ECE 2210 / 00 Capacitor Lecture Notes

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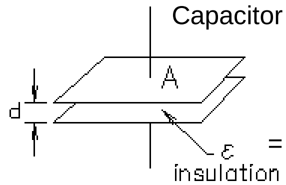
2/17/03
rev 9/16/09
12/15 & 9/19
9/28/23

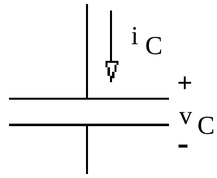
Now that we have voltages and currents which can be functions of time, it's time to introduce the capacitor and the inductor.

Electrical equivalent:

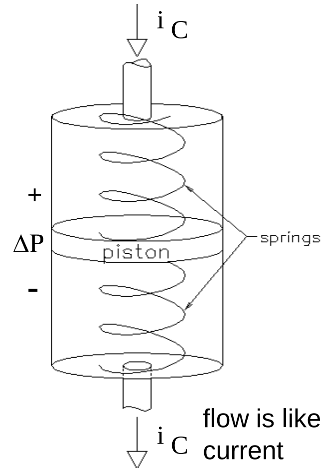


$$C = \varepsilon \cdot \frac{A}{d} = \frac{Q}{V} = \frac{dq}{dv}$$

Fluid Model:



like pressure



flow is like current

Units: $\text{farad} = \frac{\text{coul}}{\text{volt}} = \frac{\text{amp} \cdot \text{sec}}{\text{volt}}$

Basic equations & concepts you should know:

$\mu F = 1 \cdot 10^{-6} \cdot \text{farad}$ $pF = 1 \cdot 10^{-12} \cdot \text{farad}$

$$i_C = C \cdot \frac{d}{dt} v_C$$

$$v_C = \frac{1}{C} \cdot \int_{-\infty}^{t} i_C \, dt$$

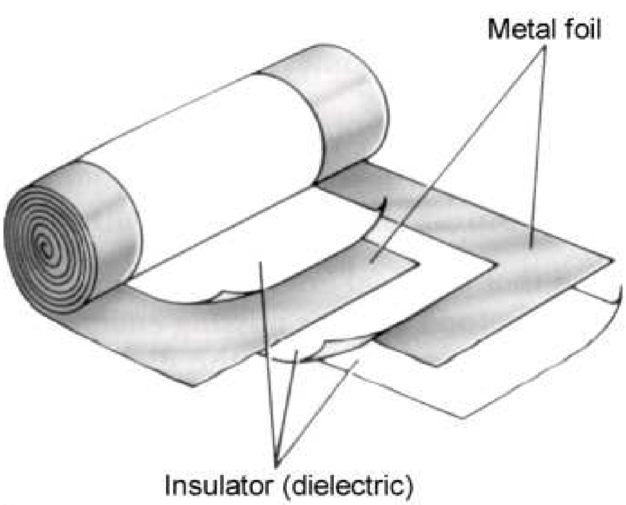
Or... $$v_C = \frac{1}{C} \cdot \int_{0}^{t} i_C \, dt + v_C(0)$$ (initial voltage)

Or... $$\Delta v_C = \frac{1}{C} \cdot \int_{t_1}^{t_2} i_C \, dt$$

Energy stored in electric field: $W_C = \frac{1}{2} \cdot C \cdot V_C^{\,2}$

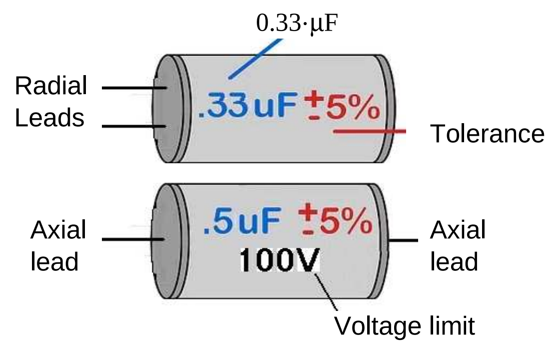
Capacitor voltage **cannot** change instantaneously

Capacitor Construction

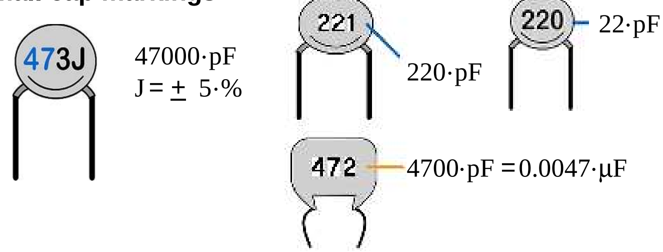


Capacitors are typically classified by the material used for insulation. The insulation determines some of the non-ideal characteristics.
See Table 3.7 in text

Large cap markings



Small cap markings



For way more about capacitors, see Section 3.6 of textbook.
Especially, see Figure 3.66 for more information about markings.

