ECE 2210 Lecture 7 notes Nodal Analysis

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9/10/08,
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General Network Analysis

In many cases you have multiple unknowns in a circuit, say the voltages across multiple resistors. Network analysis is a systematic way to generate multiple equations which can be solved to find the multiple unknowns. These equations are based on basic Kirchoff's and Ohm's laws.

Loop or Mesh Analysis You may have used these methods in previous classes, particularly in Physics. The best thing to do now is to forget all that. Loop analysis is rarely the easiest way to analyze a circuit and is inherently confusing. Hopefully I've brought you to a stage where you have some intuitive feeling for how currents flow in circuits. I don't want to ruin that now by screwing around with loop currents that don't really exist.

Nodal analysis This is a much better method. It's just as powerful, usually easier, and helps you develop your intuitive feeling for how circuits work.

Nodal Analysis

Node = all points connected by wire, all at same voltage (potential)

Ground: One node in the circuit which will be our reference node. Ground, by definition, will be the zero voltage node. All other node voltages will be referenced to ground and may be positive or negative. Think of gage pressure in a fluid system. In that case atmospheric pressure is considered zero. If there is no ground in the circuit, define one for yourself. Try to chose a node which is hooked to one side of a voltage source.

Nodal Voltage: The voltage of a node referenced to ground. The objective of nodal analysis is to find all the nodal voltages. If you know the voltage at a node then it's a "known" node. Ground is a known node (duh, it's zero). If one end of a known voltage source hooked to ground, then the node on the other end is also known (another duh).

Method: Label all the unknown nodes as; "a", "b", "c", etc. Then the unknown nodal voltages become; V_a, V_b, V_c, etc. Write a KCL equation for each unknown node, defining currents as necessary. Replace each unknown current with an Ohm's law relationship using the nodal voltages. Now you have just as many equations as unknowns. Solve.

Nodal Analysis Steps

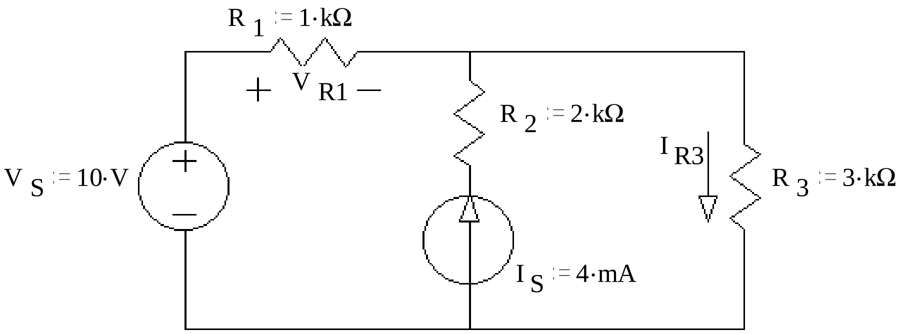
1) If the circuit doesn't already have a ground, label one node as ground (zero voltage).
If the ground can be defined as one side of a voltage source, that will make the following steps easier.
Label the remaining node, either with known voltages or with letters, a, b, ...
2) Label unknown node voltages as V_a, V_b, ... and label the current in each resistor as I_1, I_2, ...
3) Write Kirchoff's current equations for each unknown node.
4) Replace the currents in your **KCL** equations with expressions like this.
5) Solve the multiple equations for the multiple unknown voltages.

$$\frac{V_a - V_b}{R_1}$$

Ohm's law relationship using the nodal voltages.

Nodal Analysis Examples

Ex 1 Use nodal analysis to find the voltage across R_1 (V_{R1}).



1) See next page
Label one node as ground (zero voltage). By choosing the negative side of a voltage source as ground, the upper-left node is known (10V). Label the remaining nodes, either with known voltages or with letters, a, b, ...

2) Label unknown node voltages as V_a, V_b, ...
and label the current in each resistor as I_1, I_2, ...

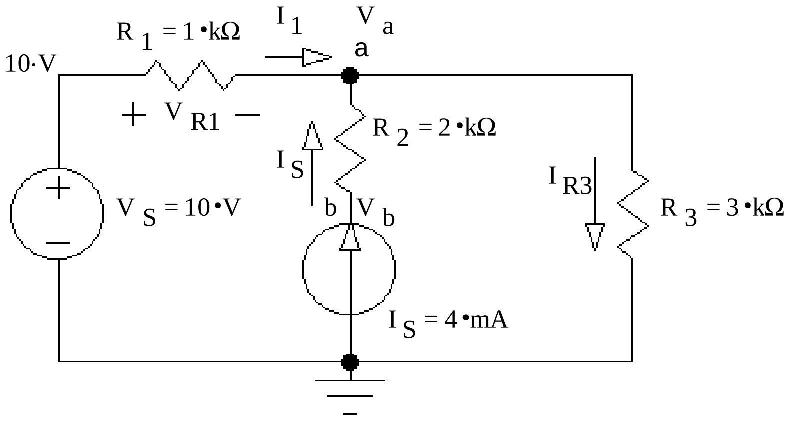
3) Write Kirchoff's current equations for node a.

$$I_1 + I_S = I_{R3}$$

4) Replace the currents in the **KCL** equations with Ohm's law relationships.

$$\frac{V_S - V_a}{R_1} + I_S = \frac{V_a - 0}{R_3}$$

$$\frac{V_S}{R_1} - \frac{V_a}{R_1} + I_S = \frac{V_a}{R_3}$$



5) Solve:

$$\frac{V_S}{R_1} + I_S = \frac{V_a}{R_3} + \frac{V_a}{R_1}$$

$$\frac{V_S}{R_1} + I_S = V_a \cdot \left(\frac{1}{R_1} + \frac{1}{R_3} \right)$$

$$V_a := \frac{\frac{V_S}{R_1} + I_S}{\left(\frac{1}{R_1} + \frac{1}{R_3} \right)} \qquad V_a = 10.5 \cdot V$$

Usually it's easier to put in the numbers at this point

$$\frac{10 \cdot V}{1 \cdot k\Omega} + 4 \cdot mA = \frac{V_a}{3 \cdot k\Omega} + \frac{V_a}{1 \cdot k\Omega}$$

Multiply both sides by a value that will clear the denominators.

$$3 \cdot k\Omega \cdot \left(\frac{10 \cdot V}{1 \cdot k\Omega} + 4 \cdot mA \right) = \left(\frac{V_a}{3 \cdot k\Omega} + \frac{V_a}{1 \cdot k\Omega} \right) \cdot 3 \cdot k\Omega$$

$$30 \cdot V + 3 \cdot k\Omega \cdot 4 \cdot mA = V_a + 3 \cdot V_a$$

$$30 \cdot V + 12 \cdot V = 4 \cdot V_a$$

$$V_a = \frac{42 \cdot V}{4} = 10.5 \cdot V$$

Either way, you still have to find V_{R1} from V_a.

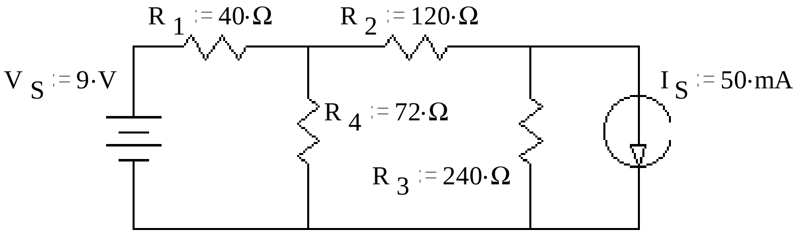
$$V_{R1} := V_S - V_a \qquad V_{R1} = -0.5 \cdot V$$

V_b doesn't matter in this case

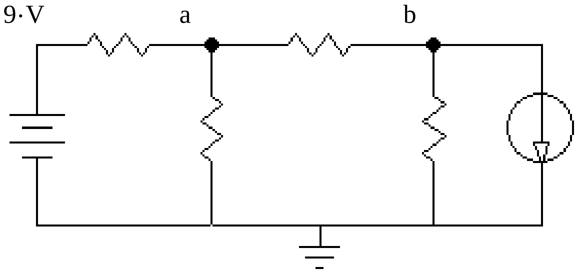
b) Find the current through R_3 (I_{R3}).

$$I_{R3} = \frac{V_a}{R_3} = 3.5 \cdot mA$$

Ex 2 Same circuit used in Thévenin example, where R_4 was R_L.

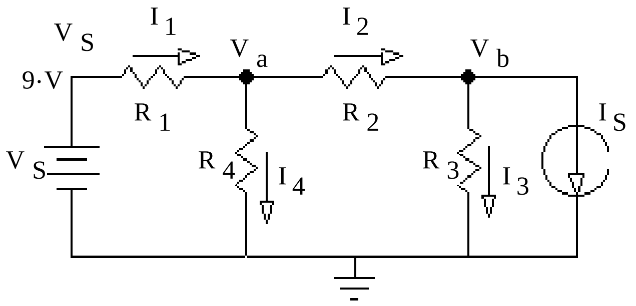


1) Define ground and nodes:



2 unknown nodes ---> will need 2 equations

2) Label unknown node voltages as V_a, V_b, ... and label the current in each resistor as I_1, I_2, ...
It doesn't matter if these currents are in the correct directions.



3) Write Kirchoff's current equations for each unknown node.

node a $\quad I_1 = I_2 + I_4$

node b $\quad I_2 = I_3 + I_S$

4) Replace the currents in your **KCL** equations with expressions like this. $\dfrac{V_a - V_b}{R_1}$

node a $\quad I_1 = I_2 + I_4$

$$\frac{V_S - V_a}{R_1} = \frac{V_a - V_b}{R_2} + \frac{V_a - 0 \cdot V}{R_4}$$

node b $\quad I_2 = I_3 + I_S$

$$\frac{V_a - V_b}{R_2} = \frac{V_b - 0 \cdot V}{R_3} + I_S$$

Now you have just as many equations as unknowns.

