AC stands for **Alternating Current** as opposed to **Direct Current**. AC refers to voltages and currents that change with time, usually the voltage is + sometimes and - at other times. This results in currents with go one direction when the voltage is + and the reverse direction when the voltage is -.

AC is important for two reasons.
Power is created and distributed as AC. Signals are AC.

**AC Power**
Power is generated by rotating magnetic fields.
This naturally produces sinusoidal AC waveforms.

It is easier to make AC motors than DC motors.

**AC Power allows use of transformers to reduce line losses**
Transformers work with AC, but not DC. Transformers can be used to raise or lower AC voltages (with an opposite change of current). This can be very useful in power distribution systems. Power is voltage times current. You can distribute the same amount of power with high voltage and low current as you can with low voltage and high current. However, the lower the current, the lower the $I^2R$ loses in the wires (all real wires have some resistance). So you'd like to distribute power at the highest possible voltage. Transformers allow you to do this with AC, but won't work with DC.

**Ideal:** power in = power out
**Ideal transformation of voltage and current:**
\[
\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}
\]

**Example:**

**Without transformers**

**With transformers**

In this example, the power lost in the transmission lines is only 1/10,000th what it is without transformers.

That's why they raise the voltage in transmission lines to the point where they crackle and buzz. That crackle is the sound of the losses into the surrounding air and can become significant if the voltage is too high.
Signals
A time-varying voltage or current that carries information. If it varies in time, then it has an AC component.

In some unpredictable fashion
DC is not a signal. Neither is a pure sine wave. If you can predict it, what information can it provide?
Neither DC nor pure sine wave have any "bandwidth". In fact, no periodic waveform is a signal & no periodic waveform has bandwidth. You need bandwidth to transmit information.

Signal sources
Microphone
Camera
Thermistor or other thermal sensor
Potentiometer
LVDT (Linear Variable Differential Transformer)
Light sensor
Computer
switch
e etc...

A transducer is a device which transforms one form of energy to another. Some sensors are transducers, many are not

Most often a signal comes from some other system.

Periodic waveforms: Waveshape repeats
T = Period = repeat time
f = frequency, cycles / second  \( f = \frac{1}{T} = \frac{\omega}{2\pi} \)
\( \omega = \text{radian frequency, radians/sec} \)  \( \omega = 2\pi f \)
A = amplitude
DC = average

Sinusoidal AC
\( y(t) = A\cos(\omega t + \phi) \)

voltage: \( v(t) = V_p\cos(\omega t + \phi) \)
current: \( i(t) = I_p\cos(\omega t + \phi) \)
Phase: \( \phi = \frac{\Delta t}{T} \cdot 360\text{-deg} \) \( \text{or: } \phi = \frac{\Delta t}{T} \cdot 2\pi \text{-rad} \)

Other common periodic waveforms

All but the square and triangle waves have a DC component as well as AC.
Now that we have voltages and currents which can be functions of time, it's time to introduce the capacitor and the inductor.

**Electrical equivalent:**

\[ C = \varepsilon \frac{A}{d} = \frac{Q}{V} = \frac{dq}{dv} \]

**Units:**

- Farad = coul/volt = amp-sec/volt
- \( \mu F = 1 \times 10^{-6} \) farad
- \( pF = 1 \times 10^{-12} \) farad

For drawings of capacitors and info about tolerances, see Ch.3 of textbook.

**Basic equations you should know:**

- \( C = \frac{Q}{V} \)
- \( i_C = C \frac{dv_C}{dt} \)

**Energy stored in electric field:**

\[ W_C = \frac{1}{2} C V_C^2 \]

Capacitor voltage **cannot** change instantaneously

**Parallel:**

\[ C_{eq} = C_1 + C_2 + C_3 + \ldots \]

**Series:**

\[ C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots} \]

Capacitors are the only "backwards" components.

**Sinusoids**

- \( i_C(t) = I_p \cos(\omega t) \)
- \( v_C(t) = \frac{1}{C} \int i_C \, dt = \frac{1}{C} \frac{1}{\omega} I_p \sin(\omega t - 90\deg) \)

Voltage "lags" current, makes sense, current has to flow in first to charge capacitor.

