ECE 2210 Lectures notes Thévenin & Norton Equivalent Circuits

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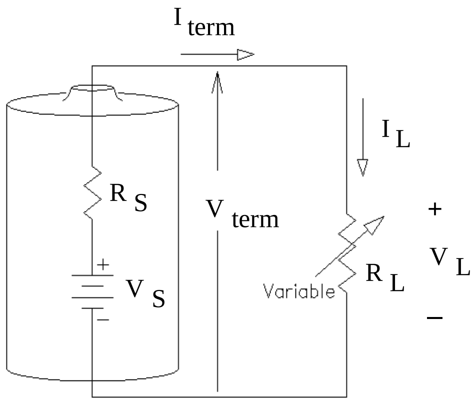
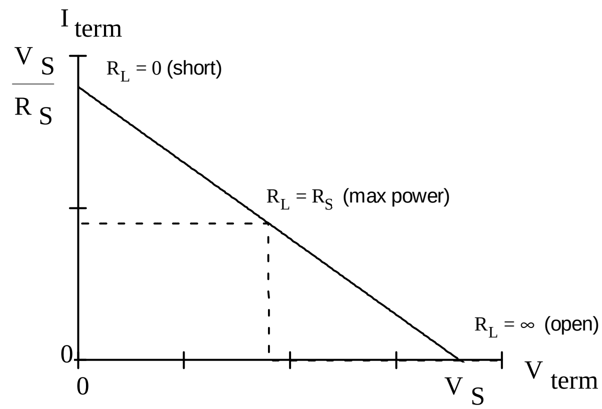
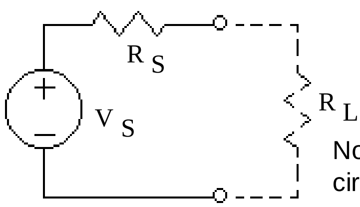
1/28/06
9/2/09

Simple Model of a Real Source

Real sources are not ideal, but we will model them with two ideal components.



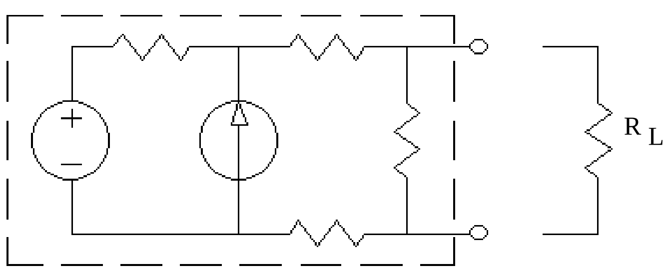


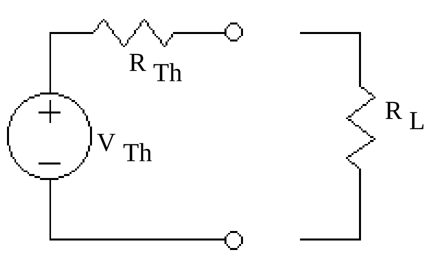


Note: R_L is NOT part of the Thévenin equivalent circuit and does not need to be shown.

Thévevin Equivalent Circuit

The same model can be used for any combination of sources and resistors.

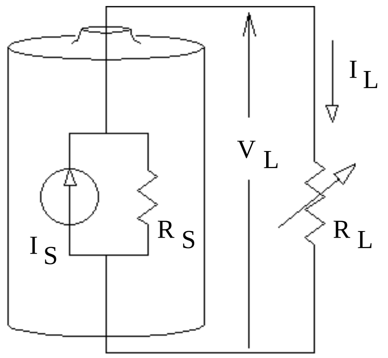


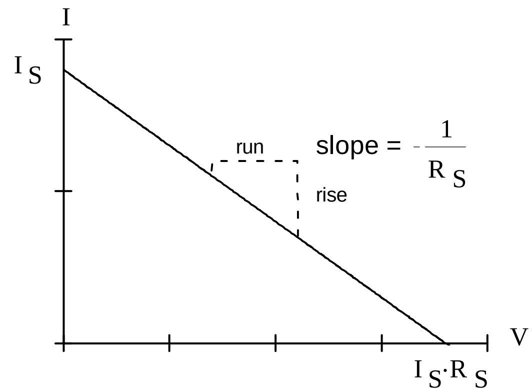


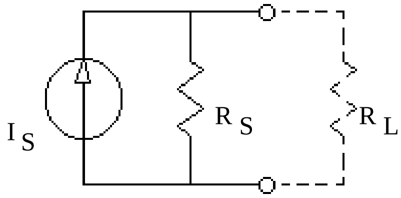
Thévenin equivalent

To calculate a circuit's Thévenin equivalent:

1) Remove the load and calculate the open-circuit voltage where the load used to be.
   This is the Thévenin voltage (V_{Th}).
2) Zero all the sources.
   (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
3) Compute the total resistance between the load terminals.
   (DO NOT include the load in this resistance.) This is the Thévenin source resistance (R_{Th}).
4) Draw the Thévenin equivalent circuit and add your values.





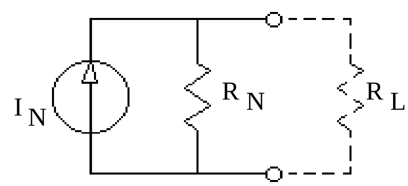


Note: R_L is not part of the Norton equivalent and does not need to be shown.

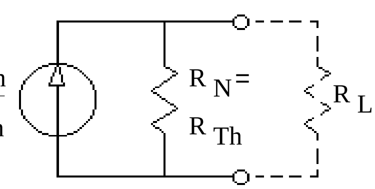
Norton equivalent

To calculate a circuit's Norton equivalent:

1) Replace the load with a short (a wire) and calculate the short-circuit current in this wire.
   This is the Norton current (I_N). Remove the short.
2) Zero all the sources.
   (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
3) Compute the total resistance between the load terminals.
   (DO NOT include the load in this resistance.) This is the Norton source resistance (R_N).
   (Exactly the same as the Thévenin source resistance (R_{Th})).
4) Draw the Norton equivalent circuit and add your values.