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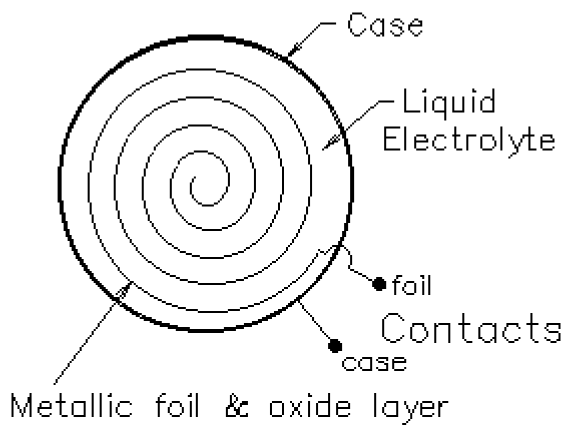
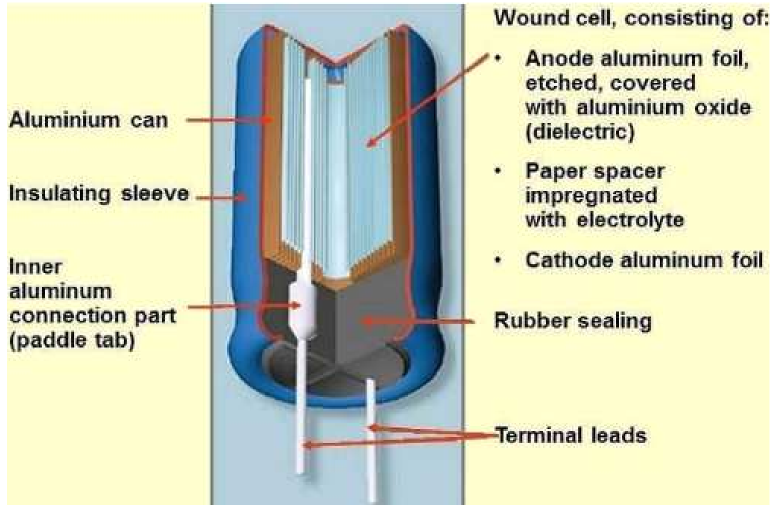
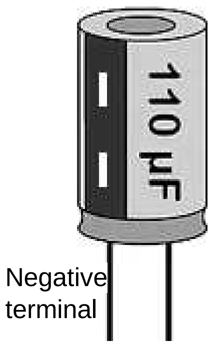
Electrolytic Capacitors Typically $1\mu F$ and larger

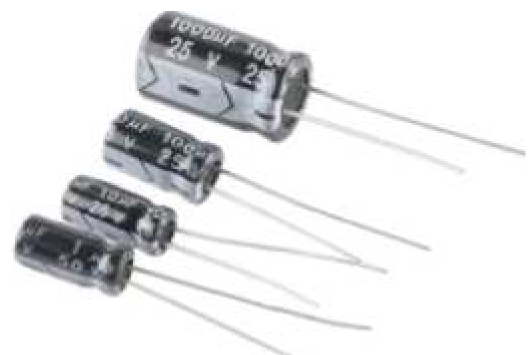
Construction (top view)





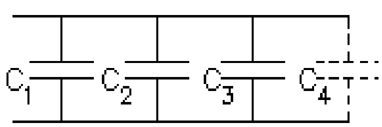
Almost all electrolytic capacitors are **polarized**. The negative terminal must always be negative with respect to the positive terminal, other wise it may be ruined.





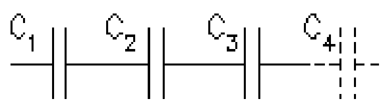
Equivalent Capacitance in series and parallel

Parallel:



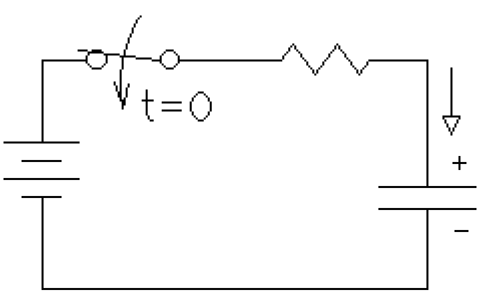
$$C_{eq} = C_1 + C_2 + C_3 + \ldots$$

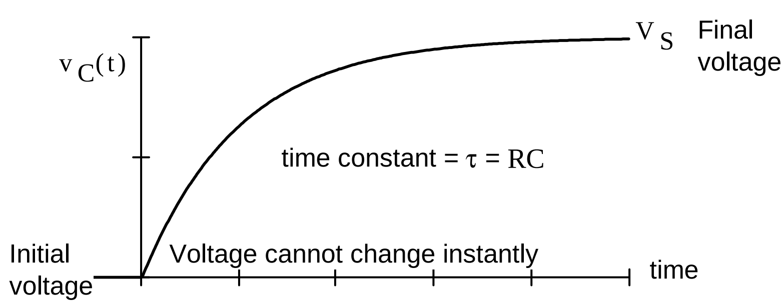
Series:

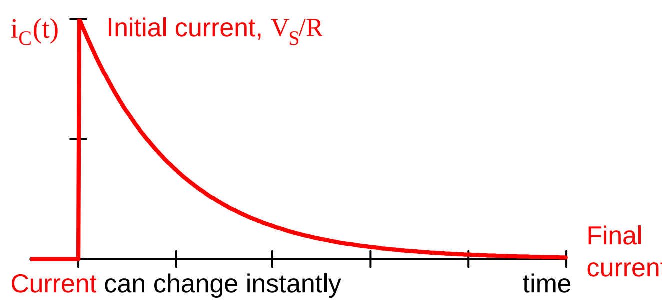


$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots}$$

Capacitors are the only "backwards" components.







Initial Condition is typically found by finding the capacitor voltage just before time $t = 0$.

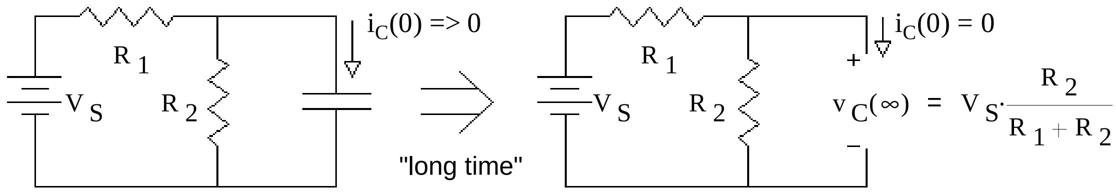
$$v_C(0^-) = v_C(0^+)$$

Voltage just before initial time = current just after initial time
Capacitor voltage cannot change instantly

Final Condition (Steady-state)

If a circuit has been connected for "a long time", then it has reached a steady state condition. That means the currents and voltages are no longer changing.

