ECE 2210 Lecture 4 notes Superposition

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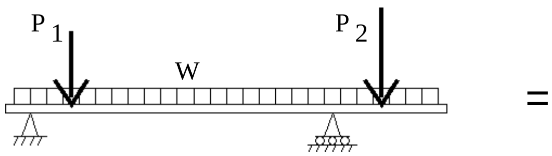
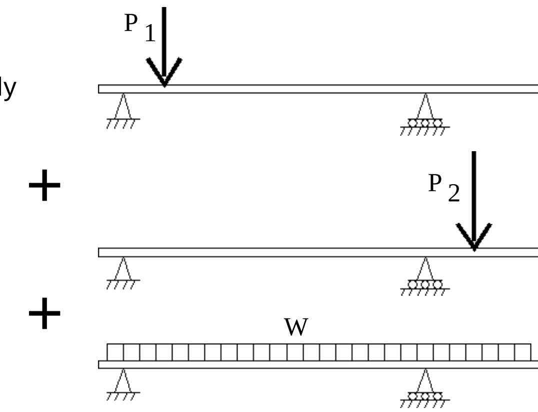
9/3/08,
?/31/09

# Circuits with more than one Source

Recall Statics. To find the reaction at each support, the reactions to each load on a beam (or anything else) can be found separately for each load. Simply add them up to find the total reactions.



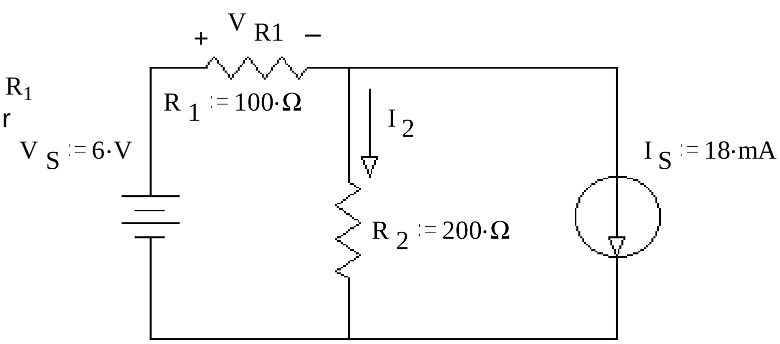




## Superposition

For circuits with more than 1 source.

1) Zero all but one source.
 (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
2) Compute your wanted voltage or current due to the remaining source. Careful, some may be negative.
3) Repeat the first two steps for all the sources.
4) Sum all the contributions from all the sources to find the actual voltage or current. **Watch your signs!**

**Ex1.** Use the method of superposition to find the current $I_2$ (through $R_2$) and the voltage across $R_1$ ($V_{R1}$). Be sure to clearly show and **circle** your intermediate results.

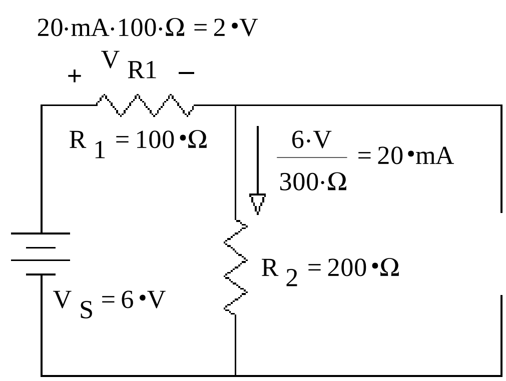



superposition:

Eliminate current source

$I_{2.Vs} := \frac{V_S}{R_1 + R_2}$    $I_{2.Vs} = 20 \cdot mA$

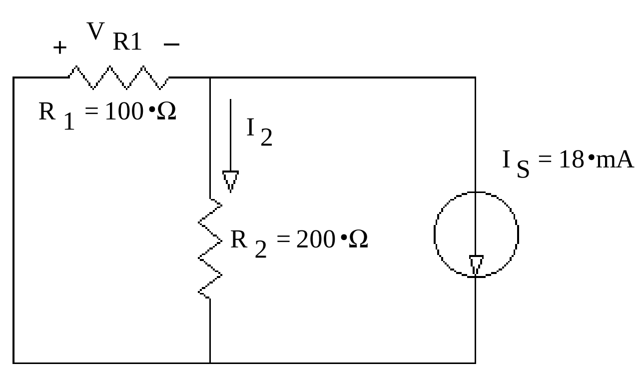
$V_{R1.Vs} := \frac{R_1}{R_1 + R_2} \cdot V_S$    $V_{R1.Vs} = 2 \cdot V$




Eliminate voltage source

$I_{2.Is} := -\frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} \cdot I_S$    $I_{2.Is} = -6 \cdot mA$

$V_{R1.Is} := -I_{2.Is} \cdot R_2$    $V_{R1.Is} = 1.2 \cdot V$

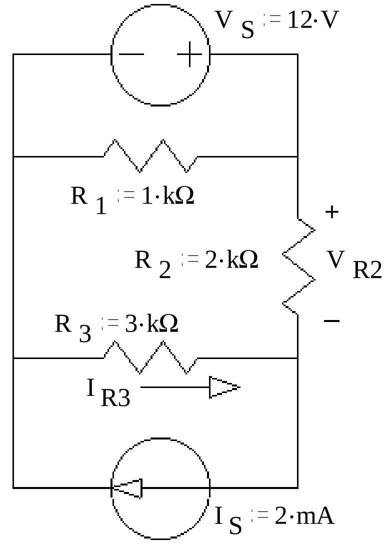



Add results

$I_2 := I_{2.Vs} + I_{2.Is}$    $I_2 = 14 \cdot mA$

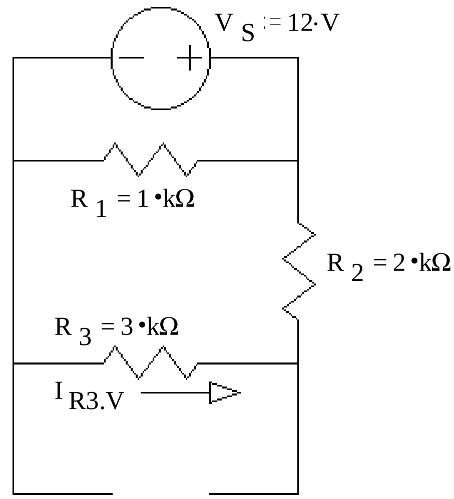
$V_{R1} := V_{R1.Vs} + V_{R1.Is}$    $V_{R1} = 3.2 \cdot V$

**Ex2.** Use the method of superposition to find the voltage accross through $R_2$ and the current through $R_3$. Be sure to clearly show and **circle** your intermediate results.




Eliminate current source

$R_1$ is a separate path and doesn't matter.

