Complex Numbers

ECE 2210 / 00

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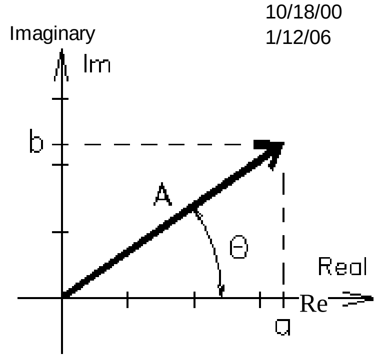
10/18/00
1/12/06



$j = \sqrt{-1}$ the imaginary number

Rectangular Form $\mathbf{A} = a + b\cdot j$

$Re(\mathbf{A}) = a \qquad Im(\mathbf{A}) = b$

Polar Form $\mathbf{A} = A\cdot e^{j\cdot\theta}$

$Re(\mathbf{A}) = A\cdot\cos(\theta) \qquad Im(\mathbf{A}) = A\cdot\sin(\theta)$

Conversions $A = |\mathbf{A}| = \sqrt{a^2 + b^2} \qquad \theta = \arg(\mathbf{A}) = \operatorname{atan}\left(\frac{b}{a}\right)$

$a = A\cdot\cos(\theta) \qquad b = A\cdot\sin(\theta)$

$\mathbf{A} = A\cdot e^{j\cdot\theta} = A\cdot\cos(\theta) + A\cdot\sin(\theta)\cdot j \qquad \mathbf{A} = a + b\cdot j = \left(\sqrt{a^2+b^2}\right)\cdot e^{j\cdot\operatorname{atan}\left(\frac{b}{a}\right)}$

Special Cases $j := \sqrt{-1} = e^{j\cdot 90\cdot deg} \qquad \frac{1}{j} = -j = e^{-j\cdot 90\cdot deg} \qquad e^{j\cdot 0\cdot deg} = 1 \qquad e^{-j\cdot 180\cdot deg} = e^{-j\cdot 180\cdot deg} = -1$

$j\cdot e^{j\cdot\theta} = e^{j\cdot(\theta + 90\cdot deg)}$

Define a 2nd number: rect: $\mathbf{D} = c + d\cdot j$ polar: $\mathbf{D} = D\cdot e^{j\cdot\phi}$

Equality $\mathbf{A} = \mathbf{D}$ if and only if $a = c$ and $b = d$ OR $A = D$ and $\theta = \phi$

Addition and Subtraction

$\mathbf{A} + \mathbf{D} = (a + b\cdot j) + (c + d\cdot j) = (a + c) + (b + d)\cdot j$

$\mathbf{A} - \mathbf{D} = (a + b\cdot j) - (c + d\cdot j) = (a - c) + (b - d)\cdot j$

Convert polars to rectangular form first

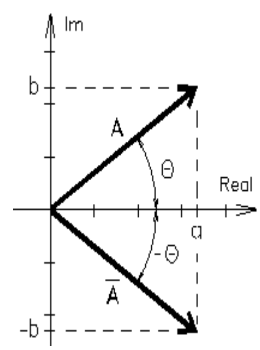
Multiplication and Division $\mathbf{A}\cdot\mathbf{D} = (a + b\cdot j)\cdot(c + d\cdot j) = (a\cdot c - b\cdot d) + (b\cdot c + a\cdot d)\cdot j$

Rectangular: $\frac{\mathbf{A}}{\mathbf{D}} = \frac{a + b\cdot j}{c + d\cdot j} = \frac{a + b\cdot j}{c + d\cdot j}\cdot\frac{c - d\cdot j}{c - d\cdot j} = \frac{a\cdot c + b\cdot d}{c^2 + d^2} + \frac{b\cdot c - a\cdot d}{c^2 + d^2}\cdot j$

Polar: $\mathbf{A}\cdot\mathbf{D} = A\cdot e^{j\cdot\theta}\cdot D\cdot e^{j\cdot\phi} = A\cdot D\cdot e^{j\cdot(\theta + \phi)}$

$\frac{\mathbf{A}}{\mathbf{D}} = \frac{A\cdot e^{j\cdot\theta}}{D\cdot e^{j\cdot\phi}} = \frac{A}{D}\cdot e^{j\cdot(\theta - \phi)}$

Powers $\mathbf{A}^n = A^n\cdot e^{j\cdot n\cdot\theta} = A^n\cdot\cos(n\cdot\theta) + A^n\cdot\sin(n\cdot\theta)\cdot j$ Convert rectangulars first, usually



Conjugates

complex number	Conjugate	
$\mathbf{A} = a + b\cdot j$	$\overline{\mathbf{A}} = a - b\cdot j$	$\overline{\overline{\mathbf{A}}} = \mathbf{A}$
$\mathbf{A} = A\cdot e^{j\cdot\theta}$	$\overline{\mathbf{A}} = A\cdot e^{-j\cdot\theta}$	
$\mathbf{F} = \frac{3 + 5\cdot j}{(2 - 6\cdot j)\cdot e^{j\cdot 40\cdot deg}}$	$\overline{\mathbf{F}} = \frac{3 - 5\cdot j}{(2 + 6\cdot j)\cdot e^{-j\cdot 40\cdot deg}}$	

Euler's equation $e^{j\cdot\alpha} = \cos(\alpha) + j\cdot\sin(\alpha)$ OR: $\cos(\alpha) = \frac{e^{j\cdot\alpha} + e^{-j\cdot\alpha}}{2} \qquad \sin(\alpha) = \frac{e^{j\cdot\alpha} - e^{-j\cdot\alpha}}{2\cdot j}$

$e^{j\cdot(\omega\cdot t + \theta)} = \cos(\omega\cdot t + \theta) + j\cdot\sin(\omega\cdot t + \theta)$

$Re\left[e^{j\cdot(\omega\cdot t + \theta)}\right] = \cos(\omega\cdot t + \theta)$

If we freeze this at time t=0, then we can represent $\cos(\omega\cdot t + \theta)$ by $e^{j\cdot\theta}$

Calculus Remember, when we write $e^{j\cdot\theta}$, we really mean $e^{j\cdot(\omega\cdot t + \theta)}$

$$\frac{d}{dt}\mathbf{A} = \frac{d}{dt}\left(A\cdot e^{j\cdot\theta}\right) = j\cdot\omega\cdot A\cdot e^{j\cdot\theta} = \omega\cdot A\cdot e^{j\cdot(\theta + 90\cdot deg)}$$

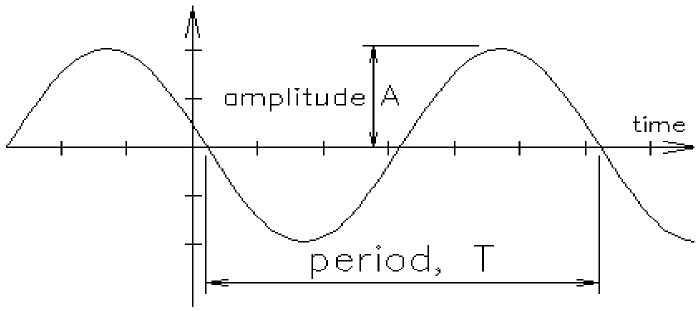
$$\int \mathbf{A}\,dt = \int A\cdot e^{j\cdot\theta}\,dt = \frac{1}{j\cdot\omega}\cdot A\cdot e^{j\cdot\theta} = \frac{1}{\omega}\cdot A\cdot e^{j\cdot(\theta - 90\cdot deg)}$$

ECE 2210 / 00 **Lecture 12 Introduction to AC Phasors**

A.Stolp 10/7/08
rev,

Phasor analysis with impedances, For steady-state sinusoidal response ONLY

Sinusoidal AC



T = Period = repeat time

f = frequency, cycles / second $\quad f = \frac{1}{T} = \frac{\omega}{2\cdot\pi}$