OR (the more common way)...
1) Find the Thévenin equivalent circuit.
2) Convert to Norton circuit, then >>> $R_N = R_{Th}$ and $I_N = \frac{V_{Th}}{R_{Th}}$



Thévevin & Norton Examples

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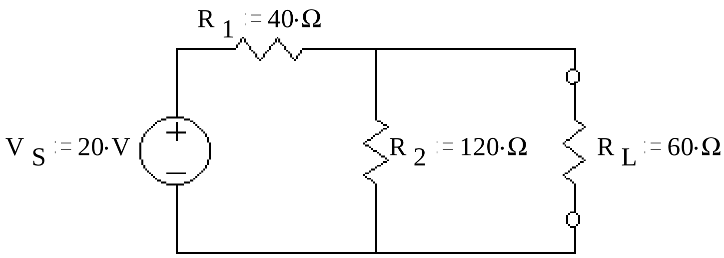
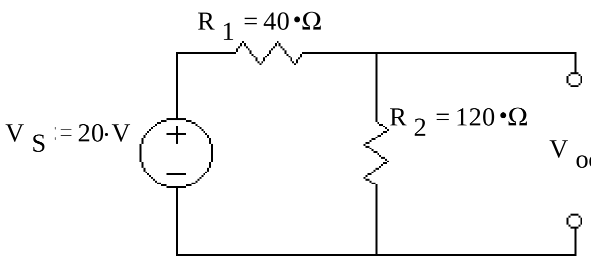
1/23/03,
1/6/13

Ex 1 Find the Thévenin equivalent:



To calculate a circuit's Thévenin equivalent:

1) Remove the load and calculate the open-circuit voltage where the load used to be.
This is the Thévenin voltage (V_{Th}).

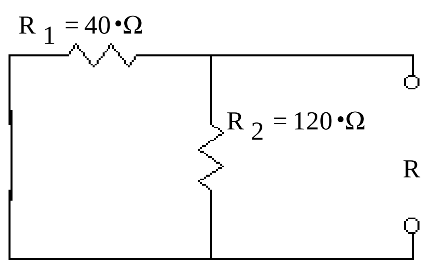


Find the open circuit voltage:

$$V_{oc} = V_{Th} := V_S \cdot \frac{R_2}{R_1 + R_2} \qquad V_{Th} = 15 \cdot V$$

2) Zero all the sources.
(To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)

Zero the source



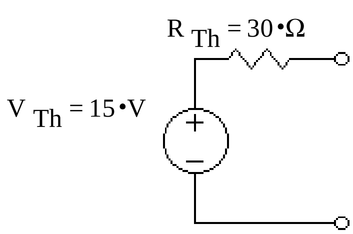
3) Compute the total resistance between the load terminals.
(DO NOT include the load in this resistance.)
This is the Thévenin source resistance (R_{Th}).

Find the Thevenin resistance:

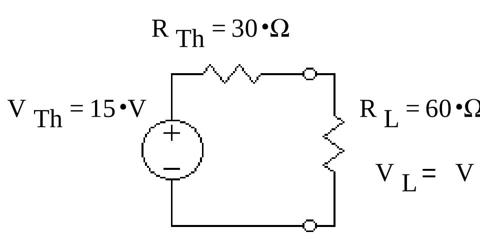
$$R_{Th} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \qquad R_{Th} = 30 \cdot \Omega$$

4) Draw the Thévenin equivalent circuit and add your values.

Thevenin equivalent circuit:



If the load were reconnected:

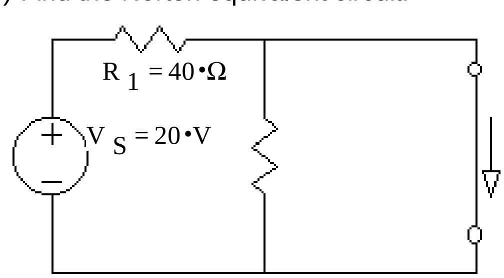


$$V_L = V_{Th} \cdot \frac{R_L}{R_{Th} + R_L} = 10 \cdot V$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = 166.7 \cdot mA$$

$$P_L = 10 \cdot V \cdot 166.7 \cdot mA = 1.667 \cdot W$$

b) Find the Norton equivalent circuit:



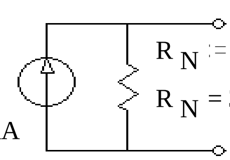
$$I_N := \frac{V_S}{R_1}$$

$$I_N = 500 \cdot mA$$

Norton equivalent circuit:

$$I_N := \frac{V_{Th}}{R_{Th}}$$

$$I_N = 500 \cdot mA$$



$$R_N := R_{Th}$$

$$R_N = 30 \cdot \Omega$$

c) Show that the Thévenin circuit is indeed equivalent to the original at several values of R_L.