So... $\frac{d}{dt}v_C = 0$ and... $i_C = C\cdot\frac{d}{dt}v_C = 0$



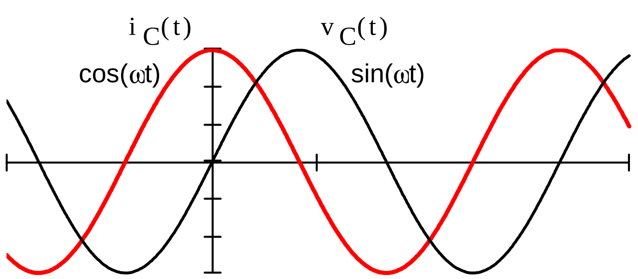
Replace the capacitor with an open and find the voltage across the open (just like finding v_{Th}).
Applies when sources are constant (DC)

Sinusoids

$$i_C(t) = I_p\cdot\cos(\omega\cdot t) \qquad v_C(t) = \frac{1}{C}\cdot\int i_C\,dt = \frac{1}{C}\cdot\frac{1}{\omega}\cdot I_p\cdot\sin(\omega\cdot t) = \frac{1}{C}\cdot\frac{1}{\omega}\cdot I_p\cdot\cos(\omega\cdot t - 90\cdot deg)$$

indefinite integral $\qquad \underbrace{\frac{1}{C}\cdot\frac{1}{\omega}\cdot I_p}_{V_p}$

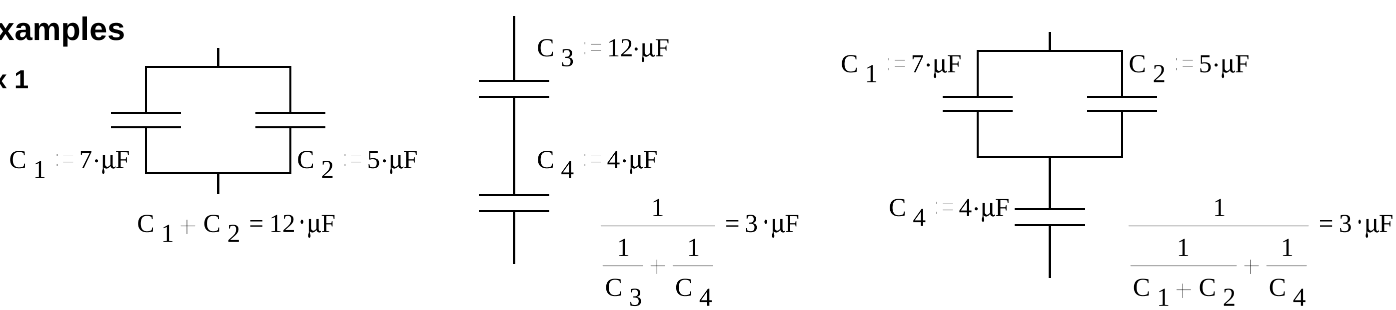
$$V_p = \frac{1}{\omega\cdot C}\cdot I_p$$



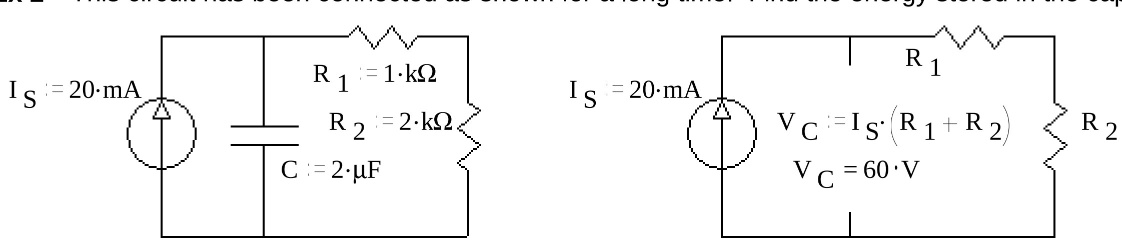
Voltage "lags" current.
This should make sense to you,
since current has to flow in first to charge capacitor.

Examples

Ex 1



Ex 2 This circuit has been connected as shown for a long time. Find the energy stored in the cap.

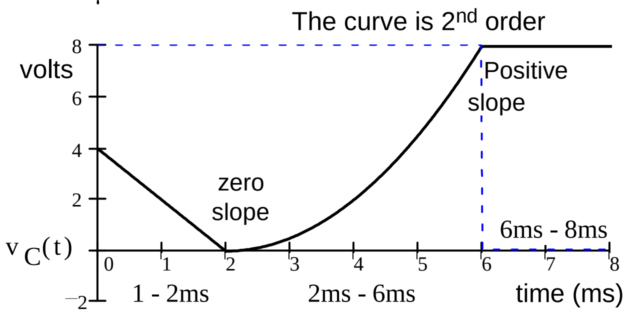


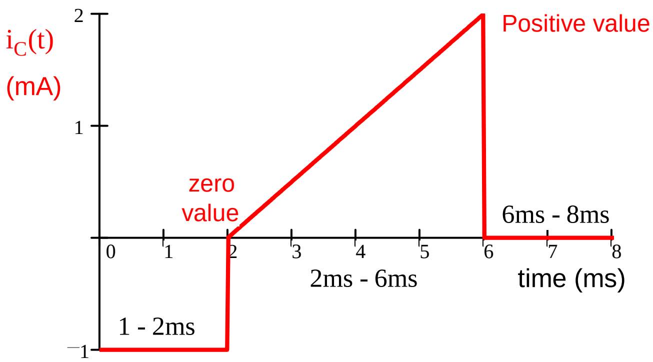
$$W_C := \frac{1}{2}\cdot C\cdot V_C^2$$

$$W_C = 3.6\cdot mJ$$

Ex 3 The voltage across a $0.5\ \mu F$ capacitor is shown below.
Make an accurate drawing of the capacitor current. Label the y-axis of your graph.