5) Solve the multiple equations for the multiple unknown voltages. Solve by any method you like:

$$\frac{V_S}{R_1} - \frac{V_a}{R_1} = \frac{V_a}{R_2} - \frac{V_b}{R_2} + \frac{V_a}{R_4}$$

$$\frac{V_a}{R_2} - \frac{V_b}{R_2} = \frac{V_b}{R_3} + I_S \qquad V_b = \frac{\frac{V_a}{R_2} - I_S}{\frac{1}{R_2} + \frac{1}{R_3}}$$

$$\frac{V_S}{R_1} - \frac{V_a}{R_1} = \frac{V_a}{R_2} - \frac{\frac{V_a}{R_2} - I_S}{R_2 \cdot \left(\frac{1}{R_2} + \frac{1}{R_3}\right)} + \frac{V_a}{R_4}$$

$$V_a := \frac{\left[\frac{V_S}{R_1} - \frac{1}{\left[R_2 \cdot \left(\frac{1}{R_2} + \frac{1}{R_3}\right)\right]} \cdot I_S\right]}{\left[\frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_2^{\,2} \cdot \left(\frac{1}{R_2} + \frac{1}{R_3}\right)} + \frac{1}{R_4}\right]} \qquad V_a = 4.6 \cdot V$$

$$V_b := \frac{\frac{V_a}{R_2} - I_S}{\frac{1}{R_2} + \frac{1}{R_3}} \qquad V_b = -0.933 \cdot V$$

Or, with numbers

node a

$$360 \cdot \Omega \cdot \left(\frac{9 \cdot V - V_a}{40 \cdot \Omega}\right) = \left(\frac{V_a - V_b}{120 \cdot \Omega} + \frac{V_a}{72 \cdot \Omega}\right) \cdot 360 \cdot \Omega$$

$$81 \cdot V - 9 \cdot V_a = 3 \cdot V_a - 3 \cdot V_b + 5 \cdot V_a$$

node b

$$240 \cdot \Omega \cdot \frac{V_a - V_b}{120 \cdot \Omega} = \left(\frac{V_b - 0 \cdot V}{240 \cdot \Omega} + 50 \cdot mA\right) \cdot 240 \cdot \Omega$$

$$2 \cdot V_a - 2 \cdot V_b = V_b + 48 \cdot mA \cdot 240 \cdot \Omega$$

$$V_a = \frac{2 \cdot V_b + V_b + 12 \cdot V}{2} = 1.5 \cdot V_b + 6 \cdot V$$

\ /

<-- substitute for V_a

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$$81 \cdot V - 9 \cdot \left(1.5 \cdot V_b + 6 \cdot V\right) = 3 \cdot \left(1.5 \cdot V_b + 6 \cdot V\right) - 3 \cdot V_b + 5 \cdot \left(1.5 \cdot V_b + 6 \cdot V\right)$$

$$81 \cdot V - 13.5 \cdot V_b - 54 \cdot V = 4.5 \cdot V_b + 18 \cdot V - 3 \cdot V_b + 7.5 \cdot V_b + 30 \cdot V$$

$$81 \cdot V - 54 \cdot V - 18 \cdot V - 30 \cdot V = -21 \cdot V = 4.5 \cdot V_b - 3 \cdot V_b + 7.5 \cdot V_b + 13.5 \cdot V_b = 22.5 \cdot V_b$$

$$V_b = \frac{-21 \cdot V}{22.5} = -0.933 \cdot V \qquad V_a = 1.5 \cdot V_b + 6 \cdot V = 4.6 \cdot V$$

Same as V_L of Ex 4 of Thévenin examples:

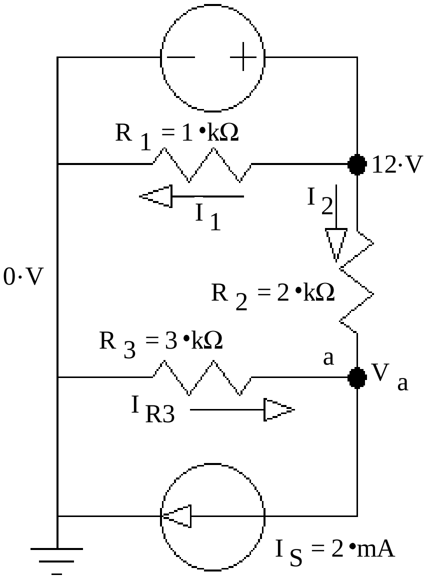
Ex 3 Like Superposition Ex.2

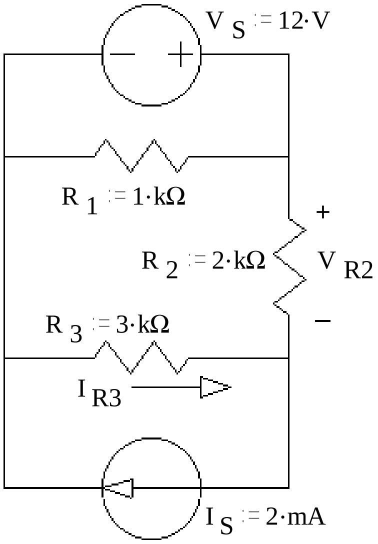
a) Use nodal analysis to find the voltage across R_2 (V_{R2}).

You **MUST** show all the steps of nodal analysis work to get credit, including drawing appropriate symbols and labels on the circuit shown.

1) Define ground and nodes:

2) Label unknown node voltages as V_a, V_b, ... and label the current in each resistor as I_1, I_2, ...





3) Write Kirchoff's current equations for each unknown node.

node a: $I_2 + I_{R3} = I_S$

4) Replace the currents in the **KCL** equations with Ohm's law relationships.

$$\frac{V_S - V_a}{R_2} + \frac{0 - V_a}{R_3} = I_S$$

5) Solve the equation for the unknown voltage.

$$\frac{V_S}{R_2} - \frac{V_a}{R_2} - \frac{V_a}{R_3} = I_S$$

$$\frac{V_S}{R_2} = \frac{V_a}{R_2} + \frac{V_a}{R_3} + I_S$$

$$\frac{V_S}{R_2} - I_S = V_a \cdot \left(\frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$V_a := \frac{\frac{V_S}{R_2} - I_S}{\left(\frac{1}{R_2} + \frac{1}{R_3}\right)} \qquad V_a = 4.8 \cdot V$$

Usually it's easier to put in the numbers at this point

$$\frac{12\cdot V - V_a}{2\cdot k\Omega} + \frac{0 - V_a}{3\cdot k\Omega} = 2\cdot mA$$

Multiply both sides by a value that will clear the denominators.

$$6\cdot k\Omega \cdot \left(\frac{12\cdot V - V_a}{2\cdot k\Omega} + \frac{0 - V_a}{3\cdot k\Omega}\right) = 2\cdot mA \cdot 6\cdot k\Omega$$

$$36\cdot V - 3\cdot V_a - 2\cdot V_a = 12\cdot V$$

$$-5\cdot V_a = -24\cdot V$$

$$V_a = \frac{-24\cdot V}{-5} = 4.8 \cdot V$$

Remember, we needed to find the voltage across R_2 (V_{R2}).

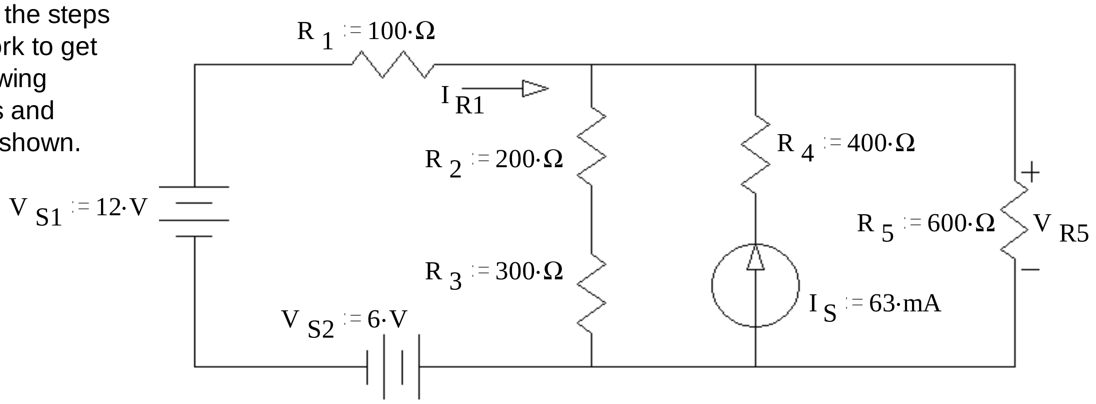
$$V_{R2} = V_S - V_a = 7.2 \cdot V$$

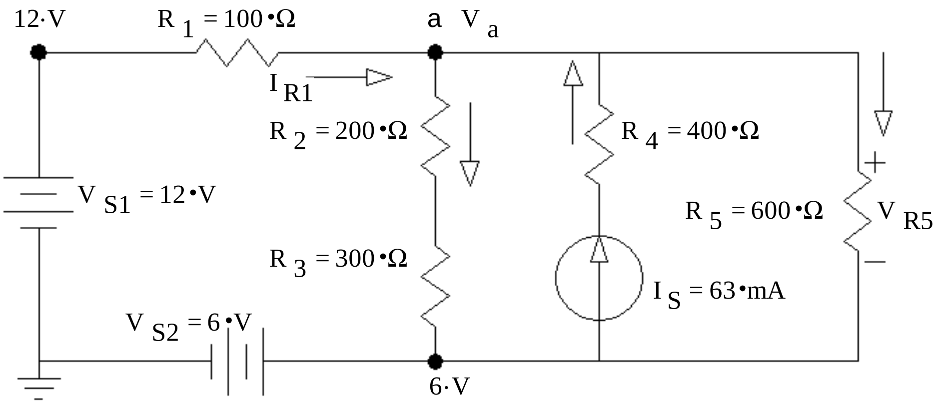
b) Find the current through R_3 (I_{R3}).

$$I_{R3} = \frac{0 - V_a}{R_3} = -1.6 \cdot mA$$ actually flows the other way

Ex 4   Use nodal analysis to find the voltage across R_5 (V_{R5}) and the current through R_1 (I_{R1}).   From exam 1, F09

You **MUST** show all the steps of nodal analysis work to get credit, including drawing appropriate symbols and labels on the circuit shown.