	Original Circuit		Thévenin Circuit	
R_L	V_L	I_L	I_L	V_L
$R_L := 0\cdot\Omega$	$0\cdot V$	$\frac{V_S}{R_1} = 500\cdot mA$	$\frac{V_{Th}}{R_{Th}+R_L} = 500\cdot mA$	$500\cdot mA\cdot 0\cdot\Omega = 0\cdot V$

Using either numbers: $P_L = V_L\cdot I_L = 0\cdot W$

$R_L := 10\cdot\Omega$ $\quad R_o := \dfrac{1}{\dfrac{1}{R_2}+\dfrac{1}{R_L}}$ $\quad R_o = 9.231\cdot\Omega$ $\quad I_L := \dfrac{V_{Th}}{R_{Th}+R_L}$ $\quad V_L := I_L\cdot R_L$

$I_L = 375\cdot mA$ $\quad V_L = 3.75\cdot V$

$$V_L = V_S\cdot\frac{R_o}{R_1+R_o} = 3.75\cdot V$$

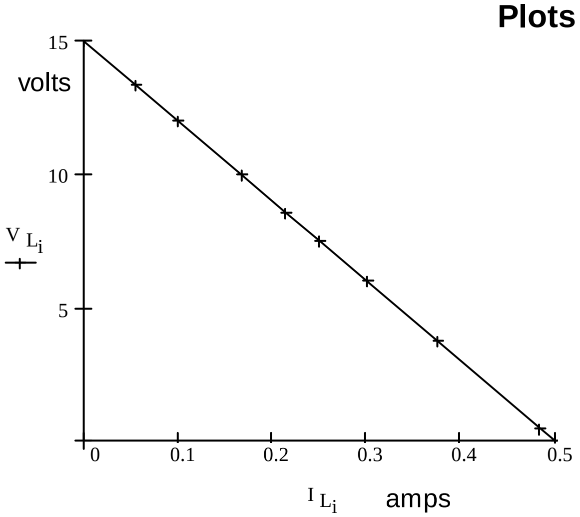
$$I_L = \frac{V_L}{R_L} = 375\cdot mA$$

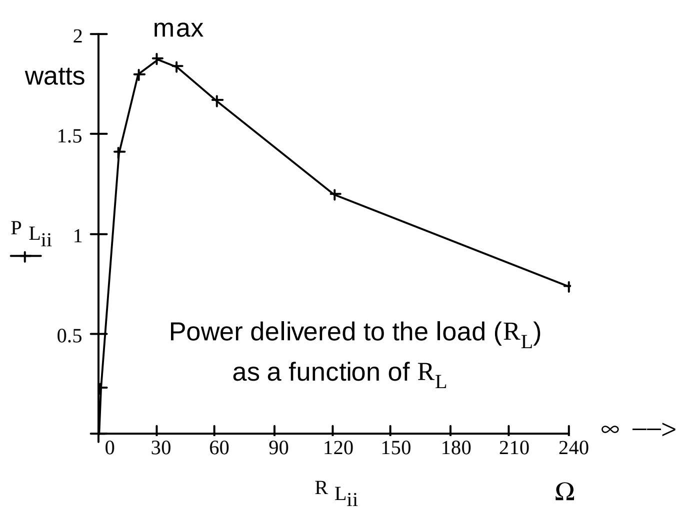
Using either numbers: $P_L = V_L\cdot I_L = 1.406\cdot W$

Repeat these calculations for a number of load resistors

$R_{L_i} :=$	R_{o_i} (Ω)	$V_L = V_S\cdot\frac{R_{o_i}}{R_1+R_{o_i}}$ (V)	$I_L = \frac{V_{L_i}}{R_{L_i}}$ (mA)	$I_L = \frac{V_{Th}}{R_{Th}+R_{L_i}}$ (mA)	$V_L = I_{L_i}\cdot R_{L_i}$ (V)	P_{L_i} (W)	
$0\cdot\Omega$	0	0	0	500	0	0	
$1\cdot\Omega$	0.992	0.484	483.871	483.871	0.484	0.234	
$10\cdot\Omega$	9.231	3.75	375	375	3.75	1.406	
$20\cdot\Omega$	17.143	6	300	300	6	1.8	
$30\cdot\Omega$	24	7.5	250	250	7.5	1.875	max
$40\cdot\Omega$	30	8.571	214.286	214.286	8.571	1.837	
$60\cdot\Omega$	40	10	166.667	166.667	10	1.667	
$120\cdot\Omega$	60	12	100	100	12	1.2	
$240\cdot\Omega$	80	13.333	55.556	55.556	13.333	0.741	
$\infty\cdot\Omega$	120	15	0	0	15	0	

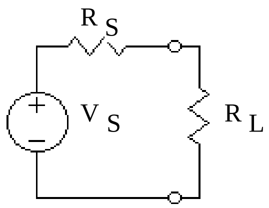
Plots





Maximum power transfer

If I wanted to maximize the power dissipated by the load, what R_L would I choose?



$$P_L = \frac{V_L^2}{R_L} = \left(\frac{R_L}{R_S + R_L}\cdot V_S\right)^2 \cdot \frac{1}{R_L} = \frac{R_L^2}{(R_S + R_L)^2}\cdot V_S^2 \cdot \frac{1}{R_L}$$

$$= \frac{R_L^2}{R_S^2 + 2\cdot R_S\cdot R_L + R_L^2}\cdot V_S^2\cdot\frac{1}{R_L} = \frac{R_L}{R_S^2 + 2\cdot R_S\cdot R_L + R_L^2}\cdot V_S^2$$

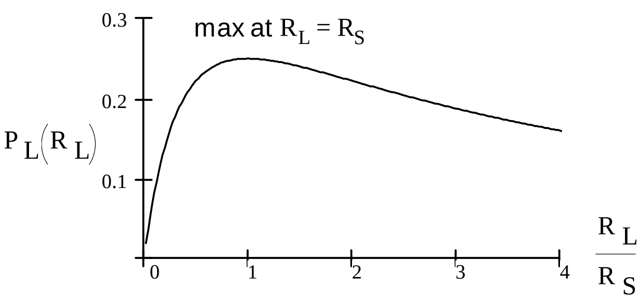
$$= \frac{1}{\dfrac{R_S^2}{R_L} + 2\cdot R_S + R_L}\cdot V_S^2$$