$C := 0.5\cdot\mu F$





1 - 2ms: $i_C = C\cdot\frac{\Delta V}{\Delta t} = 0.5\cdot\mu F\cdot\frac{-4\cdot V}{2\cdot ms} = -1\cdot mA$

2ms - 6ms: Initial slope is zero and the final slope is positive, so the current must be a triangle that starts at zero and ends at some height.

$$\Delta v_C(t) = \frac{1}{C}\cdot\int_{6ms}^{2ms} i_C(t)\,dt$$

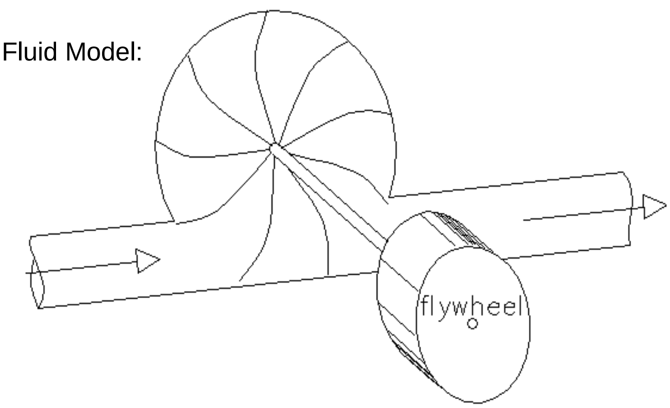
$$8\cdot V = \frac{1}{C}\cdot\left(\frac{4\cdot ms\cdot height}{2}\right)$$

$$height = 8\cdot V\cdot\frac{C\cdot 2}{4\cdot ms} = 2\cdot mA$$

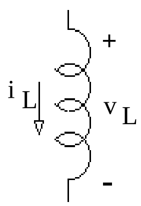
6ms - 8ms: Slope is zero, so the current must be zero.

ECE 2210 / 00 Inductor Lecture Notes

Fluid Model:



Electrical equivalent:



$L = \mu\cdot N^2\cdot K$

μ is the permeability of the inductor core

K is a constant which depends on the inductor geometry

N is the number of turns of wire

Units: $henry = \frac{volt\cdot sec}{amp} = H$ $\quad mH = 10^{-3}\cdot H \quad \mu H = 10^{-6}\cdot H$

Basic equations and Concepts you should know:

$$v_L = L\cdot\frac{d}{dt}i_L$$

$$i_L = \frac{1}{L}\cdot\int_{-\infty}^{t} v_L\,dt$$

Or... $i_L = \frac{1}{L}\cdot\int_{0}^{t} v_L\,dt + i_L(0)$ ← initial current

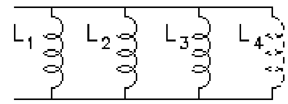
Energy stored in electric field: $W_L = \frac{1}{2}\cdot L\cdot I_L^2$

Or... $\Delta i_L = \frac{1}{L}\cdot\int_{t_1}^{t_2} v_L\,dt$

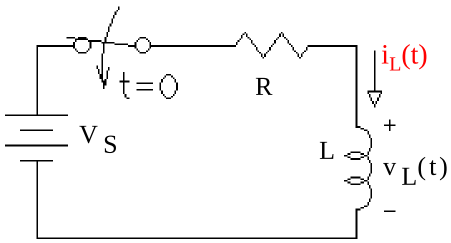
Inductor current **cannot** change instantaneously

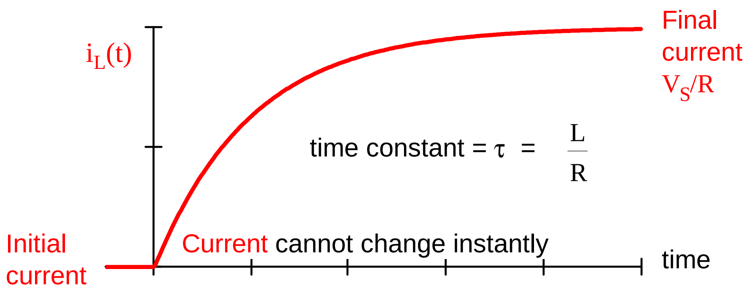
series: $L_{eq} = L_1 + L_2 + L_3 + \ldots$

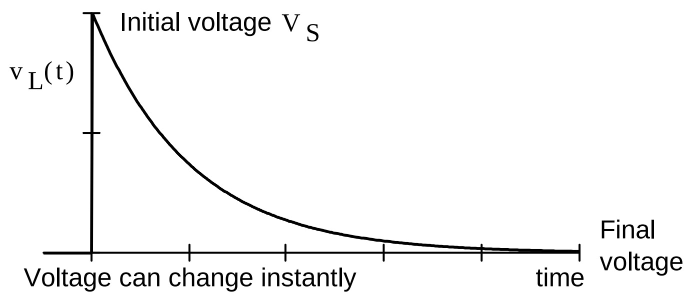
parallel: $L_{eq} = \dfrac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_2} + \dfrac{1}{L_3} + \ldots}$



"Energizing" an Inductor







Initial Condition is typically found by finding the inductor current just before time $t = 0$.

$$i_L(0^-) = i_L(0^+)$$

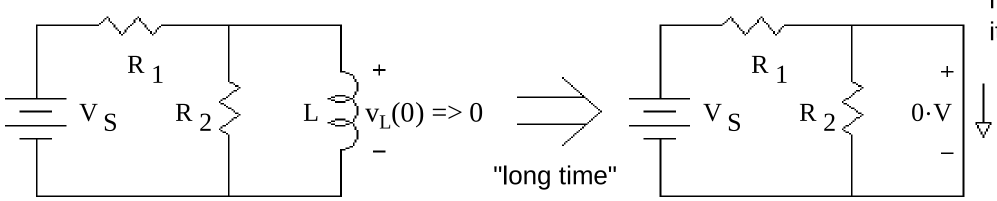
Current just before initial time = current just after initial time
Inductor current cannot change instantly

Final Condition (Steady-state)

If a circuit has been connected for "a long time", then it has reached a steady state condition. that means the currents and voltages are no longer changing.

