Simple Model of a Real Source

Real sources are not ideal, but we will model them with two ideal components.

\[ I_{\text{term}} = 0 \quad \text{(short)} \]
\[ V_{\text{term}} + I_{L} R_{L} = 0 \quad \text{max power} \]
\[ V_{L} R_{L} = \infty \quad \text{(open)} \]

Note: \( R_{L} \) is NOT part of the Thévenin equivalent circuit and does not need to be shown.

Thévenin Equivalent Circuit

The same model can be used for any combination of sources and resistors.

Thévenin equivalent

To calculate a circuit's Thévenin equivalent:
1) Remove the load and calculate the open-circuit voltage where the load used to be. This is the Thévenin voltage \( (V_{\text{th}}) \).
2) Zero all the sources.
   (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
3) Compute the total resistance between the load terminals.
   (DO NOT include the load in this resistance.) This is the Thévenin source resistance \( (R_{\text{th}}) \).
4) Draw the Thévenin equivalent circuit and add your values.
Norton equivalent
To calculate a circuit's Norton equivalent:
1) Replace the load with a short (a wire) and calculate the short-circuit current in this wire.
   This is the Norton current ($I_N$). Remove the short.
2) Zero all the sources.
   (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
3) Compute the total resistance between the load terminals.
   (DO NOT include the load in this resistance.) This is the Norton source resistance ($R_N$).
   (Exactly the same as the Thévenin source resistance ($R_{Th}$)).
4) Draw the Norton equivalent circuit and add your values.

OR (the more common way)...  
1) Find the Thévenin equivalent circuit.
2) Convert to Norton circuit, then >>> $R_N = R_{Th}$ and $I_N = \frac{V_{Th}}{R_{Th}}$
Ex 1 Find the Thévenin equivalent:

To calculate a circuit's Thévenin equivalent:
1) Remove the load and calculate the open-circuit voltage where the load used to be.
   This is the Thévenin voltage ($V_{Th}$).

   $V_{oc} = V_{Th} = V_S \frac{R_2}{R_1 + R_2}$

   $V_{Th} = 15 \cdot \text{V}$

2) Zero all the sources.
   (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)

3) Compute the total resistance between the load terminals.
   (DO NOT include the load in this resistance.)
   This is the Thévenin source resistance ($R_{Th}$).

   $R_{Th} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$

   $R_{Th} = 30 \cdot \Omega$

4) Draw the Thévenin equivalent circuit and add your values.

   Thevenin equivalent circuit:
   $V_{Th} = 15 \cdot \text{V}$

   If the load were reconnected:
   $V_L = V_{Th} \frac{R_L}{R_{Th} + R_L} = 10 \cdot \text{V}$

   $I_L = \frac{V_{Th}}{R_{Th} + R_L} = 166.7 \cdot \text{mA}$

   $P_L = 10 \cdot \text{V} \cdot 166.7 \cdot \text{mA} = 1.667 \cdot \text{W}$

b) Find the Norton equivalent circuit:

   Norton equivalent circuit:
   $I_N = \frac{V_{Th}}{R_{Th}}$

   $I_N = 500 \cdot \text{mA}$
c) Show that the Thévenin circuit is indeed equivalent to the original at several values of $R_L$.

