**Nodal Analysis**

**Node** = all points connected by wire, all at same voltage (potential)

**Ground**: One node in the circuit which will be our reference node. Ground, by definition, will be the zero voltage node. All other node voltages will be referenced to ground and may be positive or negative. Think of gage pressure in a fluid system. In that case atmospheric pressure is considered zero. If there is no ground in the circuit, define one for yourself. Try to choose a node which is hooked to one side of a voltage source.

**Nodal Voltage**: The voltage of a node referenced to ground. The objective of nodal analysis is to find all the nodal voltages. If you know the voltage at a node then it's a "known" node. Ground is a known node (duh, it's zero). If one end of a known voltage source hooked to ground, then the node on the other end is also known (another duh).

**Method**: Label all the unknown nodes as; “a”, “b”, “c”, etc. Then the unknown nodal voltages become; $V_a$, $V_b$, $V_c$, etc. Write a KCL equation for each unknown node, defining currents as necessary. Replace each unknown current with an Ohm’s law relationship using the nodal voltages. Now you have just as many equations as unknowns. Solve.

**Nodal Analysis Steps**

1) If the circuit doesn't already have a ground, label one node as ground (zero voltage). If the ground can be defined as one side of a voltage source, that will make the following steps easier.

2) Label unknown node voltages as $V_a$, $V_b$, etc and label the current in each resistor as $I_1$, $I_2$, ....

3) Write Kirchoff’s current equations for each unknown node.

4) Replace the currents in your KCL equations with expressions like this.

$$\frac{V_a - V_b}{R_1}$$

Ohm’s law relationship using the nodal voltages.

5) Solve the multiple equations for the multiple unknown voltages.

**Nodal Analysis Examples**

**Ex 1** Use nodal analysis to find the voltage across $R_1$ ($V_{R_1}$).

1) See next page

Label one node as ground (zero voltage). By choosing the negative side of a voltage source as ground, the upper-left node is known (10V). Label the remaining nodes, either with known voltages or with letters, a, b, ....
2) Label unknown node voltages as $V_a, V_b, \ldots$ and label the current in each resistor as $I_1, I_2, \ldots$.

3) Write Kirchhoff's current equations for node a.

\[
I_1 + I_S = I_{R3}
\]

4) Replace the currents in the KCL equations with Ohm's law relationships.

\[
\frac{V_S - V_a}{R_1} + I_S = \frac{V_a - 0}{R_3}
\]

\[
\frac{V_S - V_a}{R_1} + I_S = \frac{V_a}{R_3}
\]

5) Solve:

\[
\frac{V_S}{R_1} + I_S = \frac{V_a}{R_3} + \frac{V_a}{R_1}
\]

\[
\frac{V_S}{R_1} + I_S = \frac{V_a}{R_3} \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_3}\right)}
\]

\[
V_a := \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_3}\right)} V_a = 10.5 \cdot V
\]

\[
V_{R1} := V_S - V_a \quad V_{R1} = -0.5 \cdot V
\]

\[
V_b \text{ doesn't matter in this case}
\]

b) Find the current through $R_3 (I_{R3})$.

\[
I_{R3} = \frac{V_a}{R_3} = 3.5 \cdot mA
\]

**Ex 2** Same circuit used in Thévenin example, where $R_4$ was $R_L$.

1) Define ground and nodes:

2 unknown nodes ---> will need 2 equations
2) Label unknown node voltages as \( V_a \), \( V_b \), ... and label the current in each resistor as \( I_1, I_2, \ldots \). It doesn't matter if these currents are in the correct directions.

3) Write Kirchoff's current equations for each unknown node.

- **node a**
  \[
  I_1 = I_2 + I_4
  \]
  \[
  \frac{V_S - V_a}{R_1} = \frac{V_a - V_b}{R_2} + \frac{V_a - 0\text{-V}}{R_4}
  \]

- **node b**
  \[
  I_2 = I_3 + I_S
  \]
  \[
  \frac{V_a - V_b}{R_2} = \frac{V_b - 0\text{-V}}{R_3} + I_S
  \]

Now you have just as many equations as unknowns.

4) Replace the currents in your KCL equations with expressions like this.

