

## General Network Analysis

In many cases you have multiple unknowns in a circuit, say the voltages across multiple resistors. Network analysis is a systematic way to generate multiple equations which can be solved to find the multiple unknowns. These equations are based on basic Kirchoff's and Ohm's laws.

**Loop or Mesh Analysis** You may have used these methods in previous classes, particularly in Physics. The best thing to do now is to forget all that. Loop analysis is rarely the easiest way to analyze a circuit and is inherently confusing. Hopefully I've brought you to a stage where you have some intuitive feeling for how currents flow in circuits. I don't want to ruin that now by screwing around with loop currents that don't really exist.

**Nodal analysis** This is a much better method. It's just as powerful, usually easier, and helps you develop your intuitive feeling for how circuits work.

## Nodal Analysis

**Node** = all points connected by wire, all at same voltage (potential)

**Ground:** One node in the circuit which will be our reference node. Ground, by definition, will be the zero voltage node. All other node voltages will be referenced to ground and may be positive or negative. Think of gage pressure in a fluid system. In that case atmospheric pressure is considered zero. If there is no ground in the circuit, define one for yourself. Try to chose a node which is hooked to one side of a voltage source.

**Nodal Voltage:** The voltage of a node referenced to ground. The objective of nodal analysis is to find all the nodal voltages. If you know the voltage at a node then it's a "known" node. Ground is a known node (duh, it's zero). If one end of a known voltage source hooked to ground, then the node on the other end is also known (another duh).

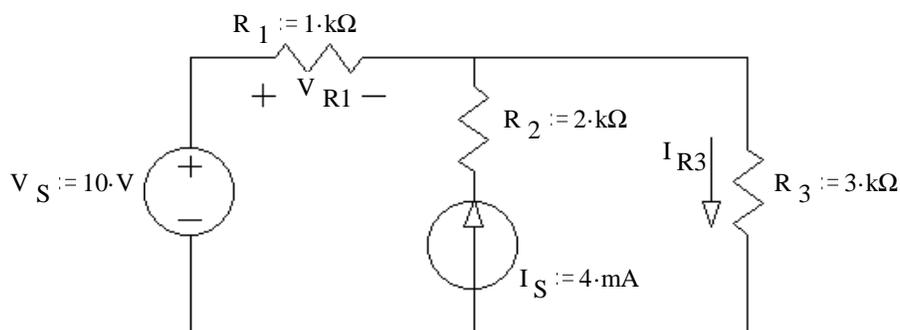
**Method:** Label all the unknown nodes as; "a", "b", "c", etc. Then the unknown nodal voltages become;  $V_a$ ,  $V_b$ ,  $V_c$ , etc. Write a KCL equation for each unknown node, defining currents as necessary. Replace each unknown current with an Ohm's law relationship using the nodal voltages. Now you have just as many equations as unknowns. Solve.

## Nodal Analysis Steps

- 1) If the circuit doesn't already have a ground, label one node as ground (zero voltage).  
If the ground can be defined as one side of a voltage source, that will make the following steps easier.  
Label the remaining node, either with known voltages or with letters, a, b, ....
- 2) Label unknown node voltages as  $V_a$ ,  $V_b$ , ... and label the current in each resistor as  $I_1$ ,  $I_2$ , ....
- 3) Write Kirchoff's current equations for each unknown node.
- 4) Replace the currents in your **KCL** equations with expressions like this.  $\frac{V_a - V_b}{R_1}$  Ohm's law relationship using the nodal voltages.
- 5) Solve the multiple equations for the multiple unknown voltages.

## Nodal Analysis Examples

**Ex 1** Use nodal analysis to find the voltage across  $R_1$  ( $V_{R1}$ ).



- 1) See next page  
Label one node as ground (zero voltage). By choosing the negative side of a voltage source as ground, the upper-left node is known (10V). Label the remaining nodes, either with known voltages or with letters, a, b, ....

2) Label unknown node voltages as  $V_a, V_b, \dots$   
and label the current in each resistor as  $I_1, I_2, \dots$

3) Write Kirchoff's current equations for node a.

$$I_1 + I_S = I_{R3}$$

4) Replace the currents in the **KCL** equations with Ohm's law relationships.

$$\frac{V_S - V_a}{R_1} + I_S = \frac{V_a - 0}{R_3}$$

$$\frac{V_S}{R_1} - \frac{V_a}{R_1} + I_S = \frac{V_a}{R_3}$$

5) Solve:

$$\frac{V_S}{R_1} + I_S = \frac{V_a}{R_3} + \frac{V_a}{R_1}$$

$$\frac{V_S}{R_1} + I_S = V_a \cdot \left( \frac{1}{R_1} + \frac{1}{R_3} \right)$$

$$V_a := \frac{\frac{V_S}{R_1} + I_S}{\left( \frac{1}{R_1} + \frac{1}{R_3} \right)} \quad V_a = 10.5 \cdot V$$

Usually it's easier to put in the numbers at this point

$$\frac{10 \cdot V}{1 \cdot k\Omega} + 4 \cdot mA = \frac{V_a}{3 \cdot k\Omega} + \frac{V_a}{1 \cdot k\Omega}$$

Multiply both sides by a value that will clear the denominators.

$$3 \cdot k\Omega \cdot \left( \frac{10 \cdot V}{1 \cdot k\Omega} + 4 \cdot mA \right) = \left( \frac{V_a}{3 \cdot k\Omega} + \frac{V_a}{1 \cdot k\Omega} \right) \cdot 3 \cdot k\Omega$$

$$30 \cdot V + 3 \cdot k\Omega \cdot 4 \cdot mA = V_a + 3 \cdot V_a$$

$$30 \cdot V + 12 \cdot V = 4 \cdot V_a$$

$$V_a = \frac{42 \cdot V}{4} = 10.5 \cdot V$$

Either way, you still have to find  $V_{R1}$  from  $V_a$ .

$$V_{R1} := V_S - V_a$$

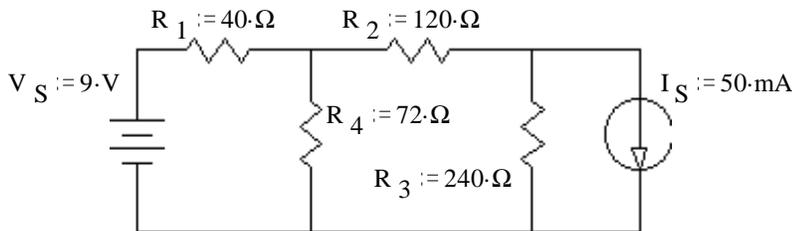
$$V_{R1} = -0.5 \cdot V$$

$V_b$  doesn't matter in this case

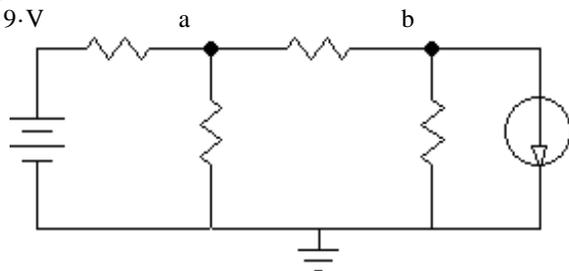
b) Find the current through  $R_3$  ( $I_{R3}$ ).