<table>
<thead>
<tr>
<th>$R_L$</th>
<th>Original Circuit</th>
<th>Thévenin Circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \cdot \Omega$</td>
<td>$0 \cdot V$</td>
<td>$V_S \frac{1}{R_1} = 500 \cdot mA$</td>
</tr>
<tr>
<td>$10 \cdot \Omega$</td>
<td>$R_o = \frac{1}{\frac{R_2}{R_L} + \frac{1}{R_L}}$</td>
<td>$R_o = 9.231 \cdot \Omega$</td>
</tr>
<tr>
<td>$V_L = V_S \frac{R_o}{R_1 + R_o} = 3.75 \cdot V$</td>
<td>$I_L = \frac{V_L}{R_L} = 375 \cdot mA$</td>
<td></td>
</tr>
<tr>
<td>$I_L = \frac{V_{Th}}{R_{Th} + R_L}$</td>
<td>$V_{L} = I_L \cdot R_L$</td>
<td></td>
</tr>
<tr>
<td>$P_L = V_L \cdot I_L = 0 \cdot W$</td>
<td></td>
<td></td>
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<tr>
<td>$1 \cdot \Omega$</td>
<td>$R_o = 0.992 \cdot \Omega$</td>
<td>$I_L = \frac{V_{Th}}{R_{Th} + R_L}$</td>
</tr>
<tr>
<td>$20 \cdot \Omega$</td>
<td>$R_o = 9.231 \cdot \Omega$</td>
<td>$V_L = 375 \cdot V$</td>
</tr>
<tr>
<td>$R_L$</td>
<td>$I_L$</td>
<td>$P_L$</td>
</tr>
<tr>
<td>$0 \cdot \Omega$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$1 \cdot \Omega$</td>
<td>$0.484$</td>
<td>$0.234$</td>
</tr>
<tr>
<td>$10 \cdot \Omega$</td>
<td>$3.75$</td>
<td>$1.406$</td>
</tr>
<tr>
<td>$20 \cdot \Omega$</td>
<td>$6$</td>
<td>$1.8$</td>
</tr>
<tr>
<td>$30 \cdot \Omega$</td>
<td>$7.5$</td>
<td>$1.875$</td>
</tr>
<tr>
<td>$40 \cdot \Omega$</td>
<td>$8.571$</td>
<td>$1.837$</td>
</tr>
<tr>
<td>$60 \cdot \Omega$</td>
<td>$10$</td>
<td>$1.667$</td>
</tr>
<tr>
<td>$120 \cdot \Omega$</td>
<td>$12$</td>
<td>$0.741$</td>
</tr>
<tr>
<td>$240 \cdot \Omega$</td>
<td>$13.333$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\infty \Omega$</td>
<td>$15$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Repeat these calculations for a number of load resistors.

**Plots**

- **Power delivered to the load ($R_L$) as a function of $R_L$**
- **Power delivered to the load ($R_L$) as a function of $R_L$**
Maximum power transfer

If I wanted to maximize the power dissipated by the load, what \( R_L \) would I choose?

\[
\begin{align*}
P_L &= \frac{V_L^2}{R_L} = \left(\frac{R_L}{R_S + R_L} \cdot rac{V_S}{R_L}\right)^2 \frac{1}{R_L} = \frac{R_L^2}{(R_S + R_L)^2} \cdot \frac{1}{R_L} ^2 \cdot \frac{1}{R_L} \\
&= \frac{R_L^2}{R_S^2 + 2R_S R_L + R_L^2} \cdot \frac{1}{R_L} = \frac{R_L^2}{R_S^2 + 2R_S R_L + R_L^2} \\
&= \frac{1}{R_S^2 + 2R_S R_L + R_L^2} \cdot \frac{V_S^2}{R_L} 
\end{align*}
\]

Next step would be to differentiate \( \frac{d}{dR_L} P_L(R_L) \),
set this equal to 0 and solve for \( R_L \) to find the maximum

Unfortunately this function is a pain to differentiate.
What if we just differentiate the denominator and find its minimum, wouldn't that work just as well?

\[
\frac{d}{dR_L} \left( \frac{R_S^2}{R_L} + 2R_S R_L + R_L^2 \right) = \frac{R_S^2}{R_L^3} + 0 + 1 = 0
\]

Maximum power transfer happens when: \( R_L = R_S \)

Just what we saw in Example 1

This is rarely important in power circuitry, where there should be plenty of power and \( R_S \) should be small. It is much more likely to be important in signal circuitry where the voltages can be very small and the source resistance may be significant -- say a microphone or a radio antenna.

All you need to remember is: \( R_L = R_S \) to maximize the power dissipation in \( R_L \)

What about efficiency?

\[
\eta (\%) = \frac{P_L(R_L)}{P_S(R_L)} = \frac{I^2 R_L}{I^2 (R_S + R_L)} = \frac{R_L}{R_S + R_L}
\]

The bigger \( R_L \) is, the higher the efficiency.
Ex 2  a) Find and draw the Thévenin equivalent circuit.

First do some simplification:

\[ V_{234} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_2 + R_4}} \]

Divide this voltage between \( R_2 \) and \( R_4 \):

\[ V_{234} = 9 \cdot V \]

\[ V_{Th} = \frac{R_4}{R_2 + R_4} \cdot V_{234} \]

\[ V_{Th} = 3 \cdot V \]

Find the Thévenin resistance:

\[ R_{Th} = \frac{1}{\frac{1}{R_4} + \frac{1}{R_2 + \frac{1}{\frac{1}{R_1} + R_3}}} \]

\[ R_{Th} = 750 \cdot \Omega \]

Thévenin equivalent circuit:

\[ V_{Th} = 3 \cdot V \]

If the load were reconnected:

\[ V_L = \frac{R_L}{R_{Th} + R_L} \cdot V_{Th} \]

\[ V_L = 1.125 \cdot V \]

\[ I_L = \frac{V_{Th}}{R_{Th} + R_L} \]

\[ I_L = 2.5 \cdot mA \]

b) Find and draw the Norton equivalent circuit.

