Simple Model of a Real Source

Real sources are not ideal, but we will model them with two ideal components.

Thévenin Equivalent Circuit

The same model can be used for any combination of sources and resistors.

Thévenin equivalent

To calculate a circuit’s Thévenin equivalent:
1) Remove the load and calculate the open-circuit voltage where the load used to be. This is the Thévenin voltage ($V_{Th}$).
2) Zero all the sources. (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
3) Compute the total resistance between the load terminals. (DO NOT include the load in this resistance.) This is the Thévenin source resistance ($R_{Th}$).
4) Draw the Thévenin equivalent circuit and add your values.
Norton equivalent
To calculate a circuit's Norton equivalent:
1) Replace the load with a short (a wire) and calculate the short-circuit current in this wire.
This is the Norton current ($I_N$). Remove the short.

2) Zero all the sources.
   (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)

3) Compute the total resistance between the load terminals.
   (DO NOT include the load in this resistance.) This is the Norton source resistance ($R_N$).
   (Exactly the same as the Thévenin source resistance ($R_{Th}$)).

4) Draw the Norton equivalent circuit and add your values.

OR (the more common way)...  
1) Find the Thévenin equivalent circuit.
2) Convert to Norton circuit, then >>>  $R_N = R_{Th}$ and $I_N = \frac{V_{Th}}{R_{Th}}$
Ex 1 Find the Thévenin equivalent:

To calculate a circuit's Thévenin equivalent:
1) Remove the load and calculate the open-circuit voltage where the load used to be. This is the Thévenin voltage ($V_{Th}$).

$$V_{oc} = \frac{V}{R_1 + R_2} V_{Th} = V_{oc} = \frac{V}{R_1 + R_2}$$

$$V_{Th} = 15\cdot V$$

2) Zero all the sources. (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)

3) Compute the total resistance between the load terminals. (DO NOT include the load in this resistance.) This is the Thévenin source resistance ($R_{Th}$).

$$R_{Th} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$R_{Th} = 30\cdot \Omega$$

4) Draw the Thévenin equivalent circuit and add your values.

Thevenin equivalent circuit:

If the load were reconnected:

$$V_{Th} = 15\cdot V$$

$$V_L = \frac{V_{Th}}{R_{Th} + R_L} = 10\cdot V$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = 166.7\cdot mA$$

$$P_L = 10\cdot V\cdot 166.7\cdot mA = 1.667\cdot W$$

b) Find the Norton equivalent circuit:

Norton equivalent circuit:

$$I_N := \frac{V}{R_{Th}}$$

$$I_N = 500\cdot mA$$
c) Show that the Thévenin circuit is indeed equivalent to the original at several values of $R_L$.

### Original Circuit

<table>
<thead>
<tr>
<th>$R_L$ (Ω)</th>
<th>$V_L$ (V)</th>
<th>$I_L$ (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$V_S$</td>
</tr>
<tr>
<td>10</td>
<td>$9.231$</td>
<td>$R_o$</td>
</tr>
<tr>
<td>20</td>
<td>$17.143$</td>
<td>$R_o$</td>
</tr>
<tr>
<td>30</td>
<td>$24$</td>
<td>$R_o$</td>
</tr>
<tr>
<td>40</td>
<td>$30$</td>
<td>$R_o$</td>
</tr>
<tr>
<td>60</td>
<td>$40$</td>
<td>$R_o$</td>
</tr>
<tr>
<td>120</td>
<td>$60$</td>
<td>$R_o$</td>
</tr>
<tr>
<td>240</td>
<td>$80$</td>
<td>$R_o$</td>
</tr>
<tr>
<td>$∞$</td>
<td>$120$</td>
<td>$R_o$</td>
</tr>
</tbody>
</table>

### Thévenin Circuit

<table>
<thead>
<tr>
<th>$R_L$ (Ω)</th>
<th>$V_L$ (V)</th>
<th>$I_L$ (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$500$</td>
<td>$0$</td>
</tr>
<tr>
<td>10</td>
<td>$483.871$</td>
<td>$375$</td>
</tr>
<tr>
<td>20</td>
<td>$300$</td>
<td>$6$</td>
</tr>
<tr>
<td>30</td>
<td>$250$</td>
<td>$7.5$</td>
</tr>
<tr>
<td>40</td>
<td>$214.286$</td>
<td>$10$</td>
</tr>
<tr>
<td>60</td>
<td>$166.667$</td>
<td>$12$</td>
</tr>
<tr>
<td>120</td>
<td>$100$</td>
<td>$13.333$</td>
</tr>
<tr>
<td>240</td>
<td>$55.556$</td>
<td>$15$</td>
</tr>
</tbody>
</table>