$$\frac{d}{dt} i_L = 0 \qquad v_L = L \cdot \frac{d}{dt} i_L = 0$$

no voltage means it looks like a short



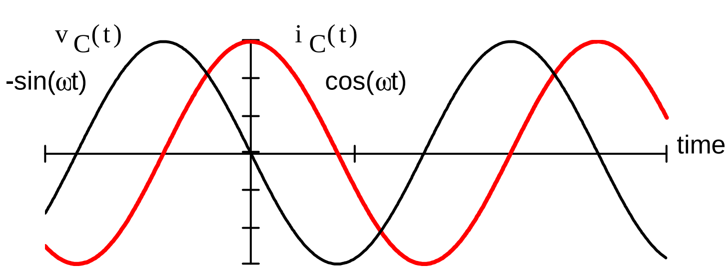
$$i_L(\infty) = \frac{V_S}{R_1}$$

Replace the inductor with a short and find the voltage across the.
Applies when sources are constant (DC)

Sinusoids

$$i_L(t) = I_p \cdot \cos(\omega \cdot t) \qquad v_L(t) = L \cdot \frac{d}{dt} i_L = L \cdot \omega \left(-I_p \cdot \sin(\omega \cdot t)\right) = \underbrace{-L \cdot \omega \cdot I_p}_{-V_p} \cdot \sin(\omega \cdot t) = \underbrace{L \cdot \omega \cdot I_p}_{V_p} \cdot \cos(\omega \cdot t + 90 \cdot \text{deg})$$

$$V_p = \omega \cdot L \cdot I_p$$



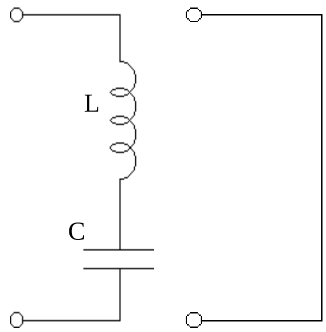
Voltage "leads" current.
This should make sense to you, since voltage has to present to make current change, so voltage comes first.

Resonance

At what frequency does $-L\cdot\omega\cdot I_p\cdot\sin(\omega\cdot t)+\frac{1}{C}\cdot\frac{1}{\omega}\cdot I_p\cdot\sin(\omega\cdot t)$ = 0 ??

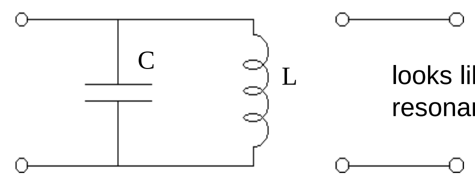
$$L\cdot\omega_o\cdot I_p = \frac{1}{C}\cdot\frac{1}{\omega_o}\cdot I_p \quad \ldots \quad \omega_o = \frac{1}{\sqrt{L\cdot C}}\left(\frac{rad}{sec}\right)$$

Series resonance



looks like a short at resonance frequency

Parallel resonance



looks like an open at resonance frequency

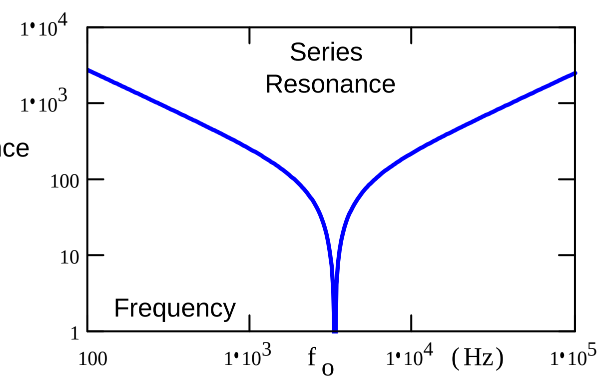
The resonance frequency is calculated the same way for either case:

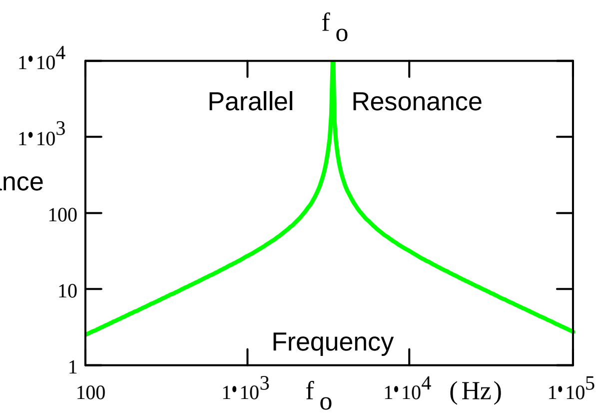
$$\omega_o = \frac{1}{\sqrt{L\cdot C}}\left(\frac{rad}{sec}\right) \quad \text{OR..} \quad \omega_o = \frac{1}{\sqrt{L_{eq}\cdot C_{eq}}}$$

If you have multiple capacitors or inductors which can be combined.

$$f_o = \frac{\omega_o}{2\cdot\pi} \ (Hz)$$

IF $C := 0.58\cdot\mu F \quad L := 4\cdot mH$ Then $f_o = \frac{1}{2\cdot\pi\cdot\sqrt{L\cdot C}} = 3.3\cdot kHz$ And you get these frequency response curves:

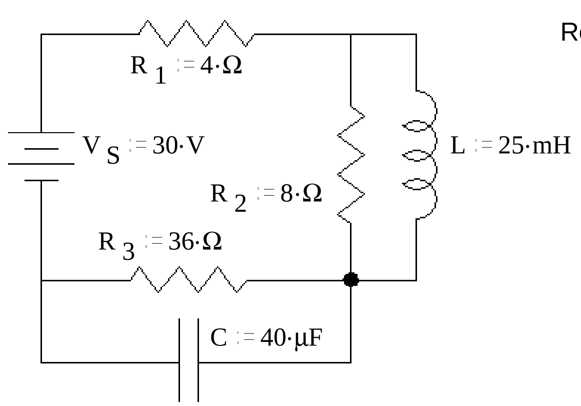




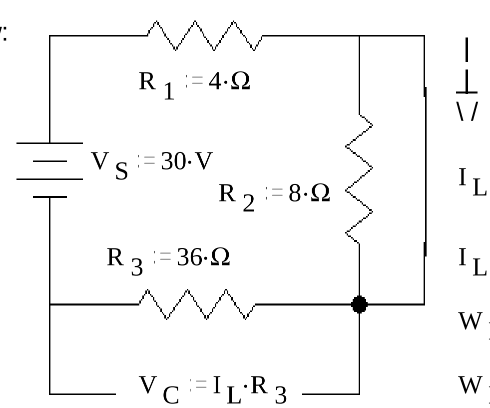
You will soon learn more about reactance and impedance.