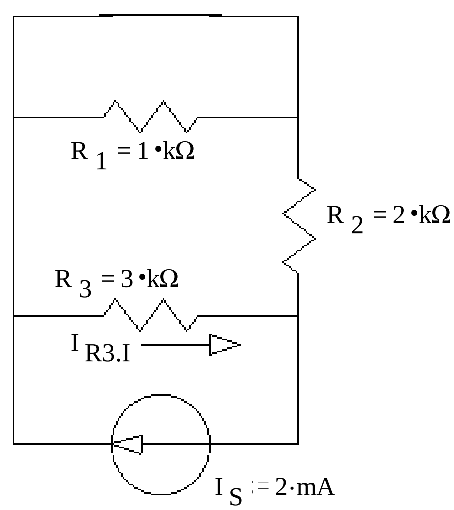



$$V_{R2.Vs} := \frac{R_2}{R_2 + R_3} \cdot V_S \qquad V_{R2.Vs} = 4.8 \cdot V$$

$$I_{R3.Vs} := -\frac{V_S}{R_2 + R_3} \qquad I_{R3.Vs} = -2.4 \cdot mA$$

Eliminate voltage source

$R_1$ is shorted and doesn't matter.




$$V_{R2.Is} := I_S \cdot \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} \qquad V_{R2.Is} = 2.4 \cdot V$$

$$I_{R3.Is} := \frac{\frac{1}{R_3}}{\frac{1}{R_2} + \frac{1}{R_3}} \cdot I_S \qquad I_{R3.Is} = 0.8 \cdot mA$$

Add results

$$V_{R2} := V_{R2.Vs} + V_{R2.Is} \qquad V_{R2} = 7.2 \cdot V$$

$$I_{R3} := I_{R3.Vs} + I_{R3.Is} \qquad I_{R3} = -1.6 \cdot mA$$

# ECE 2210 Lectures notes Thévenin & Norton Equivalent Circuits

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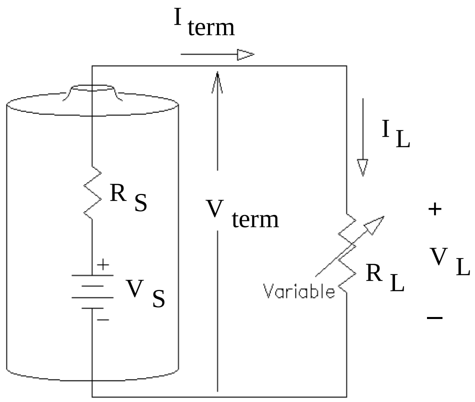
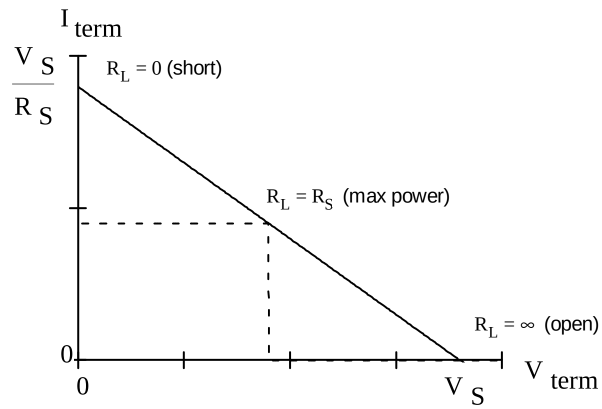
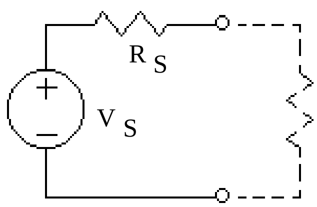
## Simple Model of a Real Source

Real sources are not ideal, but we will model them with two ideal components.



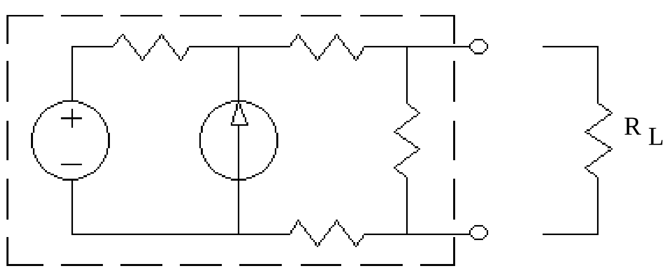



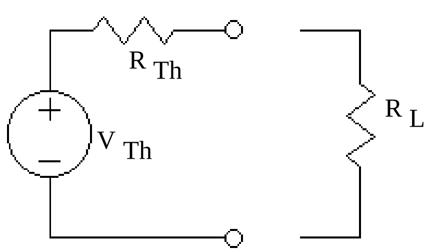




Note: $R_L$ is NOT part of the Thévenin equivalent circuit and does not need to be shown.

## Thévevin Equivalent Circuit

The same model can be used for any combination of sources and resistors.

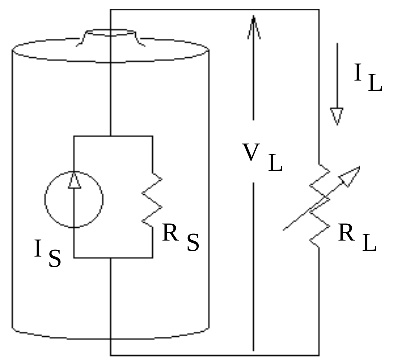


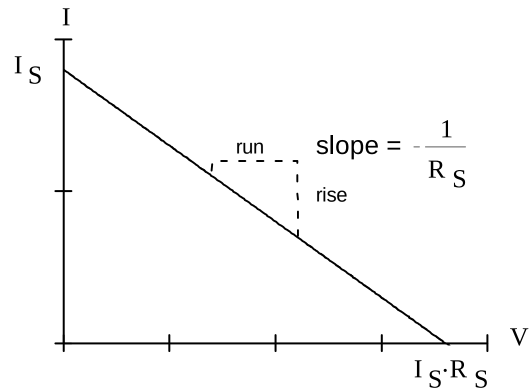



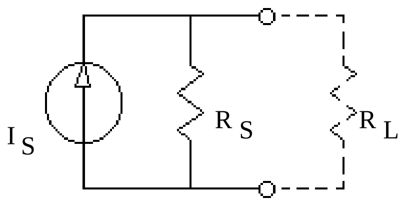

## Thévenin equivalent

To calculate a circuit's Thévenin equivalent:

1) Remove the load and calculate the open-circuit voltage where the load used to be.
   This is the Thévenin voltage ($V_{Th}$).
2) Zero all the sources.
   (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
3) Compute the total resistance between the load terminals.
   (DO NOT include the load in this resistance.) This is the Thévenin source resistance ($R_{Th}$).
4) Draw the Thévenin equivalent circuit and add your values.



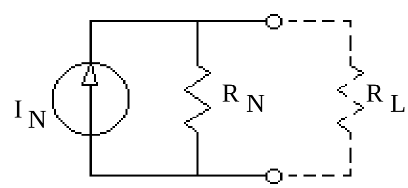




Note: $R_L$ is not part of the Norton equivalent and does not need to be shown.

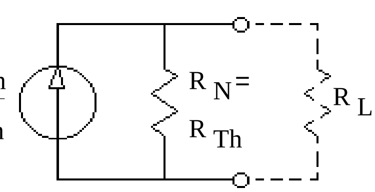
## Norton equivalent

To calculate a circuit's Norton equivalent:

1) Replace the load with a short (a wire) and calculate the short-circuit current in this wire.
   This is the Norton current ($I_N$). Remove the short.
2) Zero all the sources.
   (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
3) Compute the total resistance between the load terminals.
   (DO NOT include the load in this resistance.) This is the Norton source resistance ($R_N$).
   (Exactly the same as the Thévenin source resistance ($R_{Th}$)).
4) Draw the Norton equivalent circuit and add your values.




