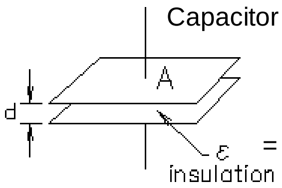
ECE 2210 / 00 Capacitor Lecture Notes

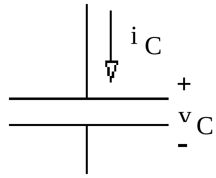
A. Stolp
2/17/03
rev 9/16/09
12/15 & 9/19

Now that we have voltages and currents which can be functions of time, it's time to introduce the capacitor and the inductor.

Electrical equivalent:

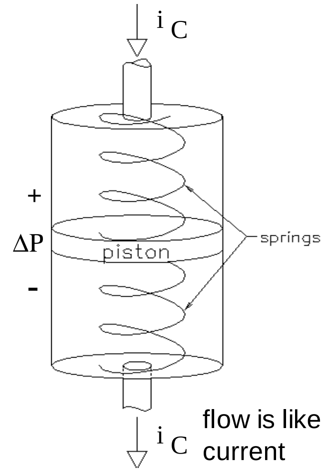


ε = permittivity



v_C like pressure

Fluid Model:



flow is like current

$$C = \varepsilon \cdot \frac{A}{d} = \frac{Q}{V} = \frac{dq}{dv}$$

Units: $\text{farad} = \frac{\text{coul}}{\text{volt}} = \frac{\text{amp}\cdot\text{sec}}{\text{volt}}$ $\qquad \mu F = 1 \cdot 10^{-6} \cdot \text{farad} \qquad pF = 1 \cdot 10^{-12} \cdot \text{farad}$

For drawings of capacitors and info about tolerances, see Ch.3 of textbook.

Basic equations you should know:

$$C = \frac{Q}{V}$$

$$v_C = \frac{1}{C} \cdot \int_{-\infty}^{t} i_C \, dt$$

$$i_C = C \cdot \frac{d}{dt} v_C$$

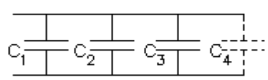
Or... $$v_C = \frac{1}{C} \cdot \int_{0}^{t} i_C \, dt + v_C(0)$$ ← initial voltage

Or... $$\Delta v_C = \frac{1}{C} \cdot \int_{t_1}^{t_2} i_C \, dt$$

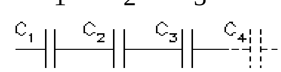
Energy stored in electric field: $W_C = \frac{1}{2} \cdot C \cdot V_C^{\,2}$

Capacitor voltage **cannot** change instantaneously

parallel: $C_{eq} = C_1 + C_2 + C_3 + \ldots$



series: $$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots}$$



Capacitors are the only "backwards" components.

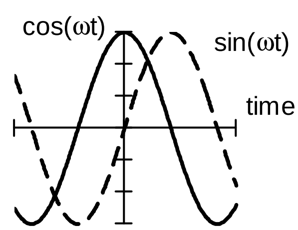
Sinusoids

$$i_C(t) = I_p \cdot \cos(\omega \cdot t)$$

$$v_C(t) = \frac{1}{C} \cdot \int i_C \, dt = \underbrace{\frac{1}{C} \cdot \frac{1}{\omega} \cdot I_p}_{V_p} \cdot \sin(\omega \cdot t) = \underbrace{\frac{1}{C} \cdot \frac{1}{\omega} \cdot I_p}_{V_p} \cdot \cos(\omega \cdot t - 90 \cdot \text{deg})$$

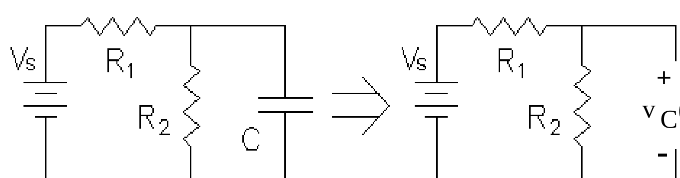
indefinite integral

Voltage "lags" current, makes sense, current has to flow in first to charge capacitor.



Steady-state or Final conditions

If a circuit has been connected for "a long time", then it has reached a steady state condition. that means the currents and voltages are no longer changing.



"long time"

$$v_C(\infty) = V_S \cdot \frac{R_2}{R_1 + R_2}$$

$$\frac{d}{dt} v_C = 0 \qquad i_C = C \cdot \frac{d}{dt} v_C = 0$$

no current means it looks like an open

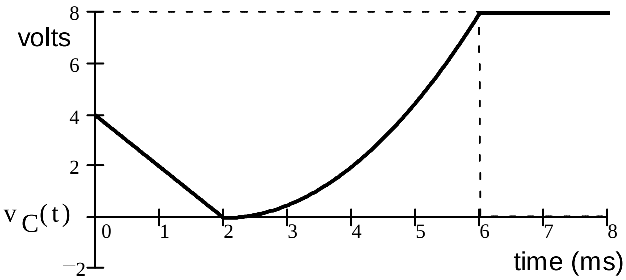
Example

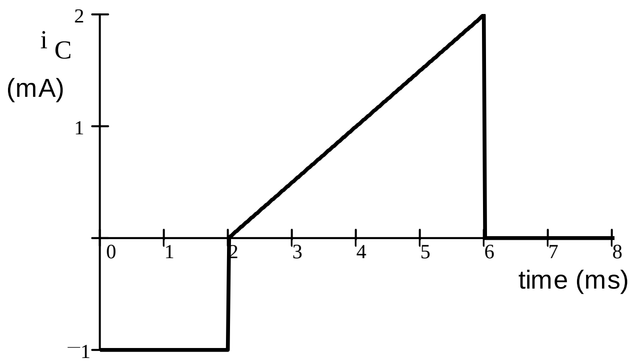
The voltage across a $0.5\ \mu F$ capacitor is shown below. Make an accurate drawing of the capacitor current. Label the y-axis of your graph (I've already done the time-axis).