ω = radian frequency, radians/sec $\quad \omega = 2\cdot\pi\cdot f$

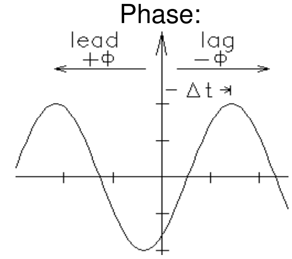
A = amplitude

Phase: $\quad \phi = -\frac{\Delta t}{T}\cdot 360\cdot deg$

or: $\quad \phi = -\frac{\Delta t}{T}\cdot 2\cdot\pi\cdot rad$

$y(t) = A\cdot\cos(\omega\cdot t + \theta)$

Phase:

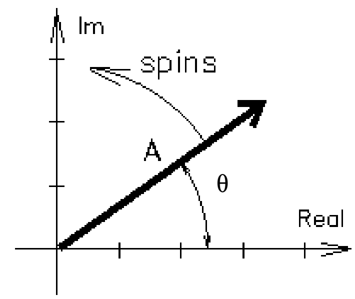


Phasor analysis

The math is all based on the Euler's equation

<u>**Euler's equation**</u> $\quad e^{j\cdot\alpha} = \cos(\alpha) + j\cdot\sin(\alpha)$

OR: $\quad \cos(\alpha) = \frac{e^{j\cdot\alpha} + e^{-j\cdot\alpha}}{2} \qquad \sin(\theta) = \frac{e^{j\cdot\alpha} - e^{-j\cdot\alpha}}{2\cdot j}$



$$e^{j\cdot(\omega\cdot t+\theta)} = \cos(\omega\cdot t + \theta) + j\cdot\sin(\omega\cdot t + \theta)$$

$$\mathrm{Re}\left[e^{j\cdot(\omega\cdot t+\theta)}\right] = \cos(\omega\cdot t + \theta)$$

If we freeze this at time t=0, then we can represent $\cos(\omega\cdot t + \theta)$ by $e^{j\cdot\theta}$ That's the phasor

<u>Phasor</u>

voltage: $\quad v(t) = V_p\cdot\cos(\omega\cdot t + \phi) \qquad \mathbf{V}(\omega) = V_p\cdot e^{j\cdot\phi}$

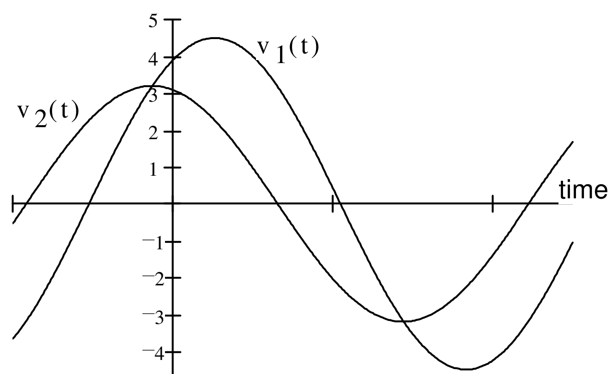
current: $\quad i(t) = I_p\cdot\cos(\omega\cdot t + \phi) \qquad \mathbf{I}(\omega) = I_p\cdot e^{j\cdot\phi}$

Phasors are drawn on a complex plane.

Phasors are used for adding and subtracting sinusoidal waveforms.

Ex1. Add the sinusoidal voltages $\quad v_1(t) = 4.5\cdot V\cdot\cos(\omega\cdot t - 30\cdot deg)$

and $\quad v_2(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 15\cdot deg)$



using phasor notation, draw a phasor diagram of the three phasors, then convert back to time domain form.

$v_1(t) = 4.5\cdot V\cdot\cos(\omega\cdot t - 30\cdot deg)$

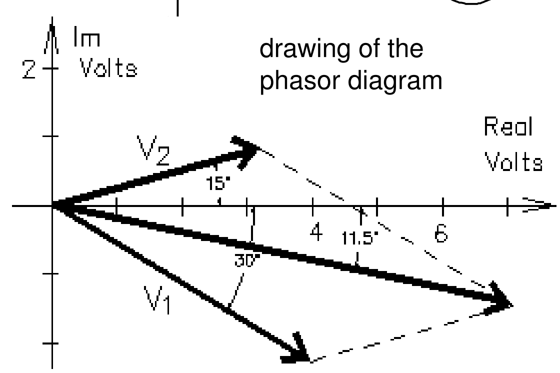
$\mathbf{V_1}(\omega) = 4.5V \angle{-30^o}$ or: $\quad \mathbf{V_1}(\omega) = 4.5\cdot V\cdot e^{-j\cdot 30\cdot deg}$

and

$v_2(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 15\cdot deg)$

$\mathbf{V_2}(\omega) = 3.2V \angle{15^o}$ or: $\quad \mathbf{V_2}(\omega) = 3.2\cdot V\cdot e^{j\cdot 15\cdot deg}$

drawing of the phasor diagram



I'm going to drop the (ω) notation from the phasor notation, it gets cumbersome, but remember that phasors are in the frequency domain..

$\mathbf{V_1} = 4.5V \angle{-30^o}$ or: $\quad \mathbf{V_1} := 4.5\cdot V\cdot e^{-j\cdot 30\cdot deg}$

$\mathbf{V_2} = 3.2V \angle{15^o}$ or: $\quad \mathbf{V_2} := 3.2\cdot V\cdot e^{j\cdot 15\cdot deg}$

Add like vectors, first change to the rectangular form

$\mathbf{V_1} = 4.5V\ \underline{/-30^o}$ $\quad 4.5\cdot V\cdot\cos(-30\cdot deg) = 3.897\cdot V$ $\quad 4.5\cdot V\cdot\sin(-30\cdot deg) = -2.25\cdot V$ $\quad \mathbf{V_1} = 3.897 - 2.25j\ \cdot V$ \

} add

$\mathbf{V_2} = 3.2V\ \underline{/15^o}$ $\quad 3.2\cdot V\cdot\cos(15\cdot deg) = 3.091\cdot V$ $\quad 3.2\cdot V\cdot\sin(15\cdot deg) = 0.828\cdot V$ $\quad \mathbf{V_2} = 3.091 + 0.828j\ \cdot V$ /

$\mathbf{V_3} := \mathbf{V_1} + \mathbf{V_2}$

Add real parts: $3.897 + 3.091 = 6.988$

Add imaginary parts: $-2.25 + 0.828 = -1.422$ $\quad \mathbf{V_3} = 6.988 - 1.422j\ \cdot V$ sum

Change $\mathbf{V_3}$ back to polar coordinates:

$$\sqrt{6.988^2 + 1.422^2} = 7.131 \qquad \operatorname{atan}\left(\frac{-1.422}{6.988}\right) = -11.502\cdot deg$$

OR, in Mathcad notation (you' ll see these in future solutions):

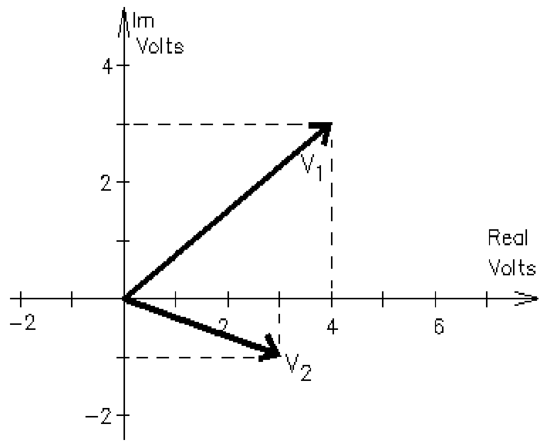
$$\left|\mathbf{V_3}\right| = 7.131\cdot V \qquad \arg\left(\mathbf{V_3}\right) = -11.5\cdot deg$$