$$I_{R3} = \frac{V_a}{R_3} = 3.5 \cdot mA$$

**Ex 2** Same circuit used in Thévenin example, where  $R_4$  was  $R_L$ .



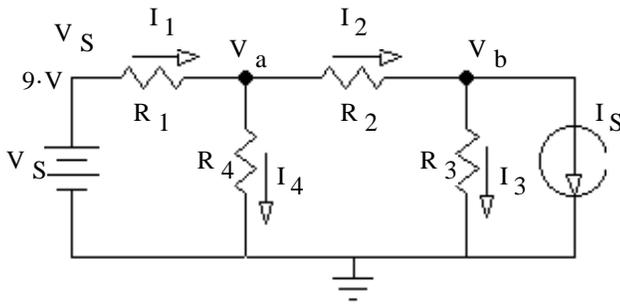
1) Define ground and nodes:



2 unknown nodes ---> will need 2 equations

- 2) Label unknown node voltages as  $V_a, V_b, \dots$  and label the current in each resistor as  $I_1, I_2, \dots$

It doesn't matter if these currents are in the correct directions.



- 3) Write Kirchoff's current equations for each unknown node.

node a  $I_1 = I_2 + I_4$

node b  $I_2 = I_3 + I_S$

- 4) Replace the currents in your **KCL** equations with expressions like this.  $\frac{V_a - V_b}{R_1}$

node a  $I_1 = I_2 + I_4$   
 $\frac{V_S - V_a}{R_1} = \frac{V_a - V_b}{R_2} + \frac{V_a - 0 \cdot V}{R_4}$

node b  $I_2 = I_3 + I_S$   
 $\frac{V_a - V_b}{R_2} = \frac{V_b - 0 \cdot V}{R_3} + I_S$

Now you have just as many equations as unknowns.

- 5) Solve the multiple equations for the multiple unknown voltages. Solve by any method you like:

$$\frac{V_S}{R_1} - \frac{V_a}{R_1} = \frac{V_a}{R_2} - \frac{V_b}{R_2} + \frac{V_a}{R_4}$$

$$\frac{V_a}{R_2} - \frac{V_b}{R_2} = \frac{V_b}{R_3} + I_S$$

$$V_b = \frac{\frac{V_a}{R_2} - I_S}{\frac{1}{R_2} + \frac{1}{R_3}}$$

$$\frac{V_S}{R_1} - \frac{V_a}{R_1} = \frac{V_a}{R_2} - \frac{\frac{V_a}{R_2} - I_S}{R_2 \cdot \left(\frac{1}{R_2} + \frac{1}{R_3}\right)} + \frac{V_a}{R_4}$$

$$V_a = \frac{\left[ \frac{V_S}{R_1} - \frac{1}{R_2 \cdot \left(\frac{1}{R_2} + \frac{1}{R_3}\right)} \cdot I_S \right]}{\left[ \frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_2^2 \cdot \left(\frac{1}{R_2} + \frac{1}{R_3}\right)} + \frac{1}{R_4} \right]}$$

$$V_a = 4.6 \cdot V$$

$$V_b = \frac{\frac{V_a}{R_2} - I_S}{\frac{1}{R_2} + \frac{1}{R_3}}$$

$$V_b = -0.933 \cdot V$$

Or, with numbers

node a  $360 \cdot \Omega \cdot \left( \frac{9 \cdot V - V_a}{40 \cdot \Omega} \right) = \left( \frac{V_a - V_b}{120 \cdot \Omega} + \frac{V_a}{72 \cdot \Omega} \right) \cdot 360 \cdot \Omega$

node b  $240 \cdot \Omega \cdot \frac{V_a - V_b}{120 \cdot \Omega} = \left( \frac{V_b - 0 \cdot V}{240 \cdot \Omega} + 50 \cdot \text{mA} \right) \cdot 240 \cdot \Omega$

$$81 \cdot V - 9 \cdot V_a = 3 \cdot V_a - 3 \cdot V_b + 5 \cdot V_a$$

$$2 \cdot V_a - 2 \cdot V_b = V_b + 48 \cdot \text{mA} \cdot 240 \cdot \Omega$$

$$81 \cdot V - 9 \cdot (1.5 \cdot V_b + 6 \cdot V) = 3 \cdot (1.5 \cdot V_b + 6 \cdot V) - 3 \cdot V_b + 5 \cdot (1.5 \cdot V_b + 6 \cdot V)$$

$$V_a = \frac{2 \cdot V_b + V_b + 12 \cdot V}{2} = 1.5 \cdot V_b + 6 \cdot V$$

<-- substitute for  $V_a$

$$81 \cdot V - 13.5 \cdot V_b - 54 \cdot V = 4.5 \cdot V_b + 18 \cdot V - 3 \cdot V_b + 7.5 \cdot V_b + 30 \cdot V$$

$$81 \cdot V - 54 \cdot V - 18 \cdot V - 30 \cdot V = -21 \cdot V = 4.5 \cdot V_b - 3 \cdot V_b + 7.5 \cdot V_b + 13.5 \cdot V_b = 22.5 \cdot V_b$$

$$V_b = \frac{-21 \cdot V}{22.5} = -0.933 \cdot V$$

$$V_a = 1.5 \cdot V_b + 6 \cdot V = 4.6 \cdot V$$

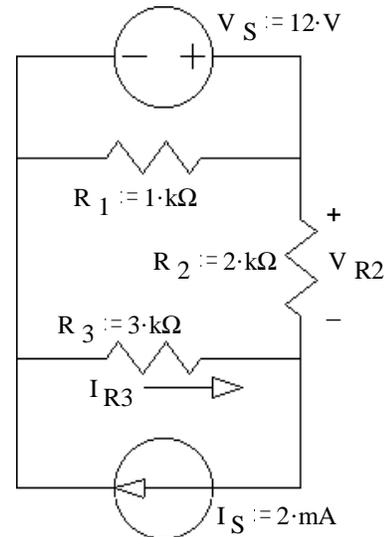
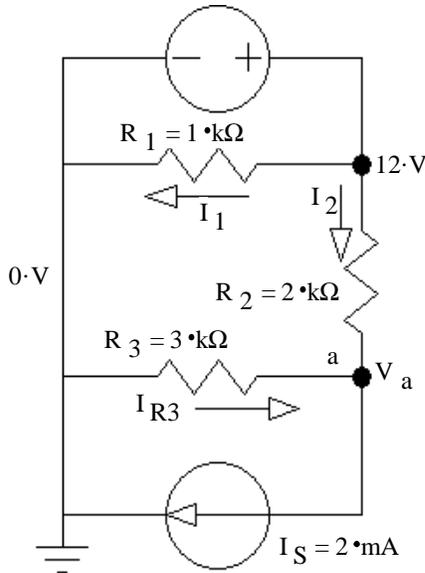
# ECE 2210 Lecture 7 notes p4

## Ex 3 Like Superposition Ex.2

a) Use nodal analysis to find the voltage across  $R_2$  ( $V_{R2}$ ).