The accuracy of your plot at 0, 2, 6, and 8 ms is important, so calculate those values and plot or label them carefully. Between those points your plot must simply be the correct shape.

$C := 0.5 \cdot \mu F$

The curve is 2nd order





1 - 2ms: $i_C = C \cdot \frac{\Delta V}{\Delta t} = 0.5 \cdot \mu F \cdot \frac{-4 \cdot V}{2 \cdot ms} = -1 \cdot mA$

2ms - 6ms: Initial slope is zero and the final slope is positive, so the current must be a triangle that starts at zero and ends at some height.

$$\Delta v_C(t) = \frac{1}{C} \cdot \int_0^t i_C(t)\, dt$$

$$8 \cdot V = \frac{1}{C} \cdot \left(\frac{4 \cdot ms \cdot height}{2} \right)$$

$$height = 8 \cdot V \cdot \frac{C \cdot 2}{4 \cdot ms} = 2 \cdot mA$$

6ms - 8ms: Slope is zero, so the current must be zero.

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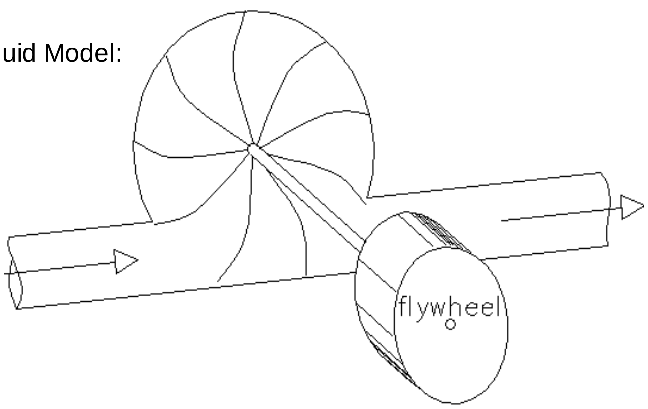
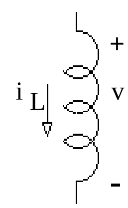
Fluid Model:



Electrical equivalent:



$L = \mu_o \cdot N^2 \cdot K$

μ is the permeability of the inductor core

K is a constant which depends on the inductor geometry

N is the number of turns of wire

Basic equations you should know: $v_L = L \cdot \frac{d}{dt} i_L$

$$i_L = \frac{1}{L} \cdot \int_{-\infty}^t v_L\, dt$$

Or... $i_L = \frac{1}{L} \cdot \int_0^t v_L\, dt + i_L(0)$ ← initial current

Or... $\Delta i_L = \frac{1}{L} \cdot \int_{t_1}^{t_2} v_L\, dt$

Energy stored in electric field: $W_L = \frac{1}{2} \cdot L \cdot I_L^2$

Inductor current **cannot** change instantaneously

Units: $henry = \frac{volt \cdot sec}{amp}$

$mH = 10^{-3} \cdot H$ $\quad \mu H = 10^{-6} \cdot H$

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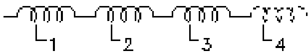
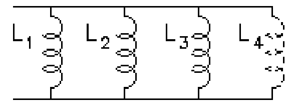
series: $L_{eq} = L_1 + L_2 + L_3 + \ldots$



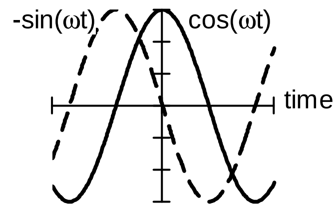
parallel: $L_{eq} = \dfrac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_2} + \dfrac{1}{L_3} + \ldots}$



Sinusoids $i_L(t) = I_p \cdot \cos(\omega \cdot t)$

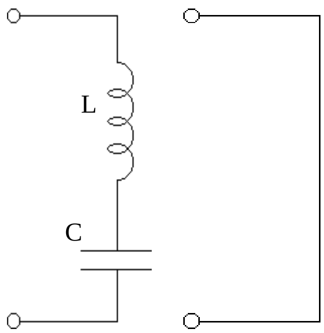
$$v_L(t) = L \cdot \frac{d}{dt} i_L = \underbrace{L \cdot \omega}_{} \cdot \left(-I_p \cdot \sin(\omega \cdot t)\right) = \underbrace{L \cdot \omega \cdot I_p}_{V_p} \cdot \cos(\omega \cdot t + 90 \cdot deg)$$

_ V_p _/ _ V_p _/



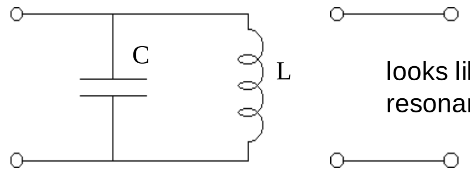
Voltage "leads" current, makes sense, voltage has to present to make current change, so voltage comes first.