Next step would be to differentiate $\frac{d}{dR_L}P_L(R_L)$,

set this equal to 0 and solve for R_L to find the maximum

Unfortunately this function is a pain to differentiate. What if we just differentiate the denominator and find its minimum, wouldn't that work just as well?

$$\frac{d}{dR_L}\left(\frac{R_S^2}{R_L} + 2\cdot R_S + R_L\right) = -1\cdot\frac{R_S^2}{R_L^2} + 0 + 1 = 0$$



Maximum power transfer happens when: $R_L = R_S$

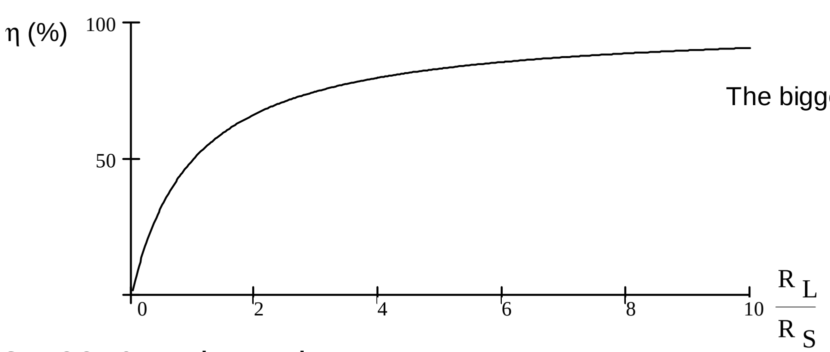
Just what we saw in Example 1

This is rarely important in power circuitry, where there should be plenty of power and R_S should be small. It is much more likely to be important in signal circuitry where the voltages can be very small and the source resistance may be significant -- say a microphone or a radio antenna.

All you need to remember is: $R_L = R_S$ to maximize the power dissipation in R_L

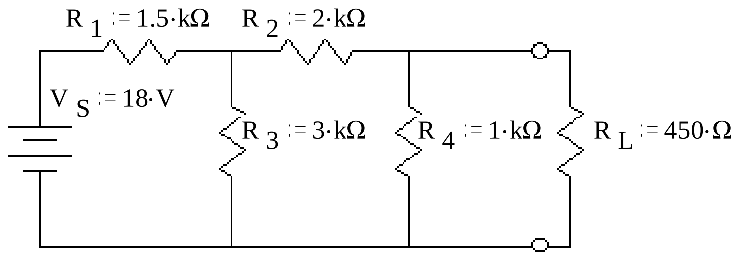
What about efficiency?

$$\frac{P_L(R_L)}{P_S(R_L)} = \frac{I^2\cdot R_L}{I^2\cdot(R_S + R_L)} = \frac{R_L}{R_S + R_L}$$

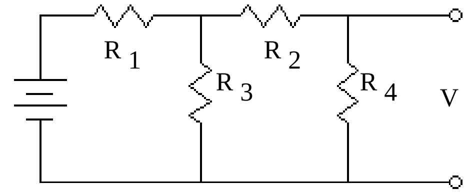


The bigger R_L is, the higher the efficiency.

Ex 2 a) Find and draw the Thévenin equivalent circuit.

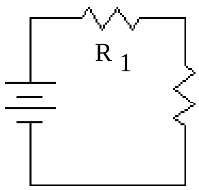


Find the open circuit voltage:



$V_{oc} = V_{Th}$

First do some simplification:



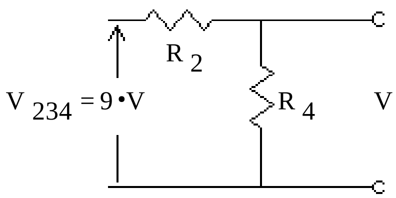
$$R_{eq234} := \frac{1}{\dfrac{1}{R_3} + \dfrac{1}{R_2 + R_4}}$$

$R_{eq234} = 1.5 \cdot k\Omega$

$$V_{234} := \frac{R_{eq234}}{R_1 + R_{eq234}} \cdot V_S$$

$V_{234} = 9 \cdot V$

Divide this voltage between R_2 and R_4:

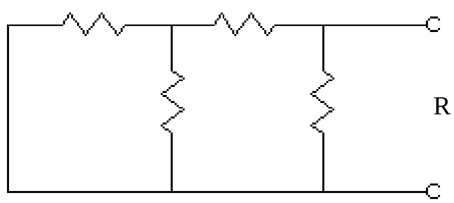


$$V_{Th} := \frac{R_4}{R_2 + R_4} \cdot V_{234}$$

$V_{Th} = 3 \cdot V$

Find the Thévenin resistance:

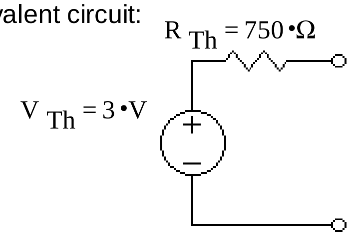
Zero the source



$$R_{Th} := \frac{1}{\dfrac{1}{R_4} + \dfrac{1}{R_2 + \dfrac{1}{\left(\dfrac{1}{R_1} + \dfrac{1}{R_3}\right)}}}$$

$R_{Th} = 750 \cdot \Omega$

Thévenin equivalent circuit:



If the load were reconnected:

$$V_L := V_{Th} \cdot \frac{R_L}{R_{Th} + R_L}$$

$V_L = 1.125 \cdot V$

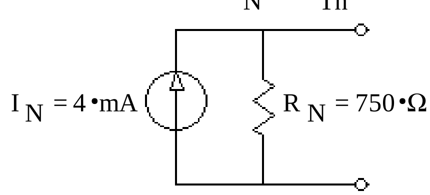
$$I_L := \frac{V_{Th}}{R_{Th} + R_L}$$

$I_L = 2.5 \cdot mA$

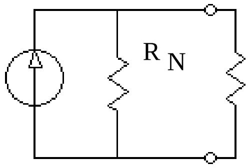
b) Find and draw the Norton equivalent circuit.

