# ECE 2210 / 00 Lecture 8 Notes Basic AC

AC stands for **A**lternating **C**urrent as opposed to DC, **D**irect **C**urrent. AC refers to voltages and currents that change with time, usually the voltage is + sometimes and - at other times. This results in currents with go one direction when the voltage is + and the reverse direction when the voltage is -.

AC is important for two reasons.

Power is created and distributed as AC. Signals are AC.

### **AC Power**

Power is generated by rotating magnetic fields. This naturally produces sinusoidal AC waveforms.

It is easier to make AC motors than DC motors.



**AC Power allows use of transformers to reduce line losses** primary pron-core transformer promary



and current: 
$$
\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}
$$

Ideal: power in  $=$  power out Ideal transformation of voltage and current in  $=$  power out

won't work with DC.

### Example:



Transformers work with AC, but not DC. Transformers can be used to raise or lower AC voltages (with an opposite change of current). This can be very useful in power distribution systems. Power is voltage times current. You can distribute the same amount of power with high voltage and low current as you can with low voltage and high current. However, the lower the current, the

lower the I2R loses in the wires (all real wires have some resistance). So you'd like to distribute power at the highest possible voltage. Transformers allow you to do this with AC, but









That's why they raise the voltage in transmission lines to the point where they crackle and buzz. That crackle is the sound of the losses into the surrounding air and can become significant if the voltage is too high.

Wire loss:  $P_W = I_w^2 \cdot 2 \cdot R_w = 2 \cdot W$ 

# **ECE 2210 / 00 Lecture Notes Basic AC p2 Signals**

A time-varying voltage or current that carriers information. If it varies in time, then it has an AC component.

 $\blacksquare$ 

Audio, video, position, temperature, digital data, etc...

In some unpredictable fashion

DC is not a signal, Neither is a pure sine wave. If you can predict it, what information can it provide? Neither DC nor pure sine wave have any "bandwidth". In fact, no periodic waveform is a signal & no periodic waveform has bandwidth. You need bandwidth to transmit information.

| |

Microphone Camera Video Thermistor or other thermal sensor Temperature Potentiometer **Position** LVDT (Linear Variable Differential Transformer) Position Most often a signal comes from some Light sensor Light sensor<br>Computer system. **Computer** switch etc...

**Signal sources**<br>
Microphone **A transducer is a device which**<br>
Audio **A** transducer is a device which transforms one form of energy to another. Some sensors are transducers, many are not

### **Periodic waveforms**: Waveshape repeats

 $T = Period = repeat time$ 1  $=\frac{\omega}{\omega}$  $f = frequency$ , cycles / second amplitude  $2 \cdot \pi$ T  $V_{ave} = V_{DC}$  $ω =$  radian frequency, radians/sec  $ω = 2 ⋅ π f$ time  $A =$ amplitude period, T DC = average Sinusoidal AC Phase: lead lag  $+\Phi$ — Ф  $y(t) = A \cdot cos(\omega \cdot t + \phi)$  $cos(\omega t)$  $\Delta t \rightarrow$ no phase and voltage:  $v(t) = V_p \cdot cos(\omega \cdot t + \phi)$ current:  $i(t) = I_p \cdot cos(\omega \cdot t + \phi)$ Phase:  $\phi = -\frac{\Delta t}{2} \cdot 360$ 360 deg or:  $\phi = -\frac{\Delta t}{2} \cdot 2 \cdot \pi$ 2 · π · rad T T

### **Other common periodic waveforms**



time









All but the square and triangle waves have a DC component as well as AC.

**∧Volts** 

## **ECE 2210 / 00 Capacitor Lecture Notes** in the control of  $\frac{1}{2}$  i<sub>C</sub>



Or..

Or..

For drawings of capacitors and info about tolerances, see Ch.3 of textbook.

r<sup>\*</sup>

Basic equations you should know:

**Sinusoids**

$$
i_C = C \frac{d}{dt} v_C
$$

Capacitors are the only "backwards" components.

$$
\frac{Q}{V}
$$
\n
$$
V_C = \frac{1}{C} \int_{-\infty}^{1} i_C dt
$$
\n
$$
C \frac{d}{dt} V_C
$$
\n
$$
V_C = \frac{1}{C} \int_{0}^{t} i_C dt + V_C(0)
$$
\n
$$
V_C = \frac{1}{C} \int_{0}^{t} i_C dt + V_C(0)
$$
\n
$$
V_C = \frac{1}{C} \int_{t}^{t} i_C dt
$$

Energy stored in electric field:  $W_C = \frac{1}{2} \cdot C$  $\frac{1}{2}$ ·C·V  $C^2$ 