Change $\mathbf{V_3}$ back to the time domain:

$$v_3(t) = v_1(t) + v_2(t) = 7.13\cdot\cos(\omega\cdot t - 11.5\cdot deg)\cdot V$$

Ex 2. Two sinusoidal voltages: $v_1(t) = 5\cdot V\cdot\cos(\omega\cdot t + 36.87\cdot deg)$ and $v_2(t) = 3.162\cdot V\cdot\cos(\omega\cdot t - 18.44\cdot deg)$

a) using phasor notation, find $v_3 = v_1 - v_2$



$\mathbf{V_1} := 5\cdot V\cdot e^{j\cdot(36.87\cdot deg)}$ $\quad 5\cdot V\cdot\cos(36.87\cdot deg) = 4\cdot V$

$5\cdot V\cdot\sin(36.87\cdot deg) = 3\cdot V$

$\mathbf{V_1} = 4 + 3j\ \cdot V$

$\mathbf{V_2} := 3.162\cdot V\cdot e^{j\cdot(-18.44\cdot deg)}$ $\quad 3.162\cdot V\cdot\cos(-18.44\cdot deg) = 3\cdot V$

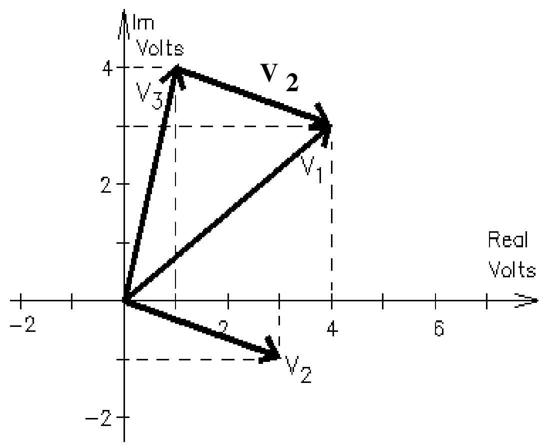
$3.162\cdot V\cdot\sin(-18.44\cdot deg) = -1\cdot V$

$\mathbf{V_2} = 3 - j\ \cdot V$

Subtract real parts: $4\cdot V - 3\cdot V = 1\cdot V$

Subtract imaginary parts: $3\cdot V - -1\cdot V = 4\cdot V$

$\mathbf{V_3} := \mathbf{V_1} - \mathbf{V_2}$ $\qquad \mathbf{V_3} = 1 + 4j\ \cdot V$



Magnitude: $\sqrt{(1\cdot V)^2 + (4\cdot V)^2} = 4.123\cdot V$

OR: $\left|\mathbf{V_3}\right| = 4.123\cdot V$

Angle: $\operatorname{atan}\left(\frac{4\cdot V}{1\cdot V}\right) = 75.96\cdot deg$ $\qquad \arg\left(\mathbf{V_3}\right) = 75.96\cdot deg$

So: $v_3(t) = v_1(t) - v_2(t) = 4.123\cdot V\cdot\cos(\omega\cdot t + 75.96\cdot deg)\cdot V$

What about Capacitors and Inductors?

Capacitors and Inductors in AC circuits cause 90^o phase shifts between voltages and currents because they integrate and differentiate. But... integration and differentiation is a piece-of-cake in phasors.

Calculus

$$\frac{d}{dt}\left[A\cdot e^{j\cdot(\omega\cdot t+\theta)}\right] = j\cdot\omega\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta)} = \omega\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta+90\cdot deg)} = \omega\cdot A\cdot e^{j\cdot(\theta+90\cdot deg)}$$

Drop the ωt (t=0) to get:

$$\int A\cdot e^{j\cdot(\omega\cdot t+\theta)}\,dt = \frac{1}{j\cdot\omega}\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta)} = \frac{1}{\omega}\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta-90\cdot deg)} = \frac{1}{\omega}\cdot A\cdot e^{j\cdot(\theta-90\cdot deg)}$$

Impedance (like resistance)

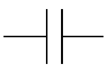
Inductor

$$v_L = L\cdot\frac{d}{dt}i_L = L\cdot\frac{d}{dt}I_p\cdot e^{j\cdot(\omega\cdot t+\theta)} = j\cdot\omega\cdot L\cdot\left[I_p\cdot e^{j\cdot(\omega\cdot t+\theta)}\right]$$

AC impedance

\ in phaser notation ----> $\mathbf{V}_{\mathbf{L}}(\omega) = j\cdot\omega\cdot L\cdot\mathbf{I}(\omega)$ $\qquad \mathbf{Z}_{\mathbf{L}} = j\cdot\omega\cdot L$

Capacitor



$$i_C = C\cdot\frac{d}{dt}v_C = C\cdot\frac{d}{dt}V_p\cdot e^{j\cdot(\omega\cdot t+\theta)} = j\cdot\omega\cdot C\cdot\left[V_p\cdot e^{j\cdot(\omega\cdot t+\theta)}\right]$$

\ in phaser notation ----> $\mathbf{I}_{\mathbf{C}}(\omega) = j\cdot\omega\cdot C\cdot\mathbf{V}(\omega)$

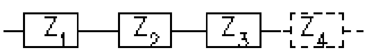
$$\mathbf{V}_{\mathbf{C}}(\omega) = \frac{1}{j\cdot\omega\cdot C}\cdot\mathbf{I}(\omega) \qquad \mathbf{Z}_{\mathbf{C}} = \frac{1}{j\cdot\omega\cdot C} = \frac{-j}{\omega\cdot C}$$

Resistor

$$v_R = i_R\cdot R \qquad \mathbf{V}_{\mathbf{R}}(\omega) = R\cdot\mathbf{I}(\omega) \qquad \mathbf{Z}_{\mathbf{R}} = R$$

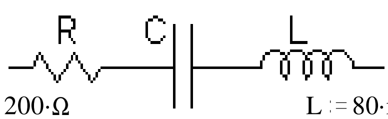
You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

series:



$$\mathbf{Z}_{\mathbf{eq}} = \mathbf{Z}_{\mathbf{1}} + \mathbf{Z}_{\mathbf{2}} + \mathbf{Z}_{\mathbf{3}} + \ldots$$

Example:



$f := 500\cdot Hz$

$\omega := 2\cdot\pi\cdot f = \omega = 3141.6\cdot\frac{rad}{sec}$

$R := 200\cdot\Omega \qquad C := 0.6\cdot\mu F \qquad L := 80\cdot mH$

$j\cdot\omega\cdot L = 251.327j\cdot\Omega$

$\frac{1}{j\cdot\omega\cdot C} = -530.516j\cdot\Omega$

$$\mathbf{Z}_{\mathbf{eq}} := R + \frac{1}{j\cdot\omega\cdot C} + j\cdot\omega\cdot L = 200\cdot\Omega - 530.5\cdot j\cdot\Omega + 251.3\cdot j\cdot\Omega = 200 - 279.2j\ \cdot\Omega$$ rectangular form

$$\sqrt{(200\cdot\Omega)^2 + (279.2\cdot\Omega)^2} = 343.4\cdot\Omega \qquad \text{atan}\left(\frac{-279.2\cdot\Omega}{200\cdot\Omega}\right) = -54.38\cdot deg$$

$\mathbf{Z}_{\mathbf{eq}} = 343.4\Omega\ \underline{/-54.4}°$ polar form

If: $\mathbf{V} := 12\cdot V\cdot e^{j\cdot 0\cdot deg}$ $\qquad \mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z}_{\mathbf{eq}}} = \frac{12\cdot V}{343.4\cdot\Omega} = 34.945\cdot mA \quad \underline{/}\ \ 0 - -54.4 = 54.4\ \ deg$