\[ I_N = \frac{V_{Th}}{R_{Th}} \]

\[ I_N = 4 \cdot mA \]

\[ R_N := R_{Th} \]

\[ R_N = 750 \cdot \Omega \]
c) Use your Norton equivalent circuit to find the current through the load.

\[ I_L = \frac{1}{R_L} \frac{1}{R_N + \frac{1}{R_L}} I_N \]

\[ V_L = I_L R_L \]

\[ I_L = 2.5 \text{ mA} \]

\[ V_L = 1.125 \text{ V} \]

\[ R_L = 750 \text{ } \Omega \]

For maximum power transfer \[ R_L = R_{Th} = 750 \text{ } \Omega \]

d) What value of \( R_L \) would result in the maximum power delivery to \( R_L \)?

\[ P_L = \frac{V_L^2}{R_L} = 3 \text{ mW} \]

e) What is the maximum power transfer?

\[ R_{Th} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \]

\[ V_{Th} = 3 \text{ V} \]

\[ R_{Th} = 750 \text{ } \Omega \]

\[ V_L = \frac{V_{Th}}{2} \]

\[ R_L = 750 \text{ } \Omega \]

\[ P_L = \frac{V_L^2}{R_L} = 3 \text{ mW} \]

**Ex 3**

a) Find and draw the Thévenin & Norton equivalent circuits.

\[ V_{S1} = 10 \text{ V} \]

\[ V_{S2} = 20 \text{ V} \]

\[ V_{Th} = 10 \text{ V} + 2.5 \text{ V} \]

\[ I = 0.5 \text{ A} \]

\[ R_1 = 5 \text{ } \Omega \]

\[ R_2 = 15 \text{ } \Omega \]

\[ V_{Th} = 3.75 \text{ V} \]

\[ R_N = 3.8 \text{ } \Omega \]

\[ I_N = 3.333 \text{ A} \]

\[ I_N = \frac{V_{Th}}{R_{Th}} \]

\[ R_{Th} = 3.75 \text{ } \Omega \]

\[ V_{Th} = 12.5 \text{ V} \]

\[ R_L = 20 \text{ } \Omega \]

\[ V_L = \frac{R_L}{R_{Th} + R_L} V_{Th} = 10.526 \text{ V} \]
Ex 4  a) Find and draw the Thévenin & Norton equivalent circuits.

ECE 2210  Thevenin notes  p8

Use superposition to find $V_{Th}$.

$$V_{S} = 9 \cdot V$$

$$V_{Th.V} = \frac{R_2 + R_3}{R_1 + R_2 + R_3} \cdot V_S$$

$$V_{Th.V} = 8.1 \cdot V$$

$$V_{Th.I} = -I_{12} R_1$$

$$V_{Th} = V_{Th.V} + V_{Th.I}$$

$$V_{Th} = 6.9 \cdot V$$

Find the Thévenin resistance

$$R_{Th} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + R_3}}$$

$$R_{Th} = 36 \cdot \Omega$$

Thévenin equivalent circuit:

$$V_{Th} = 6.9 \cdot V$$

Norton equivalent circuit:

$$I_N = \frac{V_{Th}}{R_{Th}}$$

$$I_N = 191.7 \cdot mA$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$I_L = 63.889 \cdot mA$$

$$V_L = I_L \cdot R_L = 4.6 \cdot V$$
Ex 5 A NiCad Battery pack is used to power a cell phone. When the phone is switched on the battery pack voltage drops from 4.80 V to 4.65 V and the cell phone draws 50 mA.

\[ V_S := 4.80 \text{ V} \quad V_{50} := 4.65 \text{ V} \]

a) Draw a simple, reasonable model of the battery pack using ideal parts. Find the value of each part.

\[ R_S := \frac{V_S - V_{50}}{50 \text{ mA}} \]

\[ R_S = 3 \cdot \Omega \]

\[ V_S = 4.8 \cdot \text{V} \]

\[ V_{50} = 4.65 \cdot \text{V} \]

\[ V_{50} = V_S - 50 \text{ mA} \cdot R_S = 3.9 \cdot \text{V} \]

b) The cell phone is used to make a call. Now it draws 300 mA. What is the battery pack voltage now?