Resonance Series resonance



looks like a short at resonance frequency

Parallel resonance



looks like an open at resonance frequency

The resonance frequency is calculated the same way for either case:

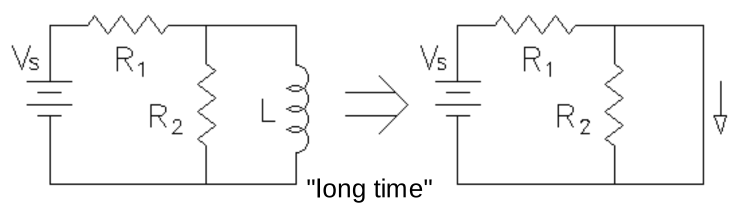
$\omega_o = \dfrac{1}{\sqrt{L \cdot C}} \left(\dfrac{rad}{sec}\right)$ OR.. $\omega_o = \dfrac{1}{\sqrt{L_{eq} \cdot C_{eq}}}$ If you have multiple capacitors or inductors which can be combined. $f_o = \dfrac{\omega_o}{2 \cdot \pi}$ (Hz)

Steady-state of Final conditions

If a circuit has been connected for "a long time", then it has reached a steady state condition. that means the currents and voltages are no longer changing.

$\dfrac{d}{dt} i_L = 0$ $\qquad v_L = L \cdot \dfrac{d}{dt} i_L = 0$

no voltage means it looks like a short



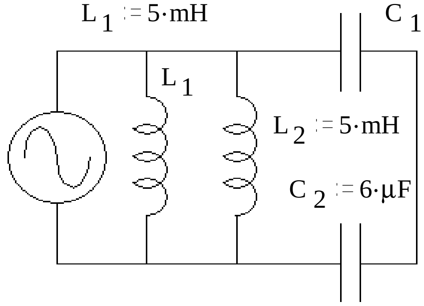
$i_L(\infty) = \dfrac{V_S}{R_1}$

Examples

Ex 1

Find the resonant frequency (or frequencies) of the circuit shown (in cycles/sec or Hz).

$L_1 := 5 \cdot mH$ $\qquad C_1 := 6 \cdot \mu F$



$L_2 := 5 \cdot mH$

$C_2 := 6 \cdot \mu F$

$L_{eq} := \dfrac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_2}}$ $\qquad L_{eq} = 2.5 \cdot mH$

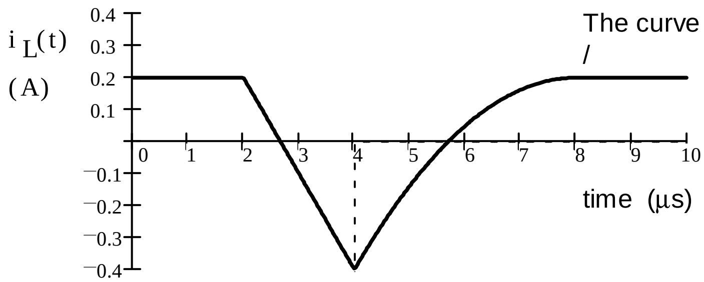
$C_{eq} := \dfrac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2}}$ $\qquad C_{eq} = 3 \cdot \mu F$

$\omega_o := \dfrac{1}{\sqrt{L_{eq} \cdot C_{eq}}}$ $\qquad \omega_o = 11547 \cdot \dfrac{rad}{sec}$ $\qquad f_o = \dfrac{\omega_o}{2 \cdot \pi} = 1838 \cdot Hz$

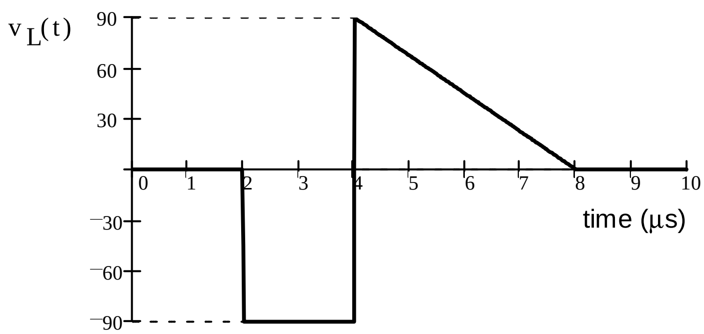
Ex 2

The current through a 0.3mH inductor is shown below. Make an accurate drawing of the inductor voltage. Make reasonable assumptions where necessary. Label your graph.

$L := 0.3\cdot mH$



The curve is 2[nd] order and ends at 8µs /



0 - 2µs: No change in current, so: $v_L = 0$

2µs - 4µs: $v_L = L\cdot\frac{\Delta I}{\Delta t} = 0.3\cdot mH\cdot\frac{-0.6\cdot A}{2\cdot\mu s} = -90\cdot V$

4µs - 8µs: Initial slope is positive and the final slope is zero, so the voltage must be a triangle that starts at some height and ends at zero.

$$\Delta i_L(t) = \frac{1}{L}\cdot\int_0^t v_L(t)\,dt$$

$$0.6\cdot A = \frac{1}{0.3\cdot mH}\cdot\left(\frac{4\cdot\mu s\cdot height}{2}\right)$$

$$height = 0.6\cdot A\cdot\frac{0.3\cdot mH\cdot 2}{4\cdot\mu s} = 90\cdot V$$

8µs - 10µs: No change in current, so: $v_L = 0$

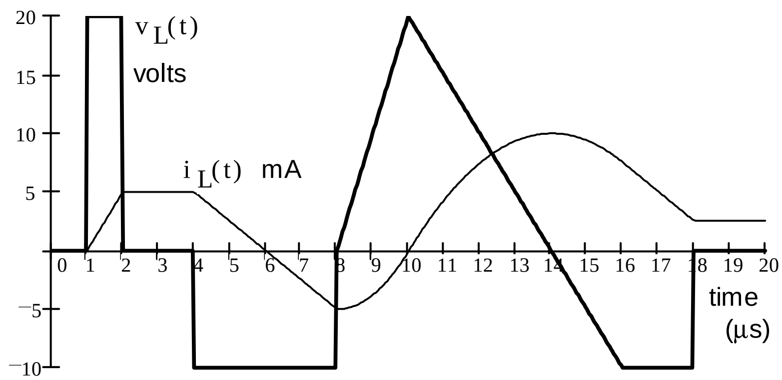
Ex 3 Given a voltage, find the current, $L := 4\cdot mH$

$$\Delta i_L(t) = \frac{1}{L}\cdot\int_{1\cdot\mu s}^{2\cdot\mu s} 20\cdot V\,dt = 5\cdot mA$$