$R_N := R_{Th}$

$$I_N := \frac{V_{Th}}{R_{Th}}$$



c) Use your Norton equivalent circuit to find the current through the load.



$I_N = 4\,\mathrm{mA}$ $\quad R_N$ $\quad R_L = 450\,\Omega$

$$I_L := \frac{\frac{1}{R_L}}{\left(\frac{1}{R_N}+\frac{1}{R_L}\right)}\cdot I_N \qquad I_L = 2.5\,\mathrm{mA}$$

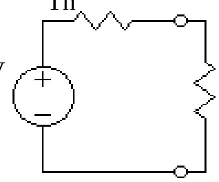
$$V_L := I_L\cdot R_L \qquad V_L = 1.125\,\mathrm{V}$$

same as above

d) What value of R_L would result in the maximum power delivery to R_L?

For maximum power transfer $R_L = R_{Th} = 750\,\Omega$

e) What is the maximum power transfer?

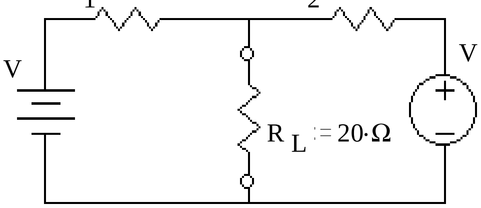


$R_{Th} = 750\,\Omega$ $\quad V_{Th} = 3\,\mathrm{V}$ $\quad R_L := 750\cdot\Omega$

$$V_L := \frac{V_{Th}}{2}$$

$$P_L = \frac{V_L^{\,2}}{R_L} = 3\,\mathrm{mW}$$

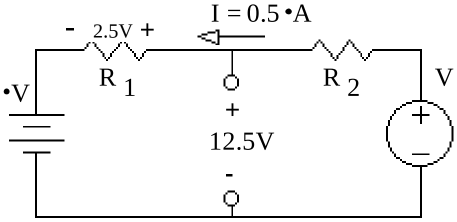
Ex 3 a) Find and draw the Thévenin & Norton equivalent circuits.



$R_1 := 5\cdot\Omega$ $\quad R_2 := 15\cdot\Omega$ $\quad V_{S1} := 10\cdot V$ $\quad V_{S2} := 20\cdot V$ $\quad R_L := 20\cdot\Omega$

$$I := \frac{V_{S2} - V_{S1}}{R_1 + R_2}$$

$I = 0.5\,\mathrm{A}$

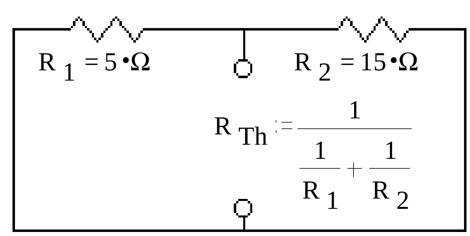


- 2.5V +

$V_{S1} = 10\,\mathrm{V}$ $\quad R_1$ $\quad R_2$ $\quad V_{S2} = 20\,\mathrm{V}$

+ 12.5V -

$V_{Th} := 10\cdot V + 2.5\cdot V$

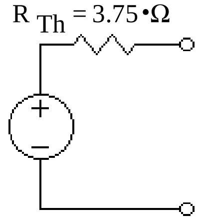


$R_1 = 5\,\Omega$ $\quad R_2 = 15\,\Omega$

$$R_{Th} := \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$$

$R_{Th} = 3.75\,\Omega$

Thévenin equivalent circuit: $R_{Th} = 3.75\,\Omega$ $\quad V_{Th} = 12.5\,\mathrm{V}$

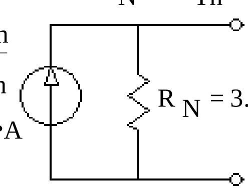


Norton equivalent circuit:

$R_N := R_{Th}$

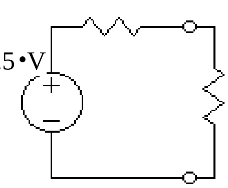
$$I_N := \frac{V_{Th}}{R_{Th}}$$

$I_N = 3.333\,\mathrm{A}$ $\quad R_N = 3.8\,\Omega$



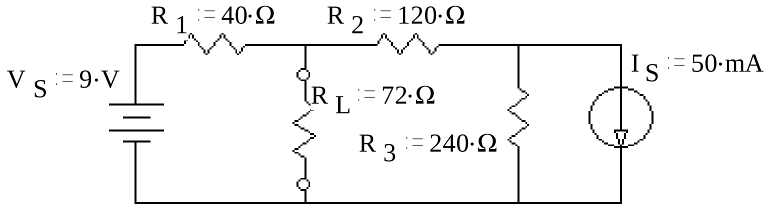
b) Use your Thévenin equivalent circuit to find the voltage across the load.

$R_{Th} = 3.75\,\Omega$ $\quad V_{Th} = 12.5\,\mathrm{V}$ $\quad R_L = 20\,\Omega$

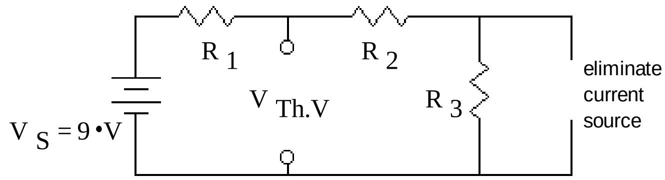


$$V_L = \frac{R_L}{R_{Th}+R_L}\cdot V_{Th} = 10.526\,\mathrm{V}$$

Ex 4 a) Find and draw the Thévenin & Norton equivalent circuits.