\[ V_B = V_S - I_{\text{call}} \cdot R_S = 3.9 \cdot \text{V} \]

\[ V_B = 4.8 \cdot \text{V} \]

\[ I_{\text{call}} := 300 \cdot \text{mA} \]

\[ R_S = 3 \cdot \Omega \]

\[ V_S = 4.8 \cdot \text{V} \]

\[ I_{\text{call}} = \frac{V_B}{R_S} = 100 \cdot \text{mA} \]

\[ V_B = V_S - I_{\text{call}} \cdot R_S = 3.9 \cdot \text{V} \]

\[ R_S = 3 \cdot \Omega \]

\[ V_S = 4.8 \cdot \text{V} \]

\[ V_{50} := 4.65 \cdot \text{V} \]

Ex 6 Consider the circuit at right.

a) What value of load resistor \( R_L \) would you choose if you wanted to maximize the power dissipation in that load resistor.

\[ R_L := R_S \]

\[ R_L = 8 \cdot \Omega \]

\[ I_S := 1 \cdot \text{A} \]

\[ R_S := 8 \cdot \Omega \]

\[ R_L \]

\[ I_L := \frac{I_S}{2} \]

\[ P_L = I_L^2 \cdot R_L = 2 \cdot \text{W} \]
Use superposition to find $V_{Th}$.

1. **Current divider:**
   
   $$I_{R4} = \frac{1}{R_2 + R_4 + R_6} I_S$$
   
   $$I_{R4} = 35.556 \text{ mA}$$

2. **Eliminate voltage source:**
   
   $$V_{Th.I} = I_{R4} R_4$$
   
   $$V_{Th.I} = 3.2 \text{ V}$$

3. **Eliminate current source:**
   
   $$V_{Th.V} = \frac{R_4}{R_1 + R_2 + R_4 + R_6} V_S$$
   
   $$V_{Th.V} = 4 \text{ V}$$

4. **Thévenin equivalent circuit:**
   
   $$V_{Th} = V_{Th.V} + V_{Th.I}$$
   
   $$V_{Th} = 7.2 \text{ V}$$

5. **Put the load back on:**
   
   $$V_{Th} = 7.2 \text{ V}$$

6. **Norton equivalent circuit:**
   
   $$I_N = \frac{V_{Th}}{R_{Th}}$$
   
   $$I_N = 120 \text{ mA}$$
   
   $$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$
   
   $$I_L = 80 \text{ mA}$$
   
   $$V_L = I_L R_L = 2.4 \text{ V}$$
Thevenin & Norton equivalent circuits
1. For each of the circuits below, find and draw the Thevenin equivalent circuit.

   a) \[ R_1 = 36 \, \text{k}\Omega \quad R_2 = 12 \, \text{k}\Omega \]

   b) The load resistor is \( R_L \), and is in a strange place in this circuit.

      Hint: use superposition to find \( V_{Th} \).

2. For the circuit of problem 1a, find the voltage across \( R_L \) (\( V_L \)) and the current through \( R_L \) (\( I_L \)) using your Thevenin equivalent circuit.

3. For each of the circuits in problem 1, find and draw the Norton equivalent circuit.

4. For the circuit of problem 1b, find \( V_L \) and \( I_L \) using your Norton equivalent circuit.

5. For the circuit shown at right, use Thevenin's theorem to find the current through the 50 \( \Omega \) resistor \( R_4 \).

6. For the circuit shown, use Norton's theorem to find the value of the current in \( R_S \). Hint: You can find \( I_N \) either by calculation of the open circuit voltage (\( V_{OC} \)) and \( R_N \) or by direct calculation of the short-circuit current (\( I_{SC} \)), however, there is something about the values of the resistors which makes the second method easier than it would at first appear.

Source resistance
7. The terminal voltage of a car's battery drops from 12.5 V to 8.5 volts when starting. The starter motor draws 60 A of current.

   a) Draw the voltage-source model (Thevenin equivalent) of this battery. Include the values of \( V_S \) and \( R_S \).

   b) Draw the current-source model (Norton equivalent) of this battery. Include the values of \( I_S \) and \( R_S \).

   c) Which of these two models is more appropriate for the car battery?

   d) What terminal voltage would you expect if this battery were being charged at 20 A?

Answers
1. a) 4.091-V , 28.4-k\Omega   b) 1.1-V , 18.3-\Omega
2. 1.69-V , 84.6-\mu A
3. a) 0.144-mA , 28.4-k\Omega   b) 60-mA , 18.3-\Omega
4. 3.16-mA, 1.042-V
5. 1.88-mA
6. 0.19-A
7. a) \( V_S = 12.5-V \quad R_S = 0.0667-\Omega \)
     b) \( I_S = 187.5-A \quad R_S = 0.0667-\Omega \)
     c) Thevenin
     d) 13.83-V