**Steady-state or Final conditions**

If a circuit has been connected for "a long time", then it has reached a steady state condition. That means the currents and voltages are no longer changing.

\[ \frac{dv_C}{dt} = 0 \quad i_C = C \frac{dv_C}{dt} = 0 \]

No current means it looks like an open.
Example

The voltage across a 0.5 \( \mu \)F capacitor is shown below. Make an accurate drawing of the capacitor current. Label the y-axis of your graph (I've already done the time-axis).

The accuracy of your plot at 0, 2, 6, and 8 ms is important, so calculate those values and plot or label them carefully. Between those points your plot must simply be the correct shape.

\[ C = 0.5 \mu \text{F} \]

\[ \text{volts} \]

\[ v_C(t) \]

\[ \text{time (ms)} \]

\[ \text{I}_C \]

\[ \text{mA} \]

\[ \text{time (ms)} \]

1 - 2ms:
\[ i_C = C \frac{\Delta V}{\Delta t} = 0.5 \mu \text{F} \times \frac{-4 \text{V}}{2 \text{ms}} = -1 \text{mA} \]

2ms - 6ms:
Initial slope is zero and the final slope is positive, so the current must be a triangle that starts at zero and ends at some height.

\[ \Delta v_C(t) = \frac{1}{C} \int_0^t i_C(t) \, dt \]

\[ 8 \cdot V = \frac{1}{C} \left( \frac{4 \text{ms} \cdot \text{height}}{2} \right) \]

\[ \text{height} = \frac{8 \cdot V \cdot C \cdot 2}{4 \text{ms}} = 2 \text{mA} \]

6ms - 8ms:
Slope is zero, so the current must be zero.

ECE 2210 / 00 Inductor Lecture Notes

Fluid Model:

\[ L = \mu_0 N^2 K \]

\( \mu \) is the permeability of the inductor core

K is a constant which depends on the inductor geometry

N is the number of turns of wire

Basic equations you should know:

\[ v_L = L \frac{di_L}{dt} \]

Energy stored in electric field:

\[ W_L = \frac{1}{2} L I_L^2 \]

Inductor current cannot change instantaneously

Units: henry \[ \frac{\text{volt-sec}}{\text{amp}} \]

\[ \text{mH} = 10^{-3} \cdot \text{H} \]

\[ \mu \text{H} = 10^{-6} \cdot \text{H} \]
**Series:***

\[ L_{eq} = L_1 + L_2 + L_3 + \ldots \]

**Parallel:***

\[ \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots \]

**Sinusoids***

\[ i_L(t) = I_p \cos(\omega t) \]

\[ v_L(t) = L \frac{di_L}{dt} = L \omega I_p \sin(\omega t) \]

Voltage "leads" current, makes sense, voltage has to present to make current changes, so voltage comes first.

**Resonance***

**Series resonance***

![Series Resonance Diagram]

- Looks like a short at resonance frequency

**Parallel resonance***

![Parallel Resonance Diagram]

- Looks like an open at resonance frequency

The resonance frequency is calculated the same way for either case:

\[ \omega_o = \frac{1}{\sqrt{L/C}} \text{ (rad/sec)} \quad \text{OR..} \quad \omega_o = \frac{1}{\sqrt{L_{eq}/C_{eq}}} \]

If you have multiple capacitors or inductors which can be combined.

\[ f_o = \frac{\omega_o}{2\pi} \text{ (Hz)} \]

**Steady-state of Final conditions***

If a circuit has been connected for "a long time", then it has reached a steady state condition. That means the currents and voltages are no longer changing.

\[ \frac{d}{dt} i_L = 0 \quad v_L = L \frac{di_L}{dt} = 0 \]

No voltage means it looks like a short.

**Examples***

**Ex 1***

Find the resonant frequency (or frequencies) of the circuit shown (in cycles/sec or Hz).

\[ \omega_o = \frac{1}{\sqrt{L_{eq}/C_{eq}}} \]

\[ \omega_o = 11547 \text{ rad/sec} \]

\[ f_o = \frac{\omega_o}{2\pi} = 1838 \text{ Hz} \]
Ex 2

The current through a 0.3mH inductor is shown below. Make an accurate drawing of the inductor voltage. Make reasonable assumptions where necessary. Label your graph.

\[ L := 0.3 \text{mH} \]

The curve is 2nd order and ends at 8\( \mu \)s

- 0 - 2\( \mu \)s: No change in current, so: \( v_L = 0 \)
- 2\( \mu \)s - 4\( \mu \)s: \( v_L = \frac{L}{\Delta t} \frac{\Delta I}{\Delta t} = \frac{0.3 \text{mH}}{2 \mu s} - 0.6 \text{A} \) = -90 \( \text{V} \)