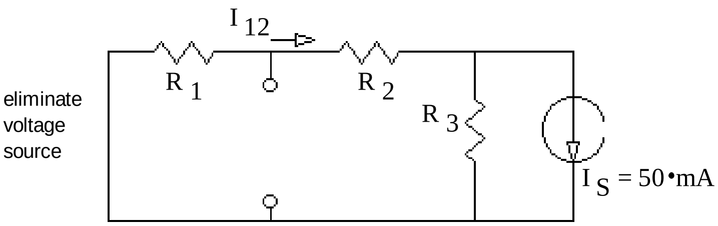


Use superposition to find V_{Th}.



$$V_{Th.V} := \frac{R_2 + R_3}{R_1 + R_2 + R_3}\cdot V_S$$

$$V_{Th.V} = 8.1\cdot V$$

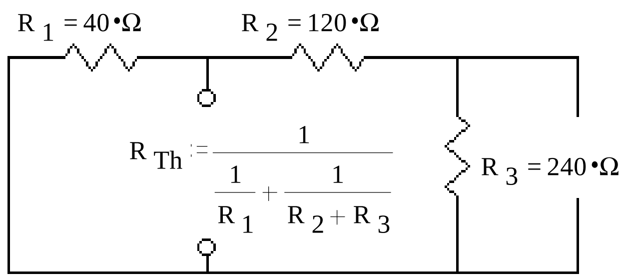


current divider: $I_{12} := \dfrac{\dfrac{1}{R_1 + R_2}}{\dfrac{1}{R_1 + R_2} + \dfrac{1}{R_3}}\cdot I_S$ $\quad I_{12} = 30\cdot mA$

$V_{Th.I} := -I_{12}\cdot R_1$ $\quad V_{Th.I} = -1.2\cdot V$

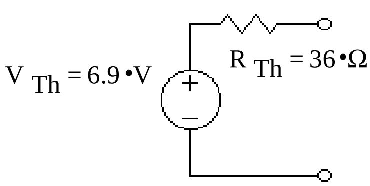
$V_{Th} := V_{Th.V} + V_{Th.I}$ $\quad V_{Th} = 6.9\cdot V$

Find the Thévenin resistance

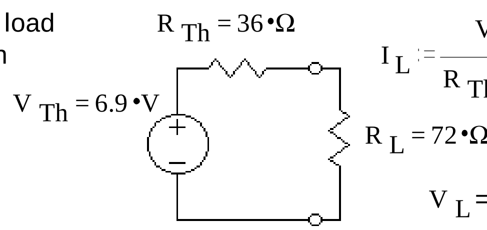


$$R_{Th} := \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2 + R_3}}$$

Thévenin equivalent circuit:



Put the load back on

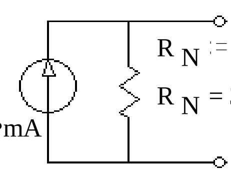


$I_L := \dfrac{V_{Th}}{R_{Th} + R_L}$ $\quad I_L = 63.889\cdot mA$

$V_L = I_L\cdot R_L = 4.6\cdot V$

Norton equivalent circuit: $I_N := \dfrac{V_{Th}}{R_{Th}}$

$I_N = 191.7\cdot mA$



$R_N := R_{Th}$

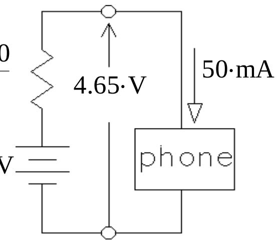
$R_N = 36\cdot\Omega$

Ex 5 A NiCad Battery pack is used to power a cell phone. When the phone is switched on the battery pack voltage drops from $4.80\ \mathrm{V}$ to $4.65\ \mathrm{V}$ and the cell phone draws $50\ \mathrm{mA}$. $V_S := 4.80\cdot V$ $V_{50} := 4.65\cdot V$

a) Draw a simple, reasonable model of the battery pack using ideal parts. Find the value of each part.

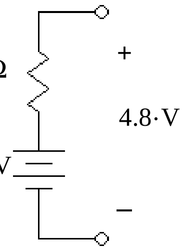
$$R_S := \frac{V_S - V_{50}}{50\cdot mA}$$

$V_S = 4.8\cdot V$



$R_S = 3\cdot\Omega$

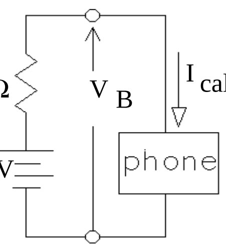
$V_S = 4.8\cdot V$



b) The cell phone is used to make a call. Now it draws $300\ \mathrm{mA}$. What is the battery pack voltage now?

$R_S = 3\cdot\Omega$

$V_S = 4.8\cdot V$



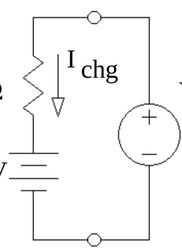
$I_{call} := 300\cdot mA$

$$V_B = V_S - I_{call}\cdot R_S = 3.9\cdot V$$

c) The battery pack is placed in a charger. The charger supplies $5.10\ \mathrm{V}$. How much current flows into the battery pack?

$R_S = 3\cdot\Omega$

$V_S = 4.8\cdot V$



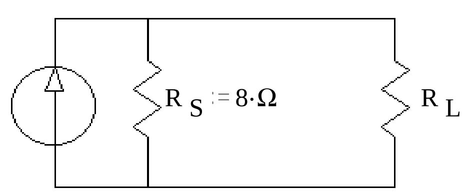
$V_{chg} := 5.10\cdot V$

$$I_{chg} = \frac{V_{chg} - V_S}{R_S} = 100\cdot mA$$

Ex 6 Consider the circuit at right.

a) What value of load resistor (R_L) would you choose if you wanted to maximize the power dissipation in that load resistor.