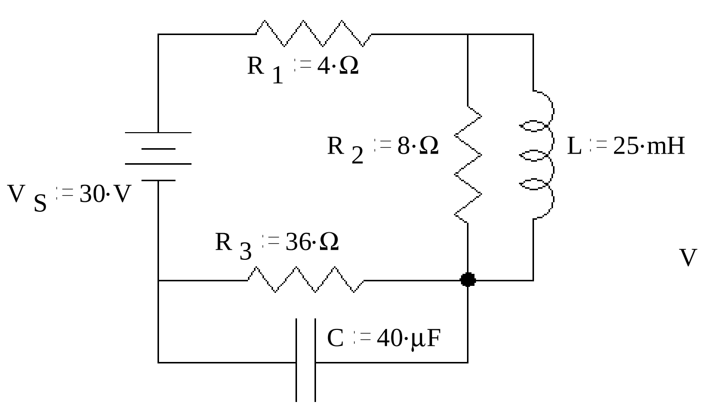
$$\frac{1}{L}\cdot\int_{4\cdot\mu s}^{8\cdot\mu s} -10\cdot V\,dt + 5\cdot mA = -5\cdot mA$$

$$\frac{1}{L}\cdot\int_{8\cdot\mu s}^{10\cdot\mu s} V(t)\,dt + -5\cdot mA$$

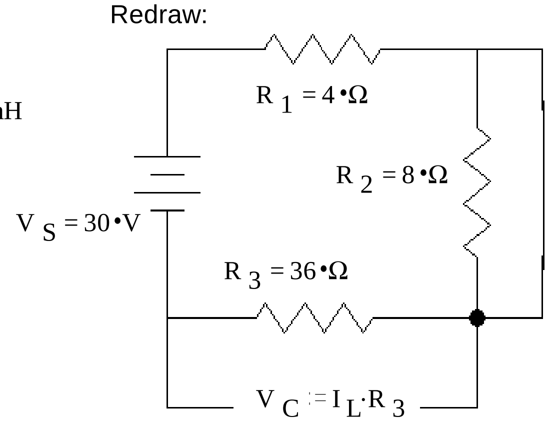
$$= \frac{1}{L}\cdot\frac{20\cdot V\cdot 2\cdot\mu s}{2} - 5\cdot mA = 0\cdot mA \quad \text{etc...}$$



Ex 4 The following circuit has been connected as shown for a long time. Find the energy stored in the capacitor and the inductor.



Redraw:



$V_C := I_L\cdot R_3$

$V_C = 27\cdot V$

$I_L := \frac{V_S}{R_1 + R_3}$

$I_L = 0.75\cdot A$

$W_L := \frac{1}{2}\cdot L\cdot I_L^2$

$W_L = 7.031\cdot mJ$

$W_C := \frac{1}{2}\cdot C\cdot V_C^2$

$W_C = 14.58\cdot mJ$

Capacitor, Inductor Notes

ECE 2210 / 00

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2/27/00,
9/13/05

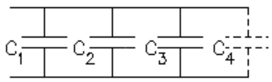
Capacitors

$C = \frac{Q}{V}$ $\quad$ $\text{farad} = \frac{\text{coul}}{\text{volt}} = \frac{\text{amp}\cdot\text{sec}}{\text{volt}}$ $\quad$ $v_C = \frac{1}{C}\cdot\int_{-\infty}^{t} i_C\,dt = \frac{1}{C}\cdot\int_{0}^{t} i_C\,dt + v_C(0)$ (initial voltage) $\quad$ $i_C = C\cdot\frac{d}{dt}v_C$

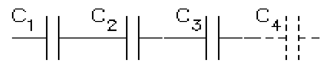
Energy stored in electric field: $W_C = \frac{1}{2}\cdot C\cdot V_C^{\,2}$

Capacitor voltage **cannot** change instantaneously

parallel: $C_{eq} = C_1 + C_2 + C_3 + \ldots$



series: $C_{eq} = \dfrac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots}$



Steady-state sinusoids:

Impedance: $Z_C = \frac{1}{j\cdot\omega\cdot C} = \frac{-j}{\omega\cdot C}$ $\quad$ Current leads voltage by 90 deg

Inductors

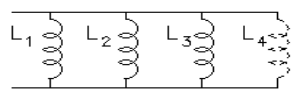
$\text{henry} = \frac{\text{volt}\cdot\text{sec}}{\text{amp}}$ $\quad$ $i_L = \frac{1}{L}\cdot\int_{-\infty}^{t} v_L\,dt = \frac{1}{L}\cdot\int_{0}^{t} v_L\,dt + i_L(0)$ (initial current) $\quad$ $v_L = L\cdot\frac{d}{dt}i_L$

Energy stored in magnetic field: $W_L = \frac{1}{2}\cdot L\cdot I_L^{\,2}$

Inductor current **cannot** change instantaneously

series: $L_{eq} = L_1 + L_2 + L_3 + \ldots$

parallel: $L_{eq} = \dfrac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots}$



Steady-state sinusoids:

Impedance: $Z_L = j\cdot\omega\cdot L$ $\quad$ Current lags voltage by 90 deg

RC and RL first-order transient circuits

For all first order transients: $v_X(t) = v_X(\infty) + \left(v_X(0) - v_X(\infty)\right)\cdot e^{-\frac{t}{\tau}}$ $\quad$ $i_X(t) = i_X(\infty) + \left(i_X(0) - i_X(\infty)\right)\cdot e^{-\frac{t}{\tau}}$

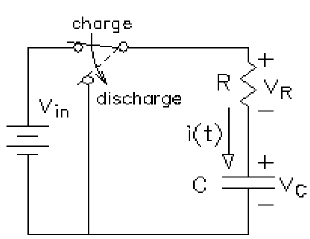
Find initial Conditions (v_C and/or i_L)

Find conditions just before time t = 0, $v_C(0\text{-})$ and $i_L(0\text{-})$. These will be the same just after time t = 0, $v_C(0+)$ and $i_L(0+)$ and will be your initial conditions. (If initial conditions are zero: Capacitors are shorts, Inductors are opens.)
Use normal circuit analysis to find your desired variable: $v_X(0)$ or $i_X(0)$

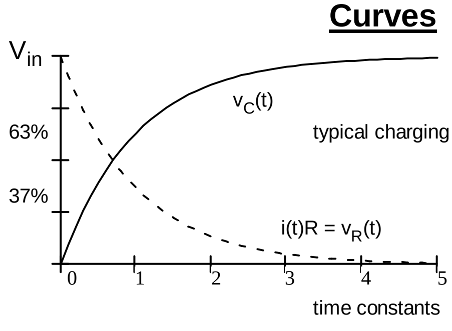
Find final conditions ("steady-state" or "forced" solution)

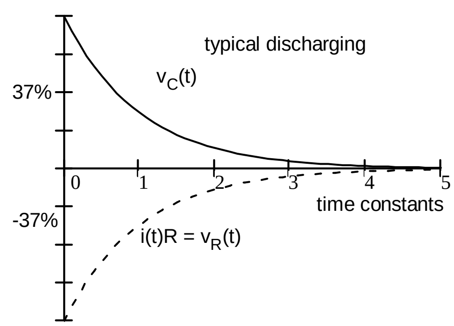
Inductors are shorts Capacitors are opens Solve by DC analysis $v_X(\infty)$ or $i_X(\infty)$

RC Time constant $= \tau = RC$

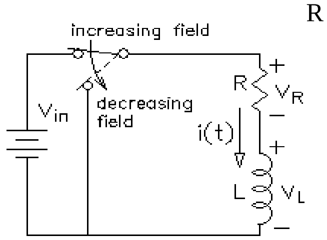


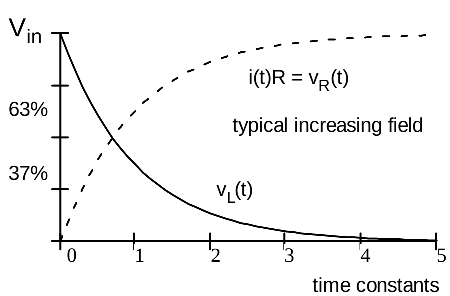
Curves

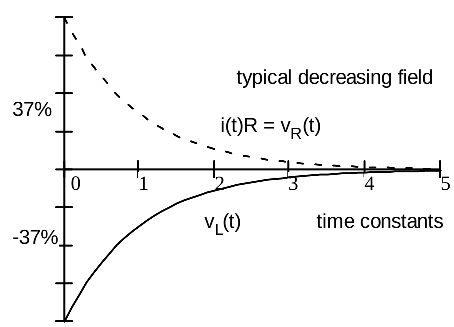




RL Time constant $= \tau = \frac{L}{R}$





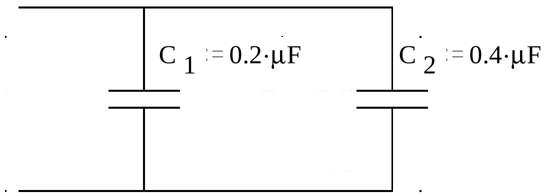


$e^{-1} = 0.368$ $\quad$ $1 - e^{-1} = 0.632$

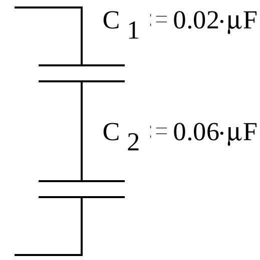
1st exam on Tue. 9/22 will include p.1 of this homework (Listen for details in class)

1) Find C_{eq} in each case

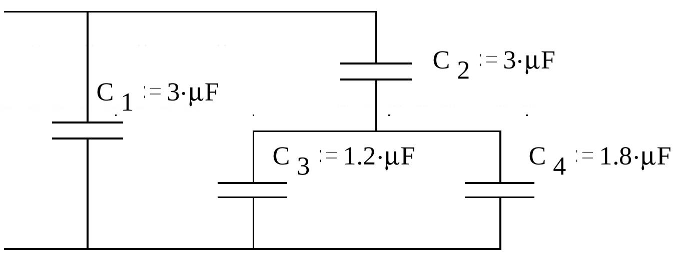
a)



b)

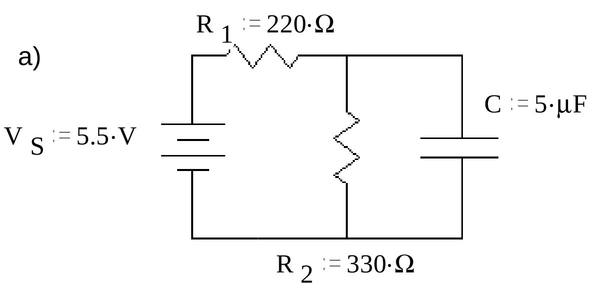


c)

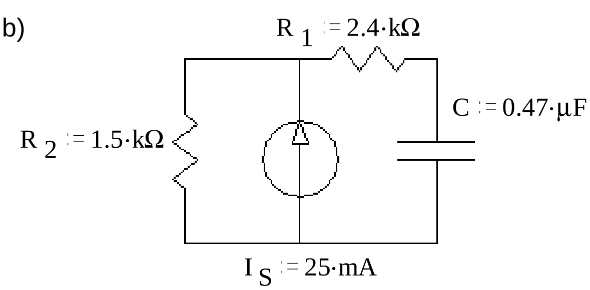


2. Each of the following circuits have been connected as shown for a long time.
Find the voltage across each capacitor and the energy stored in each.

