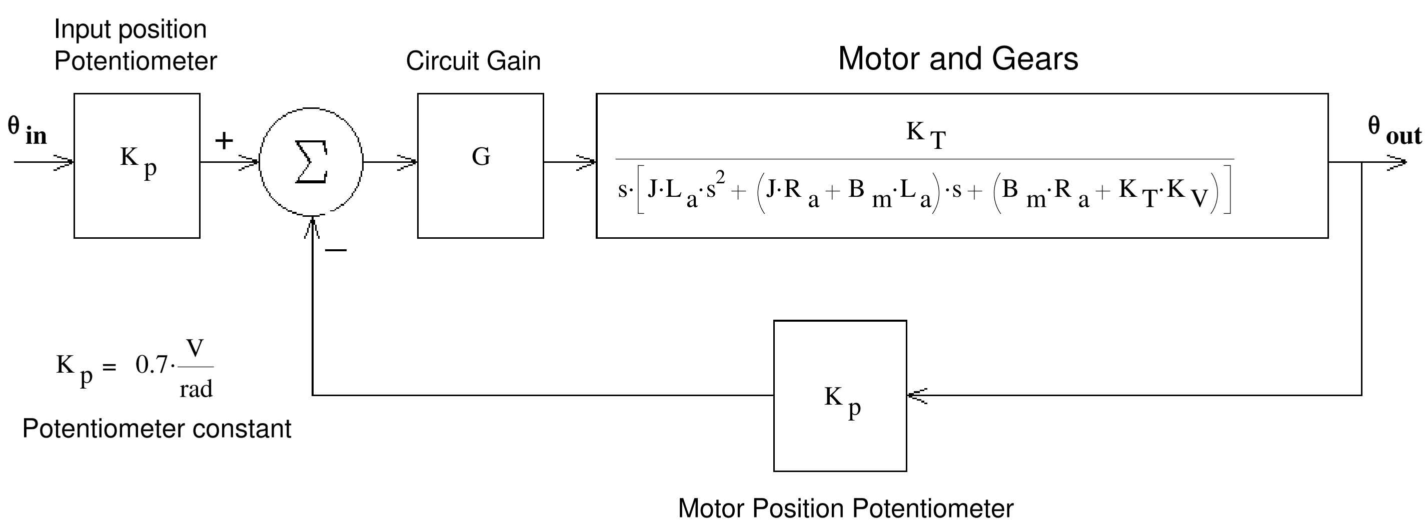
$$X_{out} \cdot (1 + A \cdot B) = A \cdot X_{in}$$

$$\frac{X_{out}}{X_{in}} = \frac{A}{1 + A \cdot B} = H(s)$$

The equivalent
transfer function

This is called a "closed loop" system, whereas a a system without feedback is called "open loop".
The term "open loop" is often used to describe a system that is out of control.

The servo used in our lab can be represented by:



$$H(s) = \frac{\theta_{out}(s)}{\theta_{in}(s)} = \frac{G \cdot K_T \cdot K_p}{s \cdot \left[J \cdot L_a \cdot s^2 + \left(J \cdot R_a + B_m \cdot L_a\right) \cdot s + \left(B_m \cdot R_a + K_T \cdot K_V\right)\right] + K_p \cdot G \cdot K_T}$$

See the appendix to lab 9 for the complete analysis