- **node a**
  \[
  I_1 = \frac{V_a - V_b}{R_1}
  \]
  \[
  \frac{V_S - V_a}{R_1} = \frac{V_a - V_b}{R_2} + \frac{V_a - 0\text{-V}}{R_4}
  \]

- **node b**
  \[
  I_2 = \frac{V_b - 0\text{-V}}{R_3} + I_S
  \]

Now you have just as many equations as unknowns.

5) Solve the multiple equations for the multiple unknown voltages. Solve by any method you like:

- **node a**
  \[
  V_a = \frac{360\Omega \cdot (9\text{-V} - V_a)}{40\Omega} = \frac{V_a - V_b}{120\Omega} + \frac{V_a}{72\Omega} \\
  \]
  \[
  V_a = 4.6\text{\text{-V}}
  \]

- **node b**
  \[
  \frac{V_a - V_b}{240\Omega} = \frac{V_b - 0\text{-V}}{240\Omega} + 50\text{-mA} \cdot 240\Omega
  \]
  \[
  \]
  \[
  V_a = \frac{2 \cdot V_b + V_b + 12\text{-V}}{2} = 1.5\text{-V}b + 6\text{-V}
  \]

Or, with numbers

- **node a**
  \[
  360\Omega \cdot \left( \frac{9\text{-V} - V_a}{40\Omega} \right) = \frac{V_a - V_b}{120\Omega} + \frac{V_a}{72\Omega}
  \]
  \[
  81\text{-V} - 9\text{-V} = 3\text{-V}a - 3\text{-V}b + 5\text{-V}a
  \]
  \[
  81\text{-V} - 9\text{-V} = 3\left(1.5\text{-V}b + 6\text{-V}\right) - 3\text{-V}b + 5\left(1.5\text{-V}b + 6\text{-V}\right)
  \]
  \[
  81\text{-V} - 13.5\text{-V}b - 54\text{-V} = 4.5\text{-V}b + 18\text{-V} - 3\text{-V}b + 7.5\text{-V}b + 30\text{-V}
  \]
  \[
  81\text{-V} - 54\text{-V} - 18\text{-V} - 30\text{-V} = -21\text{-V} = 4.5\text{-V}b - 3\text{-V}b + 7.5\text{-V}b + 13.5\text{-V}b = 22.5\text{-V}b
  \]
  \[
  V_b = \frac{-21\text{-V}}{22.5} = -0.933\text{-V}
  \]

- **node b**
  \[
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Ex 3  Like Superposition Ex.2

a) Use nodal analysis to find the voltage across $R_2$ ($V_{R2}$).

You MUST show all the steps of nodal analysis work to get credit, including drawing appropriate symbols and labels on the circuit shown.

1) Define ground and nodes:

2) Label unknown node voltages as $V_a$, $V_b$, ... and label the current in each resistor as $I_1, I_2,...$.

3) Write Kirchoff's current equations for each unknown node.

4) Replace the currents in the KCL equations with Ohm's law relationships.

5) Solve the equation for the unknown voltage.

Remember, we needed to find the voltage across $R_2$ ($V_{R2}$).

$V_{R2} = V_S - V_a = 7.2 \cdot V$

b) Find the current through $R_3$ ($I_{R3}$).

$I_{R3} = \frac{0 - V_a}{R_3} = -1.6 \cdot mA$ actually flows the other way
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Ex 4 Use nodal analysis to find the voltage across \( R_5 \) (\( V_{R5} \)) and the current through \( R_1 \) (\( I_{R1} \)). From exam 1, F09

You **MUST** show all the steps of nodal analysis work to get credit, including drawing appropriate symbols and labels on the circuit shown.

\[
\begin{align*}
V_{S1} &:= 12 \text{ V} \\
V_{S2} &:= 6 \text{ V} \\
R_1 &:= 100 \text{ Ω} \\
R_2 &:= 200 \text{ Ω} \\
R_3 &:= 300 \text{ Ω} \\
R_4 &:= 400 \text{ Ω} \\
R_5 &:= 600 \text{ Ω} \\
I_S &:= 63 \text{ mA}
\end{align*}
\]