$\mathbf{I} = 34.95mA\ \underline{/54.4}° = \mathbf{I} = 20.348 + 28.405j\ \cdot mA$

Voltage divider:

$$\mathbf{V}_{\mathbf{Zn}} = \mathbf{V}_{\mathbf{total}}\cdot\frac{\mathbf{Z}_{\mathbf{n}}}{\mathbf{Z}_{\mathbf{1}} + \mathbf{Z}_{\mathbf{2}} + \mathbf{Z}_{\mathbf{3}}} + \ldots$$

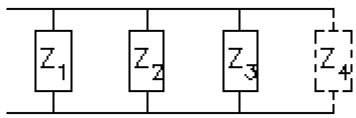
Note: $\frac{1}{j} = -j = 1\ \underline{/-90}°$

Eg: $\mathbf{V}_{\mathbf{C}} := \mathbf{V}\cdot\frac{\frac{1}{j\cdot\omega\cdot C}}{\mathbf{Z}_{\mathbf{eq}}} = 12\cdot V\cdot e^{j\cdot 0\cdot deg}\cdot\frac{530.516\cdot e^{-j\cdot 90\cdot deg}\cdot\Omega}{343.4\cdot e^{-j\cdot 54.38\cdot deg}\cdot\Omega}$

$12\cdot V\cdot\frac{530.516\cdot\Omega}{343.4\cdot\Omega} = 18.539\cdot V \qquad \underline{/}\ \ 0 + -90 - -54.4 = -35.6\ \ deg$

$\mathbf{V}_{\mathbf{C}} = 18.54V\ \underline{/-35.6}° = \mathbf{V}_{\mathbf{C}} = 15.069 - 10.795j\ \cdot V$

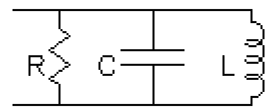
parallel:



$$\mathbf{Z_{eq}} = \frac{1}{\frac{1}{\mathbf{Z_1}} + \frac{1}{\mathbf{Z_2}} + \frac{1}{\mathbf{Z_3}} + \ldots}$$

Example:

$f := 500 \cdot Hz$ $\quad \omega := 2 \cdot \pi \cdot f$ $\quad \omega = 3141.6 \cdot \frac{rad}{sec}$



$L := 80 \cdot mH$

$j \cdot \omega \cdot L = 251.327j \cdot \Omega$

$R := 200 \cdot \Omega$ $\quad C := 0.6 \cdot \mu F$

$\frac{1}{\omega \cdot L} = 3.979 \cdot 10^{-3} \cdot \frac{1}{\Omega}$

$\frac{1}{j \cdot \omega \cdot C} = -530.516j \cdot \Omega$

$\omega \cdot C = 1.885 \cdot 10^{-3} \cdot \frac{1}{\Omega}$

$$\mathbf{Z_{eq}} := \frac{1}{\frac{1}{R} + \frac{1}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} + \frac{1}{j \cdot \omega \cdot L}} = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C - \frac{j}{\omega \cdot L}} = \frac{1}{\frac{1}{200 \cdot \Omega} + 1.885 \cdot 10^{-3} \cdot j \cdot \frac{1}{\Omega} - 3.979 \cdot 10^{-3} \cdot j \cdot \frac{1}{\Omega}}$$

$$= \frac{1}{\left(5 \cdot 10^{-3} - 2.094 \cdot 10^{-3} \cdot j\right) \cdot \frac{1}{\Omega}} \cdot \frac{5 \cdot 10^{-3} + 2.094 \cdot 10^{-3} \cdot j}{\left(5 \cdot 10^{-3} + 2.094 \cdot 10^{-3} \cdot j\right)} = 170.156 + 71.261j \cdot \Omega$$

$2.93848 \cdot 10^{-5}$

If you want the answer in polar form, it's easier to convert the denominator first.

$$\sqrt{\left(5 \cdot 10^{-3} \cdot \frac{1}{\Omega}\right)^2 + \left(2.094 \cdot 10^{-3} \cdot \frac{1}{\Omega}\right)^2} = 5.4 \cdot 10^{-3} \cdot \frac{1}{\Omega} \qquad \operatorname{atan}\left(\frac{-2.094 \cdot 10^{-3} \cdot \Omega}{5 \cdot 10^{-3} \cdot \Omega}\right) = -22.72 \cdot deg$$

$$\frac{1}{5.4 \cdot 10^{-3} \cdot \frac{1}{\Omega}} = 185.185 \cdot \Omega$$

$\mathbf{Z_{eq}} = 185.2 \underline{/22.7}^{o}$

If: $\mathbf{V} := 12 \cdot V \cdot e^{j \cdot 0 \cdot deg}$ $\quad \mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z_{eq}}} = \frac{12 \cdot V}{185.2 \cdot \Omega} = 64.795 \cdot mA \quad \underline{/} \quad 0 - 22.7 = -22.7 \ deg$

$\mathbf{I} = 60 - 25.127j \cdot mA$

Current divider:

$$\mathbf{I_{Zn}} = \mathbf{I_{total}} \cdot \frac{\frac{1}{\mathbf{Z_n}}}{\frac{1}{\mathbf{Z_1}} + \frac{1}{\mathbf{Z_2}} + \frac{1}{\mathbf{Z_3}} + \ldots}$$

Eg: $\mathbf{I_L} := \mathbf{I} \cdot \frac{\frac{1}{j \cdot \omega \cdot L}}{\frac{1}{R} + j \cdot \omega \cdot C + \frac{1}{j \cdot \omega \cdot L}} = \mathbf{I} \cdot \frac{\mathbf{Z_{eq}}}{j \cdot \omega \cdot L}$

$$= 64.795 \cdot mA \cdot e^{j \cdot 22.7 \cdot deg} \cdot \frac{185.2 \cdot e^{-j \cdot 22.7 \cdot deg} \cdot \Omega}{251.327 \cdot e^{j \cdot 90 \cdot deg} \cdot \Omega}$$

$$= 64.795 \cdot mA \cdot \frac{185.2 \cdot \Omega}{251.327 \cdot \Omega} = 47.747 \cdot mA \quad \underline{/} \quad 22.7 + -22.7 - 90 = -90 \ deg$$

$\mathbf{I_L} = -47.746j \cdot mA$

Duh... $\frac{\mathbf{V}}{j \cdot \omega \cdot L} = -47.746j \cdot mA$

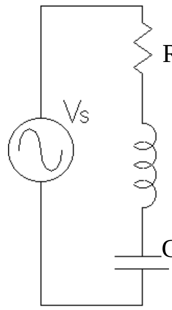
ECE 2210 / 00 Phasor Examples

Ex. 1 Find $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$ in polar phasor form. $f := 2 \cdot kHz$

$\mathbf{V}(j\omega)$

$6 \cdot V \cdot e^{j0}$

$f := 2 \cdot kHz$

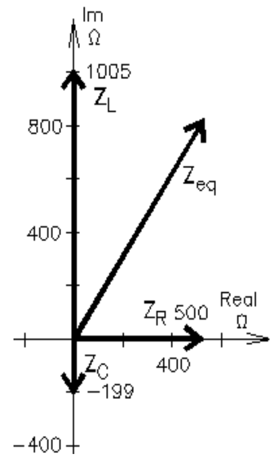


$R := 500 \cdot \Omega$ $\qquad \omega := 2 \cdot \pi \cdot f \qquad \omega = 12566.371 \cdot \frac{rad}{sec}$

$L := 80 \cdot mH$ $\qquad \mathbf{Z_L} := j \cdot \omega \cdot L \qquad \mathbf{Z_L} = 1005.31i \cdot \Omega$

$C := 0.4 \cdot \mu F$ $\qquad \mathbf{Z_C} := \frac{1}{j \cdot \omega \cdot C} \qquad \mathbf{Z_C} = -198.944j \cdot \Omega$