OR (the more common way)...
1) Find the Thévenin equivalent circuit.
2) Convert to Norton circuit, then >>> $R_N = R_{Th}$ and $I_N = \frac{V_{Th}}{R_{Th}}$

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# **Thévevin** & Norton Examples

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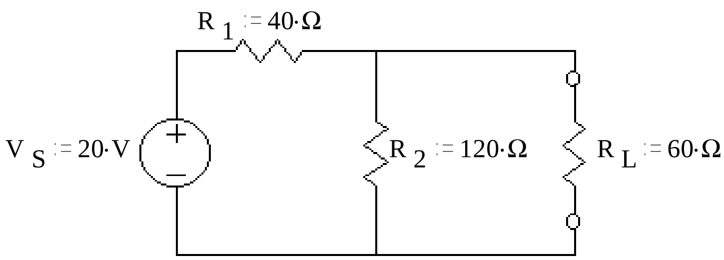
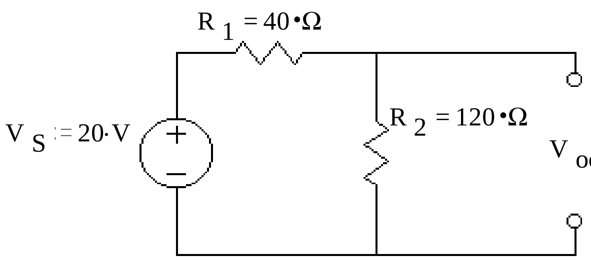
**Ex 1** Find the Thévenin equivalent:




To calculate a circuit's Thévenin equivalent:
1) Remove the load and calculate the open-circuit voltage where the load used to be.
This is the Thévenin voltage ($V_{Th}$).

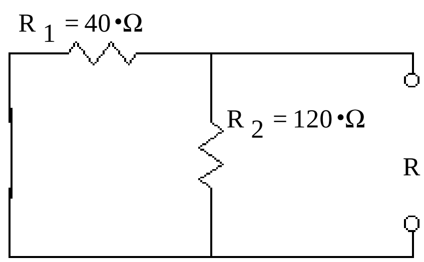



Find the open circuit voltage:

$$V_{oc} = V_{Th} := V_S\cdot\frac{R_2}{R_1 + R_2} \qquad V_{Th} = 15\cdot V$$

2) Zero all the sources.
(To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)

Zero the source



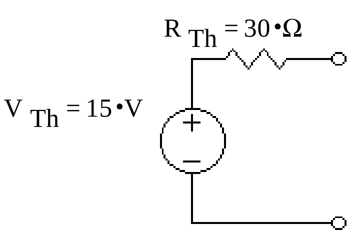

3) Compute the total resistance between the load terminals.
(DO NOT include the load in this resistance.)
This is the Thévenin source resistance ($R_{Th}$).

Find the Thevenin resistance:

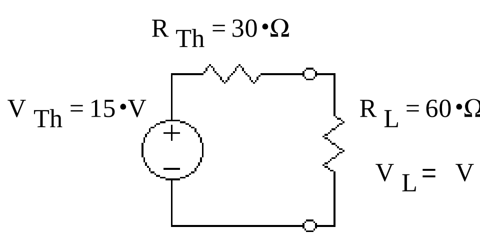
$$R_{Th} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \qquad R_{Th} = 30\cdot\Omega$$

4) Draw the Thévenin equivalent circuit and add your values.

Thevenin equivalent circuit:




If the load were reconnected:

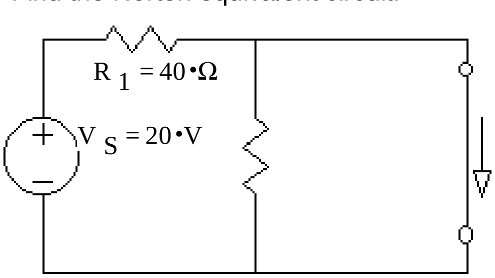



$$V_L = V_{Th}\cdot\frac{R_L}{R_{Th} + R_L} = 10\cdot V$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = 166.7\cdot mA$$

$$P_L = 10\cdot V\cdot 166.7\cdot mA = 1.667\cdot W$$

b) Find the Norton equivalent circuit:



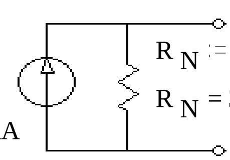

$$I_N := \frac{V_S}{R_1}$$

$$I_N = 500\cdot mA$$

Norton equivalent circuit:

$$I_N := \frac{V_{Th}}{R_{Th}}$$

$$I_N = 500\cdot mA$$



$R_N := R_{Th}$

$R_N = 30\cdot\Omega$

c) Show that the Thévenin circuit is indeed equivalent to the original at several values of $R_L$.

| $R_L$ | Original Circuit: $V_L$ | Original Circuit: $I_L$ | Thévenin Circuit: $I_L$ | Thévenin Circuit: $V_L$ |
|---|---|---|---|---|
| $R_L := 0\cdot\Omega$ | $0\cdot V$ | $\dfrac{V_S}{R_1} = 500\cdot mA$ | $\dfrac{V_{Th}}{R_{Th}+R_L} = 500\cdot mA$ | $500\cdot mA\cdot 0\cdot\Omega = 0\cdot V$ |

Using either numbers: $P_L = V_L \cdot I_L = 0\cdot W$

$R_L := 10\cdot\Omega$

$$R_o := \frac{1}{\dfrac{1}{R_2}+\dfrac{1}{R_L}} \qquad R_o = 9.231\cdot\Omega$$

$$V_L = V_S\cdot\frac{R_o}{R_1+R_o} = 3.75\cdot V$$

$$I_L = \frac{V_L}{R_L} = 375\cdot mA$$

$$I_L := \frac{V_{Th}}{R_{Th}+R_L} \qquad I_L = 375\cdot mA$$

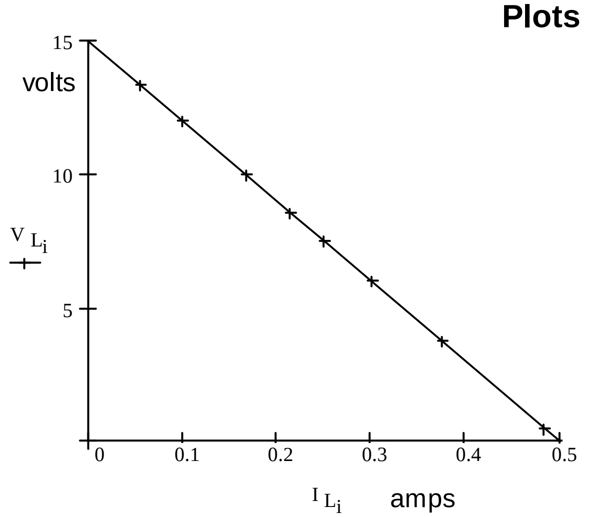
$$V_L := I_L\cdot R_L \qquad V_L = 3.75\cdot V$$

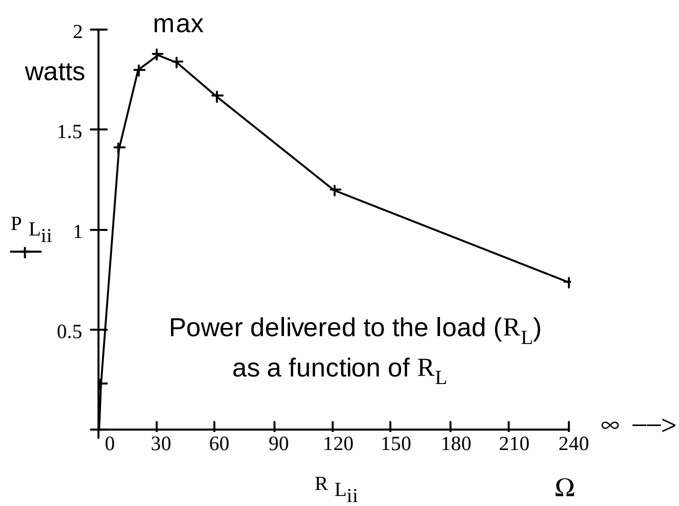
Using either numbers: $P_L = V_L\cdot I_L = 1.406\cdot W$

