Circuits with more than one Source

Recall Statics. To find the reaction at each support, the reactions to each load on a beam (or anything else) can be found separately for each load. Simply add them up to find the total reactions.

Superposition

For circuits with more than 1 source.
1) Zero all but one source.  (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
2) Compute your wanted voltage or current due to the remaining source. Careful, some may be negative.
3) Repeat the first two steps for all the sources.
4) Sum all the contributions from all the sources to find the actual voltage or current. **Watch your signs!**

**Ex1.** Use the method of superposition to find the current $I_2$ (through $R_2$) and the voltage across $R_1$ ($V_{R1}$). Be sure to clearly show and circle your intermediate results.

Superposition:

Eliminate current source

$$I_2, V_s := \frac{V_s}{R_1 + R_2}$$
$$I_2, V_s = 20 \text{mA}$$

$$V_{R1}, V_s := \frac{R_1}{R_1 + R_2} \cdot V_s$$
$$V_{R1}, V_s = 2 \cdot V$$

Eliminate voltage source

$$I_2, I_s := \frac{R_2}{\frac{1}{R_1} + \frac{1}{R_2}} \cdot I_s$$
$$I_2, I_s = -6 \text{mA}$$

$$V_{R1}, I_s := -I_2, I_s \cdot R_2$$
$$V_{R1}, I_s = 1.2 \cdot V$$

Add results

$$I_2 := I_2, V_s + I_2, I_s$$
$$I_2 = 14 \text{mA}$$

$$V_{R1} := V_{R1}, V_s + V_{R1}, I_s$$
$$V_{R1} = 3.2 \cdot V$$
Ex2. Use the method of superposition to find the voltage across through $R_2$ and the current through $R_3$. Be sure to clearly show and circle your intermediate results.

Eliminate current source

$R_1$ is a separate path and doesn't matter.

$$V_{R2.Vs} := \frac{R_2}{R_2 + R_3} \cdot V_S$$

$$V_{R2.Vs} = 4.8 \cdot V$$

$$I_{R3.Vs} := \frac{V_S}{R_2 + R_3}$$

$$I_{R3.Vs} = -2.4 \cdot mA$$

Eliminate voltage source

$R_1$ is shorted and doesn't matter.

$$V_{R2.Is} := I_S \cdot \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}$$

$$V_{R2.Is} = 2.4 \cdot V$$

$$I_{R3.Is} := I_S \cdot \frac{1}{\frac{1}{R_3} + \frac{1}{R_2}}$$

$$I_{R3.Is} = 0.8 \cdot mA$$

Add results

$$V_{R2} := V_{R2.Vs} + V_{R2.Is}$$

$$V_{R2} = 7.2 \cdot V$$

$$I_{R3} := I_{R3.Vs} + I_{R3.Is}$$

$$I_{R3} = -1.6 \cdot mA$$
ECE 2210 / 00 homework # 3 Due: Tue, 1/25/22 Warning: Long homework.
Don't leave until last day.

Answer the following problems on your own paper or tablet.
Since you have the answers, you must show the equations and work you used to arrive at the answer to get credit.

Equivalent resistance
1. Find the equivalent resistance of each of these networks, i.e. what would an ohmmeter read if hooked to the terminals. Work out and keep all your intermediate results -- they will help you in the problems to come.

   a) R₁ := 56-kΩ
   R₂ := 110-kΩ
   R₃ := 97-kΩ
   R₄ := 82-kΩ

   b) R₁ := 27-Ω
   R₂ := 100-Ω
   R₃ := 20-Ω
   R₄ := 56-Ω
   R₆ := 39-Ω
   R₇ := 51-Ω
   R₅ := 75-Ω

Note: the hard part of these problems is actually seeing which resistors are in parallel and which are in series. You may want to redraw the circuits a few times to help you figure it out.

Voltage dividers
2. a) Use the voltage divider concept to find the voltage across each of the resistors in the circuit at right. Vᵣ₁ = ? Vᵣ₂ = ? Vᵣ₃ = ?

   b) Confirm that the three resistor voltages add up to the source voltage, i.e., confirm Kirchoff’s voltage law.

   c) Without recalculating anything, what would happen to all the resistor voltages if the source voltage were doubled? Tripled?

3. The circuit at right is known as a wheatstone bridge, or simply a bridge. It is a very common measurement circuit, used with strain gauges, thermisters, and other devices whose resistance changes in response to something that you’d like to measure. Let’s assume the resistors in this circuit are 100Ω strain gauges. The resistance of these gauges changes slightly when you stretch or compress them. They are glued to a material (often steel) and are used to measure deformations of the material (called strain).

   a) Due to deformation, R₁ and R₄ decrease by 1% and R₂ and R₃ increase by 1%. Find Vᵣ₅.

   b) Due to a temperature change, the resistances of all the gages increase by 5%. Find the % change in Vᵣ₅.

   c) Why do you think the bridge circuit is used in this case?