Examples

Ex 1 The circuit has been connected as shown for a long time. Find the energy stored in the capacitor and the inductor.



Redraw:



$$I_L := \frac{V_S}{4\cdot\Omega+36\cdot\Omega}$$

$$I_L = 0.75\cdot A$$

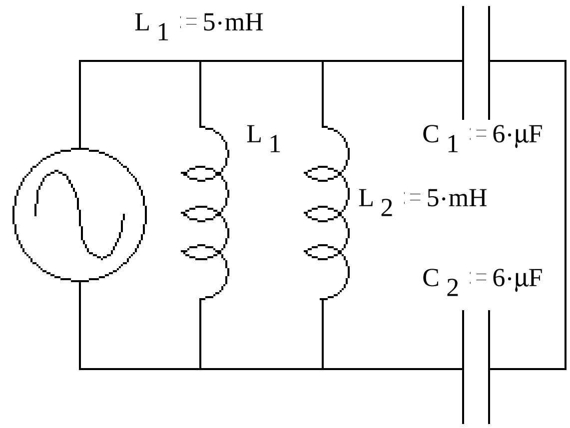
$$W_L := \frac{1}{2}\cdot L\cdot I_L^{\,2}$$

$$W_L = 7.031\cdot mJ$$

$$V_C := I_L\cdot R_3$$

$$V_C = 27\cdot V$$

$$W_C := \frac{1}{2}\cdot C\cdot V_C^{\,2} \qquad W_C = 14.58\cdot mJ$$



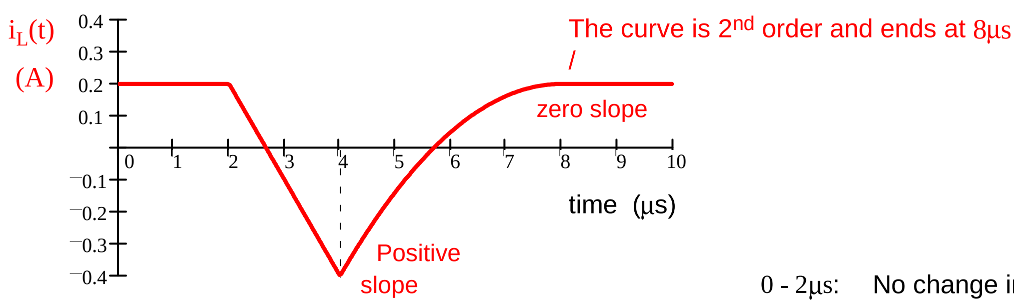
Ex 2 Find the resonant frequency (or frequencies) of the circuit shown (in cycles/sec or Hz).

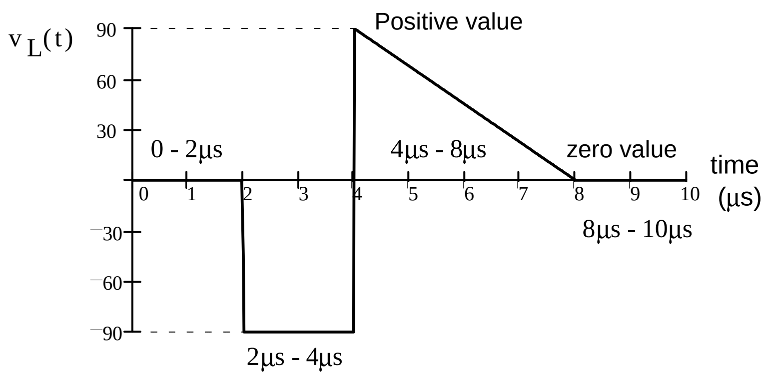
$$L_{eq} := \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} \qquad L_{eq} = 2.5 \cdot mH$$

$$C_{eq} := \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \qquad C_{eq} = 3 \cdot \mu F$$

$$\omega_o := \frac{1}{\sqrt{L_{eq} \cdot C_{eq}}} \qquad \omega_o = 11547 \cdot \frac{rad}{sec} \qquad f_o = \frac{\omega_o}{2 \cdot \pi} = 1838 \cdot Hz$$

Ex 3 The current through a 0.3mH inductor is shown below. Make an accurate drawing of the inductor voltage. Make reasonable assumptions where necessary. Label your graph. $L := 0.3 \cdot mH$





0 - 2µs: No change in current, so: $v_L = 0$

2µs - 4µs: $v_L = L \cdot \frac{\Delta I}{\Delta t} = 0.3 \cdot mH \cdot \frac{-0.6 \cdot A}{2 \cdot \mu s} = -90 \cdot V$

4µs - 8µs: Initial slope is positive and the final slope is zero, so the voltage must be a triangle that starts at some height and ends at zero.

$$\Delta i_L(t) = \frac{1}{L} \cdot \int_{4 \cdot \mu s}^{8 \cdot \mu s} v_L(t)\, dt$$