node a:

$$I_{R1} + I_S = I_R + I_5$$

$$R_3 = 300 \cdot \Omega \qquad \frac{V_{S1} - V_a}{R_1} + I_S = \frac{V_a - V_{S2}}{R_2 + R_3} + \frac{V_a - V_{S2}}{R_5}$$

$$\frac{12 \cdot V}{100 \cdot \Omega} - \frac{V_a}{100 \cdot \Omega} + 63 \cdot mA = \frac{V_a}{500 \cdot \Omega} - \frac{6 \cdot V}{500} + \frac{V_a}{600 \cdot \Omega} - \frac{6 \cdot V}{600}$$

multiply both sides by 3000Ω

$$3000 \cdot \Omega \cdot \left(\frac{12 \cdot V}{100 \cdot \Omega} - \frac{V_a}{100 \cdot \Omega} + 63 \cdot mA \right) = \left(\frac{V_a}{500 \cdot \Omega} - \frac{6 \cdot V}{500} + \frac{V_a}{600 \cdot \Omega} - \frac{6 \cdot V}{600} \right) \cdot 3000 \cdot \Omega$$

$$360 \cdot V - 30 \cdot V_a + 189 \cdot V = 6 \cdot V_a - 36 \cdot V + 5 \cdot V_a - 30 \cdot V$$

$$360 \cdot V + 189 \cdot V + 36 \cdot V + 30 \cdot V = 6 \cdot V_a + 5 \cdot V_a + 30 \cdot V_a$$

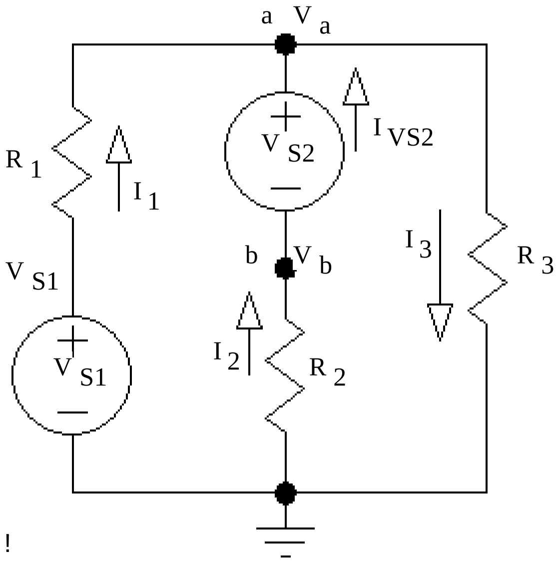
$$615 \cdot V = 41 \cdot V_a$$

$$V_a := \frac{615 \cdot V}{41} \qquad V_a = 15 \cdot V$$

$$V_{R5} = V_a - V_{S2} = 9 \cdot V$$

$$I_{R1} = \frac{V_{S1} - V_a}{R_1} = -30 \cdot mA$$

What if one side of a voltage source isn't ground?



$$I_1 + I_{VS2} = I_3$$

$$\frac{V_{S1} - V_a}{R_1} + \ ? = I_S$$

What do you put in for I_{VS2}??

Go to the other side of V_{S2}.

$$\frac{V_{S1} - V_a}{R_1} + \frac{0 - V_b}{R_2} = I_S$$

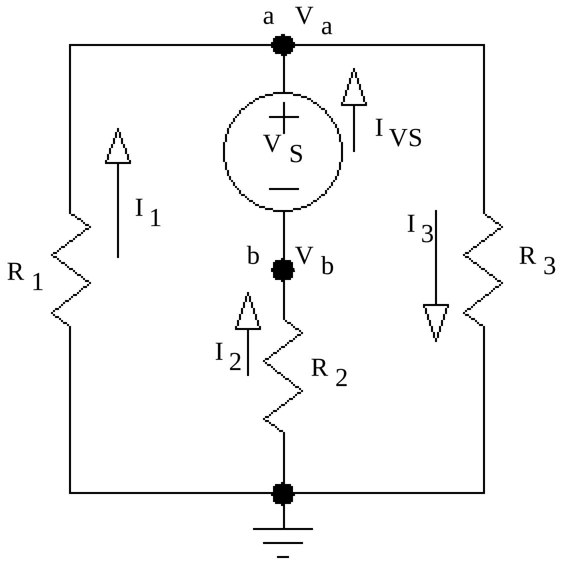
Only problem is that you get the same equation at node b !

Where does the second equation come from?

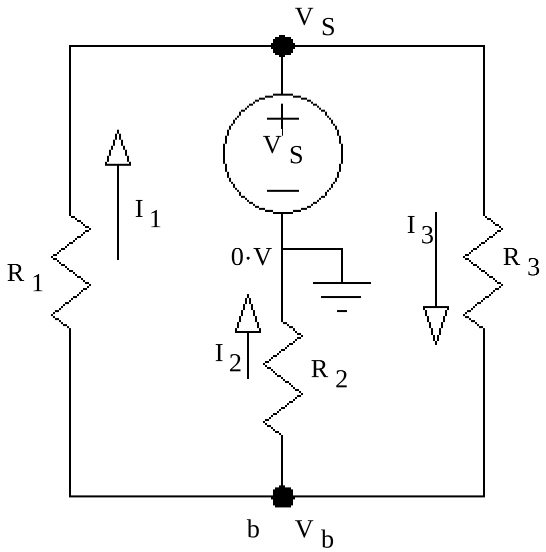
Use something like this: $V_a = V_b + V_{S2}$

Similar Circuit, but no V_{S1}.

If the ground is already at the bottom, use the same method as above.



If you can chose your ground, you can make life a little simpler.



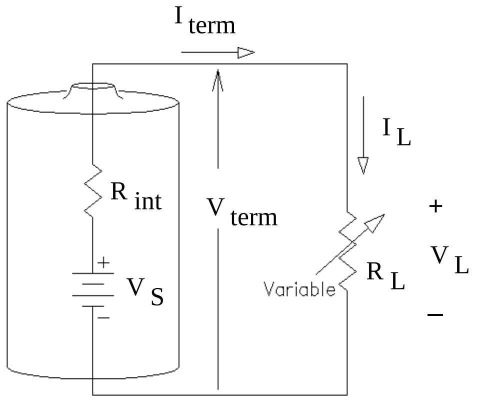
ECE 2210 Lectures notes Batteries and Solar Panels

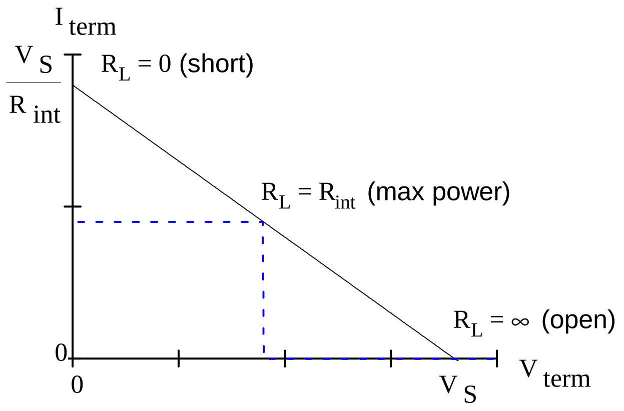
A. Stolp
9/7/20

Most of the information below comes from section 3.2 in your textbook and the following web sites:

batteryuniversity.com mpoweruk.com pveducation.org Visit these sites (& many others) for more information.

Simple Model of a Battery





In batteries the term "voltage" could apply to either the source voltage (V_S) or the terminal voltage (V_{term}), you can usually tell which by context. The source voltage (V_S) is also known as the open-circuit voltage (V_{OC}). The source resistance is called the "Internal Resistance" (R_{int}).

A real battery is an electrochemical device and is therefore much more complicated.

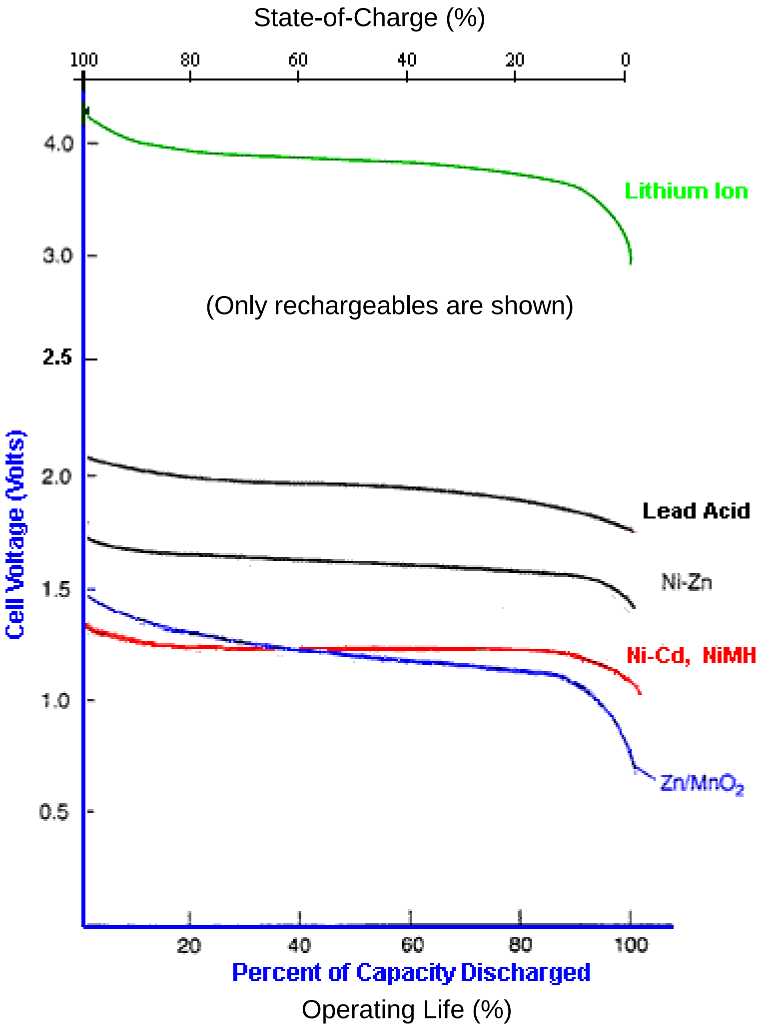
These notes will concentrate on rechargeable (secondary) batteries rather than nonrechargeable (primary) batteries.

Real Battery Parameters are Not Constant

Source Voltage

Ideally, we would like the battery source voltage (V_S) to remain constant as the battery discharges, decreasing only when the battery charge is exhausted. That would appear as a flat line on the plot shown here. As you can see, real batteries don't behave that way.