Repeat these calculations for a number of load resistors

| $R_{L_i} :=$ | $R_{o_i}$ (Ω) | $V_L = V_S\cdot\dfrac{R_{o_i}}{R_1+R_{o_i}}$ (V) | $I_L = \dfrac{V_{L_i}}{R_{L_i}}$ (mA) | $I_L = \dfrac{V_{Th}}{R_{Th}+R_{L_i}}$ (mA) | $V_L = I_{L_i}\cdot R_{L_i}$ (V) | $P_{L_i}$ (W) | |
|---|---|---|---|---|---|---|---|
| 0·Ω | 0 | 0 | 0 | 500 | 0 | 0 | |
| 1·Ω | 0.992 | 0.484 | 483.871 | 483.871 | 0.484 | 0.234 | |
| 10·Ω | 9.231 | 3.75 | 375 | 375 | 3.75 | 1.406 | |
| 20·Ω | 17.143 | 6 | 300 | 300 | 6 | 1.8 | |
| 30·Ω | 24 | 7.5 | 250 | 250 | 7.5 | 1.875 | max |
| 40·Ω | 30 | 8.571 | 214.286 | 214.286 | 8.571 | 1.837 | |
| 60·Ω | 40 | 10 | 166.667 | 166.667 | 10 | 1.667 | |
| 120·Ω | 60 | 12 | 100 | 100 | 12 | 1.2 | |
| 240·Ω | 80 | 13.333 | 55.556 | 55.556 | 13.333 | 0.741 | |
| ∞·Ω | 120 | 15 | 0 | 0 | 15 | 0 | |

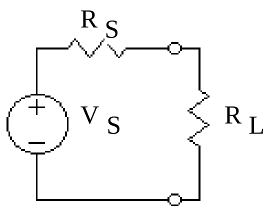
**Plots**



**Maximum power transfer**

If I wanted to maximize the power dissipated by the load, what $R_L$ would I choose?




$$P_L = \frac{V_L^2}{R_L} = \left(\frac{R_L}{R_S + R_L} \cdot V_S\right)^2 \cdot \frac{1}{R_L} = \frac{R_L^2}{(R_S + R_L)^2} \cdot V_S^2 \cdot \frac{1}{R_L}$$

$$= \frac{R_L^2}{R_S^2 + 2 \cdot R_S \cdot R_L + R_L^2} \cdot V_S^2 \cdot \frac{1}{R_L} = \frac{R_L}{R_S^2 + 2 \cdot R_S \cdot R_L + R_L^2} \cdot V_S^2$$

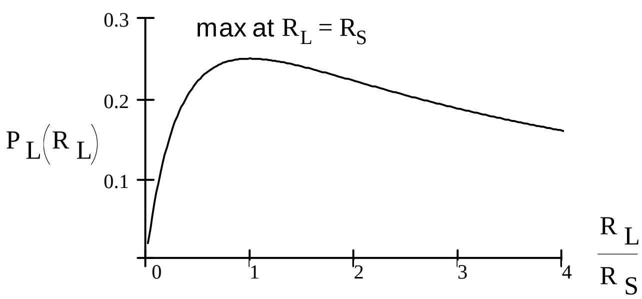
$$= \frac{1}{\frac{R_S^2}{R_L} + 2 \cdot R_S + R_L} \cdot V_S^2$$

Next step would be to differentiate $\frac{d}{dR_L} P_L(R_L)$ ,

set this equal to $0$ and solve for $R_L$ to find the maximum

Unfortunately this function is a pain to differentiate. What if we just differentiate the denominator and find its minimum, wouldn't that work just as well?

$$\frac{d}{dR_L}\left(\frac{R_S^2}{R_L} + 2 \cdot R_S + R_L\right) = -1 \cdot \frac{R_S^2}{R_L^2} + 0 + 1 = 0$$




Maximum power transfer happens when: $R_L = R_S$

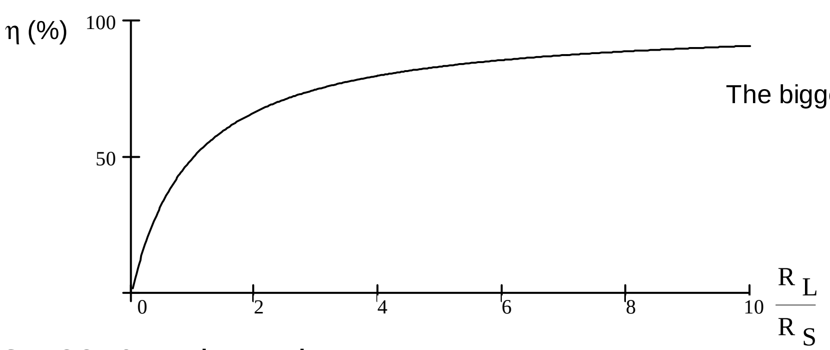
Just what we saw in Example 1

This is rarely important in power circuitry, where there should be plenty of power and $R_S$ should be small. It is much more likely to be important in signal circuitry where the voltages can be very small and the source resistance may be significant -- say a microphone or a radio antenna.

**All you need to remember** is: $R_L = R_S$ to maximize the power dissipation in $R_L$

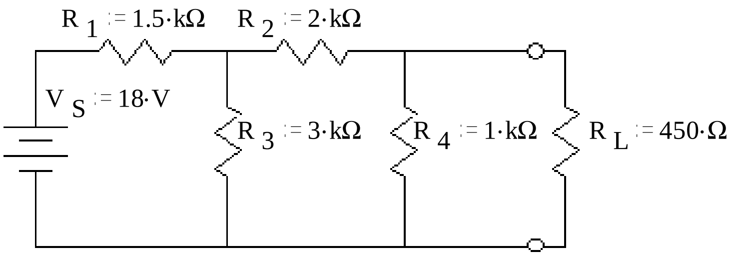
**What about efficiency?**

$$\frac{P_L(R_L)}{P_S(R_L)} = \frac{I^2 \cdot R_L}{I^2 \cdot (R_S + R_L)} = \frac{R_L}{R_S + R_L}$$

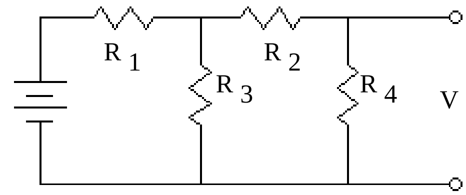



The bigger $R_L$ is, the higher the efficiency.