**Node a:**

\[
I_{R1} + I_S = I_R + I_5
\]

\[
R_3 = 300 \text{ Ω} \quad \frac{V_{S1} - V_a}{R_1} + I_S = \frac{V_a - V_{S2}}{R_2 + R_3} + \frac{V_a - V_{S2}}{R_5}
\]

\[
\begin{align*}
12 \text{ V} &- \frac{V_a}{100 \text{ Ω}} + 63 \text{ mA} = \frac{V_a}{500 \text{ Ω}} - \frac{6 \text{ V}}{500} + \frac{V_a}{600 \text{ Ω}} - \frac{6 \text{ V}}{600} \\
3000 \text{ Ω} \left( \frac{12 \text{ V} - \frac{V_a}{100 \text{ Ω}} + 63 \text{ mA}}{100 \text{ Ω}} \right) &= \left( \frac{V_a}{500 \text{ Ω}} - \frac{6 \text{ V}}{500} + \frac{V_a}{600 \text{ Ω}} - \frac{6 \text{ V}}{600} \right) \cdot 3000 \text{ Ω}
\end{align*}
\]

\[
\begin{align*}
360 \text{ V} - 30 \text{ V} a + 189 \text{ V} &= 6 \cdot V_a - 36 \text{ V} + 5 \cdot V_a - 30 \text{ V} \\
360 \text{ V} + 189 \text{ V} + 36 \text{ V} + 30 \text{ V} &= 6 \cdot V_a + 5 \cdot V_a + 30 \text{ V} a \\
615 \text{ V} &= 41 \cdot V_a \\
V_a &= \frac{615 \text{ V}}{41}
\end{align*}
\]

\[
V_a = 15 \text{ V}
\]

\[
V_{R5} = V_a - V_{S2} = 9 \text{ V}
\]

\[
I_{R1} = \frac{V_{S1} - V_a}{R_1} = -30 \text{ mA}
\]
What if one side of a voltage source isn't ground?

\[ I_1 + I_{VS2} = I_3 \]
\[ \frac{V_{S1} - V_a}{R_1} + ? = I_S \]

What do you put in for \( I_{VS2} \)?

Go to the other side of \( V_{S2} \).

\[ \frac{V_{S1} - V_a}{R_1} + \frac{0 - V_b}{R_2} = I_S \]

Only problem is that you get the same equation at node b.

Where does the second equation come from?

Use something like this: \( V_a = V_b + V_{S2} \)

Similar Circuit, but no \( V_{S1} \).

If the ground is already at the bottom, use the same method as above.

If you can choose your ground, you can make life a little simpler.
**Basic electrical quantities**

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge, actually moves</td>
<td>Q</td>
</tr>
<tr>
<td>Current, like fluid flow</td>
<td>I = ( \frac{Q}{s} )</td>
</tr>
<tr>
<td>Voltage, like pressure</td>
<td>V</td>
</tr>
<tr>
<td>Resistance</td>
<td>R = ( \frac{V}{I} )</td>
</tr>
<tr>
<td>Conductance</td>
<td>G = ( \frac{1}{R} )</td>
</tr>
<tr>
<td>Power energy/time</td>
<td>P = V \cdot I</td>
</tr>
</tbody>
</table>

**Schematic symbols**

- **Battery**
- **Voltage sources**
- **Node**
- **Ideal wire**
- **Ground**, V = 0
- **Variable potentiometer**
- **Resistors**
- **Light bulb**
- **LED**
- **Diode**
- **LED**
- **Switch**
- **Transistor**
- **Transformer**
- **Current source**
- **Meters**

**KCL, Kirchhoff’s Current Law**

\[ I_{in} = I_{out} \text{ of any point, part, or section} \]

**KVL, Kirchhoff’s Voltage Law**

\[ V_{gains} = V_{drops} \text{ around any loop} \]

**Node**

All points connected by wire, all at same voltage (potential)

**Ohm’s law** (resistors)

\[ V = I \cdot R \]

**Power**

\[ P_{IN} = P_{OUT} \text{ for resistor circuits} \]

\[ P = V \cdot I \text{ for everything} \]

\[ P = I^2 \cdot R \text{ for resistors} \]

**Resistors**

- **series:** \( R_{eq} = R_1 + R_2 + R_3 + \ldots \)
- **parallel:** \( R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots} \)

**Voltage divider**

\[ V_{Rn} = \frac{V_{total}}{R_n} \]

\[ R_1 + R_2 + R_3 + \ldots \]

**current divider**

\[ I_{Rn} = \frac{I_{total}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots} \]

**DC Notes**

1. Combine resistors into equivalents where possible.
2. Use superposition if there are multiple sources and you know all the resistors.
3. Use KCL, KVL, & Ohm’s laws to write multiple equations and solve.