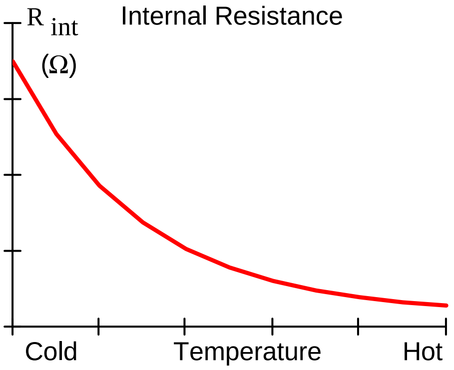
This sort of plot is also shown in your book in section 3.2.4 (p.285 in 3rd ed.) for several types of rechargeable batteries.

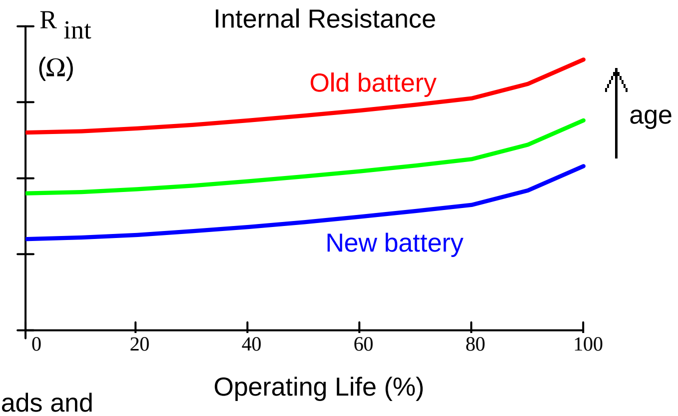


Internal Resistance

The internal resistance (R_{int}) isn't constant either.

It is affected by temperature





R_{int} generally decreases with temperature because the chemical reactions work better at higher temperatures.

R_{int} increases as the battery discharges and as the battery ages.

These curves represent common trends only. Each type of battery, especially different chemistries, will have different curves.

V_S and R_{int} can both be affected negatively by high-current loads and the battery may require some recovery time to get back to normal.

Depletion of a battery as it discharges

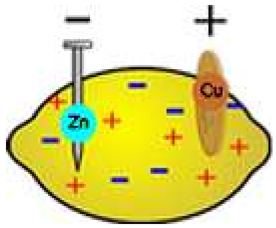
The source voltage (V_S) decreases and the internal resistance (R_{int}) increases until the battery can no longer supply the voltage and/or current required by the load.

Battery Basics

Essentially, a battery is two dissimilar metals in an acidic or salty solution. It all depends on the reduction potentials of the two metals (a measure of how willing an atom is to give up an electron). The metals are the "electrodes" and the solution is the "electrolyte".

The electrodes of the lemon battery are zinc (a galvanized nail) and a copper penny. The lemon juice acts as electrolyte to induce a chemical reaction. When current flows outside the battery, Ions move within the electrolyte to complete the circuit.

Standard reduction potential of zinc = –0.76V
Standard reduction potential of copper = 0.34V
Cell potential = 1.10V



Lemon battery

The electrolyte does not have to be a liquid. It can be a gel or even dry material. Most batteries also have physical separators to keep the electrodes from touching.

Strictly, the lemon battery is just one "cell" and a "battery" is made of multiple cells. Most people (including me) commonly use the term battery for a single cell as well.

In this class, we'll concentrate on rechargeable (secondary) batteries, particularly, lead-acid and lithium-ion.

Nominal Voltages and types of Cells

Lead Acid

The nominal voltage of lead acid is 2 volts per cell. V_{OC}, the open-circuit voltage, should be 2.1V/cell. Allowing the V_{OC} to drop below 2V/cell can cause damage (sulfation). Charge to about 2.4V/cell unless constantly connected to a charger (float charge), then hold at about 2.25V/cell. While in use, the terminal voltage can be less than 2V/cell because of R_{int} (the internal resistance). R_{int} usually is quite small for a lead acid battery.

Most lead-acid batteries are either "flooded" or "sealed". Flooded batteries have liquid electrolyte which could spill out if the battery is not kept upright (think car battery). They must be vented, can produce flammable hydrogen gas (esp. when over-charged) and may also occasionally require water. The electrolyte in **s**ealed **l**ead-**a**cid batteries (SLA or gel-cell) is a non-spillable gel and require no maintenance. They should not be 100% charged and thus have less capacity than flooded cells.

All batteries loose charge with time, even if not used. SLA batteries are among the best of the rechargeables at about 5% loss per month.

Nickel-based

Some NiCd and NiMH batteries are rated at 1.20V/cell, some at 1.25V. The cells are the same. These batteries generally have very low internal resistance and can tolerate high discharge and charge rates better than the others listed here.

Lithium-ion

The nominal voltage of most lithium-ion cells is 3.60V. Some cells are marked as 3.70V/cell but, there's only a minor internal difference. Charge to about 4.2V/cell and consider the cell discharged at about 2.8 to 3V/cell. While in use, the terminal voltage is less than V_{OC} (the open circuit voltage) because of R_{int} (the internal resistance). Cells marked at more than 3.7V/cell are constructed differently and require special chargers to reach maximum capacity. There are many different variations of lithium-ion and lithium-based cells which have somewhat different characteristics. I give general information here for only the most common. For more information, see:
https://batteryuniversity.com/learn/article/bu_216_summary_table_of_lithium_based_batteries
lithium-ion cells stored at 2V or less for any significant time can develop shorts and become dangerous to use.

Lithium-Polymer (Li-Po)

Essentially the same as lithium-ion except for how the battery layers are held together. They are usually flat, often packaged in foil pouches, and can be flexible.

Battery Capacity, C (Ah or mAh) and the C-rate (Not to be confused with a capacitor value, C, in μF)

Most rechargeable batteries are marked in Ah or mAh, which is the battery's capacity (C). This indirectly specifies how much charge or energy it can hold. Multiply C by the nominal voltage to estimate the energy stored in Wh. Multiply again by 3600sec/hr to estimate the energy stored in Joules.

$$C \cdot V_{nom} = \text{Energy}$$

The energy stored in a 3.6-V battery, rated at 3000mAh: $3000 \cdot mAh \cdot 3.6 \cdot V = 10.8 \cdot Wh$

$$3000 \cdot mAh \cdot 3.6 \cdot V \cdot 3600 \cdot \frac{sec}{hr} = 3.888 \cdot 10^{4} \cdot joule$$

If the battery supplies a lot of power, the battery will be drained of energy quickly. The battery will last longer at lower power use. All batteries will eventually die, even if not used (self discharge). The C-rate allows you to estimate the life of the battery in hours. A C-rate of 1C is also known as a one-hour discharge. Similarly, 0.5C or C/2 is a two-hour discharge and 0.2C or C/5 is a 5-hour discharge. Most batteries should not be charged or discharged above 1C to avoid shortening the battery lifetime. Exceptions are Nickel-based and some high-performance batteries. Lead acid batteries designed to start engines are designed for short periods of high-discharge.

A 3.6-V battery is rated at 3000mAh, estimate how long the battery can be discharged at 1A: $\frac{3000 \cdot mAh}{1 \cdot A} = 3 \cdot hr$

The C-rate of this 1-A discharge: $\frac{1 \cdot A}{3000 \cdot mA} = 0.333C = \frac{C}{3}$ IF the battery was fully charged to begin with.

The same battery starts fully charged, then supplies 1A for 40min. Estimate how long it can be discharged at 400mA thereafter:

$$3000 \cdot mAh - 1 \cdot A \cdot \left(40 \cdot min \cdot \frac{1 \cdot hr}{60 \cdot min}\right) = 2333 \cdot mAh$$

$\frac{2333}{3000} = 77.8 \cdot \%$ Charge remaining
State-of-charge (SoC)

$$\frac{2333 \cdot mAh}{400 \cdot mA} = 5.832 \cdot hr$$

$100 \cdot \% - 77.8 \cdot \% = 22.2 \cdot \%$ Operating Life

As a battery ages, its capacity decreases (fades). Unfortunately, after charging, most charge indicators will still indicate 100% charge for this diminished capacity.

Energy Density (Or Specific Energy)

The energy density of a battery is the energy stored at full charge per kilogram of weight (Wh/kg). For Li-ion that's about 200Wh/kg. By means of comparison, the potential energy in gasoline is over 12,000Wh/kg. However, you should also factor in the mass of internal-combustion engine and transmission and the overall efficiency (topping out at about 25%). For an automobile, this allows the battery, electronics and electric motor to compete quite well. The electrical system can top 90% efficiency and has the added advantage of energy recovery when braking or descending a grade.

Cells may be optimized for energy storage (energy density) or power delivery (power density) and there's a tradeoff. To optimize for power, a cell should have a very low internal resistance and ability to deliver over the 1C rate without undo stress.

Cycle life

The number of times a cell can be charged and discharged. This number is greatly affected by things like the depth of discharge, temperature, and other stresses placed on the battery during its lifetime.

Common standard lithium-ion cell sizes

18650 18 x 65mm 16.5mL
(~3000mAh) Commonly used in laptops, e-bikes, including Tesla EV cars.
Cells designed for individual consumer use usually also contain protection circuitry and measure about 18 x 67mm.
~$120/kWh, expected to fall to about $90 in the next 5 years.

26650 26 x 65mm 34.5mL
Some measure 26x70mm sold as 26700. Common chemistry is LiFeO4 for UPS, hobby, automotive. Fading popularity.

14500 14x 50mm
Similar size to AA. (Observe voltage incompatibility: NiCd/NiMH = 1.2V, alkaline = 1.5V, Li-ion = 3.6V)

21700 21 x 70mm 24mL (sometimes referred to as 2170)
(~6000mAh) Used for the Tesla Model 3 and other applications, made by Panasonic, Samsung, Molicel, etc..
~$100/kWh, expected to fall to about $75 in the next 5 years

32650 32 x 65mm
Primarily in LiFePO4 (Lithium Iron Phosphate)

mAh values shown above are for high-quality cells as of 2020

Energy density favors large cell sizes because packaging is, proportionally, less of the mass.
Smaller cells are easier to cool because they have a larger surface area.

Flat cells and battery packs are becoming much more common, but are not standardized.