You **MUST** show all the steps of nodal analysis work to get credit, including drawing appropriate symbols and labels on the circuit shown.

- 1) Define ground and nodes:
- 2) Label unknown node voltages as  $V_a, V_b, \dots$  and label the current in each resistor as  $I_1, I_2, \dots$



3) Write Kirchoff's current equations for each unknown node.

$$\text{node a: } I_2 + I_{R3} = I_S$$

4) Replace the currents in the **KCL** equations with Ohm's law relationships.

$$\frac{V_S - V_a}{R_2} + \frac{0 - V_a}{R_3} = I_S$$

5) Solve the equation for the unknown voltage.

$$\frac{V_S}{R_2} - \frac{V_a}{R_2} - \frac{V_a}{R_3} = I_S$$

$$\frac{V_S}{R_2} = \frac{V_a}{R_2} + \frac{V_a}{R_3} + I_S$$

$$\frac{V_S}{R_2} - I_S = V_a \cdot \left( \frac{1}{R_2} + \frac{1}{R_3} \right)$$

$$V_a := \frac{\frac{V_S}{R_2} - I_S}{\left( \frac{1}{R_2} + \frac{1}{R_3} \right)} \quad V_a = 4.8 \text{ V}$$

Usually it's easier to put in the numbers at this point

$$\frac{12 \text{ V} - V_a}{2 \text{ k}\Omega} + \frac{0 - V_a}{3 \text{ k}\Omega} = 2 \text{ mA}$$

Multiply both sides by a value that will clear the denominators.

$$6 \text{ k}\Omega \cdot \left( \frac{12 \text{ V} - V_a}{2 \text{ k}\Omega} + \frac{0 - V_a}{3 \text{ k}\Omega} \right) = 2 \text{ mA} \cdot 6 \text{ k}\Omega$$

$$36 \text{ V} - 3 \cdot V_a - 2 \cdot V_a = 12 \text{ V}$$

$$-5 \cdot V_a = -24 \text{ V}$$

$$V_a = \frac{-24 \text{ V}}{-5} = 4.8 \text{ V}$$

Remember, we needed to find the voltage across  $R_2$  ( $V_{R2}$ ).

$$V_{R2} = V_S - V_a = 7.2 \text{ V}$$

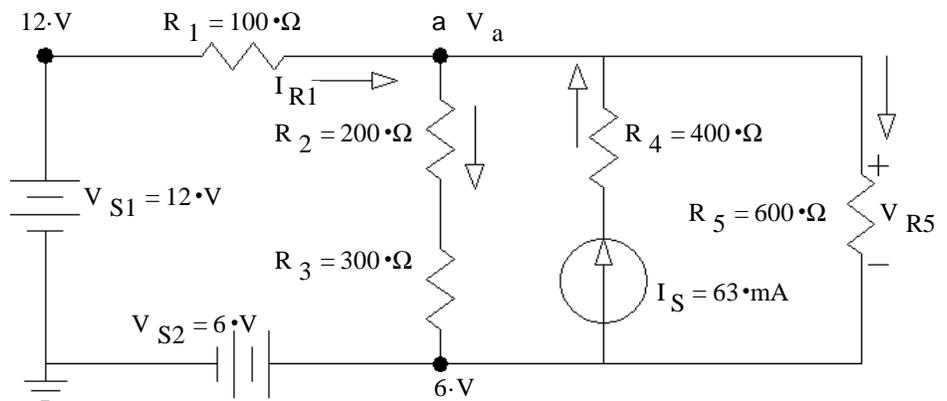
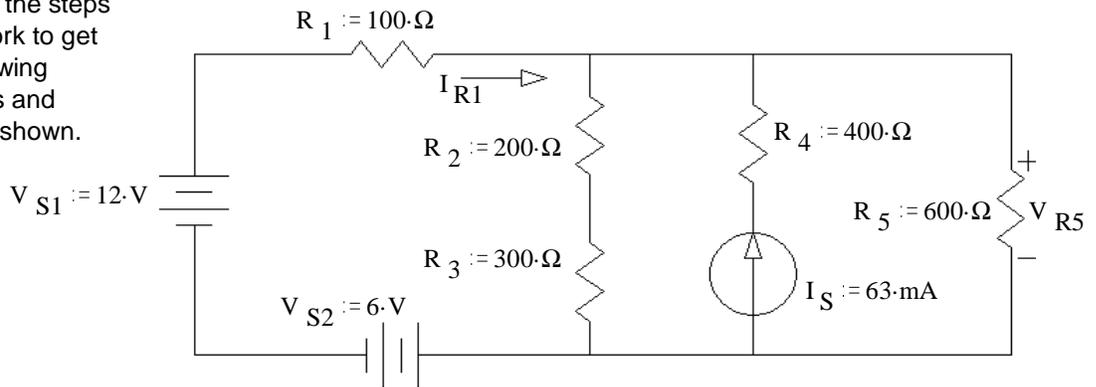
b) Find the current through  $R_3$  ( $I_{R3}$ ).

$$I_{R3} = \frac{0 - V_a}{R_3} = -1.6 \text{ mA} \quad \text{actually flows the other way}$$

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**Ex 4** Use nodal analysis to find the voltage across  $R_5$  ( $V_{R5}$ ) and the current through  $R_1$  ( $I_{R1}$ ). From exam 1, F09

You **MUST** show all the steps of nodal analysis work to get credit, including drawing appropriate symbols and labels on the circuit shown.