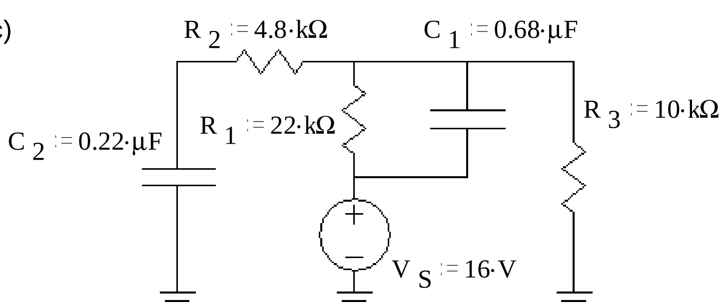
a)



b)

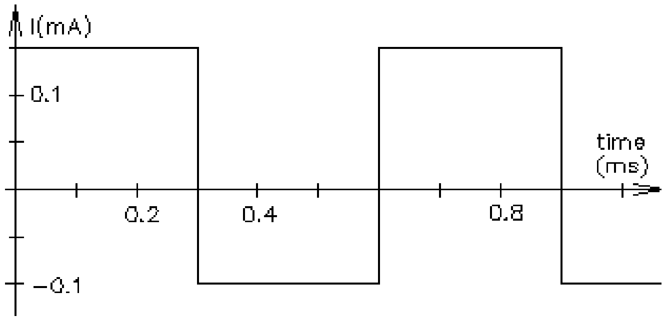


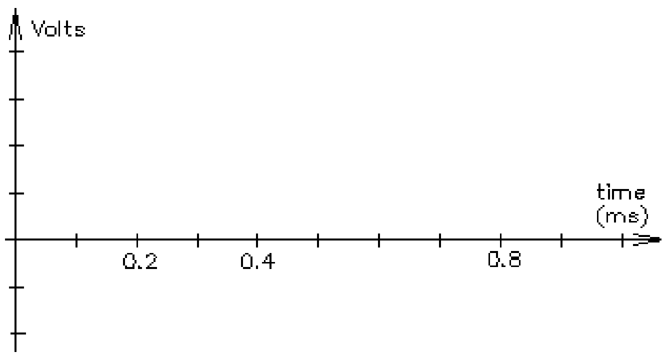
c)



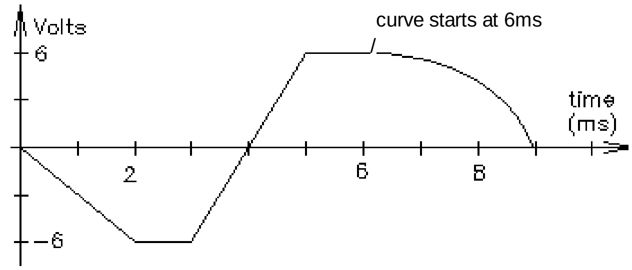
Name: ________________________________ You may want to hand in this page with answers to problems 3 & 4.

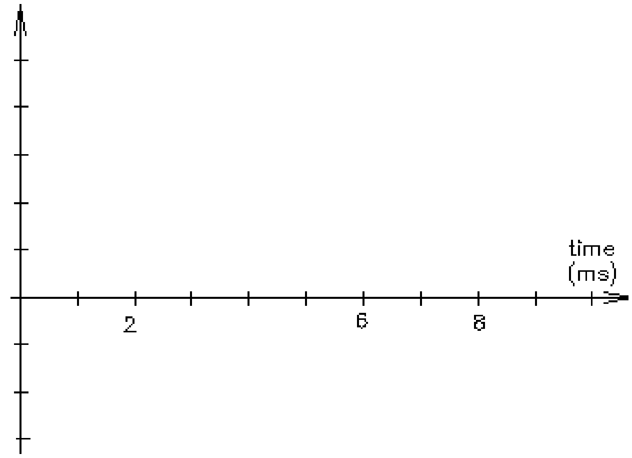
3. The current waveform shown below flows through a $0.025\ \mu F$ capacitor. Make an accurate drawing of the voltage across it. Label your graph. Assume the initial voltage across the capacitor is $0\ V$.





4. The voltage across a $2\ \mu F$ capacitor is shown below. Make an accurate drawing of the capacitor current. Label your graph.

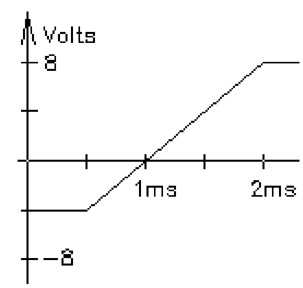


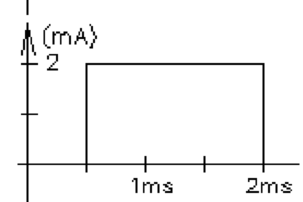


5. The voltage across a $0.68\ \mu F$ capacitor is $v_C = 6 \cdot V \cdot \cos\left(200 \cdot t + \frac{\pi}{2}\right)$ find i_c.