Battery Pack

A battery pack usually contains multiple cells and some protection circuitry.

Cells in series:	Higher voltage and energy. No increase in current, C or Ah. May suffer from "unbalenced" cells that develop over many charge cycles.
Cells in parallel:	Higher current, energy, C and Ah. No increase in voltage. Cells should be matched for nominal voltages, but will otherwise balance out at each charge.

A Tesla Model 85 that uses over 7000 18650 cells to make up the 90kWh battery pack.

Discharging

Most cells can be discharged at a 1C rate, at least for short periods. If you need higher rates, you'll need to pay more attention to battery selection. For high discharge rates, the internal resistance becomes an important factor, so does a possible need for temperature management.

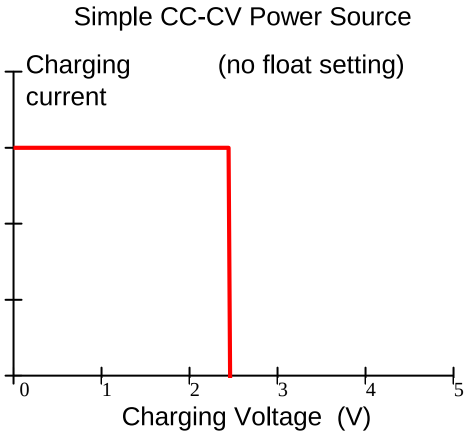
If you regularly recharge Li-ion when they reach 25% discharge, they will last a lot longer. However, an unused battery is best stored at 50% charge and at a cold temperature. Li-ion cells degrade with time, even if not used. Heat is the enemy.

All batteries, and especially rechargeable batteries, self discharge over time. Recharge them all at least twice per year.

Charging

Lead Acid

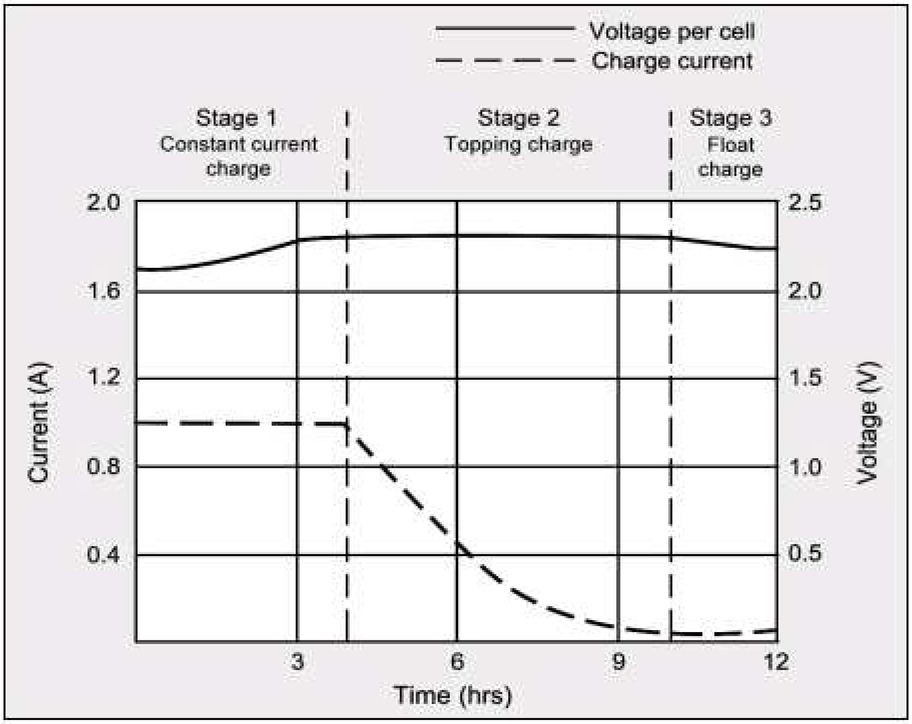
The simplest way to charge a lead-acid battery is with the constant-current - constant-voltage (CC-CV) method. Charge at a constant current of 0.1 to 0.3C until the voltage reaches 2.4V/cell (14.4V for 6 cells, a "12V" battery). Continue charging at a constant 2.4V/cell until the current drops to about 0.02C. Then remove the charger or reduce to a constant float voltage of 2.25V/cell. At temperatures above 25°C, reduce the voltages by 3mV/°C.



A healthy lead acid battery can be initially charged at up to 1.5C as long as the current is reduced when the battery reaches 2.3V/cell (14.0V for 6 cells)

Overcharging causes the battery to "bubble". The bubbles are hydrogen and oxygen caused by splitting water molecules. Needless to say, this can cause a fire and explosion hazard That's why charging should only be done in well ventilated areas.

A rested battery (No current in or out for some time) will have an open-circuit voltage of about 2.1V/cell at 90% charge. Avoid buying batteries the measure below 2.1V/cell at time of purchase. Never let the voltage drop below 2.05V/cell or the battery will suffer permanent damage.



Stage 1: Voltage rises at constant current to V-peak.

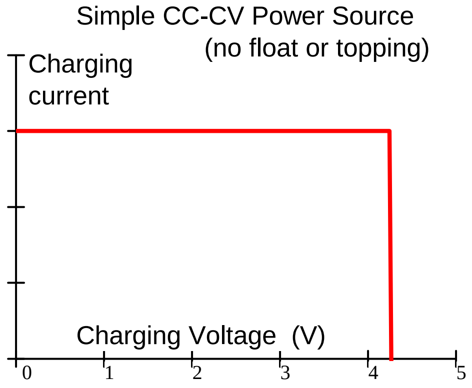
Stage 2: Current drops; full charge is reached when current levels off

Stage 3: Voltage is lowered to float charge level

A constant-current - constant-voltage power supply is not actually that rare. All power supplies have some current limit. The simplest is just a fuse, which burns out when the current limit is exceeded. A good bench power supply will be smarter than that. Many just limit the current to a maximum that you can set and that makes them behave exactly like a CC-CV supply. The supplies in our labs work that way.

Lithium-ion

Li-ion batteries may also be charged with the constant-current - constant-voltage (CC-CV) method. Charge at a constant current of 0.3 to 1C until the voltage reaches 4.2V/cell. If you can, monitor individual cells in a series battery pack. Continue charging at a constant 4.2V/cell until the current drops to about 0.02C. The charger should shut off completely at this point and only occasionally top up the battery when the voltage falls to 3.6V/cell. Never allow a cell to dwell at 4.20V for more than a few hours.

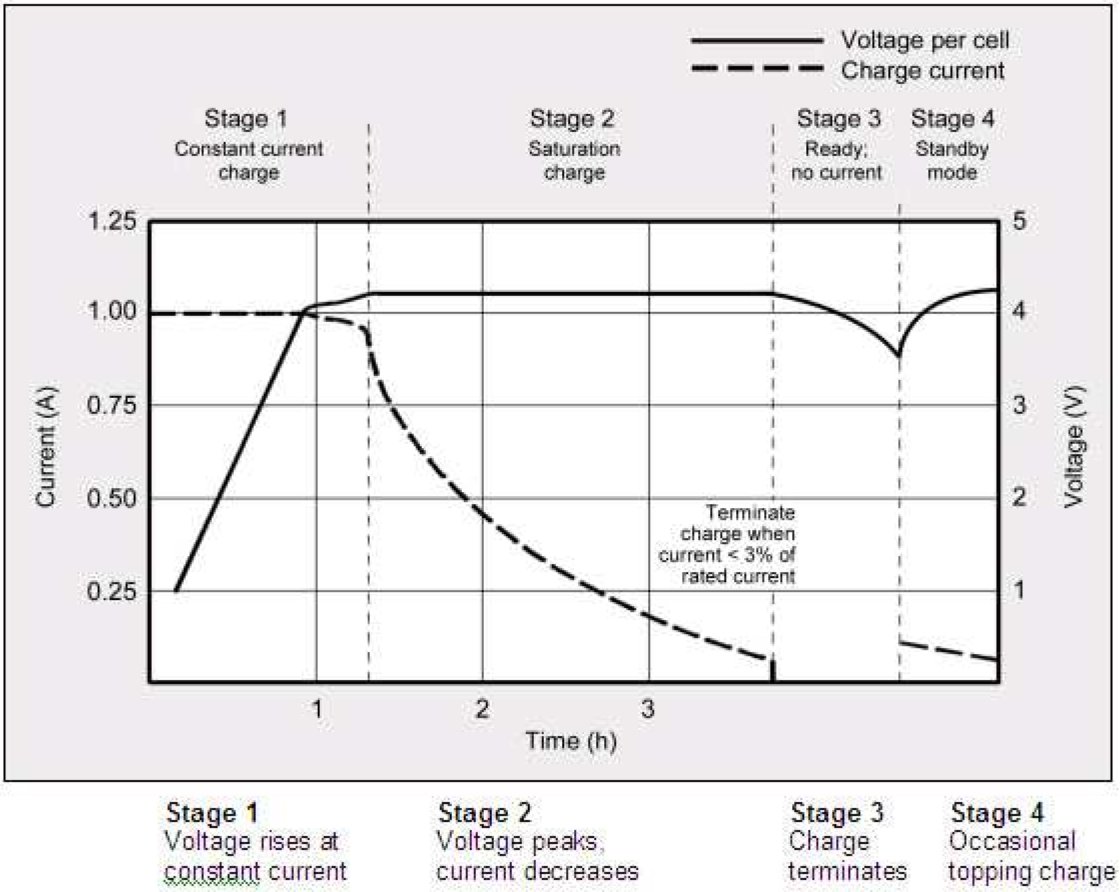


Most Li-ions are charged to 4.20V/cell, but, they will last longer if charged to a lower voltage. A lithium-ion cell charged to 4.20V/cell typically delivers 300–500 cycles. If charged to only 4.10V/cell, the life can be prolonged to 600–1,000 cycles (but the energy stored falls to about 87%). Charging to 4.0V/cell should deliver 1,200–2,000 cycles (but only about 73% energy storage). An extra advantage is significantly shorter charge times.
Don't float-charge Li-ions, not even at 3.6V/cell.

Please remember that not all Li-ion batteries charge to 4.20V/cell. Lithium iron phosphate typically charges to 3.65V/cell and lithium-titanate to 2.85V/cell. Some high-energy cells may accept 4.30V/cell or higher. If your batteries are not marked at 3.6 or 3.7V/cell, then you'd better do some investigation before charging with anything other than the manufacturer's charger.