 $C = \frac{Q}{A}$ 

Capacitor voltage **cannot** change instantaneously

**parallel:**  $C_{eq} = C_1 + C_2 + C_3$ 

 $+ \ldots$  **series:**  $C_{eq} = \frac{1}{1 - 1}$ 1 1 1  $\overline{c_3}$  +  $\cdots$  $\rm{c_{1}}$  $\rm{c_{\,2}}$  $\begin{picture}(120,115) \put(0,0){\line(1,0){15}} \put(15,0){\line(1,0){15}} \put(15,0){\line$ 

"long time"



#### **Steady-state or Final conditions**

If a circuit has been connected for "a long time", then it has reached a steady state condition. that means the currents and voltages are no longer changing.

$$
\frac{d}{dt}v_C = 0 \qquad \qquad i_C = C \frac{d}{dt}v_C = 0
$$

no current means it looks like an open

 $R_{1}$ 

 $R_2$   $\frac{2}{3}$ 

### **ECE 2210 / 00 Capacitor / Inductor Lecture Notes p1**

+

 $V_C(\infty) = V_S$ 

 $R_1 + R_2$ 

 $R_{2}$ 

# **ECE 2210 / 00 Capacitor / Inductor Lecture Notes p2 Example**

The voltage across a  $0.5 \mu$ F capacitor is shown below. Make an accurate drawing of the capacitor current. Label the y-axis of your graph (I've already done the time-axis).

The accuracy of your plot at 0, 2, 6, and 8 ms is important, so calculate those values and plot or label them carefully. Between those points your plot must simply be the correct shape.



## **ECE 2210 / 00 Inductor Lecture Notes**



Basic equations you should know:

$$
v_L = L \frac{d}{dt} i_L
$$

∆t  $= 0.5 \cdot \mu F \cdot \frac{-4 \cdot V}{4} =$  $\frac{1}{2 \cdot ms} = -1$  •mA

2ms - 6ms: Initial slope is zero and the final slope is positive, so the current must be a triangle that starts at zero and ends at some height.

$$
\Delta v_C(t) = \frac{1}{C} \int_0^t i_C(t) dt
$$
  
8-V =  $\frac{1}{C} \left( \frac{4 \cdot ms \cdot height}{2} \right)$   
height = 8-V  $\frac{C \cdot 2}{4 \cdot ms}$  = 2•mA

6ms - 8ms: Slope is zero, so the current must be zero.

Electrical equivalent: 
$$
i \underset{\forall \phi}{\bigcup_{\forall \phi} \phi} \underset{\forall L}{\overset{\longleftarrow}{\bigcup_{\forall L}}}
$$

 $L = \mu_0 \cdot N^2 \cdot K$ 

µ is the permeability of the inductor core

K is a constant which depends on the inductor geometry N is the number of turns of wire

 $i_{L} = \frac{1}{t}$ . L d t  $v_L$  dt

$$
{}^{i}L
$$
\n
$$
Or... \t iL = \frac{1}{L} \int_{0}^{t} vL dt + iL(0)
$$
\n
$$
Or... \Delta iL = \frac{1}{L} \int_{t_1}^{t_2} vL dt
$$

Energy stored in electric field:  $W_L = \frac{1}{2}L$  $rac{1}{2}$ ·L·I<sub>L</sub><sup>2</sup>

Inductor current **cannot** change instantaneously

Units: henry  $=$ volt.sec mH =  $10^{-3}$ ·H  $\mu$ H =  $10^{-6}$ ·H

amp **ECE 2210 / 00 Capacitor / Inductor Lecture Notes p2**

## ECE 2210 / 00 Capacitor / Inductor Lecture Notes p3

 $\frac{d}{dt} i$  L = L ω.  $\left(-I_p \sin(\omega \cdot t)\right)$  = L ω. I  $p \cdot \cos(\omega \cdot t + 90 \cdot \text{deg})$ 

 $v_L(t) = L\frac{d}{dt}$ 

 $L_2 + L_3 + \ldots$  **parallel:** 

**Sinusoids**  $i_{L}(t) = I_{p} \cos(\omega \cdot t)$ 

**Resonance** Series resonance

sense, voltage has to present to make current change, so voltage







comes first.

The resonance frequency is calculated the same way for either case:

$$
\omega_o = \frac{1}{\sqrt{LC}} \left( \frac{\text{rad}}{\text{sec}} \right)
$$
 OR..  $\omega_o = \frac{1}{\sqrt{L_{eq} \cdot C_{eq}}}$ 

If you have multiple capacitors or inductors which can be combined.

$$
f_{o} = \frac{\omega_{o}}{2 \cdot \pi} \quad (Hz)
$$

#### **Steady-state of Final conditions**

If a circuit has been connected for "a long time", then it has reached a steady state condition. that means the currents and voltages are no longer changing.

$$
\frac{d}{dt}i_L = 0 \qquad v_L = L \frac{d}{dt}i_L =
$$

no voltage means it looks like a short

#### **Examples**

#### **Ex 1**

Find the resonant frequency (or frequencies) of the circuit shown (in cycles/sec or Hz).





