ECE 2210 Lecture 18 notes Second order Transient examples

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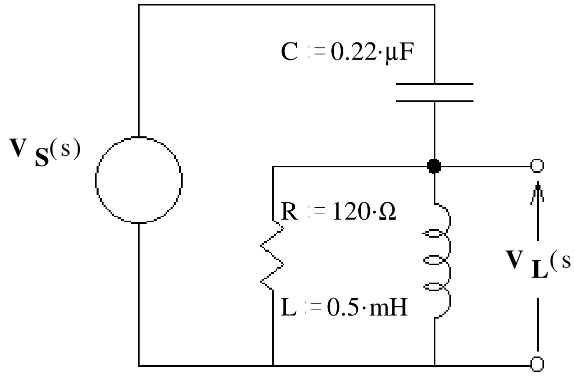
11/3/06

Ex 1. For the circuit shown:



a) Find the transfer function v_L.

$$\mathbf{V_L}(s) = \frac{\dfrac{1}{\dfrac{1}{L\cdot s}+\dfrac{1}{R}}}{\dfrac{1}{\dfrac{1}{L\cdot s}+\dfrac{1}{R}}+\dfrac{1}{C\cdot s}}\cdot \mathbf{V_S}(s)$$

$$= \frac{1}{1+\dfrac{1}{C\cdot s}\cdot\left(\dfrac{1}{L\cdot s}+\dfrac{1}{R}\right)}\cdot \mathbf{V_S}(s) = \frac{1}{1+\dfrac{1}{C\cdot s}\cdot\dfrac{1}{L\cdot s}+\dfrac{1}{C\cdot s}\cdot\dfrac{1}{R}}\cdot \mathbf{V_S}(s) = \frac{s^2}{s^2+\dfrac{1}{C\cdot R}\cdot s+\dfrac{1}{L\cdot C}}\cdot \mathbf{V_S}(s)$$

$$\mathbf{H}(s) = \frac{\mathbf{V_L}(s)}{\mathbf{V_S}(s)} = \frac{s^2}{s^2+\dfrac{1}{C\cdot R}\cdot s+\dfrac{1}{L\cdot C}}$$

$$= \frac{s^2}{s^2+\dfrac{3.788\cdot 10^4}{sec}\cdot s+\dfrac{9.091\cdot 10^9}{sec^2}}$$

$$\frac{1}{C\cdot R} = 3.788\cdot 10^4 \cdot\frac{1}{sec} \qquad \frac{1}{L\cdot C} = 9.091\cdot 10^9 \cdot\frac{1}{sec^2}$$

b) Find the characteristic equation for this circuit.

$$0 = s^2+\frac{1}{C\cdot R}\cdot s+\frac{1}{L\cdot C} = s^2+\frac{3.788\cdot 10^4}{sec}\cdot s+\frac{9.091\cdot 10^9}{sec^2}$$

Just the denominator set to zero. The solutions of the characteristic equation are the poles of the transfer function.

c) Find the differential equation for v_L.

Cross-multiply the transfer function

$$s^2\cdot \mathbf{V_S}(s) = \left(s^2+\frac{1}{C\cdot R}\cdot s+\frac{1}{L\cdot C}\right)\cdot \mathbf{V_L}(s)$$

$$s^2\cdot \mathbf{V_S}(s) = s^2\cdot \mathbf{V_L}(s)+\frac{1}{C\cdot R}\cdot s\cdot \mathbf{V_L}(s)+\frac{1}{L\cdot C}\cdot \mathbf{V_L}(s)$$

$$\frac{d^2}{dt^2}v_S(t) = \frac{d^2}{dt^2}v_L(t)+\frac{1}{C\cdot R}\cdot\frac{d}{dt}v_L(t)+\frac{1}{L\cdot C}\cdot v_L(t)$$

$$\frac{d^2}{dt^2}v_S(t) = \frac{d^2}{dt^2}v_L(t)+\frac{3.788\cdot 10^4}{sec}\cdot\frac{d}{dt}v_L(t)+\frac{9.091\cdot 10^9}{sec^2}\cdot v_L(t)$$

d) What are the solutions to the characteristic equation?

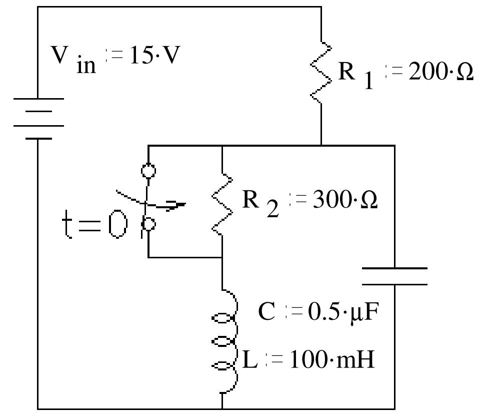
$$s_1 = \frac{-3.788\cdot 10^4}{2}+\frac{1}{2}\cdot\sqrt{\left(3.788\cdot 10^4\right)^2-4\cdot\left(9.091\cdot 10^9\right)} = -1.894\cdot 10^4 + 9.345\cdot 10^4 j$$

$$s_2 = \frac{-3.788\cdot 10^4}{2}-\frac{1}{2}\cdot\sqrt{\left(3.788\cdot 10^4\right)^2-4\cdot\left(9.091\cdot 10^9\right)} = -1.894\cdot 10^4 - 9.345\cdot 10^4 j$$

e) What type of response do you expect from this circuit?