**Ex 2** a) Find and draw the Thévenin equivalent circuit.

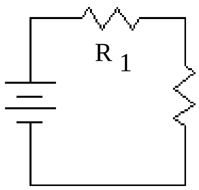



Find the open circuit voltage:




$V_{oc} = V_{Th}$

First do some simplification:



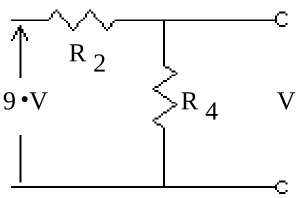

$$R_{eq234} := \frac{1}{\dfrac{1}{R_3} + \dfrac{1}{R_2 + R_4}} \qquad R_{eq234} = 1.5\cdot k\Omega$$

$$V_{234} := \frac{R_{eq234}}{R_1 + R_{eq234}} \cdot V_S$$

$$V_{234} = 9\cdot V$$

Divide this voltage between $R_2$ and $R_4$:

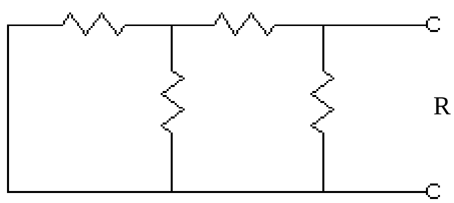
$V_{234} = 9\cdot V$




$$V_{Th} := \frac{R_4}{R_2 + R_4} \cdot V_{234} \qquad V_{Th} = 3\cdot V$$

Find the Thévenin resistance:

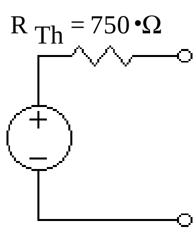
Zero the source



$$R_{Th} := \frac{1}{\dfrac{1}{R_4} + \dfrac{1}{R_2 + \dfrac{1}{\left(\dfrac{1}{R_1} + \dfrac{1}{R_3}\right)}}} \qquad R_{Th} = 750\cdot\Omega$$

Thévenin equivalent circuit:

$V_{Th} = 3\cdot V$




If the load were reconnected:

$$V_L := V_{Th} \cdot \frac{R_L}{R_{Th} + R_L} \qquad V_L = 1.125\cdot V$$

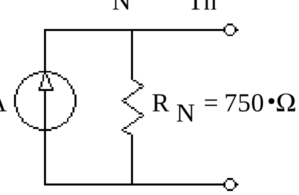
$$I_L := \frac{V_{Th}}{R_{Th} + R_L} \qquad I_L = 2.5\cdot mA$$

b) Find and draw the Norton equivalent circuit.

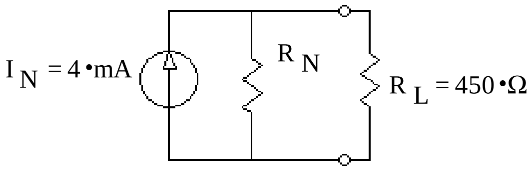
$R_N := R_{Th}$

$$I_N := \frac{V_{Th}}{R_{Th}}$$

$I_N = 4\cdot mA$



c) Use your Norton equivalent circuit to find the current through the load.




$$I_L := \frac{\frac{1}{R_L}}{\left(\frac{1}{R_N}+\frac{1}{R_L}\right)}\cdot I_N \qquad I_L = 2.5\cdot mA$$

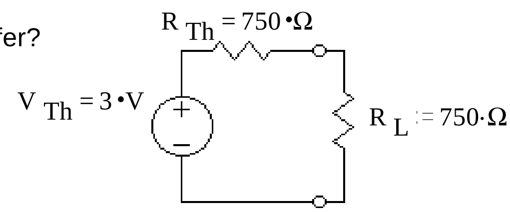
$$V_L := I_L\cdot R_L \qquad V_L = 1.125\cdot V$$

same as above

d) What value of $R_L$ would result in the maximum power delivery to $R_L$?

For maximum power transfer $R_L = R_{Th} = 750\cdot\Omega$

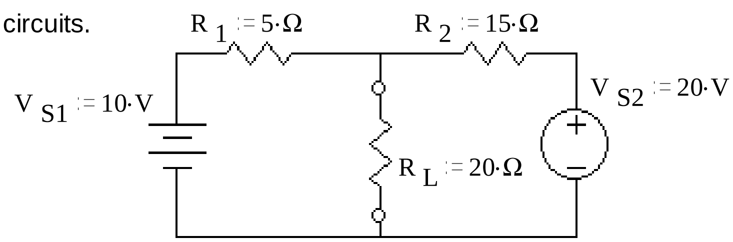
e) What is the maximum power transfer?




$$V_L := \frac{V_{Th}}{2}$$

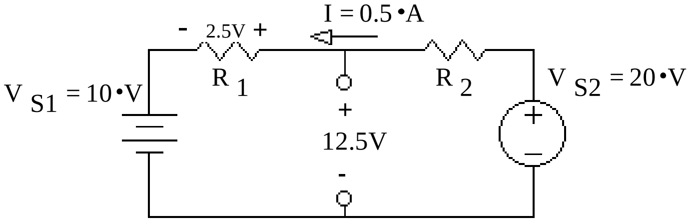
$$P_L = \frac{V_L^2}{R_L} = 3\cdot mW$$

**Ex 3** a) Find and draw the Thévenin & Norton equivalent circuits.

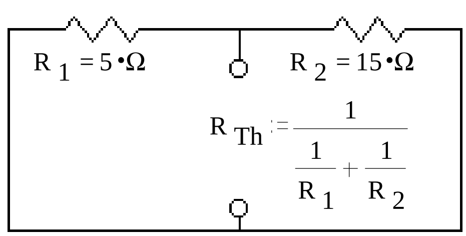



$$I := \frac{V_{S2}-V_{S1}}{R_1+R_2}$$

$I = 0.5\cdot A$



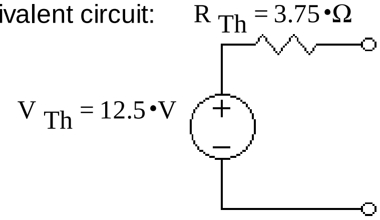

$V_{Th} := 10\cdot V + 2.5\cdot V$




$$R_{Th} := \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$$

$R_{Th} = 3.75\cdot\Omega$

Thévenin equivalent circuit:

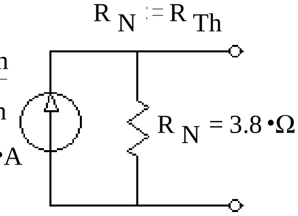



Norton equivalent circuit:

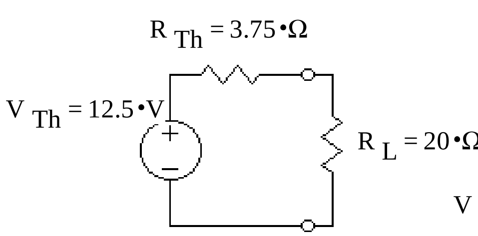
$R_N := R_{Th}$

$$I_N := \frac{V_{Th}}{R_{Th}}$$

$I_N = 3.333\cdot A$

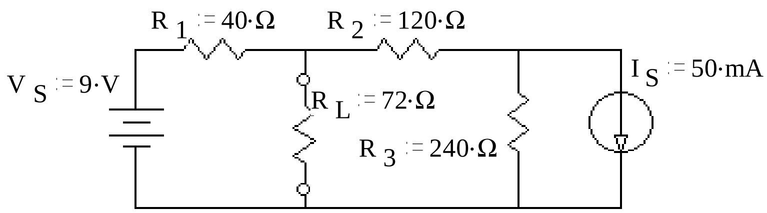



b) Use your Thévenin equivalent circuit to find the voltage across the load.