Lithium-ion Charging Cycles

Only a full cycle (charge to 100%, discharge to 0%) provides the full specific energy of a battery. For a good Li-ion, this is about 250Wh/kg. If you only charge to 85% (4.1V/cell) and recharge at 25% percent, the specific energy density would be reduced from 250Wh/kg to 150Wh/kg, but you would double the lifetime of the cell. Consumer devices typically utilize the full energy of a battery. Industrial devices and electric vehicles, typically limit the charge to 85% and discharge to 25%, or 60 percent energy usability, to prolong battery lifetime.

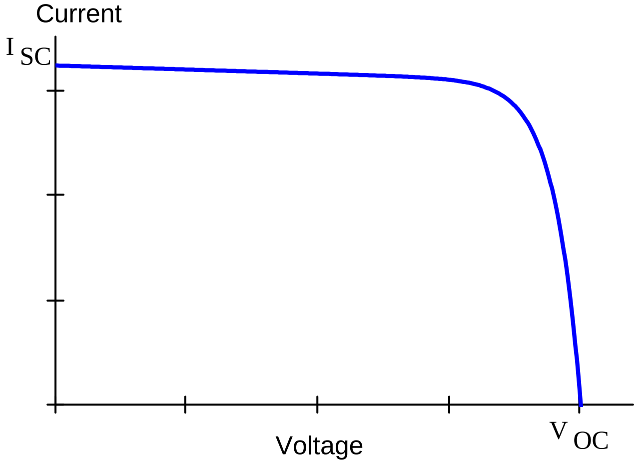


Solar Panels

Basic Characteristics

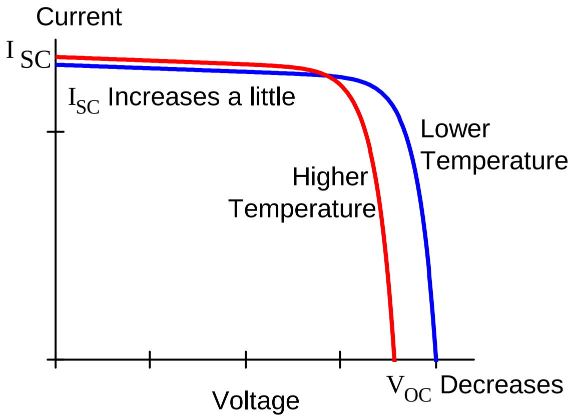
A solar panel is DC power source which derives it's power from light, usually sunlight. The most important electrical characteristic is the Current vs Voltage (IV) curve.

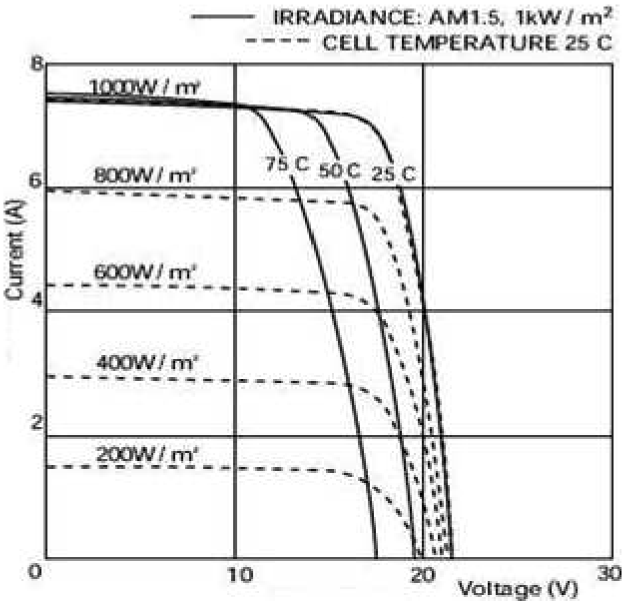
I_{SC} is the short-circuit current



V_{OC} is the open-circuit voltage

This curve is affected by temperature and light conditions.





Solar panels also degrade a little with time.

The information sticker on the back of a solar panel

HQST

HIGH QUALITY SOLAR TECHNOLOGY

Module Type:	**HQST-110D**
Max Power at STC (P_{max})	110 W
Open-Circuit Voltage (V_{oc})	22.2 V
Optimum Operating Voltage (V_{mp})	17.9 V
Optimum Operating Current (I_{mp})	6.16 A
Short-Circuit Current (I_{sc})	6.59 A
Temp Coefficient of P_{max}	-0.41%/°C
Temp Coefficient of V_{oc}	-0.32%/°C
Temp Coefficient of I_{sc}	0.05%/°C
Max System Voltage	600VDC (UL)
Max Series Fuse Rating	15 A
Fire Rating	Class C
Weight	7.5kgs / 16.5lbs
Dimensions	1020x670x35mm / 40.2x26.4x1.4in
STC	Irradiance 1000 W/m², T = 25°C, AM=1.5

WARNING: This module produces electricity when exposed to light. Please follow all applicable electrical safety precautions. Only qualified personnel should install or perform maintenance work on these modules. Beware of dangerously high DC voltages when connecting modules. Do not damage or scratch the rear surface of the module. Follow your battery manufacturer's recommendation.

Assembled in Mexico with solar cells from Vietnam

These numbers are supposed to be for **S**tandard **T**est **C**onditions (STC) which is:

1,000 Watts/m^2 Irradiance (sunlight intensity or power)

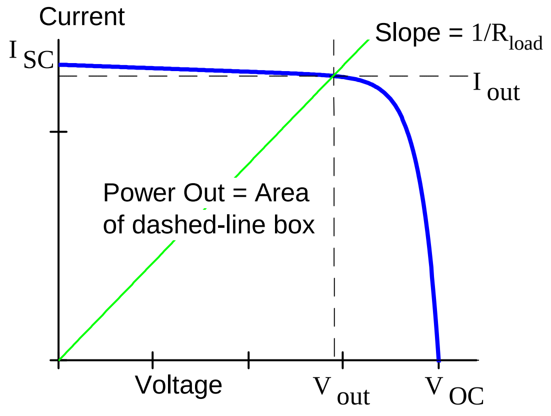
AM = 1.5, **A**ir **M**ass is a measurement of the clarity of the air above the panel.

25^{o}C Cell temperature

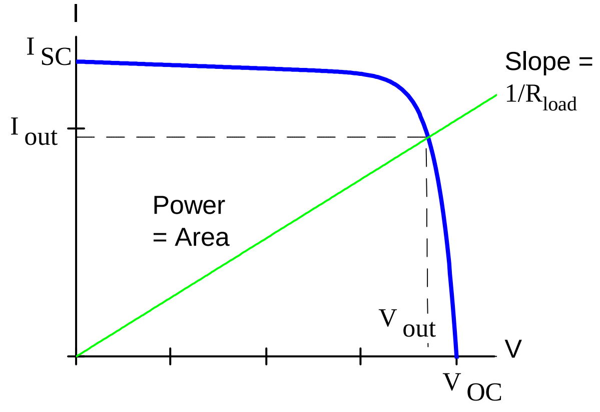
Power Output

The power supplied by any power source depends on the load.

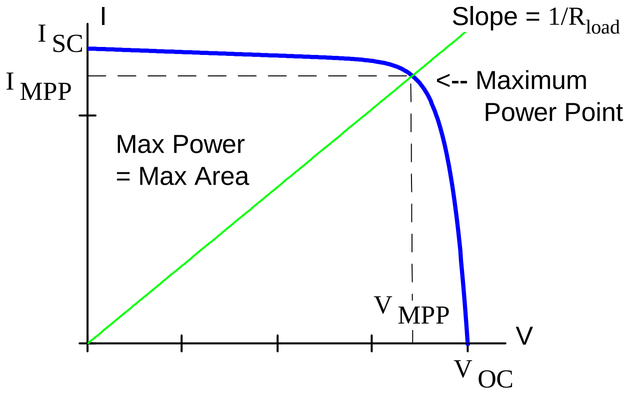
$$R_{load} = \frac{V}{I} = \frac{V_{out}}{I_{out}}$$



Replace R_{load} with a larger value --->



Replace R_{load} with the optimal value



The power supplied by any power source depends on the load. For a simple power supply with just V_S and R_S, if the load resistance equals R_S, you'll get the maximum output power. It turns out that maximum power is only 50% efficient and is rarely the goal. Solar panels are very different. Maximum power output is *very often* the goal and results in the *highest* power efficiency. Seems easy enough, to maximize P_{out} simply maximize the product of V_{out} and I_{out} and select a load resistor:

$$R_{load} = \frac{V_{MPP}}{I_{MPP}}$$

Just a few problems. You almost never hook a resistor up to a solar panel. That just makes heat, and, well, couldn't the sun do that directly? And, every time the curve changes (see previous page) the optimal value of resistance would also have to change. This is where the Maximum Power Point Tracker (MPPT) comes in. The MPPT (or Power Optimizer) is a computer- controlled device which finds the maximum power point of a solar panel and harvests power at just that point, converting the power to a voltage and current suitable for the load. To do this it must utilize a DC to DC power converter or a DC to AC power inverter.

Efficiency

$$\text{Efficiency} = \eta = \frac{\text{Power out}}{\text{Power in}} = \frac{P_{out}}{P_{in}}$$

The rated power is found at Standard Test Conditions (STC) and the Maximum Power Point (MMP).

At Standard Test Conditions (STC) the input power is ($1000W/m^2$) x (Area of the Panel).