node a:

$$\begin{aligned}
 I_{R1} + I_S &= I_R + I_5 \\
 R_3 = 300 \cdot \Omega \cdot \frac{V_{S1} - V_a}{R_1} + I_S &= \frac{V_a - V_{S2}}{R_2 + R_3} + \frac{V_a - V_{S2}}{R_5} \\
 \frac{12 \cdot V}{100 \cdot \Omega} - \frac{V_a}{100 \cdot \Omega} + 63 \cdot \text{mA} &= \frac{V_a}{500 \cdot \Omega} - \frac{6 \cdot V}{500} + \frac{V_a}{600 \cdot \Omega} - \frac{6 \cdot V}{600} && \text{multiply both sides by } 3000\Omega \\
 3000 \cdot \Omega \cdot \left( \frac{12 \cdot V}{100 \cdot \Omega} - \frac{V_a}{100 \cdot \Omega} + 63 \cdot \text{mA} \right) &= \left( \frac{V_a}{500 \cdot \Omega} - \frac{6 \cdot V}{500} + \frac{V_a}{600 \cdot \Omega} - \frac{6 \cdot V}{600} \right) \cdot 3000 \cdot \Omega \\
 360 \cdot V - 30 \cdot V_a + 189 \cdot V &= 6 \cdot V_a - 36 \cdot V + 5 \cdot V_a - 30 \cdot V \\
 360 \cdot V + 189 \cdot V + 36 \cdot V + 30 \cdot V &= 6 \cdot V_a + 5 \cdot V_a + 30 \cdot V_a \\
 615 \cdot V &= 41 \cdot V_a \\
 V_a &:= \frac{615 \cdot V}{41} && V_a = 15 \cdot V \\
 V_{R5} &= V_a - V_{S2} = 9 \cdot V \\
 I_{R1} &= \frac{V_{S1} - V_a}{R_1} = -30 \cdot \text{mA}
 \end{aligned}$$

# ECE 2210 Lecture 7 notes p6

What if one side of a voltage source isn't ground?

$$I_1 + I_{VS2} = I_3$$

$$\frac{V_{S1} - V_a}{R_1} + ? = I_S$$

What do you put in for  $I_{VS2}$ ??

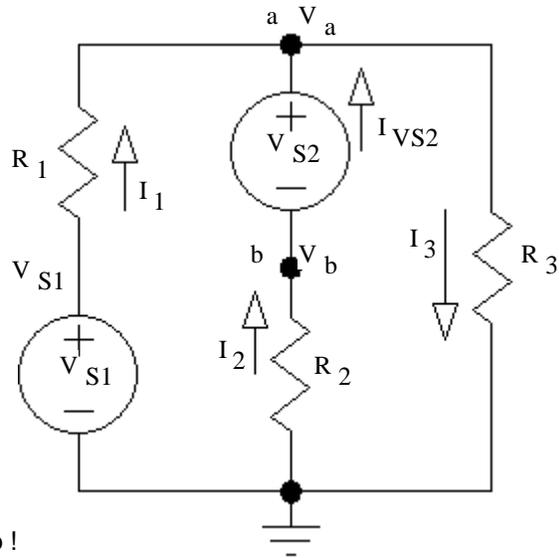
Go to the other side of  $V_{S2}$ .

$$\frac{V_{S1} - V_a}{R_1} + \frac{0 - V_b}{R_2} = I_S$$

Only problem is that you get the same equation at node b!

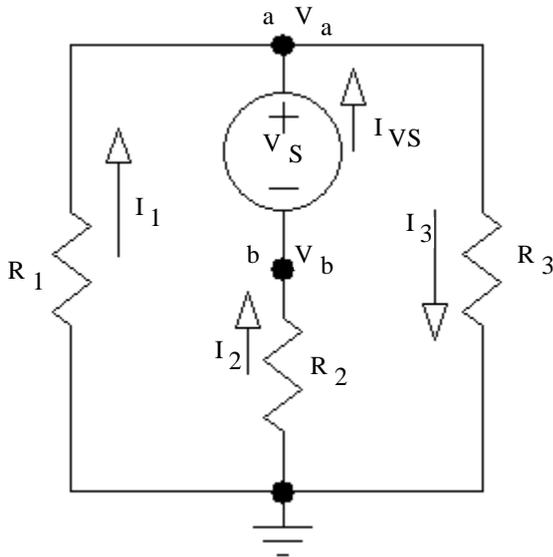
Where does the second equation come from?

Use something like this:  $V_a = V_b + V_{S2}$

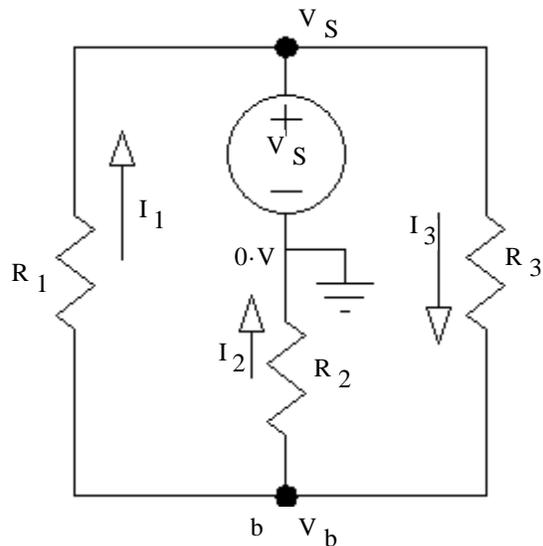


## Similar Circuit, but no $V_{S1}$ .

If the ground is already at the bottom, use the same method as above.



If you can choose your ground, you can make life a little simpler.