**ECE 2210 / 00 Capacitor / Inductor Lecture Notes p3**



The current through a 0.3mH inductor is shown below. Make an accurate drawing of the inductor voltage. Make reasonable assumptions where necessary. Label your graph.



The curve is  $2^{nd}$  order and ends at  $8\mu s$ 

$$
0 - 2\mu s
$$
: No change in current, so:  $v_{\text{I}} = 0$ 

∆t  $= 0.3 \cdot mH \cdot \frac{0.6 \cdot A}{0.6 \cdot A} =$  $\frac{0.0 \text{ Hz}}{2 \cdot \mu s} = -90 \text{ V}$ 

4µs - 8µs: Initial slope is positive and the final slope is zero, so the voltage must be a triangle that starts at some height and ends at zero.

$$
\Delta i_{L}(t) = \frac{1}{L} \int_{0}^{t} v_{L}(t) dt
$$
  
0.6 A =  $\frac{1}{0.3 \cdot mH} \left\langle \frac{4 \cdot \mu s \cdot \text{height}}{2} \right\rangle$   
height = 0.6 A· $\frac{0.3 \cdot mH \cdot 2}{2} = 90 \cdot V$ 

$$
4 \cdot \mu s
$$

 $8\mu s$  - 10 $\mu s$ : No change in current, so:  $v_L = 0$ 



**Ex 4** The following circuit has been connected as shown for a long time. Find the energy stored in the

capacitor and the inductor.





**Find final conditions** ("steady-state" or "forced" solution) Inductors are shorts Capacitors are opens  $\,$  Solve by DC analysis  $\,$  v  $_{\rm X}$ ( $\scriptstyle \infty )$  or  $\,$  i  $_{\rm X}$ ( $\scriptstyle \infty )$ 



## ECE 2210 / 00 homework # 7 Due: Tue, 2/16/21 a

Answer the following problems on your own paper.

- 
- 1. For each of the following sinusoidal waves, find:<br>
1) peak-to-peak voltage or current,  $V_{pp}$  or  $I_{pp}$ 
	- 2) amplitude, A,  $\rm V_p$ , or  $\rm I_p$
	- 3) period, T
	- 4) frequency f in cycles/sec or Hz
	- 5) an expression for  $v(t)$  or  $i(t)$  in terms of Acos( $\omega t + \phi$ ) the frequency ω is in radians/sec the phase angle  $\phi$  is in rad/sec or degrees



- 2. For each of the following waveforms, find: 2a)
	- 1) Peak-to-peak voltage or current,  $\rm V_{pp}$  or  $\rm I_{pp}$
	- 2) Average,  $(V_{\text{DC}}, I_{\text{DC}}, V_{\text{ave}},$  or  $I_{\text{ave}})$
	- 3) Period, T
	- 4) Frequency f in cycles/sec or Hz





3. For problem 2a above, write a full expression for v(t) in terms of v(t) = Acos( $\omega t + \phi$ ) + V<sub>DC</sub>

#### **Answers**

- 1. a)  $0.2 \cdot V$   $0.1 \cdot V$   $12 \cdot ms$   $83.3 \cdot Hz$   $0.1 \cdot V \cdot cos(523.6 \cdot t)$ 
	- b)  $24\cdot V$  12 $\cdot V$  0.018 $\cdot$ ms 55.6 $\cdot$ kHz  $v(t) = 12 \cdot V \cdot \cos(349100 \cdot t - 90 \cdot \text{deg})$
	- c) 16.mA 8.mA 0.3.ms 3333.Hz  $8·mA·cos(20940·t + 150·deg)$



 $t + 150 \text{ deg}$  3. v(t) = 6.V.cos(  $1047 \text{·t} - 90 \text{ deg}$ ) + 3.V

### **ECE 2210 / 00 homework # 7**

## ECE 2210 / 00 homework # 8 Due: Sat, 2/20/21 a2





2. Each of the following circuits have been connected as shown for a long time. Find the voltage across each capacitor and the energy stored in each.







### **ECE 2210 / 00 homework # 8**

Name: \_You may want to hand in this page with answers to problems 3 & 4.