6. The current through a $0.0047\ \mu F$ capacitor is $i_C = 18 \cdot \mu A \cdot \cos\left(628 \cdot t - \frac{\pi}{4}\right)$ find v_c.

7. A capacitor voltage and current are shown at right. What value is the capacitor?





Answers

1. a) $0.6 \cdot \mu F$ b) $0.015 \cdot \mu F$ c) $4.5 \cdot \mu F$

2. a) $3.3V\ \ 0.027 \cdot mJ$ b) $37.5 \cdot V\ \ 0.33 \cdot mJ$ c) $11 \cdot V\ \ 0.0411 \cdot mJ\ \ 5 \cdot V\ \ 2.75 \cdot \mu J$

3. $1.8 \cdot V\ \ 0.6 \cdot V\ \ 2.4 \cdot V$ 4. $-6 \cdot mA\ \ 12 \cdot mA$ ramp to $-8mA$

5. $i_c = 0.816 \cdot mA \cdot \cos(200 \cdot t + \pi)$ 6. $v_c = 6.1 \cdot V \cdot \cos\left(628 \cdot t - \frac{3 \cdot \pi}{4}\right)$ 7. $0.25 \cdot \mu F$

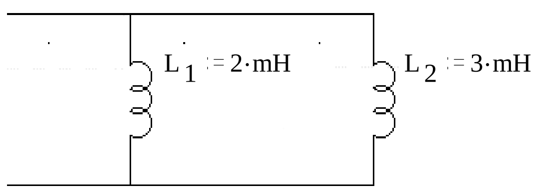
Name: ________________________________ Due: Fri, 9/25/20

ECE 2210 / 00 hw # 9 ~~(Because I forgot to handout Tue, this may be turned in Mon 9/30 for full credit)~~

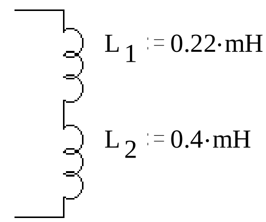
You will need another paper for your calculations, but you may want to hand this sheet in with your drawings.

1. Find L_{eq} in each case

a)

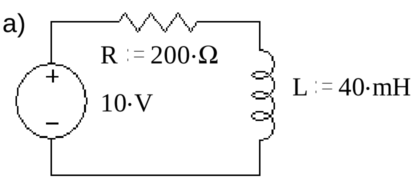


b)

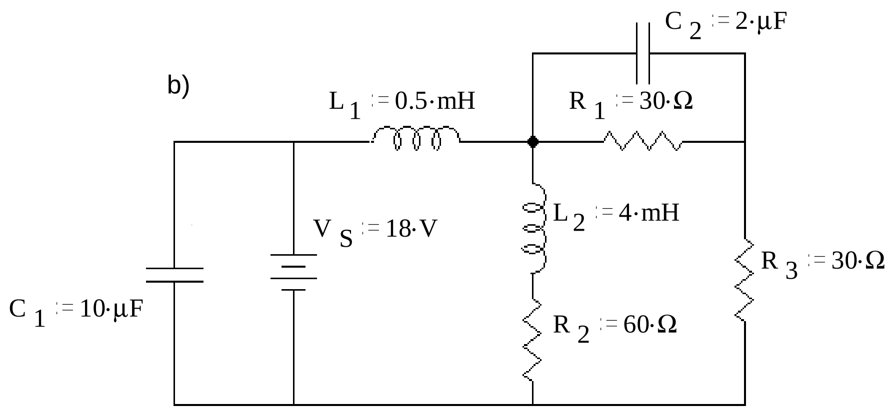


2. Find the stored energy in each capacitor and/or inductor under steady-state conditions. Note: Treat caps as opens and inductors as shorts to find DC voltages and currents.

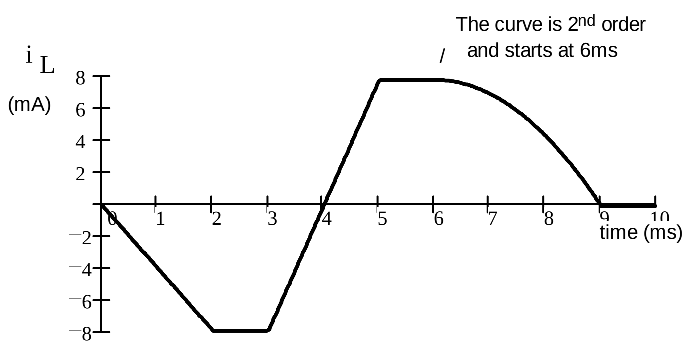
a)

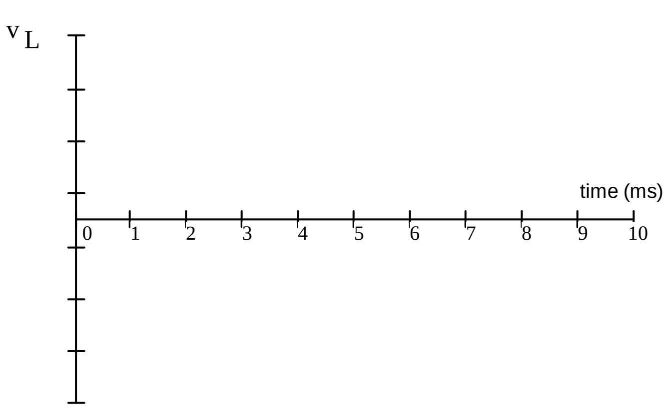


b)