# DC Notes

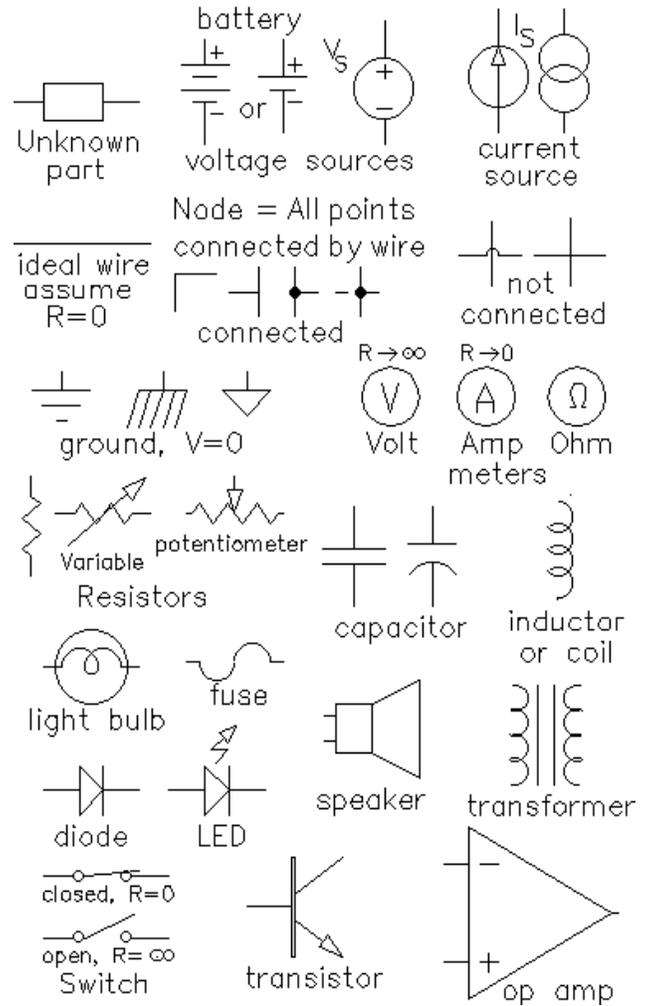
ECE 2210 / 00

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2/8/00, rev 1/28/06

## Basic electrical quantities

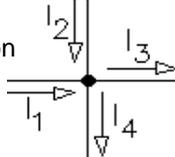
Quantity	Unit
Charge, actually moves $Q$	Coulomb (C)
Current, like fluid flow $I = \frac{Q}{s}$	Amp (A, mA, $\mu$ A,...)
Voltage, like pressure $V$	volt (V, mV, kV,...)
Resistance $R = \frac{V}{I}$	Ohm ( $\Omega$ , k $\Omega$ , M $\Omega$ ,...)
Conductance $G = \frac{1}{R}$	Siemens (S, old unit mho)
Power energy/time $P = V \cdot I$	Watt (W, mW, kW, MW,...)

## Schematic symbols



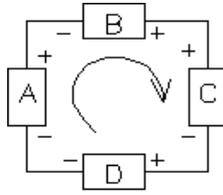
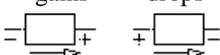
## KCL, Kirchoff's Current Law

$I_{in} = I_{out}$  of any point, part, or section



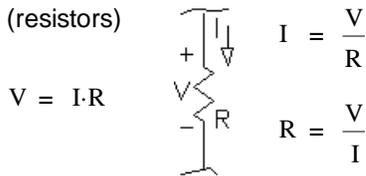
## KVL, Kirchoff's Voltage Law

$V_{gains} = V_{drops}$  around any loop

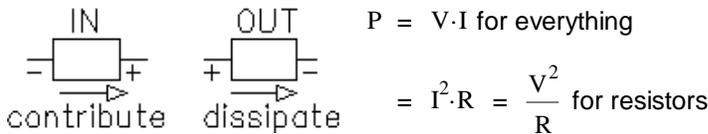


**Node** = all points connected by wire, all at same voltage (potential)

## Ohm's law (resistors)



**Power**  $P_{IN} = P_{OUT}$  for resistor circuits



Maximum power transfer:  $R_L = R_{Th}$   
Load = Thevenin's

## Resistors

**series:**  $R_{eq} = R_1 + R_2 + R_3 + \dots$

Exactly the **same current** through each resistor

## Voltage divider:

$$V_{Rn} = V_{total} \cdot \frac{R_n}{R_1 + R_2 + R_3 + \dots}$$

**parallel:**  $R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$

Exactly the **same voltage** across each resistor

## current divider:

$$I_{Rn} = I_{total} \cdot \frac{\frac{1}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$

Multiple unknowns:

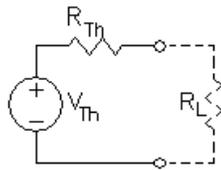
1. Combine resistors into equivalents where possible.
2. Use superposition if there are multiple sources and you know all the resistors.
3. Use KCL, KVL, & Ohm's laws to write multiple equations and solve.

## DC Notes

### Thévenin equivalent

To calculate a circuit's Thévenin equivalent:

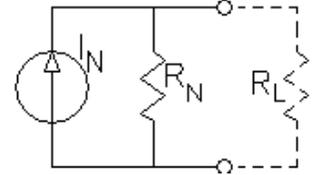
- 1) Remove the load and calculate the open-circuit voltage where the load used to be. This is the Thévenin voltage ( $V_{Th}$ ).
- 2) Zero all the sources. (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
- 3) Compute the total resistance between the load terminals. (DO NOT include the load in this resistance.) This is the Thévenin source resistance ( $R_{Th}$ ).
- 4) Draw the Thévenin equivalent circuit and add your values.



### Norton equivalent

To calculate a circuit's Norton equivalent:

- 1) Replace the load with a short (a wire) and calculate the short-circuit current in this wire. This is the Norton current ( $I_N$ ). Remove the short.
- 2) Zero all the sources. (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
- 3) Compute the total resistance between the load terminals. (DO NOT include the load in this resistance.) This is the Norton source resistance ( $R_N$ ). (Exactly the same as the Thévenin source resistance ( $R_{Th}$ )).
- 4) Draw the Norton equivalent circuit and add your values.

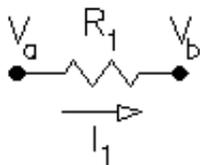


### OR (the more common way)...

- 1) Find the Thévenin equivalent circuit.
- 2) Convert to Norton circuit,  $R_N = R_{Th}$  and  $I_N = V_{Th}/R_{Th}$ .

### Nodal Analysis

- 1) If the circuit doesn't already have a ground, label one node as ground (zero voltage). If the ground can be defined as one side of a voltage source, that will make the following steps easier.
- 2) Label unknown node voltages as  $V_a$ ,  $V_b$ , ... and label the current in each resistor as  $I_1$ ,  $I_2$ , ....
- 3) Write Kirchoff's current equations for each unknown node.
- 4) Replace the currents in your KCL equations with expressions like the one below.



$$I_1 = \frac{V_a - V_b}{R_1}$$

- 5) Solve the multiple equations for the multiple unknown voltages

### Superposition

For circuits with **more than 1 source**.

- 1) Zero all but one source. (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
- 2) Compute your wanted voltage or current due to the remaining source. Careful, some may be negative.
- 3) Repeat the first two steps for all the sources.
- 4) Sum all the contributions from all the sources to find the actual voltage or current. **Watch your signs!**

# ECE 2210 / 00    Lecture 8 Notes    Basic AC

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1/27/03,  
9/9/20

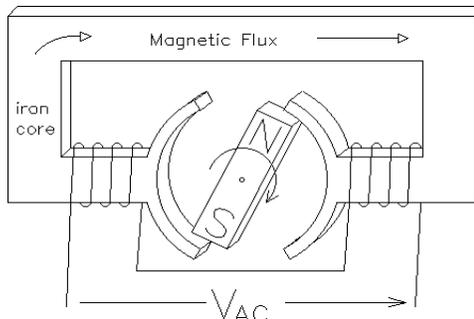
AC stands for **A**lternating **C**urrent as opposed to DC, **D**irect **C**urrent. AC refers to voltages and currents that change with time, usually the voltage is + sometimes and - at other times. This results in currents with go one direction when the voltage is + and the reverse direction when the voltage is -.

AC is important for two reasons.  
Power is created and distributed as AC. Signals are AC.

## AC Power

Power is generated by rotating magnetic fields.  
This naturally produces sinusoidal AC waveforms.