$\mathbf{Z_{eq}} := R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C} \qquad \mathbf{Z_{eq}} = 500 + 806.366j \cdot \Omega$



find the current: (The hard way)

$$\mathbf{I} := \frac{6 \cdot V \cdot e^{j \cdot 0}}{\mathbf{Z_{eq}}} = \frac{6 \cdot V}{500 + 806.37 \cdot j} \cdot \left(\frac{500 - 806.37 \cdot j}{500 - 806.37 \cdot j} \right) = \frac{6 \cdot V}{(500 + 806.37 \cdot j) \cdot \Omega} \cdot \left(\frac{500 - 806.37 \cdot j}{500 - 806.37 \cdot j} \right)$$

$$= \frac{6 \cdot V \cdot (500 - 806.37 \cdot j)}{\left(500^2 + 806.37^2\right) \cdot \Omega} = \frac{3000 - 4838.2 \cdot j}{900232.6} \cdot A = \left(\frac{3000}{900232.6} - \frac{4838.2 \cdot j}{900232.6} \right) \cdot A = 3.332 - 5.374j \cdot mA$$

It's much easier to convert the impedance to polar form first $\sqrt{500^2 + 806.37^2} = 948.806 \qquad \operatorname{atan}\left(\frac{806.37}{500}\right) = 58.198 \cdot deg$

$\mathbf{Z_{eq}} = 948.8\Omega \ \underline{/ \ 58.2}^o$

find the current: $\mathbf{I} := \frac{6 \cdot V \cdot e^{j \cdot 0}}{\mathbf{Z_{eq}}}$ magnitude: $\frac{6 \cdot V}{948.8 \cdot \Omega} = 6.324 \cdot mA$

angle: $0 \cdot deg - 58.2 \cdot deg = -58.2 \cdot deg \qquad \mathbf{I} = 6.324mA \ \underline{/ -58.2}^o$

	<u>find the magnitude</u>	<u>find the angle</u>	
$\mathbf{V_R} := \mathbf{I} \cdot R$	$6.324 \cdot mA \cdot 500 \cdot \Omega = 3.162 \cdot V$	$-58.2 \cdot deg + 0 \cdot deg = -58.2 \cdot deg$	$\mathbf{V_R} = 3.162V \ \underline{/ -58.2}^o$
$\mathbf{V_L} := \mathbf{I} \cdot \mathbf{Z_L}$	$6.324 \cdot mA \cdot 1005 \cdot \Omega = 6.356 \cdot V$	$-58.2 \cdot deg + 90 \cdot deg = 31.8 \cdot deg$	$\mathbf{V_L} = 6.356V \ \underline{/ \ 31.8}^o$
$\mathbf{V_C} := \mathbf{I} \cdot \mathbf{Z_C}$	$6.324 \cdot mA \cdot (-199) \cdot \Omega = -1.258 \cdot V$	$-58.2 \cdot deg + (90) \cdot deg = 31.8 \cdot deg$	$\mathbf{V_C} = -1.258V \ \underline{/ \ 31.8}^o$
OR:	$6.324 \cdot mA \cdot (199) \cdot \Omega = 1.258 \cdot V$	$-58.2 \cdot deg + (-90) \cdot deg = -148.2 \cdot deg$	$\mathbf{V_C} = 1.258V \ \underline{/ -148.2}^o$

OR, you can also find these voltages directly, using a voltage divider. I.E. to find $\mathbf{V_C}$ directly:

$$\mathbf{V_C} := \frac{\frac{1}{j \cdot \omega \cdot C}}{R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C}} \cdot 6 \cdot V = \frac{1}{R \cdot (j \cdot \omega \cdot C) + j \cdot \omega \cdot L \cdot (j \cdot \omega \cdot C) + 1} \cdot 6 \cdot V = \frac{1}{R \cdot (j \cdot \omega \cdot C) - \omega^2 \cdot L \cdot C + 1} \cdot 6 \cdot V$$

$$= \frac{1}{\left(1 - \omega^2 \cdot L \cdot C\right) + j \cdot \omega \cdot R \cdot C} \cdot 6 \cdot V \qquad \left(1 - \omega^2 \cdot L \cdot C\right) = -4.053 \qquad j \cdot \omega \cdot R \cdot C = 2.513j$$

$$= \frac{6 \cdot V}{-4.053 + 2.513 \cdot j}$$ Let's convert denominator to polar first

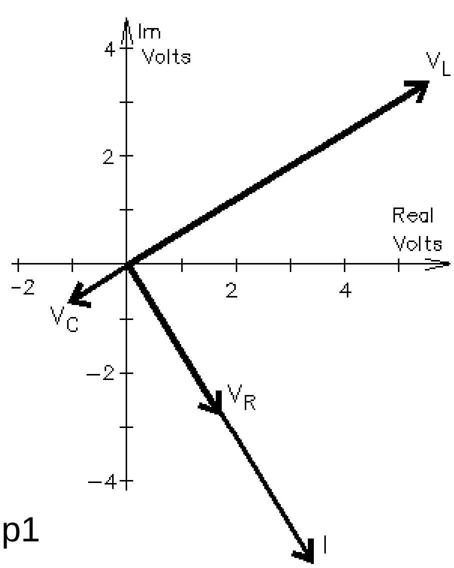
$\sqrt{(-4.053)^2 + 2.513^2} = 4.769 \qquad \operatorname{atan}\left(\frac{2.513}{-4.053}\right) = -31.8 \cdot deg$ NO!!

but this is actually in the third quadrant, so modify your calculator's results:

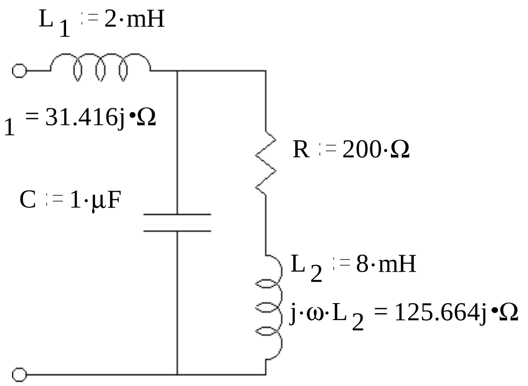
add 180º
$148.2 \cdot deg$

magnitude: $\frac{6 \cdot V}{4.769} = 1.258 \cdot V$ angle: $0 \cdot deg - 148.2 \cdot deg = -148.2 \cdot deg$

$= 1.258V \ \underline{/ -148.2}^o$



Ex. 2 a) Find $\mathbf{Z_{eq}}$. $f := 2.5\cdot kHz$ $\omega := 2\cdot\pi\cdot f$ $\omega = 15708 \cdot \frac{rad}{sec}$



$$\mathbf{Z_{eq}} = j\cdot\omega\cdot L_1 + \frac{1}{\frac{1}{\left(\frac{1}{j\cdot\omega\cdot C}\right)} + \frac{1}{R + j\cdot\omega\cdot L_2}}$$

But it's easier to split the problem up

Left branch

$$\mathbf{Z_l} := \frac{1}{j\cdot\omega\cdot C} \qquad \mathbf{Z_l} = -63.662j\cdot\Omega$$

$$\frac{1}{\left(\frac{1}{j\cdot\omega\cdot C}\right)} = j\cdot\omega\cdot C = 0.01571i\cdot\frac{1}{\Omega}$$