4\( \mu \)s - 8\( \mu \)s: Initial slope is positive and the final slope is zero, so the voltage must be a triangle that starts at some height and ends at zero.

\[ \Delta i_L(t) = \frac{1}{L} \int_0^t v_L(t) \, dt \]

\[ 0.6 \cdot A = \frac{1}{0.3 \text{mH}} \left( -\frac{4 \mu \text{s} \cdot \text{height}}{2} \right) \]

Height = 0.6 \cdot A \cdot 0.3 \text{mH} \cdot 2 \mu \text{s} = 90 \text{V} \]

8\( \mu \)s - 10\( \mu \)s: No change in current, so: \( v_L = 0 \)

Ex 3

Given a voltage, find the current, \( L := 4 \cdot \text{mH} \)

\[ \Delta i_L(t) = \frac{1}{L} \begin{cases} 
2\mu s & \text{20-V dt = 5 mA} \\
1\mu s & \text{-10-V dt + 5 mA = -5 mA} \\
8\mu s & \text{4-\mu s} \\
10\mu s & V(t) dt + 5 mA \\
8\mu s & 
\end{cases} \]

\[ = \frac{1}{L} \begin{cases} 
20 \cdot \text{V} \cdot 2 \mu \text{s} = 5 \text{mA} \\
0 \text{mA} \end{cases} \text{ etc...} \]

Ex 4

The following circuit has been connected as shown for a long time. Find the energy stored in the capacitor and the inductor.

Redraw:

\[ E_C := \frac{1}{2} C \cdot V_C^2 \]

\( W_C = 14.58 \text{ mJ} \)
Capacitors

$$C = \frac{Q}{V} \text{ farad} = \frac{\text{coul}}{\text{volt}} = \frac{\text{amp} \cdot \text{sec}}{\text{volt}} \quad V_C = \frac{1}{C} \int_0^t i_C \, dt + v_C(0) \quad i_C = C \frac{dv_C}{dt}$$

Energy stored in electric field: $W_C = \frac{1}{2} C \cdot V_C^2$

$$\text{parallel: } C_{eq} = C_1 + C_2 + C_3 + \ldots$$

$$\text{series: } C_{eq} = \frac{1}{C_1 + \frac{1}{C_2} + \frac{1}{C_3} + \ldots}$$

Steady-state sinusoids:

Impedance: $Z_C = \frac{1}{j \omega C} = -\frac{j}{\omega C}$

Capacitor voltage cannot change instantaneously

Inductors

$$\text{henry} = \frac{\text{volt} \cdot \text{sec}}{\text{amp}} \quad i_L = \frac{1}{L} \int_{-\infty}^t v_L \, dt = \frac{1}{L} \int_0^t v_L \, dt + i_L(0) \quad v_L = L \frac{di_L}{dt}$$

Energy stored in magnetic field: $W_L = \frac{1}{2} L I_L^2$

$$\text{series: } L_{eq} = L_1 + L_2 + L_3 + \ldots$$

$$\text{parallel: } L_{eq} = \frac{1}{L_1 + \frac{1}{L_2} + \frac{1}{L_3} + \ldots}$$

Steady-state sinusoids:

Impedance: $Z_L = j \omega L$ Current lags voltage by 90 deg

Inductor current cannot change instantaneously

RC and RL first-order transient circuits

For all first order transients: $v_X(t) = v_X(\infty) + \left[v_X(0) - v_X(\infty)\right] e^{-\frac{t}{\tau}}$

$$v_C(t) = v_C(\infty) + \left[v_C(0) - v_C(\infty)\right] e^{-\frac{t}{RC}}$$

$$i_L(t) = i_L(\infty) + \left[i_L(0) - i_L(\infty)\right] e^{-\frac{t}{L/R}}$$

Find initial Conditions ($v_C$ and/or $i_L$)

Find conditions just before time $t = 0$, $v_C(0-)$ and $i_L(0-)$. These will be the same just after time $t = 0$, $v_C(0+)$ and $i_L(0+)$ and will be your initial conditions. (If initial conditions are zero: Capacitors are shorts, Inductors are opens.)