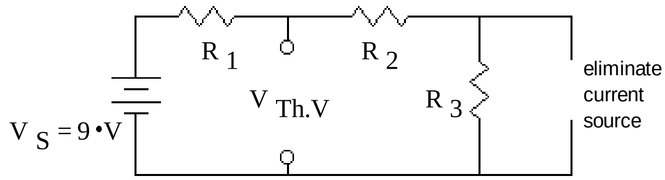



$$V_L = \frac{R_L}{R_{Th}+R_L}\cdot V_{Th} = 10.526\cdot V$$

**Ex 4** a) Find and draw the Thévenin & Norton equivalent circuits.

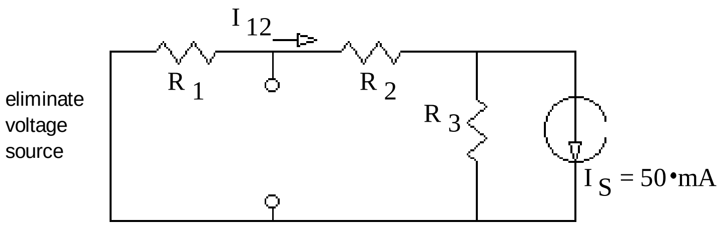



Use superposition to find $V_{Th}$.




$$V_{Th.V} := \frac{R_2 + R_3}{R_1 + R_2 + R_3} \cdot V_S$$

$$V_{Th.V} = 8.1 \cdot V$$

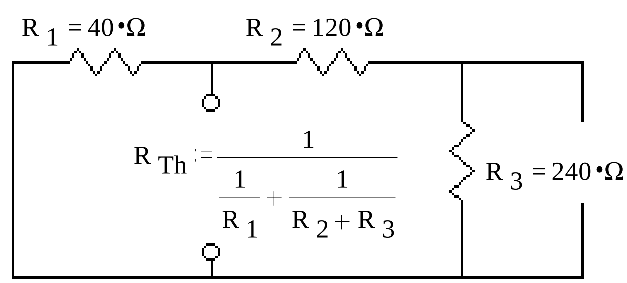



current divider: $$I_{12} := \frac{\frac{1}{R_1 + R_2}}{\frac{1}{R_1 + R_2} + \frac{1}{R_3}} \cdot I_S \qquad I_{12} = 30 \cdot mA$$

$$V_{Th.I} := -I_{12} \cdot R_1 \qquad V_{Th.I} = -1.2 \cdot V$$

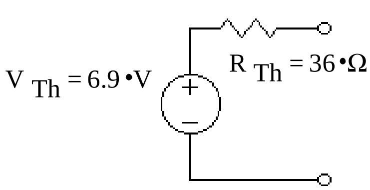
$$V_{Th} := V_{Th.V} + V_{Th.I} \qquad V_{Th} = 6.9 \cdot V$$

Find the Thévenin resistance

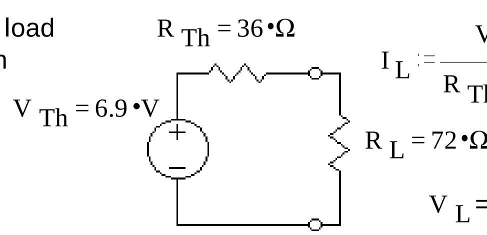



$$R_{Th} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + R_3}}$$

Thévenin equivalent circuit:



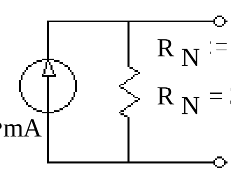

Put the load back on




$$I_L := \frac{V_{Th}}{R_{Th} + R_L} \qquad I_L = 63.889 \cdot mA$$

$$V_L = I_L \cdot R_L = 4.6 \cdot V$$

Norton equivalent circuit: $I_N := \frac{V_{Th}}{R_{Th}}$ $\quad I_N = 191.7 \cdot mA$



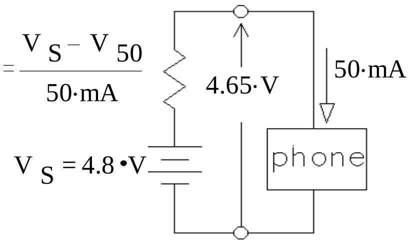
$R_N := R_{Th}$

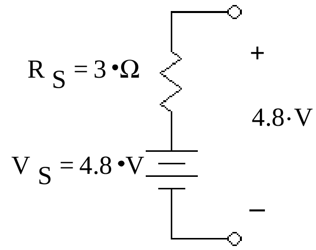
$R_N = 36 \cdot \Omega$

**Ex 5** A NiCad Battery pack is used to power a cell phone. When the phone is switched on the battery pack voltage drops from 4.80 V to 4.65 V and the cell phone draws 50 mA. $V_S := 4.80\cdot V$ $V_{50} := 4.65\cdot V$

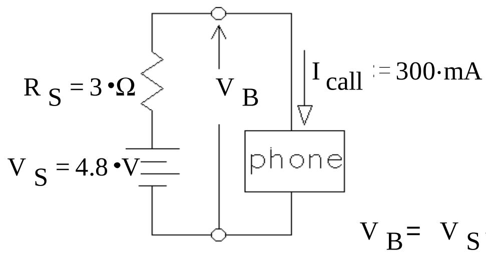
a) Draw a simple, reasonable model of the battery pack using ideal parts.
Find the value of each part.

$$R_S := \frac{V_S - V_{50}}{50\cdot mA}$$



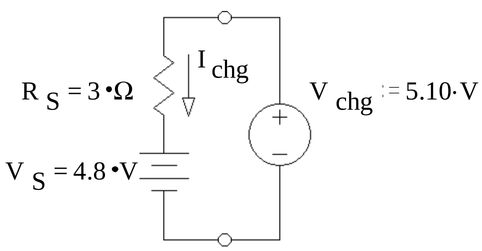




b) The cell phone is used to make a call. Now it draws 300 mA.
What is the battery pack voltage now?




$$V_B = V_S - I_{call}\cdot R_S = 3.9\cdot V$$

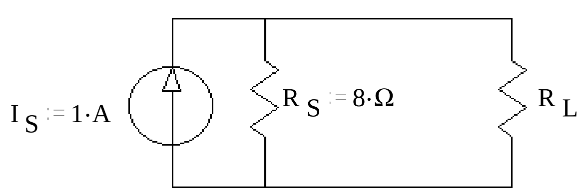
c) The battery pack is placed in a charger. The charger supplies 5.10 V. How much current flows into the battery pack?




$$I_{chg} = \frac{V_{chg} - V_S}{R_S} = 100\cdot mA$$

**Ex 6** Consider the circuit at right.

a) What value of load resistor ($R_L$) would you choose if you wanted to maximize the power dissipation in that load resistor.