Right branch

$$\mathbf{Z_r} := j\cdot\omega\cdot L_2 + R \qquad \mathbf{Z_r} = 200 + 125.664j \cdot\Omega$$

$$\frac{1}{200 + 125.664\cdot j} = 3.585\cdot 10^{-3} - 2.252\cdot 10^{-3} j$$

denominator: $j\cdot\omega\cdot C + \frac{1}{R + j\cdot\omega\cdot L_2} = 0.01571\cdot j + \left(3.585\cdot 10^{-3} - 2.252\cdot 10^{-3}\cdot j\right) = 3.585\cdot 10^{-3} + 1.346\cdot 10^{-2} i \quad \frac{1}{\Omega}$

rectangular division:

$$\frac{1}{\left(3.585\cdot 10^{-3} + 1.346\cdot 10^{-2}\cdot j\right)}\cdot\frac{\left(3.585\cdot 10^{-3} - 1.346\cdot 10^{-2}\cdot j\right)}{\left(3.585\cdot 10^{-3} - 1.346\cdot 10^{-2}\cdot j\right)} = \frac{3.585\cdot 10^{-3} - 1.346\cdot 10^{-2}\cdot j}{1.94\cdot 10^{-4}} = 18.479 - 69.381j \quad \Omega$$

$$\left(3.585\cdot 10^{-3}\right)^2 + \left(1.346\cdot 10^{-2}\right)^2 = 1.94\cdot 10^{-4}$$

add: $j\cdot\omega\cdot L_1 = 31.416j\cdot\Omega$ $31.416\cdot j + (18.479 - 69.381\cdot j) = 18.479 - 37.965j \quad \Omega$

convert to polar (if needed): $\sqrt{18.48^2 + 37.97^2} = 42.228$ $\operatorname{atan}\left(\frac{-37.97}{18.48}\right) = -64.048\cdot deg$ $\mathbf{Z_{eq}} = 42.23\Omega$ <u>/-64.05º</u>

Another Way

Sometimes you might simplify a little before putting in numbers.

$$\mathbf{Z_{eq}} := j\cdot\omega\cdot L_1 + \frac{1}{\frac{1}{R + j\cdot\omega\cdot L_2} + \frac{1}{\frac{1}{j\cdot\omega\cdot C}}} = j\cdot\omega\cdot L_1 + \frac{1}{\frac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C} = j\cdot\omega\cdot L_1 + \frac{R + j\cdot\omega\cdot L_2}{1 + j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right)}$$

$$= j\cdot\omega\cdot L_1 + \frac{R + j\cdot\omega\cdot L_2}{1 - \omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}$$

$$\mathbf{Z_{eq}} = 31.416\cdot j\cdot\Omega + \frac{(200 + 125.664\cdot j)\cdot\Omega}{-0.974 + 3.142\cdot j}\cdot\left(\frac{-0.974 - 3.142\cdot j}{-0.974 - 3.142\cdot j}\right) = 31.416\cdot j\cdot\Omega + \frac{(200 + 125.664\cdot j)\cdot(-0.974 - 3.142\cdot j)}{0.974^2 + 3.142^2}$$

$$= 31.416\cdot j\cdot\Omega + \frac{((200\cdot(-0.974) - 125.664\cdot(-3.142)) + (125.664\cdot(-0.974) - 200\cdot 3.142)\cdot j)\cdot\Omega}{0.974^2 + 3.142^2}$$

$$= 31.416\cdot j\cdot\Omega + \frac{(200.036288 - 750.796736\cdot j)\cdot\Omega}{10.82084} = 31.416\cdot j\cdot\Omega + 18.486\cdot\Omega - 69.384\cdot j\cdot\Omega = 18.486 - 37.968j\cdot\Omega$$

$\sqrt{18.49^2 + 37.97^2} = 42.233$ $\operatorname{atan}\left(\frac{-37.97}{18.49}\right) = -64.036\cdot deg$ $\mathbf{Z_{eq}} = 42.23\Omega$ <u>/-64.04º</u>

a little roundoff difference

b) $\mathbf{V}_{in} := 12\cdot V\cdot e^{j\cdot 20\cdot deg}$ Find $\mathbf{I}_{L1}$, $\mathbf{V}_C$ $\mathbf{I}_{L1} := \dfrac{\mathbf{V}_{in}}{\mathbf{Z}_{eq}}$ $\dfrac{12\cdot V}{42.23\cdot\Omega} = 284.16\cdot mA$ $20\cdot deg - (-64.04)\cdot deg = 84.04\cdot deg$

$\mathbf{I}_{L1}$ = 284mA / 84.04°

$\mathbf{V}_C := \mathbf{I}_{L1}\cdot(18.479 - 69.381\cdot j)\cdot\Omega$ $284\cdot mA\cdot\sqrt{18.479^2 + 69.381^2}\cdot\Omega = 20.391\cdot V$

$84.04\cdot deg + \operatorname{atan}\left(\dfrac{-69.381}{18.479}\right) = 8.954\cdot deg$ $\mathbf{V}_C$ = 20.4V / 8.95°

You could then use another voltage divider to find $\mathbf{V}_R$ or $\mathbf{V}_{L2}$.

convert to rectangular (if needed): $20.391\cdot V\cdot\cos(8.954\cdot deg) = 20.143\cdot V$

$20.391\cdot V\cdot\sin(8.954\cdot deg) = 3.174\cdot V$ $\mathbf{V}_C = 20.14 + 3.174\cdot j \quad V$

Another Way

To find V_C directly:

$$\mathbf{V}_C := \frac{\dfrac{1}{\dfrac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C}}{j\cdot\omega\cdot L_1 + \dfrac{1}{\dfrac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C}}\cdot\mathbf{V}_{in}$$

--> math --> $\mathbf{V}_C = 20.153 + 3.178j \cdot V$ Same but for a little roundoff difference

c) Let's find $\mathbf{I}_{L2}$. $\mathbf{Z}_r = 200 + 125.664j \cdot\Omega$ $\sqrt{200^2 + 125.664^2} = 236.202$ $\operatorname{atan}\left(\dfrac{125.664}{200}\right) = 32.142\cdot deg$

$$\mathbf{I}_{L2} := \frac{\mathbf{V}_C}{\mathbf{Z}_r} = \frac{20.4\cdot V\cdot e^{j\cdot 8.95\cdot deg}}{236.202\cdot\Omega\cdot e^{j\cdot 32.142\cdot deg}} = \frac{20.4\cdot V}{236.202\cdot\Omega}$$ / 8.95 - 32.142° = 86.4mA /-23.19

Another Way

Directly by Current divider:

$$\mathbf{I}_{L2} := \frac{\dfrac{1}{R + j\cdot\omega\cdot L_2}}{j\cdot\omega\cdot C + \dfrac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I}_{L1} = \frac{1}{j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right) + 1}\cdot\mathbf{I}_{L1} = \frac{\mathbf{I}_{L1}}{1 - \omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}$$

denominator: $\sqrt{\left(1 - \omega^2\cdot C\cdot L_2\right)^2 + (\omega\cdot C\cdot R)^2} = 3.289$ $\operatorname{atan}\left(\dfrac{\omega\cdot C\cdot R}{1 - \omega^2\cdot C\cdot L_2}\right) + 180\cdot deg = 107.224\cdot deg$ (real part is negative /)

$$\mathbf{I}_{L2} = \frac{284\cdot mA\cdot e^{j\cdot 84.04\cdot deg}}{3.289\cdot e^{j\cdot 107.224\cdot deg}} = \frac{284\cdot mA}{3.289}$$ / 84.04 - 107.224° = 86.4mA /-23.18°

d) How about $\mathbf{I}_C$? $\mathbf{I}_C := \dfrac{\mathbf{V}_C}{\left(\dfrac{1}{j\cdot\omega\cdot C}\right)} = \mathbf{V}_C\cdot j\cdot\omega\cdot C$ = 20.4V / 8.95° 0.015708 / 90° $\dfrac{1}{\Omega}$ = 320mA / 98.95°

Another Way Could also be found directly by current divider: $\mathbf{I}_C := \dfrac{j\cdot\omega\cdot C}{j\cdot\omega\cdot C + \dfrac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I}_{L1}$ = 320mA / 98.95°

Something Weird

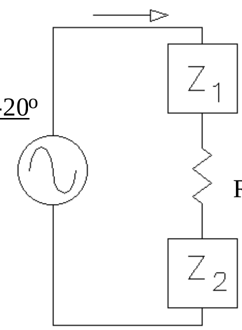
$\mathbf{I}_C$ is greater than the input current ($\mathbf{I}_{L1}$) . What's going on?