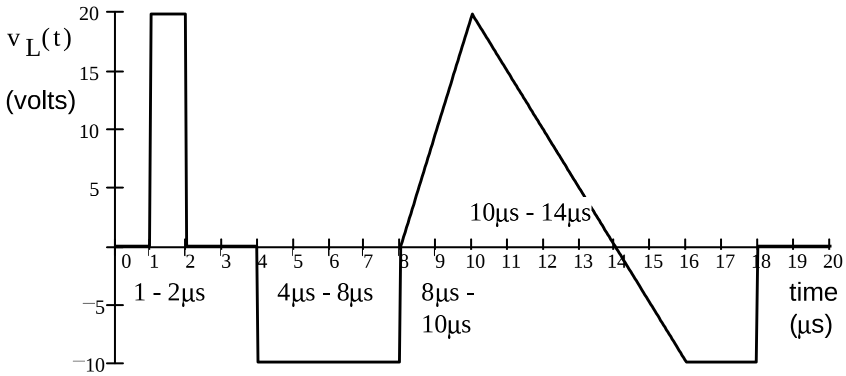
$$0.6 \cdot A = \frac{1}{0.3 \cdot mH} \cdot \left(\frac{4 \cdot \mu s \cdot height}{2} \right)$$

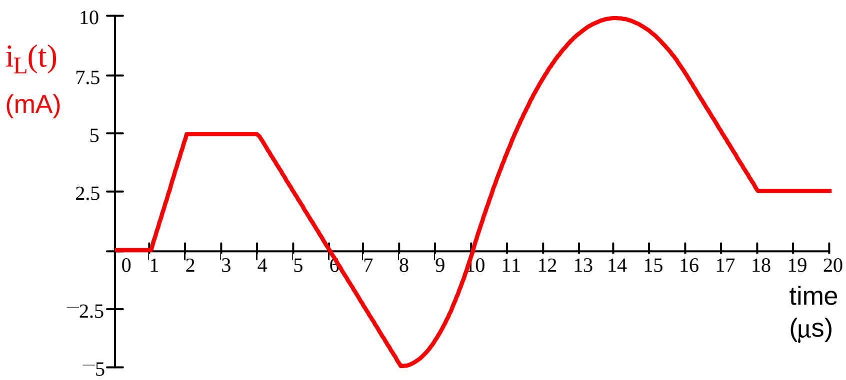
$$height = 0.6 \cdot A \cdot \frac{0.3 \cdot mH \cdot 2}{4 \cdot \mu s} = 90 \cdot V$$

8µs - 10µs: No change in current, so: $v_L = 0$

Ex 4 Given an inductor voltage, find the current.

$L := 4 \cdot mH$





1 - 2µs:

$$\Delta i_L(t) = \frac{1}{L} \cdot \int_{1 \cdot \mu s}^{2 \cdot \mu s} 20 \cdot V \, dt = 5 \cdot mA$$ = change from 1 - 2µs

4µs - 8µs:

$$\Delta i_L(t) = \frac{1}{L} \cdot \int_{4 \cdot \mu s}^{8 \cdot \mu s} -10 \cdot V \, dt = -10 \cdot mA$$ = change from 4µs - 8µs

8µs - 10µs:

$$\Delta i_L(t) = \frac{1}{L} \cdot \int_{8 \cdot \mu s}^{10 \cdot \mu s} V(t) \, dt$$

$$= \frac{1}{L} \cdot \frac{20 \cdot V \cdot 2 \cdot \mu s}{2} = 5 \cdot mA$$ = change from 8µs - 10µs

Voltage ramps from 0 to +20V, so current curve progresses from 0 slope to a positive slope

10µs - 14µs:

$$\Delta i_L(t) = \frac{1}{L} \cdot \int_{10 \cdot \mu s}^{14 \cdot \mu s} V(t) \, dt + 0 \cdot mA$$

$$= \frac{1}{L} \cdot \frac{20 \cdot V \cdot 4 \cdot \mu s}{2} = 10 \cdot mA$$

Current curves from a positive slope to 0 slope

etc...

Ex 5 The current through and the voltage across an unknown component are shown below.

a) What type of component is it?
Give a good reason for your choice.

inductor ? $v(t) := L \cdot \frac{d}{dt} i(t)$

Doesn't fit graphs, still have current even when voltage isn't changing.

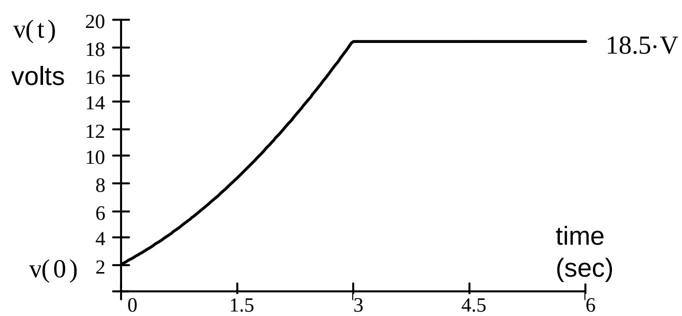
Also Inductor current can't change instantly.

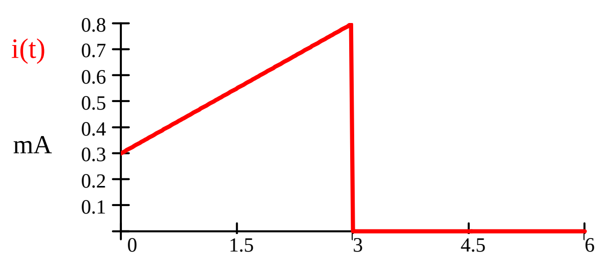
Can't be a Inductor

capacitor ? $i(t) := C \cdot \frac{d}{dt} v(t)$

This fits the graphs, $i(t)$ corresponds to slope of $v(t)$

Must be a Capacitor





b) What is the value of the component?

$$v(3 \cdot sec) = \frac{1}{C} \cdot \int_0^{3 \cdot sec} i(t) \, dt + v(0)$$

OR: $\Delta v = \frac{1}{C} \cdot (current\_area)$ $\Delta v := 16.5 \cdot V$ $current\_area := 0.3 \cdot mA \cdot 3 \cdot sec + \frac{1}{2} \cdot 0.5 \cdot mA \cdot 3 \cdot sec$