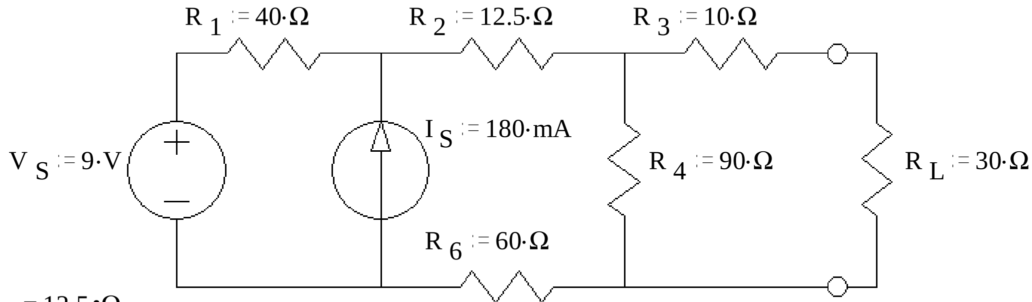



$R_L := R_S$ $R_L = 8\cdot\Omega$

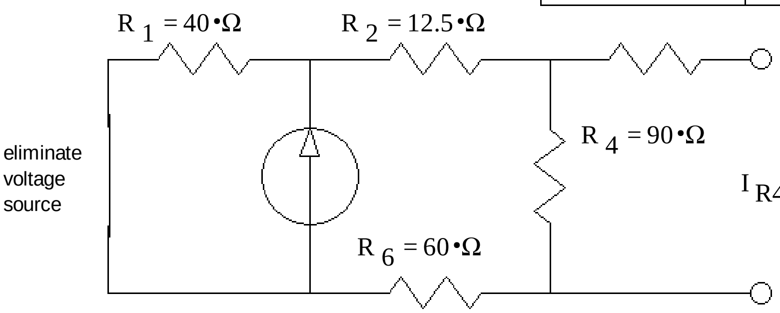
b) With that load resistor ($R_L$) find the power dissipation in the load.

$I_L := \frac{I_S}{2}$ $P_L = I_L^2\cdot R_L = 2\cdot W$

**Ex 7**




Use superposition to find $V_{Th}$.



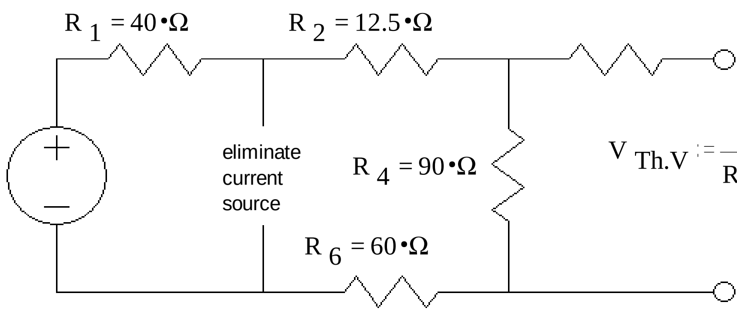

current divider:

$$I_{R4} := \frac{\frac{1}{R_2 + R_4 + R_6}}{\frac{1}{R_2 + R_4 + R_6} + \frac{1}{R_1}} \cdot I_S \qquad I_{R4} = 35.556\cdot mA$$

$$V_{Th.I} := I_{R4}\cdot R_4$$

$$V_{Th.I} = 3.2\cdot V$$

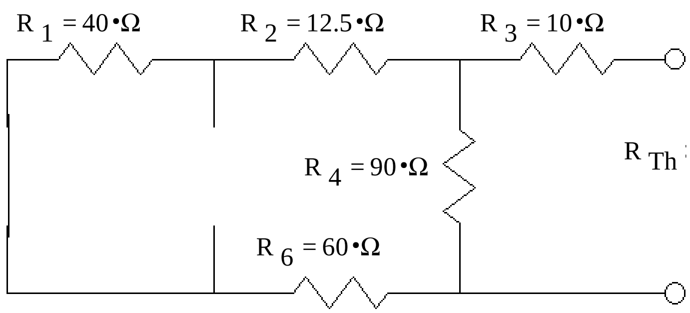
$$R_{246} := R_2 + R_4 + R_6 \qquad R_{246} = 162.5\cdot\Omega$$




$$V_{Th.V} := \frac{R_4}{R_1 + R_2 + R_4 + R_6}\cdot V_S \qquad V_{Th.V} = 4\cdot V$$

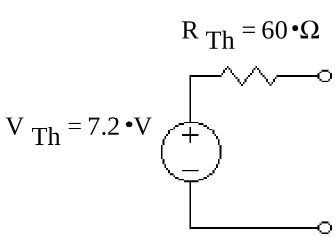
$$V_{Th} := V_{Th.V} + V_{Th.I}$$

$$V_{Th} = 7.2\cdot V$$

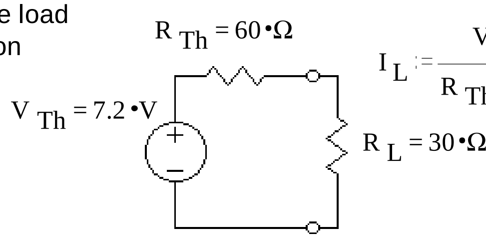



$$R_{Th} := \frac{1}{\frac{1}{R_4} + \frac{1}{R_1 + R_2 + R_6}} + R_3 \qquad R_{Th} = 60\cdot\Omega$$

Thévenin equivalent circuit:




Put the load back on



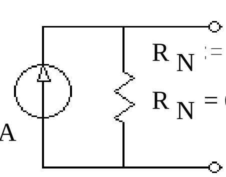

$$I_L := \frac{V_{Th}}{R_{Th} + R_L} \qquad I_L = 80\cdot mA$$

$$V_L = I_L\cdot R_L = 2.4\cdot V$$

Norton equivalent circuit:

$$I_N := \frac{V_{Th}}{R_{Th}}$$

$$I_N = 120\cdot mA$$



$$R_N := R_{Th}$$

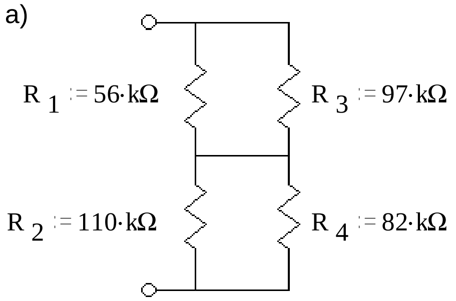
$$R_N = 60\cdot\Omega$$

Answer the following problems on your own paper or tablet.
Since you have the answers, you **must show** the equations and work you used to arrive at the answer to get credit.

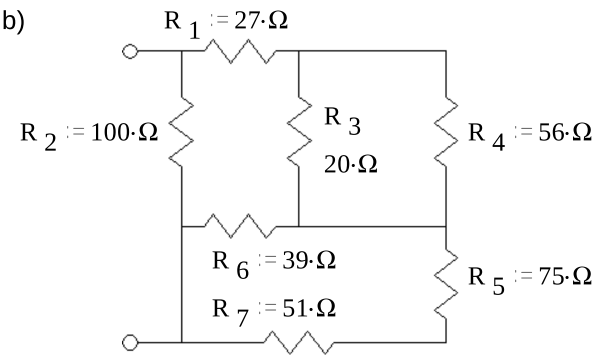
**Equivalent resistance**

1. Find the equivalent resistance of each of these networks, i.e. what would an ohmmeter read if hooked to the terminals. Work out and keep all your intermediate results -- they will help you in the problems to come.

a)




b)



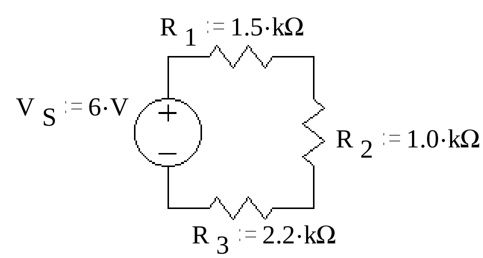

Note: the hard part of these problems is actually seeing which resistors are in parallel and which are in series. You may want to redraw the circuits a few times to help you figure it out.