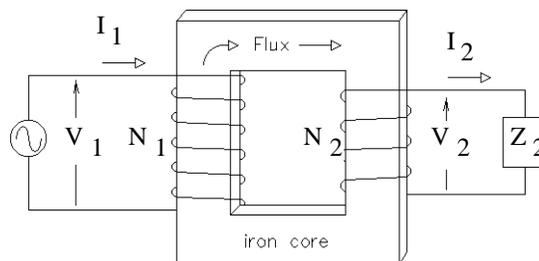
It is easier to make AC motors than DC motors.



## AC Power allows use of transformers to reduce line losses

Transformers work with AC, but not DC. Transformers can be used to raise or lower AC voltages (with an opposite change of current). This can be very useful in power distribution systems. Power is voltage times current. You can distribute the same amount of power with high voltage and low current as you can with low voltage and high current. However, the lower the current, the lower the  $I^2R$  losses in the wires (all real wires have some resistance). So you'd like to distribute power at the highest possible voltage. Transformers allow you to do this with AC, but won't work with DC.

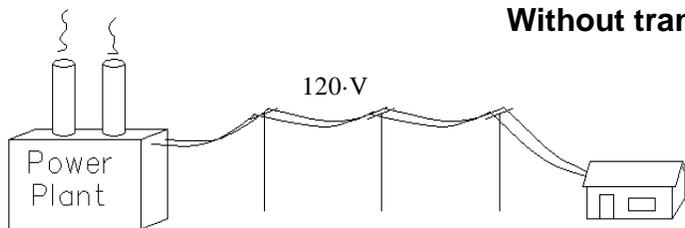
iron-core transformer



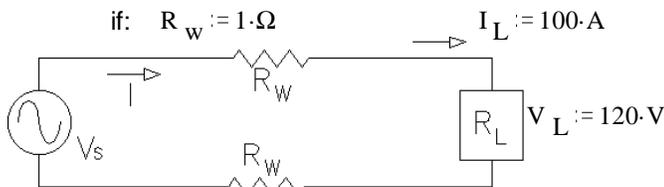
Ideal: power in = power out

$$\text{Ideal transformation of voltage and current: } \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

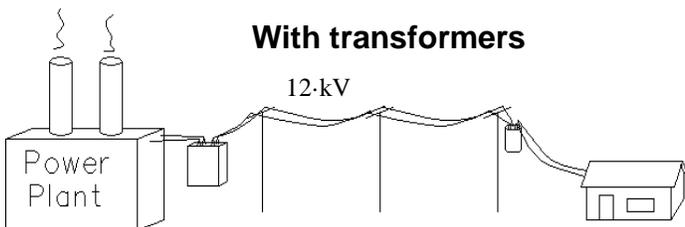
Example:



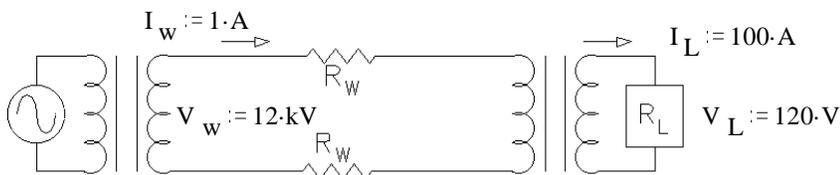
Without transformers



Wire loss:  $P_W = I_L^2 \cdot 2 \cdot R_W = 20 \cdot \text{kW}$



With transformers



Wire loss:  $P_W = I_w^2 \cdot 2 \cdot R_W = 2 \cdot \text{W}$

In this example, the power lost in the transmission lines is only 1/10,000<sup>th</sup> what it is without transformers.

That's why they raise the voltage in transmission lines to the point where they crackle and buzz. That crackle is the sound of the losses into the surrounding air and can become significant if the voltage is too high.

## Signals

A time-varying voltage or current that carries information. If it varies in time, then it has an AC component.



|  
Audio, video, position, temperature, digital data, etc...

In some unpredictable fashion

DC is not a signal, Neither is a pure sine wave. If you can predict it, what information can it provide?

Neither DC nor pure sine wave have any "bandwidth". In fact, no periodic waveform is a signal & no periodic waveform has bandwidth. You need bandwidth to transmit information.

### Signal sources

- |   |             |
|---|-------------|
| Microphone                                      | Audio       |
| Camera  | Video       |
| Thermistor or other thermal sensor              | Temperature |
| Potentiometer                                   | Position    |
| LVDT (Linear Variable Differential Transformer) | Position    |
| Light sensor                                    |             |
| Computer switch                                 |             |
| etc...  |             |

A transducer is a device which transforms one form of energy to another. Some sensors are transducers, many are not

Most often a signal comes from some other system.

### Periodic waveforms: Waveshape repeats

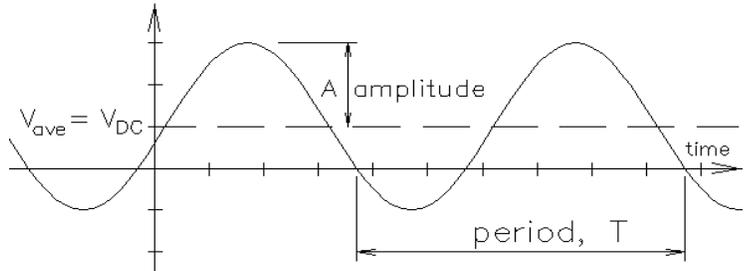
T = Period = repeat time

$$f = \text{frequency, cycles / second} \quad f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$\omega = \text{radian frequency, radians/sec} \quad \omega = 2\pi \cdot f$$