Use normal circuit analysis to find your desired variable: $v_X(\infty)$ or $i_X(\infty)$

Find final conditions ("steady-state" or "forced" solution)

Inductors are shorts Capacitors are opens Solve by DC analysis $v_X(\infty)$ or $i_X(\infty)$

RC Time constant $= \tau = RC$

RL Time constant $= \tau = \frac{L}{R}$

$$e^{-1} = 0.368 \quad 1 - e^{-1} = 0.632$$
Answer the following problems on your own paper.

1. For each of the following sinusoidal waves, find:
   1) peak-to-peak voltage or current, \( V_{pp} \) or \( I_{pp} \)
   2) amplitude, \( A \), \( V_p \), or \( I_p \)
   3) period, \( T \)
   4) frequency \( f \) in cycles/sec or Hz
   5) an expression for \( V(t) \) or \( I(t) \) in terms of \( A \cos(\omega t + \phi) \)
   - the frequency \( \omega \) is in radians/sec
   - the phase angle \( \phi \) is in rad/sec or degrees

2. For each of the following waveforms, find:
   1) Peak-to-peak voltage or current, \( V_{pp} \) or \( I_{pp} \)
   2) Average, \( (V_{DC}, I_{DC}, V_{ave}, \text{ or } I_{ave}) \)
   3) Period, \( T \)
   4) Frequency \( f \) in cycles/sec or Hz

3. For problem 2a above, write a full expression for \( V(t) \) in terms of \( V(t) = A \cos(\omega t + \phi) + V_{DC} \)

**Answers**

1. a) 0.2-V 0.1-V 12-ms 83.3-Hz 0.1-V-cos(523.6-t)
   b) 24-V 12-V 0.018-ms 55.6-kHz
   \( v(t) := 12 \cdot \cos(349100 \cdot t - 90 \text{-deg}) \)
   c) 16-mA 8-mA 0.3-ms 3333-Hz
   8-mA-cos(20940-t + 150-deg)

2. a) 12-V 3-V 6-ms 167-Hz
   b) 12-V 6-V 4-ms 250-Hz
   c) 250-mA 25-mA 0.6-ms 1.667-kHz

3. \( v(t) := 6 \cdot \cos(1047 \cdot t - 90 \text{-deg}) + 3-V \)
1) Find $C_{eq}$ in each case

a) $C_1 = 0.2 \mu F$, $C_2 = 0.4 \mu F$

2) Each of the following circuits have been connected as shown for a long time. Find the voltage across each capacitor and the energy stored in each.

a) $R_1 = 220 \Omega$, $C = 5 \mu F$, $V_S = 5.5 \text{ V}$, $R_2 = 330 \Omega$

b) $R_1 = 2.4 \text{k} \Omega$, $C = 0.47 \mu F$, $I_S = 25 \text{ mA}$

c) $R_2 = 4.8 \text{k} \Omega$, $C_1 = 0.68 \mu F$, $C_2 = 0.22 \mu F$, $R_1 = 22 \text{k} \Omega$, $R_3 = 10 \text{k} \Omega$, $V_S = 16 \text{ V}$
3. The current waveform shown below flows through a 0.025 \( \mu \)F capacitor. Make an accurate drawing of the voltage across it. Label your graph. Assume the initial voltage across the capacitor is 0 V.

![Current Waveform](image)

4. The voltage across a 2 \( \mu \)F capacitor is shown below. Make an accurate drawing of the capacitor current. Label your graph.

![Voltage Waveform](image)

5. The voltage across a 0.68 \( \mu \)F capacitor is \( v_c = 6 \cdot V \cdot \cos \left( \frac{200 \cdot t \cdot \pi}{2} \right) \) find \( i_c \).

6. The current through a 0.0047 \( \mu \)F capacitor is \( i_c = 18 \cdot \mu A \cdot \cos \left( \frac{628 \cdot t \cdot \pi}{4} \right) \) find \( v_c \).

7. A capacitor voltage and current are shown at right. What value is the capacitor?

![Capacitor Waveform](image)

**Answers**

1. a) 0.6-\( \mu \)F  b) 0.015-\( \mu \)F  c) 4.5-\( \mu \)F

2. a) 3.3V 0.027-mJ  b) 37.5V 0.33-mJ  c) 11-V 0.0411-mJ  5-V 2.75-\( \mu \)J

3. 1.8-V 0.6-V 2.4-V  4. -6-mA 12-mA ramp to -8mA

4. \( i_c = 0.816-\mu A \cdot \cos(200 \cdot t \cdot \pi) \)  6. \( v_c = 6.1-V \cdot \cos \left( \frac{628 \cdot t \cdot 3 \cdot \pi}{4} \right) \)

7. 0.25-\( \mu \)F
You will need another paper for your calculations, but you may want to hand this sheet in with your drawings.