Using either numbers: $P_L = V_L \cdot I_L = 0 \cdot W$

Repeat these calculations for a number of load resistors

### Plots

- Power delivered to the load ($R_L$) as a function of $R_L$
- $V_L$ vs $I_L$
Maximum power transfer  If I wanted to maximize the power dissipated by
the load, what $R_L$ would I choose?

$$P_L = \frac{V_L^2}{R_L} = \frac{R_L}{R_S + R_L} \cdot \frac{V_S}{R_L} = \frac{R_L^2}{(R_S + R_L)^2} \cdot \frac{V_S^2}{R_L}$$

$$= \frac{R_L^2}{R_S^2 + 2R_S R_L + R_L^2} \cdot \frac{V_S^2}{R_L} = \frac{R_L}{R_S^2 + 2R_S R_L + R_L^2} \cdot \frac{V_S^2}{R_L}$$

Next step would be to differentiate $\frac{d}{dR_L} P_L(R_L)$,
set this equal to 0 and solve for $R_L$ to find the maximum

Unfortunately this function is a pain to differentiate.
What if we just differentiate the denominator and find
its minimum, wouldn’t that work just as well?

$$\frac{d}{dR_L} \left( \frac{R_S^2}{R_L} + 2R_S R_L + R_L^2 \right) = -R_S^2 + 0 + 1 = 0$$

Maximum power transfer happens when: $R_L = R_S$
Just what we saw in Example 1

This is rarely important in power circuitry, where
there should be plenty of power and $R_S$ should be
small. It is much more likely to be important in
signal circuitry where the voltages can be very
small and the source resistance may be
significant -- say a microphone or a radio antenna.

All you need to remember is: $R_L = R_S$ to maximize the power dissipation in $R_L$

What about efficiency?

$$\eta = \frac{P_L(R_L)}{P_S(R_L)} = \frac{i^2 R_L}{i^2 (R_S + R_L)} = \frac{R_L}{R_S + R_L}$$

The bigger $R_L$ is, the higher the efficiency.
Ex 2 a) Find and draw the Thévenin equivalent circuit.

\[ V_{Th} = 3 \cdot V \]

First do some simplification:

\[ R_{eq234} = \frac{1}{R_1 + \frac{1}{R_3 + R_4}} \]

Divide this voltage between \( R_2 \) and \( R_4 \):

\[ V_{234} = 9 \cdot V \]

Find the Thévenin resistance:

\[ R_{Th} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{R_3 + R_4}}} \]

\[ R_{Th} = 750 \cdot \Omega \]

Thévenin equivalent circuit:

\[ V_{Th} = 3 \cdot V \]

b) Find and draw the Norton equivalent circuit.

\[ I_N = 4 \cdot mA \]

\[ R_N := R_{Th} \]

\[ R_N = 750 \cdot \Omega \]

If the load were reconnected:

\[ V_L := V_{Th} \cdot \frac{R_L}{R_{Th} + R_L} \]

\[ V_L = 1.125 \cdot V \]

\[ I_L := \frac{V_{Th}}{R_{Th} + R_L} \]

\[ I_L = 2.5 \cdot mA \]
c) Use your Norton equivalent circuit to find the current through the load.

\[ I_\text{N} = 4 \text{ mA} \]
\[ R_N = 450 \cdot \Omega \]
\[ R_L = 450 \cdot \Omega \]
\[ I_L = \frac{1}{R_L} I_N \]
\[ I_L = 2.5 \text{ mA} \]

\[ V_L := I_L R_L \]
\[ V_L = 1.125 \text{ V} \]

\[ \text{same as above} \]

d) What value of \( R_L \) would result in the maximum power delivery to \( R_L \)?

For maximum power transfer \( R_L = R_{\text{Th}} = 750 \cdot \Omega \)

\[ P_L = \frac{V_L^2}{R_L} = 3 \text{ mW} \]

e) What is the maximum power transfer?

\[ V_{\text{Th}} = 3 \text{ V} \]
\[ R_{\text{L}} = 750 \cdot \Omega \]

\[ V_L := \frac{V_{\text{Th}}}{2} \]

\[ P_L = \frac{V_L^2}{R_L} = 3 \text{ mW} \]

### Ex 3

a) Find and draw the Thévenin & Norton equivalent circuits.