**Maximum power transfer:**

\[ R_L = R_{Th} \]

Load = Thevenin's
**Thévenin equivalent**

To calculate a circuit’s Thévenin equivalent:

1) Remove the load and calculate the open-circuit voltage where the load used to be. This is the Thévenin voltage ($V_{Th}$).

2) Zero all the sources. (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)

3) Compute the total resistance between the load terminals. (DO NOT include the load in this resistance.) This is the Thévenin source resistance ($R_{Th}$).

4) Draw the Thévenin equivalent circuit and add your values.

**Norton equivalent**

To calculate a circuit’s Norton equivalent:

1) Replace the load with a short (a wire) and calculate the short-circuit current in this wire. This is the Norton current ($I_N$). Remove the short.

2) Zero all the sources. (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)

3) Compute the total resistance between the load terminals. (DO NOT include the load in this resistance.) This is the Norton source resistance ($R_N$). (Exactly the same as the Thévenin source resistance ($R_{Th}$)).

4) Draw the Norton equivalent circuit and add your values.

**OR (the more common way)...**

1) Find the Thévenin equivalent circuit.

2) Convert to Norton circuit, $R_N = R_{Th}$ and $I_N = V_{Th}/R_{Th}$.

**Nodal Analysis**

1) If the circuit doesn’t already have a ground, label one node as ground (zero voltage). If the ground can be defined as one side of a voltage source, that will make the following steps easier.

2) Label unknown node voltages as $V_a$, $V_b$, ... and label the current in each resistor as $I_1$, $I_2$, ....

3) Write Kirchoff’s current equations for each unknown node.

4) Replace the currents in your KCL equations with expressions like the one below.

$$ I_1 = \frac{V_a - V_b}{R_1} $$

5) Solve the multiple equations for the multiple unknown voltages.

**Superposition**

For circuits with **more than 1 source**.

1) Zero all but one source. (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)

2) Compute your wanted voltage or current due to the remaining source. Careful, some may be negative.

3) Repeat the first two steps for all the sources.

4) Sum all the contributions from all the sources to find the actual voltage or current. **Watch your signs!**
AC stands for **Alternating Current** as opposed to **DC, Direct Current**. AC refers to voltages and currents that change with time, usually the voltage is + sometimes and - at other times. This results in currents with go one direction when the voltage is + and the reverse direction when the voltage is -.

AC is important for two reasons.
Power is created and distributed as AC. Signals are AC.

**AC Power**

Power is generated by rotating magnetic fields.
This naturally produces sinusoidal AC waveforms.

It is easier to make AC motors than DC motors.

**AC Power allows use of transformers to reduce line losses**

Transformers work with AC, but not DC. Transformers can be used to raise or lower AC voltages (with an opposite change of current). This can be very useful in power distribution systems. Power is voltage times current. You can distribute the same amount of power with high voltage and low current as you can with low voltage and high current. However, the lower the current, the lower the $I^2R$ losses in the wires (all real wires have some resistance). So you’d like to distribute power at the highest possible voltage. Transformers allow you to do this with AC, but won't work with DC.

Ideal: power in = power out

Ideal transformation of voltage and current:

$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

**Example:**

**Without transformers**

- **Power Plant**
  - 120-V
  - $V_s$ = 12kV
  - $I_L$ = 100-A
  - Wire loss: $P_W = I_L^2 \cdot R_w = 20 \cdot kW$

**With transformers**

- **Power Plant**
  - 12kV
  - $V_w$ = 12kV
  - $I_w$ = 1-A
  - $R_w$ = 1-$\Omega$

In this example, the power lost in the transmission lines is only 1/10,000th what it is without transformers.

That’s why they raise the voltage in transmission lines to the point where they crackle and buzz. That crackle is the sound of the losses into the surrounding air and can become significant if the voltage is too high.
Signals
A time-varying voltage or current that carries information. If it varies in time, then it has an AC component.

In some unpredictable fashion
DC is not a signal. Neither is a pure sine wave. If you can predict it, what information can it provide?
Neither DC nor pure sine wave have any "bandwidth". In fact, no periodic waveform is a signal & no periodic waveform has bandwidth. You need bandwidth to transmit information.