1. Find $L_{eq}$ in each case
   
   - **a)**
     
     \[ L_1 := 2 \text{ mH}, \quad L_2 := 3 \text{ mH} \]
   
   - **b)**
     
     \[ L_1 := 0.22 \text{ mH}, \quad L_2 := 0.4 \text{ mH} \]

2. Find the stored energy in each capacitor and/or inductor under steady-state conditions. Note: Treat caps as opens and inductors as shorts to find DC voltages and currents.
   
   - **a)**
     
     \[ R := 200 \Omega, \quad 10 \text{ V}, \quad L := 40 \text{ mH}, \quad C_1 := 10 \mu \text{F} \]
   
   - **b)**
     
     \[ L_1 := 0.5 \text{ mH}, \quad R_1 := 30 \Omega, \quad V_S := 18 \text{ V}, \quad L_2 := 4 \text{ mH}, \quad R_2 := 60 \Omega, \quad C_2 := 2 \mu \text{F} \]

3. The current waveform shown below flows through a 2 mH inductor. Make an accurate drawing of the voltage across it. Label your graph.
   
   **The curve is 2nd order and starts at 6 ms**

4. The voltage across a 0.5 mH inductor is shown below. Make an accurate drawing of the inductor current. Label your graph. Assume the initial current is 0 mA.
5. The voltage across a 1.2 mH inductor is \( v_L = 4 \text{ mV} \cos(300t) \) find \( i_L \).

6. The current through a 0.08 mH inductor is \( i_L = 20 \text{ mA} \cos(628t - \frac{\pi}{4}) \) find \( v_L \).

7. Refer to the circuit shown. Assume that \( V_s \) is a sinusoidal input voltage whose frequency can be adjusted. At some frequency of \( V_s \) this circuit can resonate. At that frequency \( i_C(t) = -i_L(t) \). \( (i_C(t) \) is 180 degrees out-of-phase with \( i_L(t) \).

Show that resonance occurs at this frequency:

\[
\omega_0 = \frac{1}{\sqrt{L \cdot C}}, \quad f_0 = \frac{1}{2 \pi \sqrt{L \cdot C}}
\]

8. Find the resonant frequency, \( f_0 \) in each case.

a) \( L_1 := 0.5 \text{ mH} \)

\[
\begin{align*}
\text{C}_1 &: = 0.1 \mu \text{F} \\
\text{C}_2 &: = 0.22 \mu \text{F} \\
\text{L}_2 &: = 0.5 \text{ mH}
\end{align*}
\]

b) \( L_1 := 10 \text{ mH} \)

\[
\begin{align*}
\text{C}_1 &: = 0.12 \mu \text{F} \\
\text{C}_2 &: = 0.12 \mu \text{F} \\
\text{L}_2 &: = 5 \text{ mH}
\end{align*}
\]

Answers

1. 1.2 mH 0.62 mH

2. a) 0.05 mJ b) 1.62 mJ 0.081 mJ 0.09 mJ 0.18 mJ

3. Straight lines between the following points: (0 ms, -8 mV), (2 ms, -8 mV), (2 ms, 0 mV), (3 ms, 0 mV), (3 ms, 16 mV), (5 ms, 16 mV), (5 ms, 0 mV), (6 ms, 0 mV), (9 ms, -10.67 mV), (9 ms, 0 mV), (10 ms, 0 mV)

4. Straight lines between the following points: (0 ms, 0 A), (0.2 ms, 1.2 A), (0.6 ms, -0.4 A), curves until it's flat at (0.76 ms, -0.72 A), continues to curve up to (1 ms, 0 A), (1.1 ms, 0 A)

5. \( i_L = 11.1 \text{ mA} \cos(300t - 90 \text{ deg}) \)

6. \( v_L = 1 \text{ mV} \cos\left(628t + \frac{1}{4} \pi\right) \)

7. Assume a sinusoidal voltage, find \( i_C \) and \( i_L \) by integration and differentiation, and show that they are equal and opposite at the resonant frequency.

8. a) 17.79 kHz b) 5305 Hz