Example: Using numbers from the information sticker shown earlier.

$$P_{in} := \left(\frac{1000\cdot W}{m^2}\right)\cdot(1.020\cdot m\cdot 0.670\cdot m) \qquad P_{in} = 683.4\cdot W$$

$P_{out} := 110\cdot W$ (rated) $V_{MMP} := 17.9\cdot V$ $I_{MMP} := 6.16\cdot A$ (from sticker)

$V_{MMP}\cdot I_{MMP} = 110.264\cdot W$ (calculated P_{out})

$$\text{Efficiency} = \eta = \frac{P_{out}}{P_{in}}\cdot 100\cdot\% = 16.096\cdot\%$$

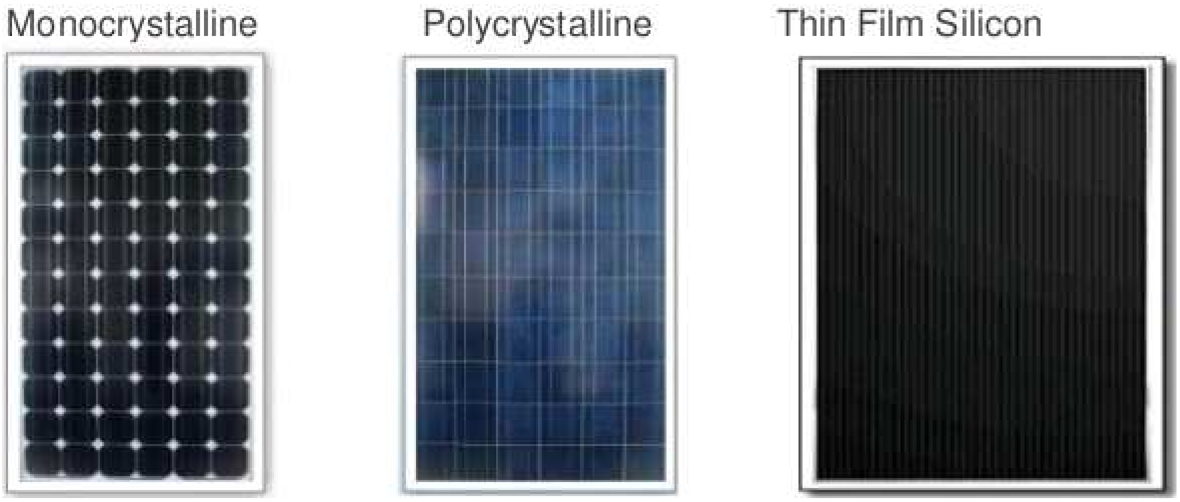
Probably a little optimistic

Common Types

Amorphous Silicon is a "thin film" technology where all the layers and connections needed are deposited layer-by-layer onto a substrate of glass or plastic. Those made on plastic can be flexible. Cheap, but usually less (often way less) than 8% efficient.

Monocrystalline Silicon solar cells are made on silicon "wafers" similar to those used to make integrated circuits. Typically 15-18% efficient.

Polycrystalline Silicon is cheaper and less efficient than monocrystalline. Typically 13-16% efficient

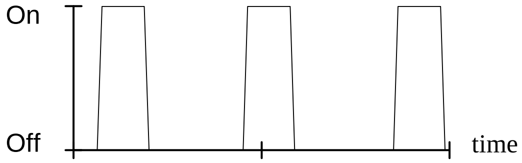


Using Solar Panels to Charge Batteries

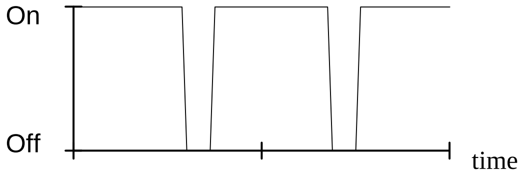
It is not a good idea to hook a solar panel directly to a battery. You need to use a charge controller to prevent overcharging. These controllers usually also control current to the load so as to prevent overly discharging the battery.

PWM: The simplest of these uses **P**ulse-**W**idth **M**odulation (PWM) to limit the battery current. That means they simply make and break the connection between the panel and the battery get the correct average current. They require a solar panel whose V_{OC} is several volts higher than the fully-charged battery voltage. They are not designed to harvest power from the panel in the most efficient way, at the maximum power point. They are inexpensive and work well if the panel and battery are matched.

Lower Current



Higher Current



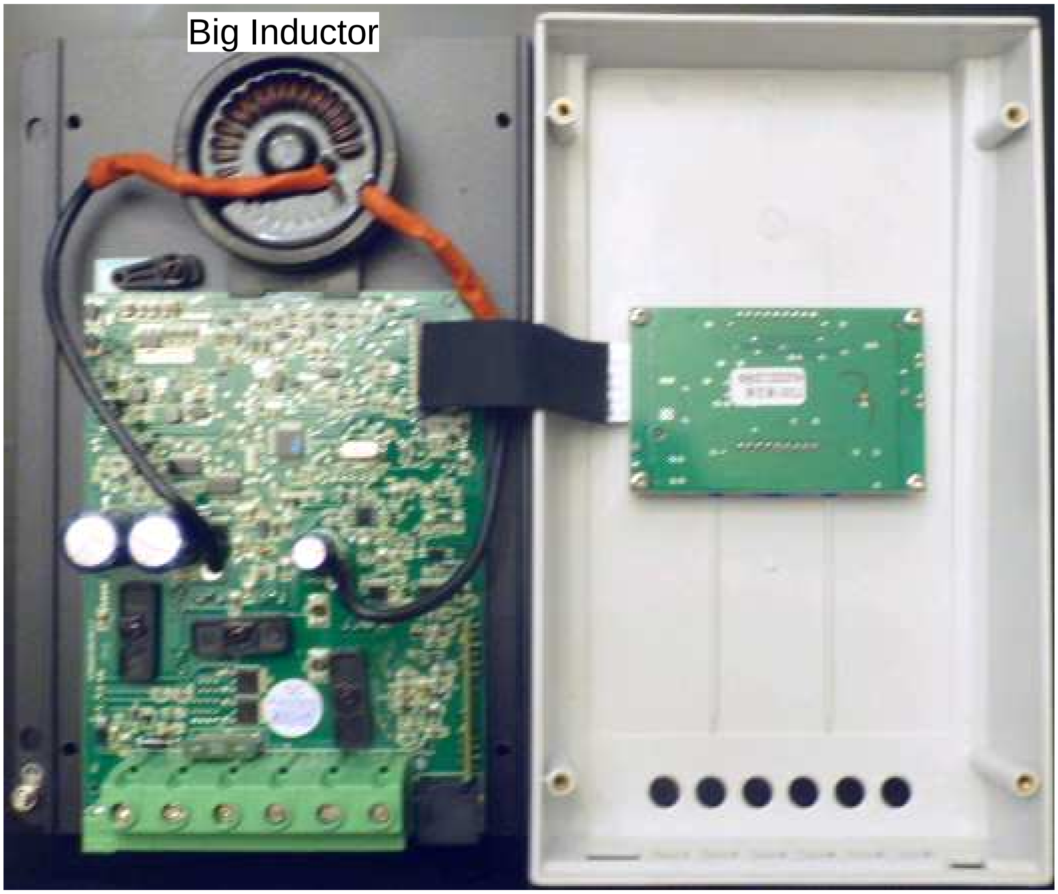
A PWM controller.
A deceptive use of MPPT in the model name.

MPPT: The Maximum Power Point Tracker (MPPT) has an internal computer which finds the maximum power point of a solar panel and harvests power at just that point. An internal DC to DC power converter matches up the voltage and current to the battery's needs. The MPPT must designed for the battery type and voltage that you have. Often they must be connected to the battery first, before connecting to the panel, in order to detect the battery characteristics.

Not all controllers sporting an MPPT label actually are MPPT controllers. Check customer feedback or check internally to see if it has the large inductor needed for the DC to DC power converter.

Using Solar Panels to Produce AC Power

To connect solar panels to the AC power grid requires a more complex device. It must control the panels, Invert the DC to AC power, and control the connection to the power line. These come in MPPT and non MPPT types. Some will allow you to power local items off-grid, serving as backup power, and many will not, working only when connected to the power company. They must also confirm to electrical codes and be approved by you power provider. You will also need a special power meter and a net-metering agreement with the power company.

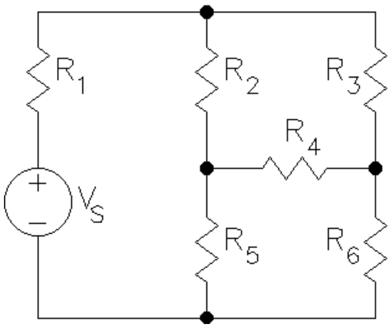


ECE 2210 / 00 homework # 6A Due: Mon, 2/8/21 a

Nodal Analysis

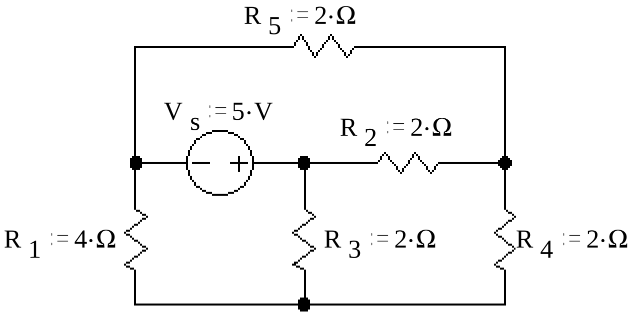
1. a) If you select the bottom node as ground, how many unknown node voltages remain? (Assume V_S is a known quantity.) How many simultaneous equations would you need to solve to analyze this circuit?

   b) Use nodal analysis to find all the necessary simultaneous equations.

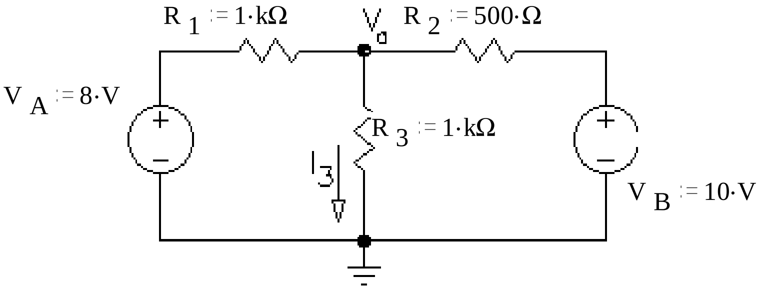


2. a) Use nodal analysis to find all the node voltages.

   b) Your node voltages will depend on your selection of a reference node (ground) as well as your arbitrary node labels, so the grader won't look at these specifically. Use your node voltages to find the potential (voltage) across each resistor. Report the magnitude and polarity of each.