- 3. The current waveform shown below flows through a  $0.025 \mu F$  capacitor. Make an accurate drawing of the voltage across it. Label your graph. Assume the initial voltage across the capacitor is 0 V.
- 4. The voltage across a  $2 \mu$ F capacitor is shown below. Make an accurate drawing of the capacitor current. Label your graph.



5. The voltage across a 0.68 
$$
\mu
$$
F capacitor is  $v_c = 6 \cdot V \cdot \cos\left(200 \cdot t + \frac{\pi}{2}\right)$  find  $i_c$ .

6. The current through a 0.0047  $\mu$ F capacitor is  $i\text{ }_{c}$  =  $18\cdot \mu$ A $\cdot$ cos $\left(628\cdot t-\frac{\pi}{4}\right)$  $\begin{pmatrix} \frac{\pi}{4} \end{pmatrix}$  find v<sub>c</sub>.

7. A capacitor voltage and current are shown at right. What value is the capacitor?

#### **Answers**

**ECE 2210 / 00 homework # 8**

1. a)  $0.6 \cdot \mu$ F b)  $0.015 \cdot \mu$ F c)  $4.5 \cdot \mu$ F 2. a) 3.3V 0.027.mJ b) 37.5.V 0.33.mJ c) 11.V 0.0411.mJ 5.V 2.75.µJ 3. 1.8 V 0.6 V 2.4 V 4. - 6 mA 12 mA ramp to - 8mA 5. i<sub>c</sub> =  $0.816 \cdot \text{mA} \cdot \text{cos}(200 \cdot t + \pi)$ t +  $\pi$ ) 6. v<sub>c</sub> = 6.1 · V · cos 628 · t -  $\frac{3 \cdot \pi}{4}$ 4 7. 0.25.µF

# ∧ Volts 8  $1ms$  $2ms$ -8  $(mA)$ 2  $2ms$  $1<sub>ms</sub>$

#### Name:

# <u>Due:</u> Tue, 2/23/21<br>ECE 2210 / 00 hw # 9

You will need another paper for your calculations, but you may want to hand this sheet in with your drawings.

1. Find  $L_{eq}$  in each case b)



2. Find the stored energy in each capacitor and/or inductor under steady-state conditions. Note: Treat caps as opens and inductors as shorts to find DC voltages and currents.







3. The current waveform shown below flows through a 2 mH inductor. Make an accurate drawing of the voltage across it. Label your graph.



4. The voltage across a 0.5 mH inductor is shown below. Make an accurate drawing of the inductor current. Label your graph. Assume the initial current is 0 mA.



### **ECE 2210 / 00 homework # 9**

5. The voltage across a  $1.2 \text{ mH}$  inductor is  $= 4 \cdot mV \cdot cos(300 \cdot t)$  find  $i_L$ .

6. The current through a 0.08 mH inductor is 
$$
i_L = 20 \cdot mA \cdot \cos\left(628 \cdot t - \frac{\pi}{4}\right)
$$
 find  $v_L$ .

7. Refer to the circuit shown. Assume that  $V_s$  is a sinusoidal input voltage whose frequency can be adjusted. At some frequency of  $V_s$  this circuit can resonate. At that frequency  $i_C(t)$  = - $i_L(t)$ .  $(i_C(t)$  is 180 degrees out-of-phase with  $i_L(t)$ ).

Show that resonance occurs at this frequency:



8. Find the resonant frequency,  $\rm f_o$  in each case.



#### **Answers**

- 1. 1.2.mH 0.62.mH 2. a) 0.05.mJ b) 1.62.mJ 0.081.mJ 0.09.mJ 0.18.mJ 3. Straight lines between the following points: (0ms,-8mV), (2ms,-8mV), (2ms,0mV), (3ms,0mV), (3ms,16mV), (5ms,16mV), (5ms,0mV), (6ms,0mV), (9ms,-10.67mV), (9ms,0mV), (10ms,0mV)
- 4. Straight lines between the following points: (0ms,0A), (0.2ms,1.2A), (0.6ms,-0.4A), curves until it's flat at (0.76ms, -0.72A), continues to curve up to (1ms, 0A), (1.1ms,0A)
- 5.  $i_L = 11.1 \cdot mA \cdot \cos(300 \cdot t 90 \cdot deg)$  6.  $v_L$

$$
6. \quad v_L = 1 \cdot mV \cdot \cos\left(628 \cdot t + \frac{1}{4} \cdot \pi\right)
$$

7. Assume a sinusoidal voltage, find  $i<sub>C</sub>$  and  $i<sub>L</sub>$  by integration and differentiation, and show that they are equal and opposite at the resonant frequency.

8. a) 17.79.kHz b) 5305.Hz **ECE 2210 / 00 homework # 9**