The solutions to the characteristic equation are complex so the response will be **underdamped**.

2. Analysis of the circuit shown yields the characteristic equation below. The switch has been in the open position for a long time and is closed (as shown) at time $t = 0$. Find the initial and final conditions and write the full expression for $i_L(t)$, including all the constants that you find.



$$s^2 + \left(\frac{1}{C\cdot R_1}\right)\cdot s + \left(\frac{1}{L\cdot C}\right) = 0$$

$$\left(\frac{1}{C\cdot R_1}\right) = 2.273\cdot 10^4 \cdot sec^{-1} \qquad \left(\frac{1}{L\cdot C}\right) = 9.091\cdot 10^9 \cdot sec^{-2}$$

$$s^2 + 10000\cdot\frac{1}{sec}\cdot s + 2\cdot 10^7\cdot\frac{1}{sec^2} = 0$$

$$s_1 := \left[\frac{-10000}{2} + \frac{1}{2}\cdot\sqrt{(10000)^2 - 4\cdot\left(2\cdot 10^7\right)}\right]\cdot sec^{-1} \qquad s_2 := \left[\frac{-10000}{2} - \frac{1}{2}\cdot\sqrt{(10000)^2 - 4\cdot\left(2\cdot 10^7\right)}\right]\cdot sec^{-1}$$

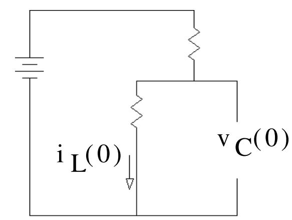
$$s_1 = -2764\cdot sec^{-1} \qquad s_2 = -7236\cdot sec^{-1}$$

s_1 and s_2 are both real and distinct, overdamped

Find the initial conditions:

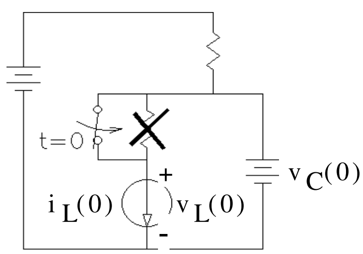
Before the switch closed, the inductor current was: $\dfrac{15\cdot V}{R_1 + R_2} = 30\cdot mA = i_L(0)$

Before the switch closed, the capacitor voltage was: $v_C(0) = \dfrac{R_2}{R_1 + R_2}\cdot(15\cdot V) = 9\cdot V$ so: $v_{C0} := 9\cdot V$



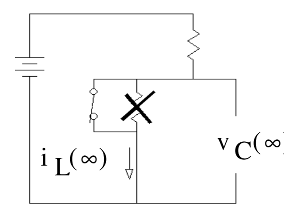
When the switch is opened, the new voltage across the inductor is: $v_{L0} := v_{C0}$

$$\frac{d}{dt}i_L(0) = \frac{1}{L}\cdot v_{L0} = \frac{1}{L}\cdot v_{C0} = 90\cdot\frac{A}{sec}$$



Find the final condition:

$$i_L(\infty) = \frac{15\cdot V}{R_1} = 75\cdot mA$$



General solution for the overdamped condition: $i_L(t) = i_L(\infty) + B\cdot e^{s_1\cdot t} + D\cdot e^{s_2\cdot t}$

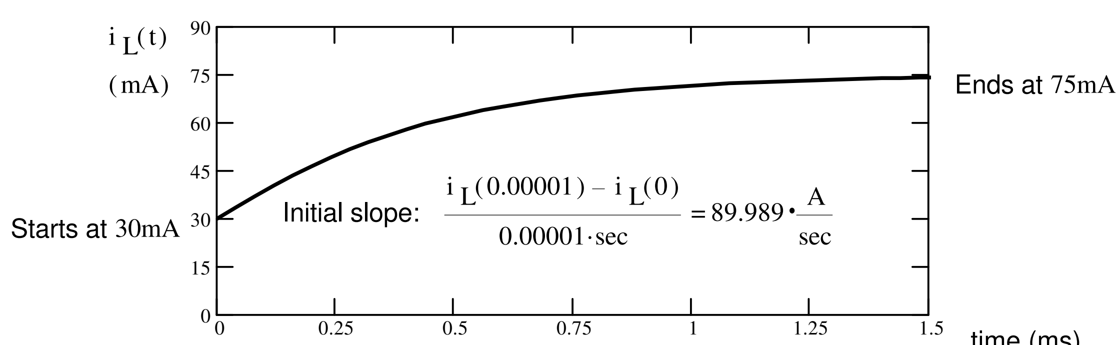
Initial conditions: $i_{L0} := \dfrac{15\cdot V}{R_1 + R_2} = i_L(\infty) + B + D$, so $B = i_{L0} - i_L(\infty) - D = 30\cdot mA - 75\cdot mA - D$