$C := \frac{current\_area}{\Delta v}$ $C = 100 \cdot \mu F$

Capacitor, Inductor Notes

ECE 2210 / 00

A.Stolp
2/27/00,
9/13/05

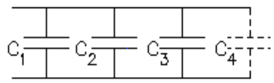
Capacitors

$C = \frac{Q}{V}$ $\quad$ $\text{farad} = \frac{\text{coul}}{\text{volt}} = \frac{\text{amp}\cdot\text{sec}}{\text{volt}}$ $\quad$ $v_C = \frac{1}{C}\cdot\int_{-\infty}^{t} i_C\,dt = \frac{1}{C}\cdot\int_{0}^{t} i_C\,dt + v_C(0)$ (initial voltage) $\quad$ $i_C = C\cdot\frac{d}{dt}v_C$

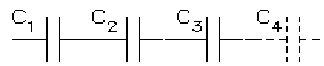
Energy stored in electric field: $W_C = \frac{1}{2}\cdot C\cdot V_C^2$

Capacitor voltage **cannot** change instantaneously

parallel: $C_{eq} = C_1 + C_2 + C_3 + \ldots$



series: $C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots}$



Steady-state sinusoids:

Impedance: $Z_C = \frac{1}{j\cdot\omega\cdot C} = \frac{-j}{\omega\cdot C}$ $\quad$ Current leads voltage by 90 deg

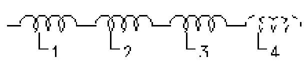
Inductors

$\text{henry} = \frac{\text{volt}\cdot\text{sec}}{\text{amp}}$ $\quad$ $i_L = \frac{1}{L}\cdot\int_{-\infty}^{t} v_L\,dt = \frac{1}{L}\cdot\int_{0}^{t} v_L\,dt + i_L(0)$ (initial current) $\quad$ $v_L = L\cdot\frac{d}{dt}i_L$

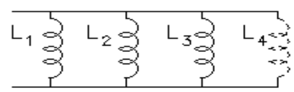
Energy stored in magnetic field: $W_L = \frac{1}{2}\cdot L\cdot I_L^2$

Inductor current **cannot** change instantaneously

series: $L_{eq} = L_1 + L_2 + L_3 + \ldots$



parallel: $L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots}$



Steady-state sinusoids:

Impedance: $Z_L = j\cdot\omega\cdot L$ $\quad$ Current lags voltage by 90 deg

RC and RL first-order transient circuits

For all first order transients: $v_X(t) = v_X(\infty) + \left(v_X(0) - v_X(\infty)\right)\cdot e^{-\frac{t}{\tau}}$ $\quad$ $i_X(t) = i_X(\infty) + \left(i_X(0) - i_X(\infty)\right)\cdot e^{-\frac{t}{\tau}}$

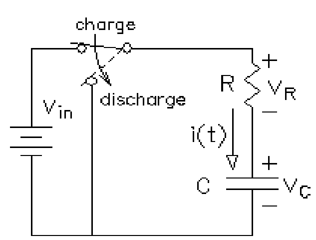
Find initial Conditions $\quad$ (v_C and/or i_L)

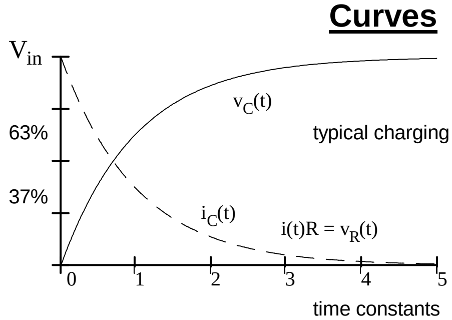
Find conditions just before time $t = 0$, $v_C(0-)$ and $i_L(0-)$. These will be the same just after time $t = 0$, $v_C(0^+)$ and $i_L(0^+)$ and will be your initial conditions. (If initial conditions are zero: Capacitors are shorts, Inductors are opens.)
Use normal circuit analysis to find your desired variable: $v_X(0)$ or $i_X(0)$

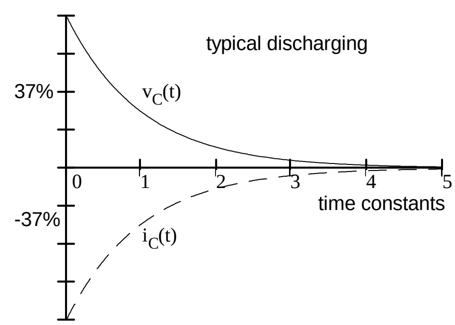
Find final conditions ("steady-state" or "forced" solution)
Inductors are shorts Capacitors are opens $\quad$ Solve by DC analysis $\quad$ $v_X(\infty)$ or $i_X(\infty)$

Curves

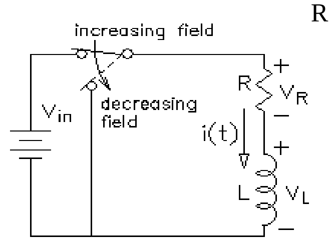
RC Time constant $= \tau = RC$

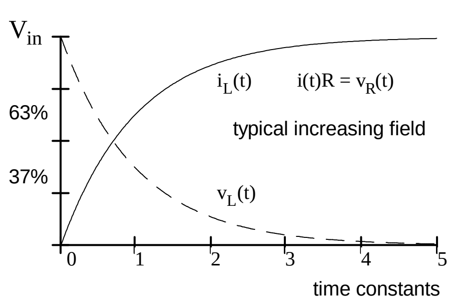


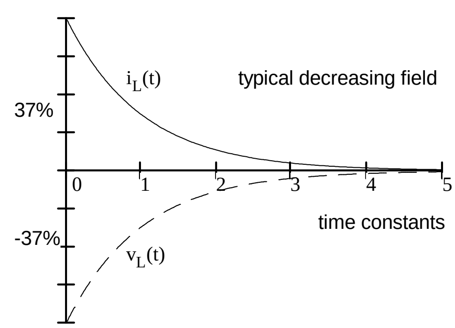




RL Time constant $= \tau = \frac{L}{R}$







$e^{-1} = 0.368$ $\quad$ $1 - e^{-1} = 0.632$