$I_S := 1\cdot A$

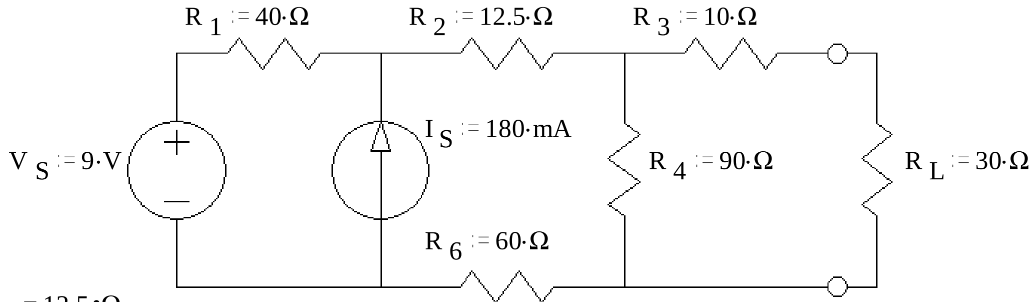


$R_L := R_S$ $R_L = 8\cdot\Omega$

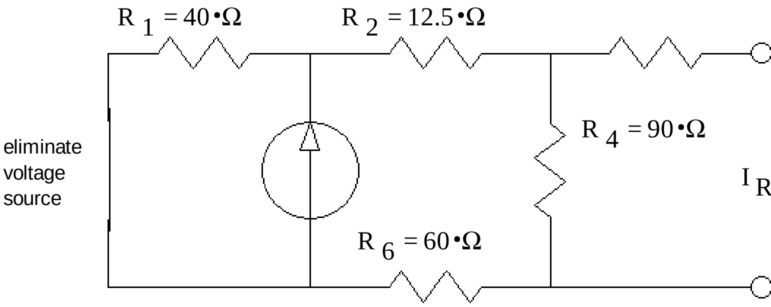
b) With that load resistor (R_L) find the power dissipation in the load.

$$I_L := \frac{I_S}{2} \qquad P_L = I_L^2\cdot R_L = 2\cdot W$$

Ex 7



Use superposition to find V_{Th}.



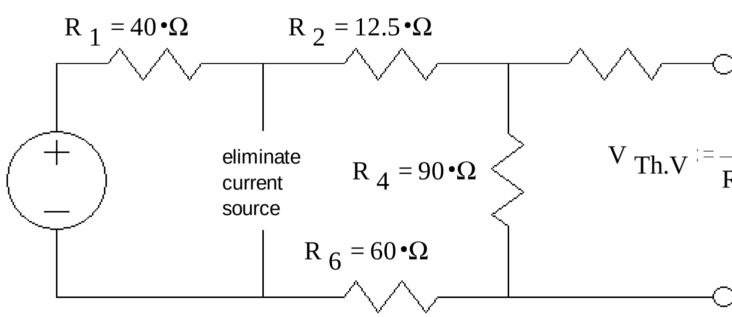
current divider:

$$I_{R4} := \frac{\dfrac{1}{R_2 + R_4 + R_6}}{\dfrac{1}{R_2 + R_4 + R_6} + \dfrac{1}{R_1}} \cdot I_S \qquad I_{R4} = 35.556 \cdot mA$$

$$V_{Th.I} := I_{R4} \cdot R_4$$

$$V_{Th.I} = 3.2 \cdot V$$

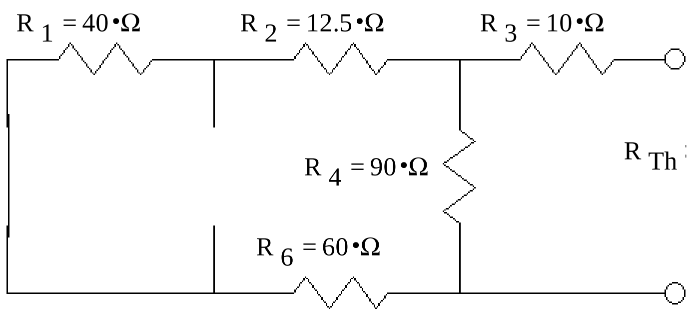
$$R_{246} := R_2 + R_4 + R_6 \qquad R_{246} = 162.5 \cdot \Omega$$



$$V_{Th.V} := \frac{R_4}{R_1 + R_2 + R_4 + R_6} \cdot V_S \qquad V_{Th.V} = 4 \cdot V$$

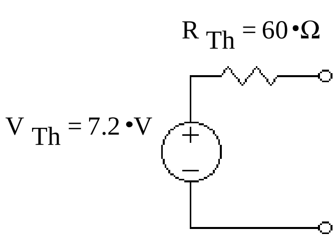
$$V_{Th} := V_{Th.V} + V_{Th.I}$$

$$V_{Th} = 7.2 \cdot V$$

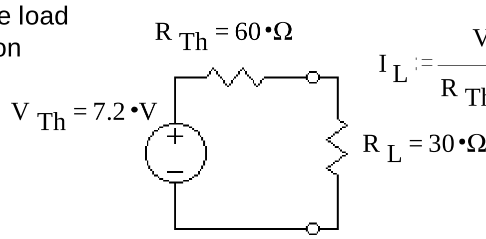


$$R_{Th} := \frac{1}{\dfrac{1}{R_4} + \dfrac{1}{R_1 + R_2 + R_6}} + R_3 \qquad R_{Th} = 60 \cdot \Omega$$

Thévenin equivalent circuit:



Put the load back on



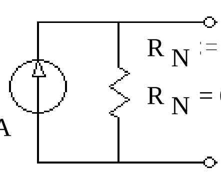
$$I_L := \frac{V_{Th}}{R_{Th} + R_L} \qquad I_L = 80 \cdot mA$$

$$V_L = I_L \cdot R_L = 2.4 \cdot V$$

Norton equivalent circuit:

$$I_N := \frac{V_{Th}}{R_{Th}}$$

$$I_N = 120 \cdot mA$$



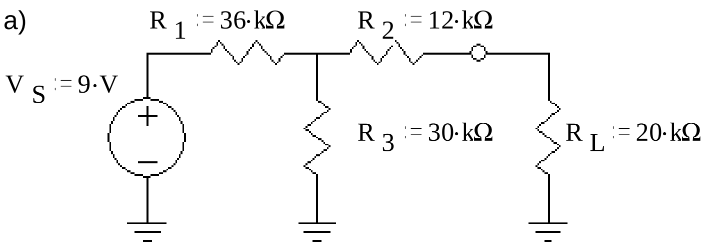
$$R_N := R_{Th}$$

$$R_N = 60 \cdot \Omega$$

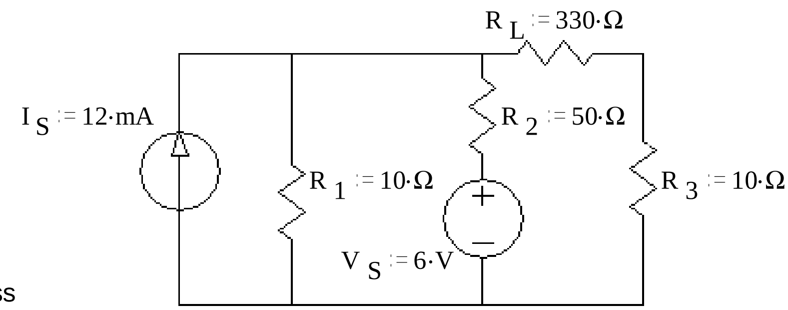
Thevenin & Norton equivalent circuits

1. For each of the circuits below, find and draw the Thevenin equivalent circuit.

   a)



   b) The load resistor is R_L, and is in a strange place in this circuit. Hint: use superposition to find V_{Th}.



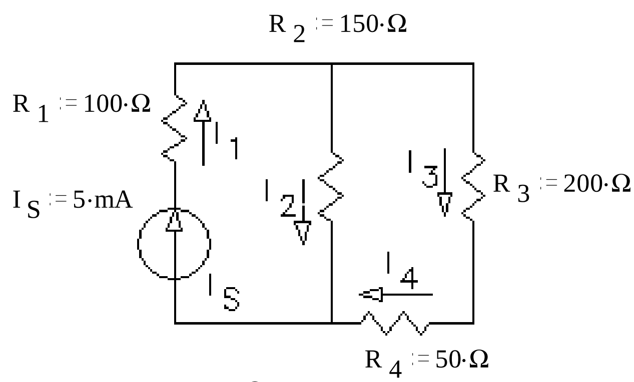
   2nd hint: Nodal analysis is even easier.

2. For the circuit of problem 1a, find the voltage across R_L (V_L) and the current through R_L (I_L) using your Thevenin equivalent circuit.

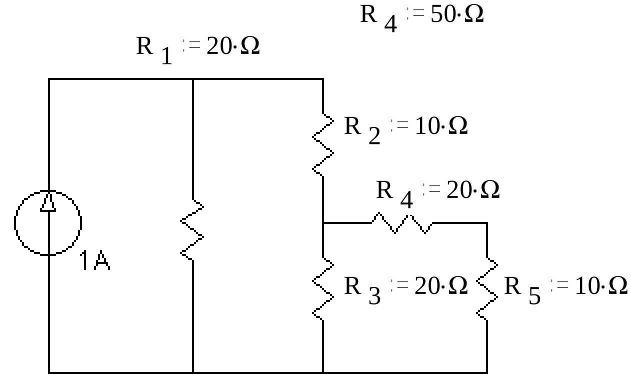
3. For each of the circuits in problem 1, find and draw the Norton equivalent circuit.

4. For the circuit of problem 1b, find V_L and I_L using your Norton equivalent circuit.

5. For the circuit shown at right, use Thevenin's theorem to find the current through the 50 Ω resistor R_4.



6. For the circuit shown, use Norton's theorem to find the value of the current in R_5. Hint: You can find I_N either by calculation of the open circuit voltage (V_{OC}) and R_N or by direct calculation of the short-circuit current (I_{SC}), however, there is something about the values of the resistors which makes the second method easier than it would at first appear.



Source resistance

7. The terminal voltage of a car's battery drops from 12.5 V to 8.5 volts when starting. The starter motor draws 60 A of current.

   a) Draw the voltage-source model (Thevenin equivalent) of this battery. Include the values of V_S and R_S.

   b) Draw the current-source model (Norton equivalent) of this battery. Include the values of I_S and R_S.

   c) Which of these two models is more appropriate for the car battery?

   d) What terminal voltage would you expect if this battery were being charged at 20 A?

<u>Answers</u>

1. a) $4.091\cdot V$, $28.4\cdot k\Omega$ b) $1.1\cdot V$, $18.3\cdot\Omega$ 2. $1.69\cdot V$, $84.6\cdot\mu A$

3. a) $0.144\cdot mA$, $28.4\cdot k\Omega$ b) $60\cdot mA$, $18.3\cdot\Omega$ 4. $3.16\cdot mA$, $1.042\cdot V$ 5. $1.88\cdot mA$

6. $0.19\cdot A$ 7. a) $V_S = 12.5\cdot V$ $R_S := 0.0667\cdot\Omega$ b) $I_S = 187.5\cdot A$ $R_S := 0.0667\cdot\Omega$

c) Thevenin   d) $13.83\cdot V$