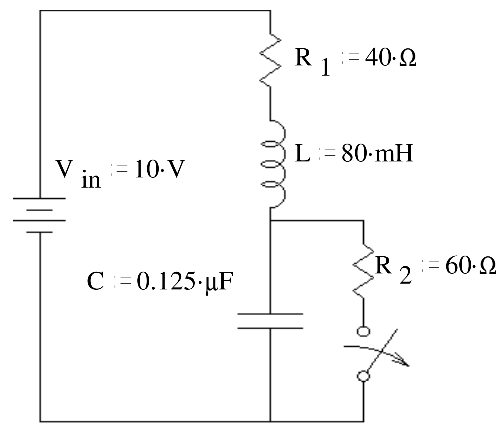
$= -45\cdot mA - D$

$$\frac{d}{dt}i_L(0) = 90\cdot\frac{A}{sec} = s_1\cdot B + s_2\cdot D = s_1\cdot(-45\cdot mA - D) + s_2\cdot D = s_1\cdot(-45\cdot mA) - s_1\cdot D + s_2\cdot D$$

solve for D & B: $D := \dfrac{90\cdot\frac{A}{sec} - s_1\cdot(-45\cdot mA)}{-s_1 + s_2}$ $\qquad D = 7.69\cdot mA \qquad B := -45\cdot mA - D \qquad B = -52.7\cdot mA$

Plug numbers back in: $i_L(t) := 75\cdot mA - 52.7\cdot mA\cdot e^{-2764t} + 7.69\cdot mA\cdot e^{-7236t}$





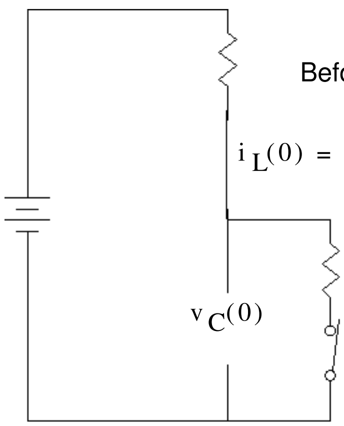
4. Analysis of the circuit shown yields the characteristic equation and s values below. The switch has been in the closed position for a long time and is opened (as shown) at time $t = 0$. Find the initial and final conditions and write the full expression for $v_C(t)$, including all the constants.

$$0 = s^2 + \frac{R_1}{L}\cdot s + \frac{1}{L\cdot C}$$

$$s_1 := \left(-250 + 10^4\cdot j\right)\cdot\frac{1}{sec} \quad , \qquad s_2 := \left(-250 - 10^4\cdot j\right)\cdot\frac{1}{sec}$$

Solution: $\alpha := -250\cdot\frac{1}{sec}$ $\qquad \omega := 10000\cdot\frac{1}{sec}$

Initial conditions:

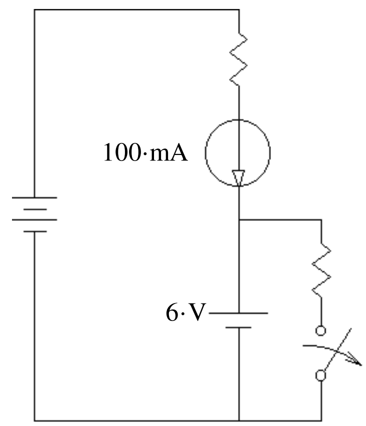


Before switch opens

$$i_L(0) = \frac{V_{in}}{R_1 + R_2} = 100\cdot mA$$

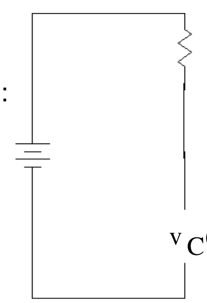
$$v_C(0) = V_{in}\cdot\frac{R_2}{R_1 + R_2} = 6\cdot V$$

Just after switch opens



$$\frac{d}{dt}v_C(0) = \frac{i_C(0)}{C} = \frac{100\cdot mA}{C} = 8\cdot 10^5 \cdot\frac{V}{sec}$$

Find final condition:



$$v_C(\infty) = V_{in} = 10\cdot V$$

Find constants: $v_C(0) = v_C(\infty) + B$ $\qquad B = v_C(0) - v_C(\infty)$ $\qquad B := 6\cdot V - 10\cdot V$ $\qquad B = -4\cdot V$

$$\frac{d}{dt}v_C(0) = \alpha\cdot B + D\cdot\omega \qquad D := \frac{8\cdot 10^5\cdot\frac{V}{sec} - \alpha\cdot B}{\omega} \qquad D = 79.9\cdot V$$

Write the full expression for $v_C(t)$, including all the constants that you find.

$$v_C(t) = e^{\alpha\cdot t}\cdot(B\cdot\cos(\omega\cdot t) + D\cdot\sin(\omega\cdot t)) + v_C(\infty)$$

$$v_C(t) := e^{-250\,t}\cdot\left(-4\cdot V\cdot\cos\left(10^4\cdot t\right) + 79.9\cdot V\cdot\sin\left(10^4\cdot t\right)\right) + 10\cdot V$$

The following graph is **not required**