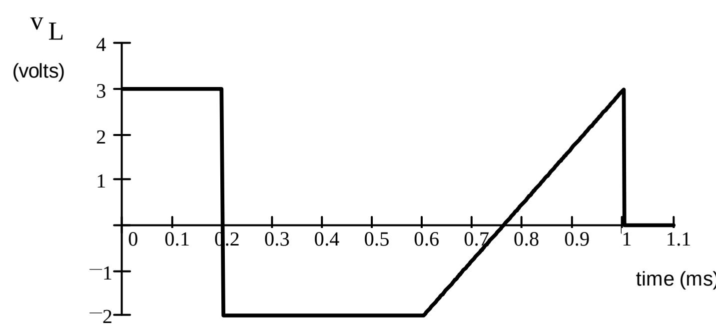


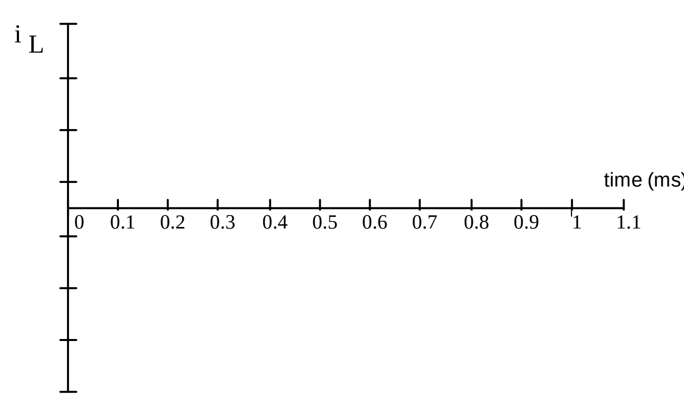
3. The current waveform shown below flows through a 2 mH inductor. Make an accurate drawing of the voltage across it. Label your graph.





4. The voltage across a 0.5 mH inductor is shown below. Make an accurate drawing of the inductor current. Label your graph. Assume the initial current is 0 mA.





ECE 2210 / 00 homework # 9

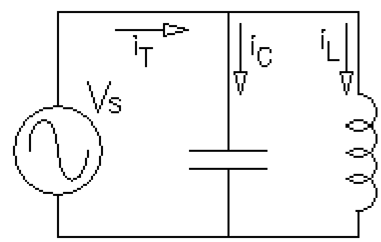
5. The voltage across a 1.2 mH inductor is $v_L = 4 \cdot mV \cdot \cos(300 \cdot t)$ find i_L.

6. The current through a 0.08 mH inductor is $i_L = 20 \cdot mA \cdot \cos\left(628 \cdot t - \frac{\pi}{4}\right)$ find v_L.

7. Refer to the circuit shown. Assume that V_s is a sinusoidal input voltage whose frequency can be adjusted. At some frequency of V_s this circuit can resonate. At that frequency $i_C(t) = -i_L(t)$. ($i_C(t)$ is 180 degrees out-of-phase with $i_L(t)$).

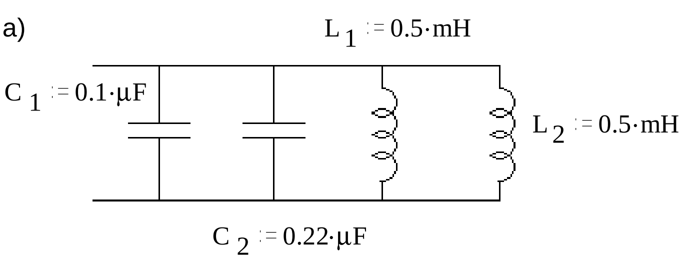
   Show that resonance occurs at this frequency:

$$\omega_o = \frac{1}{\sqrt{L \cdot C}} \quad , \qquad f_o = \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}}$$

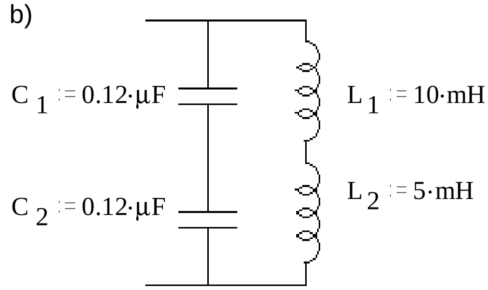


8. Find the resonant frequency, f_o in each case.

   a)



   b)



<u>Answers</u>

1. $1.2 \cdot mH$ $\quad 0.62 \cdot mH$ 2. a) $0.05 \cdot mJ$ $\quad$ b) $1.62 \cdot mJ$ $\quad 0.081 \cdot mJ$ $\quad 0.09 \cdot mJ$ $\quad 0.18 \cdot mJ$
3. Straight lines between the following points: (0ms,-8mV), (2ms,-8mV), (2ms,0mV), (3ms,0mV), (3ms,16mV), (5ms,16mV), (5ms,0mV), (6ms,0mV), (9ms,-10.67mV), (9ms,0mV), (10ms,0mV)
4. Straight lines between the following points: (0ms,0A), (0.2ms,1.2A), (0.6ms,-0.4A), curves until it's flat at (0.76ms, -0.72A), continues to curve up to (1ms, 0A), (1.1ms,0A)
5. $i_L = 11.1 \cdot mA \cdot \cos(300 \cdot t - 90 \cdot deg)$ 6. $v_L = 1 \cdot mV \cdot \cos\left(628 \cdot t + \frac{1}{4} \cdot \pi\right)$
7. Assume a sinusoidal voltage, find i_C and i_L by integration and differentiation, and show that they are equal and opposite at the resonant frequency.
8. a) $17.79 \cdot kHz$ $\quad$ b) $5305 \cdot Hz$

ECE 2210 / 00 homework # 9