\[ R_1 := 5 \cdot \Omega \]
\[ R_2 := 15 \cdot \Omega \]
\[ R_L := 20 \cdot \Omega \]
\[ V_{S1} := 10 \cdot \text{V} \]
\[ V_{S2} := 20 \cdot \text{V} \]
\[ V_{\text{Th}} := 10 \cdot \text{V} + 2.5 \cdot \text{V} \]

\[ \text{Thévenin equivalent circuit:} \]
\[ R_{\text{Th}} := 3.75 \cdot \Omega \]
\[ V_{\text{Th}} := 12.5 \text{ V} \]

\[ \text{Norton equivalent circuit:} \]
\[ R_N := R_{\text{Th}} \]
\[ I_N := \frac{V_{\text{Th}}}{R_{\text{Th}}} \]
\[ I_N = 3.333 \text{ mA} \]

b) Use your Thévenin equivalent circuit to find the voltage across the load.

\[ V_{\text{Th}} := 12.5 \text{ V} \]
\[ R_{\text{L}} := 20 \cdot \Omega \]

\[ V_L = \frac{R_L}{R_{\text{Th}} + R_L} V_{\text{Th}} = 10.526 \text{ V} \]
Ex 4  a) Find and draw the Thévenin & Norton equivalent circuits.

ECE 2210  Thevenin notes  p8

Use superposition to find $V_{Th}$.

$$V_{Th.V} = \frac{R_2 + R_3}{R_1 + R_2 + R_3} \cdot V_S$$

$$V_{Th.V} = 8.1 \cdot V$$

$$V_{Th.I} = -I_{12} \cdot R_1 = -1.2 \cdot V$$

$$V_{Th} = V_{Th.V} + V_{Th.I} = 6.9 \cdot V$$

Find the Thévenin resistance

$$R_{Th} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + R_3}}$$

Thévenin equivalent circuit:

$$V_{Th} = 6.9 \cdot V$$

$$R_{Th} = 36 \cdot \Omega$$

Put the load back on

$$I_L := \frac{V_{Th}}{R_{Th} + R_L} = 63.889 \cdot mA$$

$$V_L = I_L \cdot R_L = 4.6 \cdot V$$

Norton equivalent circuit:

$$I_N := \frac{V_{Th}}{R_{Th}} = 191.7 \cdot mA$$

$$R_N := R_{Th} = 36 \cdot \Omega$$
Ex 5 A NiCad Battery pack is used to power a cell phone. When the phone is switched on the battery pack voltage drops from 4.80 V to 4.65 V and the cell phone draws 50 mA.

\[ V_S = 4.80 \text{ V} \quad V_{50} = 4.65 \text{ V} \]

a) Draw a simple, reasonable model of the battery pack using ideal parts. Find the value of each part.

\[ R_S = \frac{V_S - V_{50}}{50 \text{ mA}} = 4.65 \text{ V} \]
\[ V_S = 4.8 \text{ V} \]

b) The cell phone is used to make a call. Now it draws 300 mA. What is the battery pack voltage now?

\[ V_B = V_S - I_{call} R_S = 3.9 \text{ V} \]

Ex 6 Consider the circuit at right.

a) What value of load resistor \((R_L)\) would you choose if you wanted to maximize the power dissipation in that load resistor.

\[ R_L = R_S \quad R_L = 8 \text{ \Omega} \]

b) With that load resistor \((R_L)\) find the power dissipation in the load.

\[ I_L = \frac{I_S}{2} \quad P_L = I_L^2 R_L = 2 \text{ W} \]
Use superposition to find $V_{\text{Th}}$.

**Current Divider:**

$$I_R4 := \frac{1}{R_2 + R_4 + R_6} I_S$$

$$I_{R4} = 35.556 \text{ mA}$$

$$V_{\text{Th}.I} := I_{R4} R_4$$

$$V_{\text{Th}.I} = 3.2 \text{ V}$$

**Eliminate Voltage Source:**

$$V_{\text{Th}.V} := \frac{R_4}{R_1 + R_2 + R_4 + R_6} V_S$$

$$V_{\text{Th}.V} = 4 \text{ V}$$

$$V_{\text{Th}} := V_{\text{Th}.V} + V_{\text{Th}.I}$$

$$V_{\text{Th}} = 7.2 \text{ V}$$

**Eliminate Current Source:**

$$R_{\text{Th}} := \frac{1}{\frac{1}{R_4} + \frac{1}{R_1 + R_2 + R_6} + R_3}$$

$$R_{\text{Th}} = 60 \text{ }$$

**Thévenin Equivalent Circuit:**

$$V_{\text{Th}} = 7.2 \text{ V}$$

$$R_{\text{Th}} = 60 \text{ }$$

**Put the Load Back on:**

$$I_L := \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

$$I_L = 80 \text{ mA}$$

$$V_L = I_L R_L = 2.4 \text{ V}$$

**Norton Equivalent Circuit:**

$$I_N := \frac{V_{\text{Th}}}{R_{\text{Th}}}$$

$$I_N = 120 \text{ mA}$$