A = amplitude

DC = average



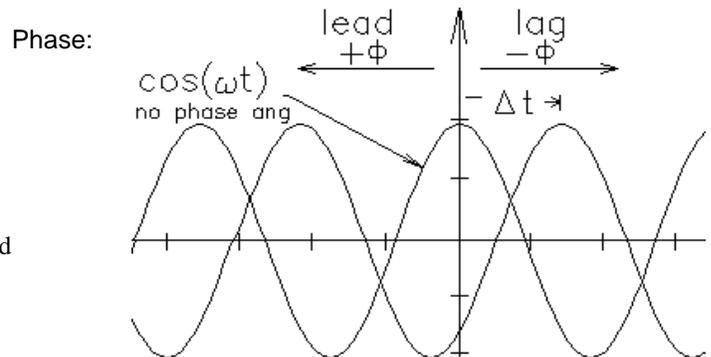
### Sinusoidal AC

$$y(t) = A \cdot \cos(\omega \cdot t + \phi)$$

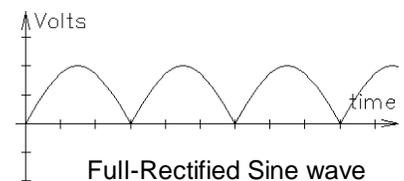
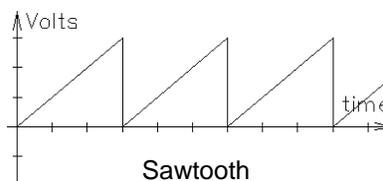
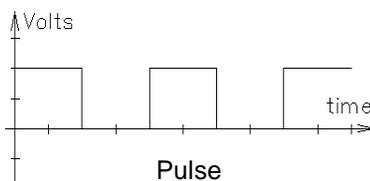
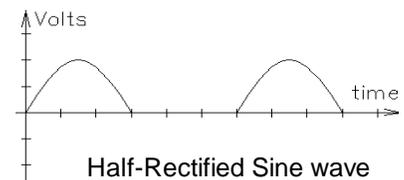
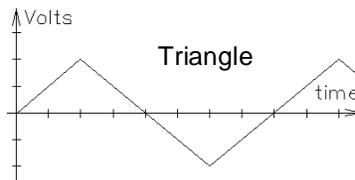
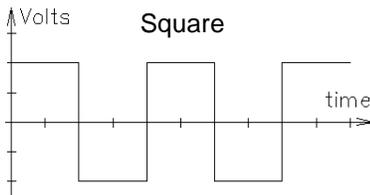
$$\text{voltage: } v(t) = V_p \cdot \cos(\omega \cdot t + \phi)$$

$$\text{current: } i(t) = I_p \cdot \cos(\omega \cdot t + \phi)$$

$$\text{Phase: } \phi = -\frac{\Delta t}{T} \cdot 360\text{-deg} \quad \text{or:} \quad \phi = -\frac{\Delta t}{T} \cdot 2\pi\text{-rad}$$



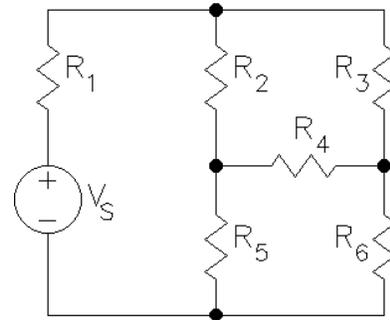
### Other common periodic waveforms



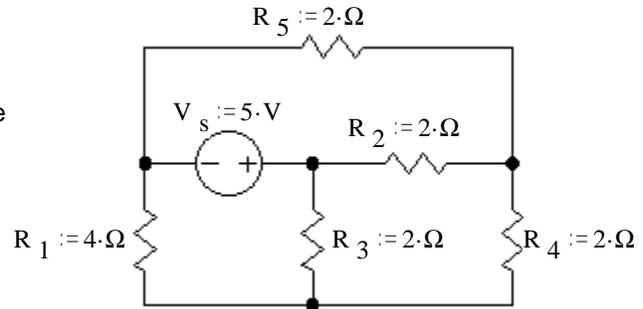
All but the square and triangle waves have a DC component as well as AC.

**Nodal Analysis**

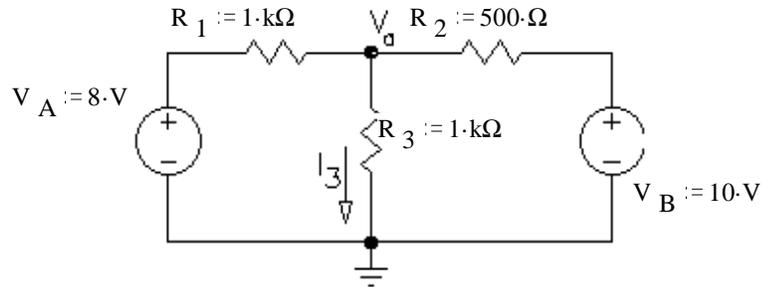
1. a) If you select the bottom node as ground, how many unknown node voltages remain? (Assume  $V_S$  is a known quantity.) How many simultaneous equations would you need to solve to analyze this circuit?
- b) Use nodal analysis to find all the necessary simultaneous equations.



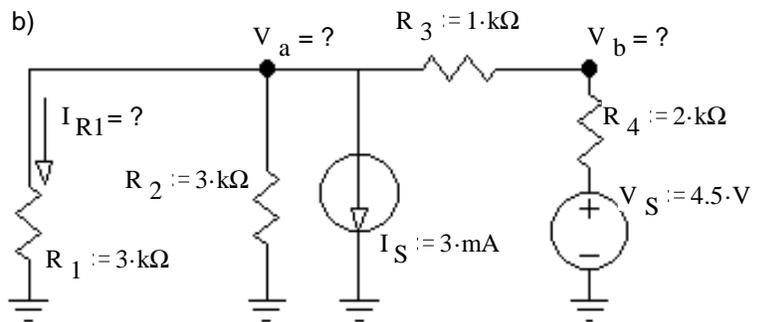
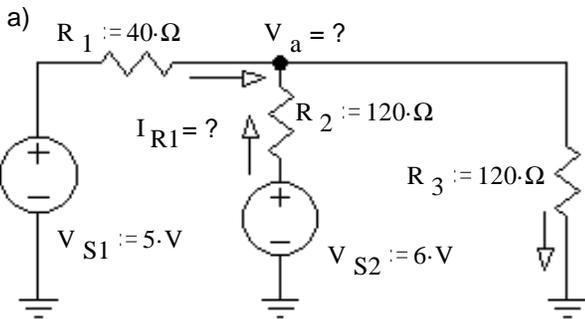
2. a) Use nodal analysis to find all the node voltages.
- b) Your node voltages will depend on your selection of a reference node (ground) as well as your arbitrary node labels, so the grader won't look at these specifically. Use your node voltages to find the potential (voltage) across each resistor. Report the magnitude and polarity of each.



3. Use Nodal analysis to find  $V_a$  and use  $V_a$  to find  $I_3$ .



4. Use Nodal analysis to solve following problems: Each problem asks for at least 1 voltage and a current. Use the voltage(s) to find the current.



**Don't forget your folder number.**

hint: you may be able to eliminate one unknown node for the initial calculation.