Signal sources
Microphone
Audio
Camera
Video
Thermistor or other thermal sensor
Temperature
Potentiometer
Position
LVDT (Linear Variable Differential Transformer)
LVDT
Light sensor
Computer
switch
etc...

A transducer is a device which transforms one form of energy to another. Some sensors are transducers, many are not

Most often a signal comes from some other system.

Periodic waveforms: Waveshape repeats
T = Period = repeat time
f = frequency, cycles / second
ω = radian frequency, radians/sec
A = amplitude
DC = average

Sinusoidal AC
y(t) = A \cdot \cos(\omega t + \phi)
voltage: v(t) = V_p \cdot \cos(\omega t + \phi)
current: i(t) = I_p \cdot \cos(\omega t + \phi)
Phase: \phi = \frac{\Delta t}{T} \cdot 360\text{-deg} \quad \text{or:} \quad \phi = \frac{\Delta t}{T} \cdot 2\pi\text{-rad}

Other common periodic waveforms
All but the square and triangle waves have a DC component as well as AC.
Nodal Analysis
1. a) If you select the bottom node as ground, how many unknown node voltages remain? (Assume \( V_S \) is a known quantity.) How many simultaneous equations would you need to solve to analyze this circuit?
   b) Use nodal analysis to find all the necessary simultaneous equations.

2. a) Use nodal analysis to find all the node voltages.
   b) Your node voltages will depend on your selection of a reference node (ground) as well as your arbitrary node labels, so the grader won't look at these specifically. Use your node voltages to find the potential (voltage) across each resistor. Report the magnitude and polarity of each.

3. Use Nodal analysis to find \( V_a \) and use \( V_a \) to find \( I_3 \).

4. Use Nodal analysis to solve following problems: Each problem asks for at least 1 voltage and a current. Use the voltage(s) to find the current.
   a) \( R_1 := 40\,\Omega \) \( V_{a} = ? \) \( I_{R1} = ? \)
   b) \( V_{a} = ? \) \( R_3 := 3\,\kappa\Omega \)

Don't forget your folder number.

Answers
1. a) 3, 3  
   b) \( \frac{V_a}{R_1} + \frac{V_b}{R_2} + \frac{V_c}{R_3} - \frac{V_a}{R_2} - \frac{V_c}{R_3} = 0\,\text{A} \)
2. a) Answer will depend on your choice of ground, so check your answers to part b to see if you did part a right.
   b) 3.077-V, + bottom, 2.308-V, + left, 1.923-V, + top, 0.385-V, + bottom, 2.692-V, + right
3. 7-V, 7-mA

You may not get this homework back before the 1st exam. Photocopy it if you want to be sure to have it.
1. For each of the following sinusoidal waves, find:
   1) peak-to-peak voltage or current, \( V_{pp} \) or \( I_{pp} \)
   2) amplitude, \( A \), \( V_p \), or \( I_p \)
   3) period, \( T \)
   4) frequency \( f \) in cycles/sec or Hz
   5) an expression for \( v(t) \) or \( i(t) \) in terms of \( A \cos(\omega t + \phi) \)
      the frequency \( \omega \) is in radians/sec
      the phase angle \( \phi \) is in rad/sec or degrees

2. For each of the following waveforms, find:
   1) Peak-to-peak voltage or current, \( V_{pp} \) or \( I_{pp} \)
   2) Average, \( (V_{DC}, I_{DC}, V_{ave}, \text{or} \ I_{ave}) \)
   3) Period, \( T \)
   4) Frequency \( f \) in cycles/sec or Hz

3. For problem 2a above, write a full expression for \( v(t) \) in terms of \( v(t) = A \cos(\omega t + \phi) + V_{DC} \)

Answers
1. a) 0.2-V 0.1-V 12-ms 83.3-Hz 0.1-V-cos(523.6-t)
   b) 24-V 12-V 0.018-ms 55.6-kHz
   c) 16-mA 8-mA 0.3-ms 3333-Hz
      8-mA-cos(20940-t + 150-deg)
2. a) 12-V 3-V 6-ms 167-Hz
   b) 12-V 6-V 4-ms 250-Hz
   c) 250-mA 25-mA 0.6-ms 1.667-kHz
3. \( v(t) = 6-V \cdot \cos(1047-t - 90-deg) + 3-V \)