The angle between $\mathbf{I}_C$ & $\mathbf{I}_{L2}$ is big enough that they somewhat cancel each other out (partially resonate).

Check Kirchoff's Current Law: $\mathbf{I}_C + \mathbf{I}_{L2} = 29.485 + 282.569j \cdot mA \overset{?}{=} \mathbf{I}_{L1} = 29.485 + 282.569j \cdot mA$ yes

Ex. 3 a) Find $\mathbf{Z_2}$.

$i(t) = 25 \cdot mA \cdot \cos\left(377 \cdot \frac{rad}{sec} \cdot t + 10 \cdot deg\right)$



$V_{in} = 10V\ \underline{/-20^o}$

$\mathbf{Z_1} := (120 - 60 \cdot j) \cdot \Omega$

$R := 50 \cdot \Omega$

$\mathbf{Z_2} = ?$

$\mathbf{I} := 25 \cdot mA \cdot e^{j \cdot 10 \cdot deg}$

$\mathbf{V_{in}} := 10 \cdot V \cdot e^{-j \cdot 20 \cdot deg}$

$\mathbf{Z_T} := \frac{\mathbf{V_{in}}}{\mathbf{I}} = \frac{10 \cdot V}{25 \cdot mA}\ \underline{/-20 - 10^o} = 400\Omega\ \underline{/-30^o}$

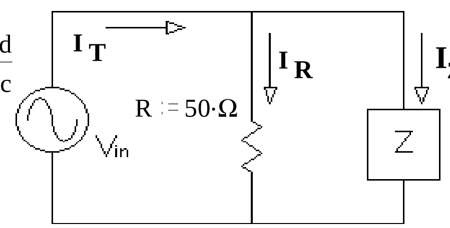
$\mathbf{Z_T} = 346.41 - 200j\ \cdot\Omega$

$\mathbf{Z_2} := \mathbf{Z_T} - R - \mathbf{Z_1} = (346.41 - 200 \cdot j) \cdot \Omega - 50 \cdot \Omega - (120 - 60 \cdot j) \cdot \Omega = 176.41 - 140j\ \cdot\Omega$

b) Circle 1: i) The source current leads the source voltage <--- answer, because $10^o > -20^o$.

ii) The source voltage leads the source current

Ex 4. a) Find $\mathbf{V_{in}}$ in polar form.



$\mathbf{I_T}$, $\mathbf{I_R}$, $R := 50 \cdot \Omega$

$\mathbf{I_Z} = 100\ \underline{/0^o}$ mA

$\mathbf{Z} := (80 - 60 \cdot j) \cdot \Omega$

$\mathbf{I_Z} := 100 \cdot mA \qquad \mathbf{Z} := (80 - 60 \cdot j) \cdot \Omega \qquad \omega := 1000 \cdot \frac{rad}{sec}$

$\mathbf{V_{in}} := \mathbf{I_Z} \cdot \mathbf{Z} \qquad \mathbf{V_{in}} = 8 - 6j\ \cdot V$

$\sqrt{8^2 + 6^2} = 10 \qquad \operatorname{atan}\left(\frac{-6}{8}\right) = -36.87\ \cdot deg$

$\mathbf{V_{in}} = 10V\ \underline{/-36.9^o}$

b) Find $\mathbf{I_T}$ in polar form. $\mathbf{I_R} := \frac{\mathbf{V_{in}}}{R} = \frac{10 \cdot V}{50 \cdot \Omega}\ \underline{/-36.9^o} = \frac{10 \cdot V}{50 \cdot \Omega} \cdot \cos(-36.9 \cdot deg) + j \cdot \frac{10 \cdot V}{50 \cdot \Omega} \cdot \sin(-36.9 \cdot deg) = 160 - 120i\ \cdot mA$

$\mathbf{I_T} := \mathbf{I_R} + \mathbf{I_Z} = (160 - 120 \cdot j) \cdot mA + 100 \cdot mA = 260 - 120j\ \cdot mA$

$\sqrt{260^2 + 120^2} = 286.356 \qquad \operatorname{atan}\left(\frac{-120}{260}\right) = -24.78\ \cdot deg \qquad \mathbf{I_T} = 286mA\ \underline{/-24.8^o}$

c) Circle 1: i) The source current leads the source voltage answer i), $-24.8^o > -36.9^o$

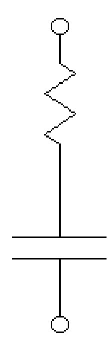
ii) The source voltage leads the source current

d) The impedance $\mathbf{Z}$ (above) is made of two components in series. What are they and what are their values?

$\mathbf{Z} = 80 - 60j\ \cdot\Omega$

Must have a resistor because there is a real part.

$R := \mathrm{Re}(\mathbf{Z}) \qquad R = 80\ \cdot\Omega$



Must have a capacitor because the imaginary part is negative.

$\mathrm{Im}(\mathbf{Z}) = -60\ \cdot\Omega = \frac{-1}{\omega \cdot C} \qquad C := \frac{-1}{\omega \cdot \mathrm{Im}(\mathbf{Z})} \qquad C = 16.667\ \cdot\mu F$

Ex. 5 The impedance $\mathbf{Z} = 80 - 60j \cdot \Omega$ is made of two components in parallel. What are they and what are their values?

Must have a resistor because there is a real part.

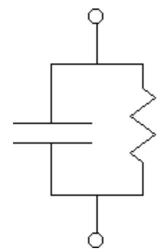
Must have an capacitor because the imaginary part is negative.

$$\mathbf{Z} = \frac{1}{\frac{1}{R} + j\cdot\omega\cdot C} \qquad \frac{1}{\mathbf{Z}} = \frac{1}{(80 - 60\cdot j)\cdot\Omega}\cdot\left(\frac{80 + 60\cdot j}{80 + 60\cdot j}\right) = \frac{80 + 60\cdot j}{80^2 + 60^2} = \frac{80 + 60\cdot j}{10,000}\cdot\frac{1}{\Omega}$$

$$\frac{1}{\mathbf{Z}} = 0.008 + 0.006i \cdot \Omega^{-1} = \frac{1}{R} + j\cdot\omega\cdot C$$

$$\frac{1}{R} = .008\cdot\frac{1}{\Omega} \qquad R := \frac{1}{.008\cdot\Omega^{-1}} \qquad R = 125\cdot\Omega$$

$$\omega\cdot C = .006\cdot\frac{1}{\Omega} \qquad C := \frac{.006\cdot\Omega^{-1}}{\omega} \qquad C = 6\cdot\mu F$$



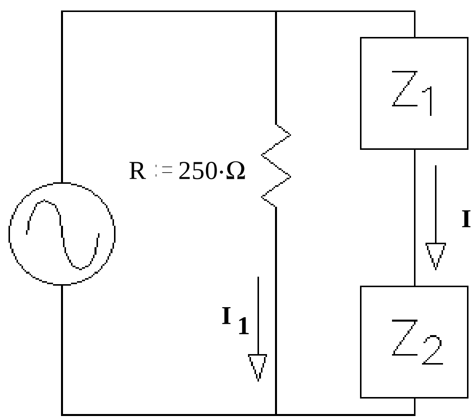
$R = 125\cdot\Omega$

Positive imaginary parts would require inductors

Ex. 6 a) Find $\mathbf{I_1}$

$$\omega := 20000\cdot\frac{rad}{sec}$$

$$\mathbf{V_{in}} := 20\cdot V\cdot e^{j\cdot 30\cdot deg}$$



$R := 250\cdot\Omega$

voltage $\mathbf{V_{Z1}} := (8 - 5\cdot j)\cdot V$

$\mathbf{I_2} := 20\cdot mA\cdot e^{j\cdot 20\cdot deg}$

$$\mathbf{I_1} := \frac{\mathbf{V_{in}}}{R} = \frac{20\cdot V}{250\cdot\Omega}\cdot e^{j\cdot 30\cdot deg} = 80\cdot mA\cdot e^{j\cdot 30\cdot deg}$$

polar division

b) Circle 1: i) $\mathbf{V_{in}}$ leads $\mathbf{I_2}$ ii) $\mathbf{V_{in}}$ lags $\mathbf{I_2}$