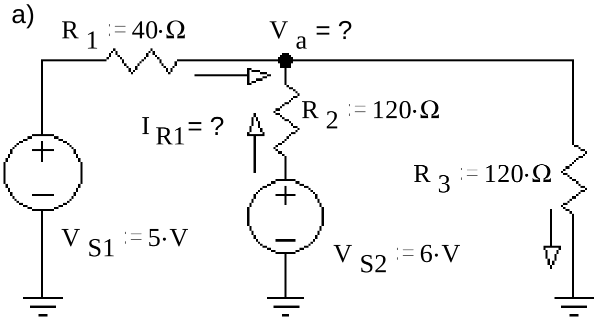


3. Use Nodal analysis to find V_a and use V_a to find I_3.

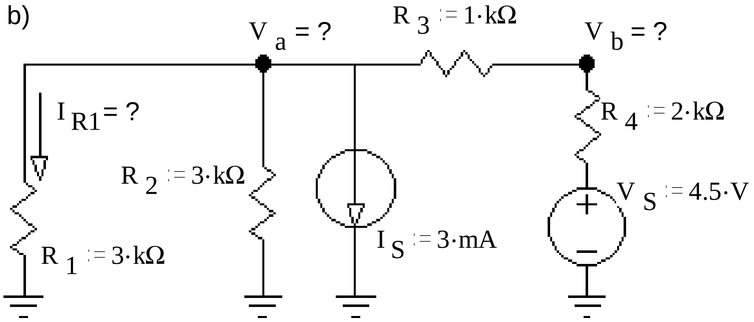


4. Use Nodal analysis to solve following problems: Each problem asks for at least 1 voltage and a current. Use the voltage(s) to find the current.

a)



b)



Don't forget your folder number.

hint: you may be able to eliminate one unknown node for the initial calculation.

Answers

1. a) 3,3 b) $V_a\cdot\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)-\frac{V_b}{R_2}-\frac{V_c}{R_3} = \frac{V_S}{R_1}$, $\frac{V_a}{R_2}-V_b\cdot\left(\frac{1}{R_2}+\frac{1}{R_5}+\frac{1}{R_4}\right)+\frac{V_c}{R_4} = 0\cdot A$

$$\frac{V_a}{R_3}+\frac{V_b}{R_4}-V_c\cdot\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_6}\right) = 0\cdot A$$

2. a) Answer will depend on your choice of ground, so check your answers to part b to see if you did part a right.
   b) $3.077\cdot V$, + bottom , $2.308\cdot V$, + left , $1.923\cdot V$, + top , $0.385\cdot V$, + bottom , $2.692\cdot V$, + right

3. $7\cdot V$, $7\cdot mA$ 4. a) $4.2\cdot V$, $20\cdot mA$ b) $V_a := -1.5\cdot V$ $V_b := 0.5\cdot V$ $I_{R1} := -0.5\cdot mA$

You may not get this homework back before the 1st **exam.** Photocopy it if you want to be sure to have it.

ECE 2210 / 00 homework # 6B Due: Thur, 2/11/21 a

Name ______________________________

Batteries

1. As a battery discharges, the source voltage (V_S) ______________________
   and the internal resistance (R_{int}) ______________________

2. a) The nominal voltage of a lead acid battery is _____________ per cell.
   Is this V_S or V_{term} ? ______________

   b) Over-charging a flooded, or wet cell lead acid battery creates what danger"

   c) How many cells does a 12-V car battery ?

3. a) The nominal voltage of a li-ion battery is _____________ per cell.

   b) How many cells does a 14.4-V laptop battery have?

4. A 14.4-V laptop battery is rated at 3Ah.

   a) How much energy does it store? Give the answer in two different units.

   b) The battery is dischared at 0.28C. How much current is that?

   c) How long should a full charge last at this rate of discharge?

5. You have 6 18650 li-ion cells, each rated at 2800mAh.

   a) The 6 cells are wired in series. What's the rated voltage?
   What's the rated mAh?

   b) The 6 cells are wired in parallel. What's the rated voltage?
   What's the rated mAh?

   c) Could you wire them up to get a 10.8V battery? If yes how?

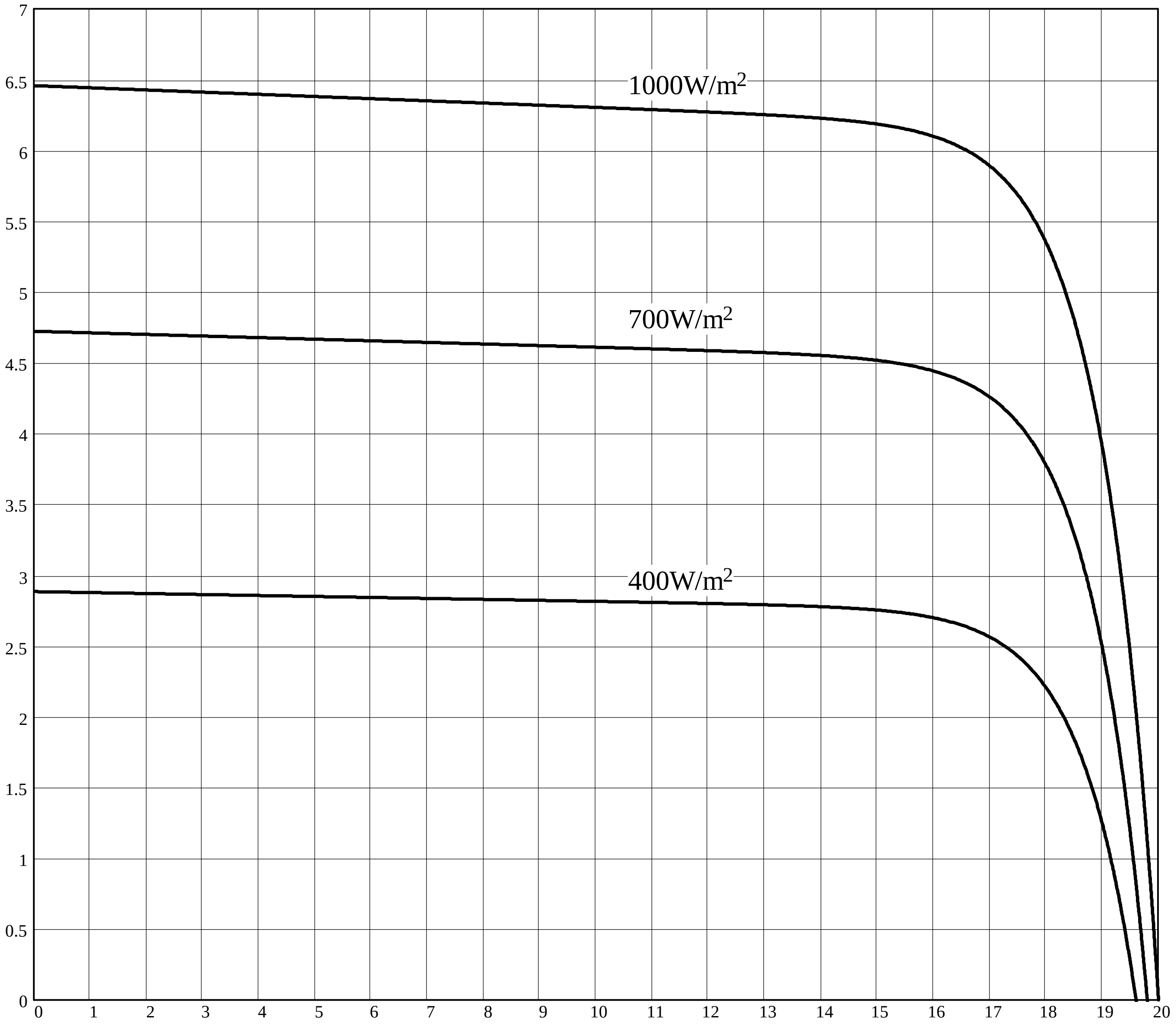
   What's the rated mAh?

6. a) How does a CC-CV power supply operate?

   b) Is it OK to leave a CC-CV power supply hooked to a lead acid or li-ion battery indefinitely? If not, why not?

Solar panels

Problems 7 - 14 refer to the IV curves for a solar panel shown below.



7. Find I_{SC} Find V_{OC}

8. Do your best to estimate the maximum-power point for the $1000W/m^2$ curve. Find the current (I_{MPP}), voltage (V_{MPP}), and power (P_{MPP}) and the load resistor (R_{load}) that could be hooked to the panel to make it operate at this point.

9. The panel measurements are $0.8m$ x $0.8m$, find the efficiency of this panel

10. What is the likely rated power of this panel?

11. Do your best to estimate the maximum-power point for the other two curves and find that maximum power.

12. The optimum load resistor (R_{load}) you found above is left in place while the irradiance is reduced to $700W/m^2$. Estimate the power produced by the panel.

    Note: there isn't a nice mathematical way to calculate this, so use this graphical method:
    1. Draw a current vs voltage line for the resistor, a line that starts at 0,0 and has a slope of $1/R_{load}$. Incidentally, this line passes through the maximum-power point for the $1000W/m^2$ curve.

    2. With an irradiance of $700W/m^2$ the panel will operate where this line crosses the $700W/m^2$ curve. Find the voltage, current and power at that point.

13. Using the same load resistor (R_{load}) and irradiance of $400W/m^2$. Estimate the power power produced by the panel.

14. Compare your answers to problems 12 and 13 to your answers to problem 11.

Answers

1. decreases increases
2. a) $2\cdot V$ V_{tern} b) fire c) 6
3. a) $3.6\cdot V$ b) 4
4. a) $43.2\cdot Wh$ $1.56\cdot 10^5\cdot J$ b) $0.84\cdot A$ c) $3.57\cdot hr$
5. a) $21.6\cdot V$ $2800\cdot mAh$ b) $3.6\cdot V$ $16800\cdot mAh$
   c) Two sets of 3 in series. The two sets are then wired in parallel. $5600\cdot mAh$

6. a) A CC-CV source acts like a constant current source up to a set voltage level.
   Then it acts as a constant voltage source at that level.
   b) No, a charger needs to shut off once the current decreases to some small value.

7. $6.45\cdot A$ $20\cdot V$

8. $5.87\cdot A$ $17.1\cdot V$ $100\cdot 4\cdot W$ $2.91\cdot\Omega$

9. $15.7\cdot\%$ 10. $100\cdot W$ 11. $72.5\cdot W$ $43.8\cdot W$

12. $60.5\cdot W$ 13. $23.4\cdot W$

14. Powers are significantly lower because the panel is not operating at the maximum power points.