**Answers**

1. a) 3,3      b) 
$$V_a \cdot \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) - \frac{V_b}{R_2} - \frac{V_c}{R_3} = \frac{V_S}{R_1}, \quad \frac{V_a}{R_2} - V_b \cdot \left( \frac{1}{R_2} + \frac{1}{R_5} + \frac{1}{R_4} \right) + \frac{V_c}{R_4} = 0 \cdot A$$

$$\frac{V_a}{R_3} + \frac{V_b}{R_4} - V_c \cdot \left( \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_6} \right) = 0 \cdot A$$

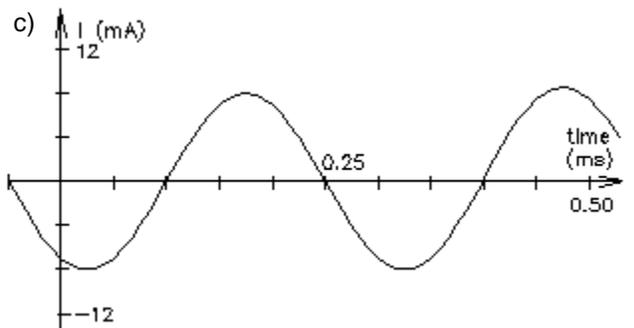
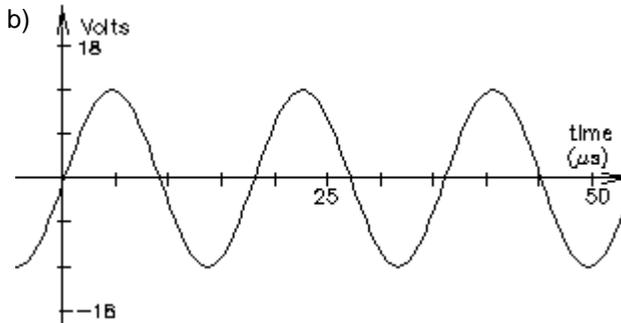
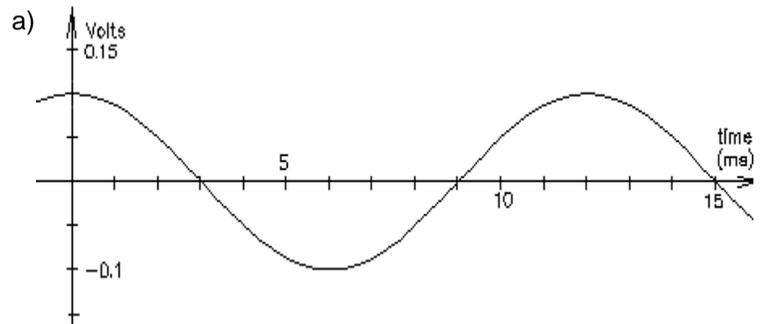
2. a) Answer will depend on your choice of ground, so check your answers to part b to see if you did part a right.
- b) 3.077·V , + bottom ,      2.308·V , + left ,      1.923·V , + top ,      0.385·V , + bottom ,      2.692·V , + right
3. 7·V , 7·mA      4. a) 4.2·V , 20·mA      b)  $V_a := -1.5 \cdot V$      $V_b := 0.5 \cdot V$      $I_{R1} := -0.5 \cdot mA$

You may not get this homework back before the **1st exam**. Photocopy it if you want to be sure to have it.

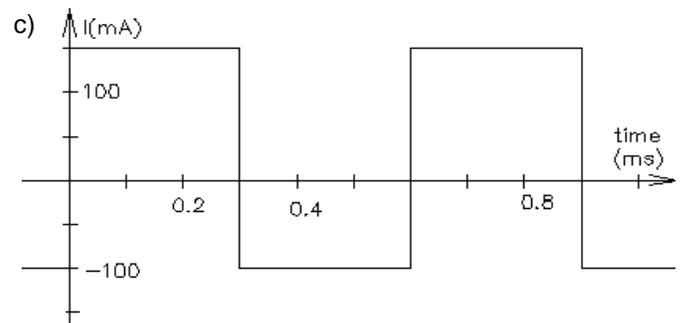
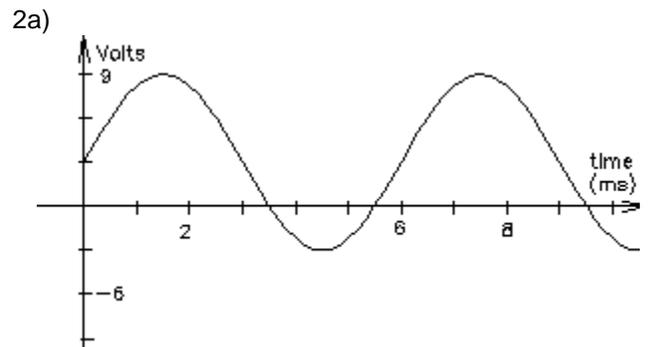
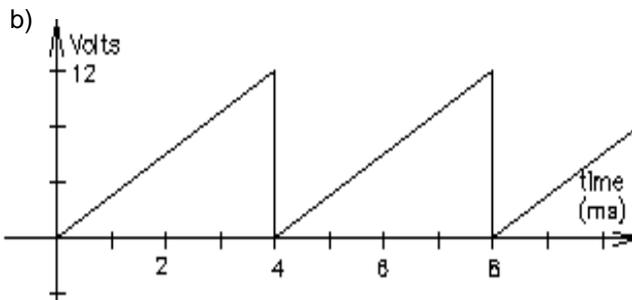
**1st exam on Tue. 9/21/21 will include this material**

Answer the following problems on your own paper.

- For each of the following sinusoidal waves, find:
  - peak-to-peak voltage or current,  $V_{pp}$  or  $I_{pp}$
  - amplitude,  $A$ ,  $V_p$ , or  $I_p$
  - period,  $T$
  - frequency  $f$  in cycles/sec or Hz
  - an expression for  $v(t)$  or  $i(t)$  in terms of  $A\cos(\omega t + \phi)$   
 the frequency  $\omega$  is in radians/sec  
 the phase angle  $\phi$  is in rad/sec or degrees



- For each of the following waveforms, find:
  - Peak-to-peak voltage or current,  $V_{pp}$  or  $I_{pp}$
  - Average, ( $V_{DC}$ ,  $I_{DC}$ ,  $V_{ave}$ , or  $I_{ave}$ )
  - Period,  $T$
  - Frequency  $f$  in cycles/sec or Hz



- For problem 2a above, write a full expression for  $v(t)$  in terms of  $v(t) = A\cos(\omega t + \phi) + V_{DC}$

**Answers**

- 0.2-V 0.1-V 12-ms 83.3-Hz  $0.1 \cdot V \cdot \cos(523.6 \cdot t)$
  - 24-V 12-V 0.018-ms 55.6-kHz  
 $v(t) := 12 \cdot V \cdot \cos(349100 \cdot t - 90 \cdot \text{deg})$
  - 16-mA 8-mA 0.3-ms 3333-Hz  
 $8 \cdot \text{mA} \cdot \cos(20940 \cdot t + 150 \cdot \text{deg})$

- 12-V 3-V 6-ms 167-Hz
  - 12-V 6-V 4-ms 250-Hz
  - 250-mA 25-mA 0.6-ms 1.667-kHz
  - $v(t) := 6 \cdot V \cdot \cos(1047 \cdot t - 90 \cdot \text{deg}) + 3 \cdot V$