Why? Show numbers: __30__ > __20__ ______ < ______

c) Find $\mathbf{Z_2}$ in polar form

Convert $\mathbf{V_{in}}$ to rectangular coordinates

$20\cdot V\cdot\cos(30\cdot deg) = 17.321\cdot V$ $\qquad 20\cdot V\cdot\sin(30\cdot deg) = 10\cdot V$ pol to rect

$\mathbf{V_{in}} = 17.321 + 10j \cdot V$

$\mathbf{V_{Z2}} := \mathbf{V_{in}} - \mathbf{V_{Z1}}$ $\qquad \mathbf{V_{Z2}} = 9.321 + 15j \cdot V$ subtract

rect to pol $\sqrt{9.321^2 + 15^2} = \left|\mathbf{V_{Z2}}\right| = 17.66\cdot V$

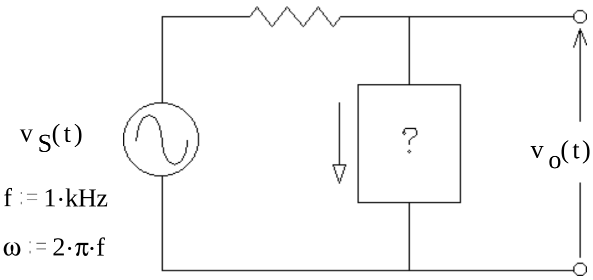
$\operatorname{atan}\left(\frac{15}{9.321}\right) = \arg(\mathbf{V_{Z2}}) = 58.145\cdot deg$

div $\mathbf{Z_2} := \frac{\mathbf{V_{Z2}}}{\mathbf{I_2}}$ $\qquad \frac{17.66\cdot V}{20\cdot mA} = 883\cdot\Omega$ $\qquad \angle\ 58.145\cdot deg - 20\cdot deg = 38.145\cdot deg$ $\qquad \mathbf{Z_2} = 883\ \angle\ 38.15^{\circ}\ \Omega$

$\mathbf{Z_2} = 694.436 + 545.379j \cdot\Omega$

Ex. 7 You need to design a circuit in which the "output" voltage leads the input voltage ($v_S(t)$) by 40º of phase.

$R := 400 \cdot \Omega$



$f := 1 \cdot \text{kHz}$

$\omega := 2 \cdot \pi \cdot f$

$\omega = 6.283 \cdot 10^3 \cdot \frac{\text{rad}}{\text{sec}}$

a) What should go in the box: R, L, C?

$$\mathbf{V_o} = \frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}} \cdot \mathbf{V_S}$$

angle of $\frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}}$ is 40º.

This can only happen if the angle of $\mathbf{Z_{box}}$ is positive, so $\mathbf{Z_{box}}$ is a inductor

b) Find its value. $\mathbf{V_o} =$ $\mathbf{V_o} = \frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L} \cdot \mathbf{V_S}$ angle $\frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L}$ is $90 - \text{atan}\left(\frac{\omega \cdot L}{R}\right) = 40^o$.

So: $\text{atan}\left(\frac{\omega \cdot L}{R}\right) = 50^o$ $\quad \frac{\omega \cdot L}{R} = \tan(50 \cdot \text{deg}) = 1.192$ $\quad L = \frac{R \cdot 1.192}{\omega} = 75.9 \cdot \text{mH}$

c) Repeat if the "output" voltage should lag the input voltage ($v_S(t)$) by 20º of phase.

angle of $\frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}}$ is -20º. This can only happen if the angle of $\mathbf{Z_{box}}$ is negative, so $\mathbf{Z_{box}}$ is a capacitor

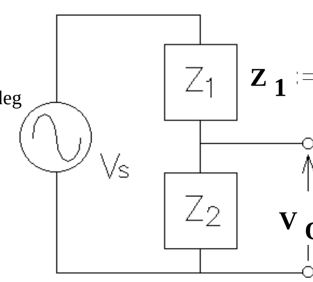
$$\mathbf{V_o} = \frac{\frac{1}{j \cdot \omega \cdot C}}{R + \frac{1}{j \cdot \omega \cdot C}} \cdot \mathbf{V_S}$$

angle $\frac{\frac{1}{j \cdot \omega \cdot C}}{R + \frac{1}{j \cdot \omega \cdot C}}$ is $-90 - \text{atan}\left(\frac{-\frac{1}{\omega \cdot C}}{R}\right) = -90 - \text{atan}\left(-\frac{1}{\omega \cdot C \cdot R}\right)$

$\text{atan}\left(-\frac{1}{\omega \cdot C \cdot R}\right) = -70^o$. $\quad -\frac{1}{\omega \cdot C \cdot R} = \tan(-70 \cdot \text{deg}) = -2.747$ $\quad C = \frac{1}{\omega \cdot R \cdot 2.747} = 0.145 \cdot \mu\text{F}$

Ex. 8 Find $\mathbf{V_O}$ in the circuit shown. Express it as a magnitude and phase angle (polar).

$\mathbf{V_S} := 6 \cdot V \cdot e^{j \cdot 18 \cdot \text{deg}}$



$\mathbf{Z_1} := 25 \cdot \Omega + 35j \cdot \Omega$

$\mathbf{Z_2} := 80 \cdot \Omega \cdot e^{-j \cdot 60 \cdot \text{deg}}$

$\mathbf{V_O} := \frac{\mathbf{Z_2}}{\mathbf{Z_1} + \mathbf{Z_2}} \cdot \mathbf{V_S}$ Simple voltage divider

$|\mathbf{Z_2}| \cdot \cos(-60 \cdot \text{deg}) = 40 \cdot \Omega$ $\quad |\mathbf{Z_2}| \cdot \sin(-60 \cdot \text{deg}) = -69.282 \cdot \Omega$ $\quad \mathbf{Z_2} = 40 - 69.282j \cdot \Omega$

$\mathbf{Z_1} + \mathbf{Z_2} = 25 \cdot \Omega + 35j \cdot \Omega + 40 \cdot \Omega - 69.282 \cdot j \cdot \Omega = 65 - 34.282j \cdot \Omega = 73.486 \cdot \Omega \cdot e^{-j \cdot 27.81 \cdot \text{deg}}$

$$\mathbf{V_O} := \frac{\mathbf{Z_2}}{\mathbf{Z_1} + \mathbf{Z_2}} \cdot \mathbf{V_S} = \frac{80 \cdot \Omega \cdot e^{-j \cdot 60 \cdot \text{deg}}}{73.486 \cdot \Omega \cdot e^{-j \cdot 27.81 \cdot \text{deg}}} \cdot \left(6 \cdot V \cdot e^{j \cdot 18 \cdot \text{deg}}\right) = \frac{80 \cdot \Omega}{73.486 \cdot \Omega} \cdot 6 \cdot V \cdot e^{j \cdot (-60 - (-27.81) + 18) \cdot \text{deg}} = 6.53 \cdot V \cdot e^{-j \cdot 14.2 \cdot \text{deg}}$$