4. Use voltage divider concepts to find the voltages indicated in the following circuits. You may want to use some of your results from problem 1. You may need to use the voltage divider equation more than once.

   a) R₁ := 56-kΩ
   R₂ := 110-kΩ
   R₃ := 97-kΩ
   R₄ := 82-kΩ
   Vₛ := 36-V
   Vᵣ₅ = ? Vᵣ₄ = ? Vᵣ₁ = ?

   b) R₈ := 12-Ω
   R₁ := 27-Ω
   R₃ := 20-Ω
   R₄ := 56-Ω
   R₂ := 100-Ω
   R₆ := 39-Ω
   R₇ := 51-Ω
   R₅ := 75-Ω
   Vₛ := 10-V
   Vᵣ₈ = ? Vᵣ₂ = ? Vᵣ₁ = ?
ECE 2210 / 00  homework # 3  p.2

Current Dividers

5. The circuit at right shows a current source hooked to a resistor network. Remember that the grounds are all connected together. You can draw lines between them if it helps you.

   \[ R_1 = 1.1 \text{ k}\Omega \]
   \[ R_2 = 0.68 \text{ k}\Omega \]
   \[ R_3 = 0.82 \text{ k}\Omega \]
   \[ I_S = 35 \text{ mA} \]

   a) Use the current divider concept to find the current through each of the resistors in the circuit at right.

   \[ I_{R1} = ? \]
   \[ I_{R2} = ? \]
   \[ I_{R3} = ? \]
   \[ I_{R4} = ? \]

   b) Confirm that \( I_{R2} + I_{R3} = I_{R1} \) and that \( I_{R1} + I_{R4} = I_S \), ie, confirm Kirchoff's current law twice.

   c) Without recalculating anything, what would happen to all the currents if the source current were doubled? Tripled?

6. Refer back to the circuit of problem 4b.

   a) Find the equivalent resistance as seen by the source \( (R_S + \text{your answer for problem 1b}) \) and use that to find the source current \( (I_S \text{ or } I_R) \).

   b) Find these currents by current divider methods.

   \[ I_{R2} = ? \]
   \[ I_{R1} = ? \]
   \[ I_{R4} = ? \]

   c) Using Ohm's law and the currents you found in this problem, confirm the voltages found in problem 4b.

Power

7. Refer to the circuit of problem 2.

   a) How much power is dissipated by each resistor?

   \[ P_{R1} = ? \]
   \[ P_{R2} = ? \]
   \[ P_{R3} = ? \]

   b) Independently determine the power that the source is contributing to the circuit.

   \[ P_S = V_S I_S = ? \]

   c) Show that power is conserved (\( \Sigma \) answers to a = answer to b).

8. Refer to the circuit of problem 5.

   a) How much power is dissipated by each resistor?

   \[ P_{R1} = ? \]
   \[ P_{R2} = ? \]
   \[ P_{R3} = ? \]
   \[ P_{R4} = ? \]

   b) Independently determine the power that the source is contributing to the circuit.

   \[ P_S = V_S I_S = ? \]

   c) Show that power is conserved.

9. The circuit at right has five unknown components labeled A through E.

   a) Which of the components are absorbing power from the circuit?

   \[ V_B = 2 \text{ V} \]
   \[ V_E = 4 \text{ V} \]
   \[ I_D = 0.2 \text{ A} \]
   \[ V_D = 8 \text{ V} \]
   \[ I_C = 0.3 \text{ A} \]

   b) Which of the components are contributing power to the circuit?

   c) Show that power is conserved.

Answers

1. a) \( R_{eq} = 82.5 \text{ k}\Omega \)  b) \( R_{eq} = 41.7 \text{ k}\Omega \)

2. a) 1.91 V, 1.28 V, 2.81 V  b) 1.91 V + 1.28 V + 2.81 V = 6 V  c) double, triple

3. a) 100 mV  b) 0% change  c) Reading won't be affected by temperature.

4. a) 5.54 V, 17.35 V, 13.11 V  b) 2.23 V, 7.77 V, 2.93 V

5. a) 17.67 mA, 9.66 mA, 8.01 mA, 17.33 mA  b) both check  c) double, triple

6. a) 53.7 \Omega , 0.186 A  b) 77.65 mA, 108.6 mA, 28.6 mA  c) all agree

7. a) 2.44 mW, 1.63 mW, 3.59 mW  b) 7.66 mW  c) \( P_S = P_{R1} + P_{R2} + P_{R3} \)

8. a) 0.343 W, 0.0634 W, 0.0526 W, 0.451 W  b) 0.910 W  c) \( P_{R1} + P_{R2} + P_{R3} + P_{R4} = P_S \)

9. a) C, D, E  b) A, B  c) 6 W = 6 W
Answer the following problems on your own paper. Show your equations and work to get credit on this and all future homeworks.

Superposition

1. Use superposition to find $I_3$. Circle your intermediate solutions on your paper. Your intermediate solutions show how much of $I_3$ is due to $V_{S1}$, and how much is due to $V_{S2}$.

2. Use superposition to solve following problems: Each problem asks for both a current and a voltage. Clearly indicate your intermediate answers, the grader will look for those.

\[ I_{R2} = ? \quad R_2 = 12\Omega \]

\[ V_{R1} = ? \quad V_S = 12\text{V} \]

a) $V_a = ?$ The letter "a" is the name of the "node" at the black dot. $V_a$ is a node voltage, referenced to ground.

\[ V_a = V_{R3} \]

These are ground symbols. They are all connected together, although that connection is not explicitly shown.

b) $I_{R2} = ?$ $V_{R1} = ?$

$R_1 = 11\Omega$ $R_2 = 22\Omega$ $I_S = 0.5\text{A}$

\[ V_S = 12\text{V} \]

c) Watch your signs.

$I_{R1} = ?$ $R_1 = 3\text{k}\Omega$

$R_2 = 3\text{k}\Omega$ $R_3 = 1\text{k}\Omega$

$V_a = ?$

$V_S = 4.5\text{V}$ $R_4 = 2\text{k}\Omega$

$I_S = 3\text{mA}$

Answers

1. $2\text{mA} + 5\text{mA} = 7\text{mA}$

2. a) $4.2\text{V}$, $20\text{mA}$  b) $7.67\text{V}$, $197\text{mA}$

3. $0.5\text{V}$, $-0.5\text{mA}$