**Voltage dividers**

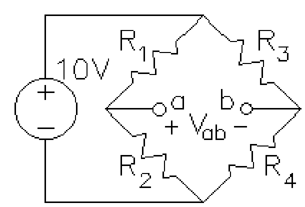
2. a) Use the voltage divider concept to find the voltage across each of the resistors in the circuit at right. $V_{R1} = ?$ $V_{R2} = ?$ $V_{R3} = ?$

   b) Confirm that the three resistor voltages add up to the source voltage, ie, confirm Kirchoff's voltage law.

   c) Without recalculating anything, what would happen to all the resistor voltages if the source voltage were doubled? Tripled?




3. The circuit at right is known as a *wheatstone bridge*, or simply a *bridge*. It is a very common measurement circuit, used with strain gauges, thermisters, and other devices whose resistance changes in response to something that you'd like to measure. Let's assume the resistors in this circuit are 100Ω strain gauges. The resistance of these gauges changes slightly when you stretch or compress them. They are glued to a material (often steel) and are used to measure deformations of the material (called strain).



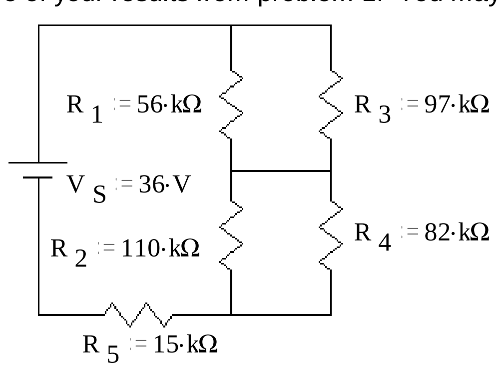

   a) Due to deformation, $R_1$ and $R_4$ decrease by 1% and $R_2$ and $R_3$ increase by 1%. Find $V_{ab}$.

   b) Due to a temperature change, the resistances of all the gages increase by 5%. Find the % change in $V_{ab}$.

   c) Why do you think the bridge circuit is used in this case?

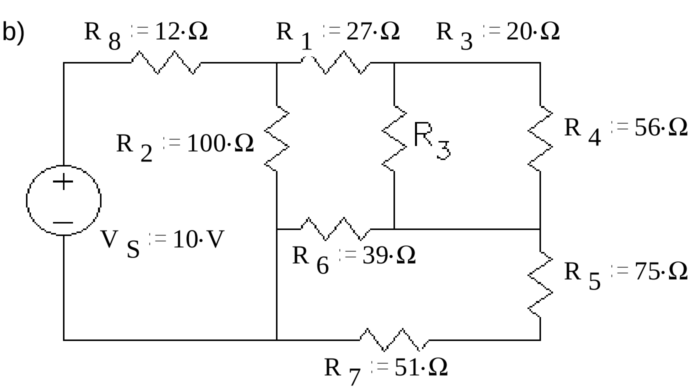
4. Use voltage divider concepts to find the voltages indicated in the following circuits. You may want to use some of your results from problem 1. You may need to use the voltage divider equation more than once.

a)




$V_{R5} = ?$ $V_{R4} = ?$ $V_{R1} = ?$

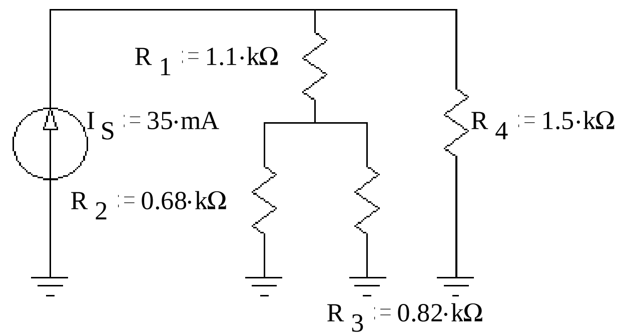
b)




$V_{R8} = ?$ $V_{R2} = ?$ $V_{R1} = ?$

**Current Dividers**

5. The circuit at right shows a current source hooked to a resistor network. Remember that the grounds are all connected together. You can draw lines between them if it helps you.




   a) Use the current divider concept to find the current through each of the resistors in the circuit at right.

   $I_{R1} = ?$ $I_{R2} = ?$ $I_{R3} = ?$ $I_{R4} = ?$

   b) Confirm that $I_{R2} + I_{R3} = I_{R1}$ and that $I_{R1} + I_{R4} = I_S$, ie, confirm Kirchoff's current law twice.

   c) Without recalculating anything, what would happen to all the currents if the source current were doubled? Tripled?

6. Refer back to the circuit of problem 4b.
   a) Find the equivalent resistance as seen by the source ($R_8$ + your answer for problem 1b) and use that to find the source current ($I_S$ or $I_{R8}$).

   b) Find these currents by current divider methods. $I_{R2} = ?$ $I_{R1} = ?$ $I_{R4} = ?$

   c) Using Ohm's law and the currents you found in this problem, confirm the voltages found in problem 4b.

**Power**

7. Refer to the circuit of problem 2.

   a) How much power is dissipated by each resistor? $P_{R1} = ?$ $P_{R2} = ?$ $P_{R3} = ?$

   b) Independently determine the power that the source is contributing to the circuit. $P_S = V_S \cdot I_S = ?$

   c) Show that power is conserved (Σ answers to a = answer to b).

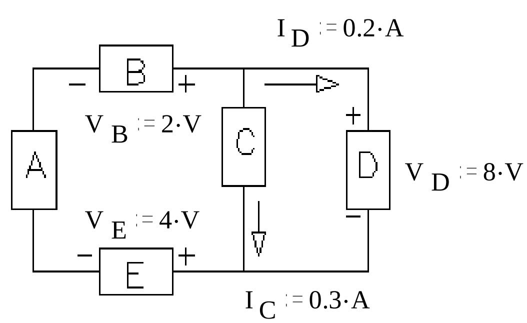
8. Refer to the circuit of problem 5.

   a) How much power is dissipated by each resistor? $P_{R1} = ?$ $P_{R2} = ?$ $P_{R3} = ?$ $P_{R4} = ?$

   b) Independently determine the power that the source is contributing to the circuit. $P_S = V_S \cdot I_S = ?$

   c) Show that power is conserved.

9. The circuit at right has five unknown components labeled A through E.




   a) Which of the components are absorbing power from the circuit?

   b) Which of the components are contributing power to the circuit?

   c) Show that power is conserved.

**<u>Answers</u>**

1. a) $R_{eq} := 82.5\cdot k\Omega$ b) $R_{eq} := 41.7\cdot\Omega$
2. a) $1.91\cdot V$, $1.28\cdot V$, $2.81\cdot V$ b) $1.91\cdot V + 1.28\cdot V + 2.81\cdot V = 6\cdot V$ c) double, triple
3. a) $100\cdot mV$ b) 0% change c) Reading won't be affected by temperature.
4. a) $5.54\cdot V$, $17.35\cdot V$, $13.11\cdot V$ b) $2.23\cdot V$, $7.77\cdot V$, $2.93\cdot V$
5. a) $17.67\cdot mA$, $9.66mA$, $8.01\cdot mA$, $17.33\cdot mA$ b) both check c) double, triple
6. a) $53.7\cdot\Omega$, $0.186\cdot A$ b) $77.65\cdot mA$, $108.6\cdot mA$, $28.6\cdot mA$ c) all agree
7. a) $2.44\cdot mW$, $1.63\cdot mW$, $3.59\cdot mW$ b) $7.66\cdot mW$ c) $P_S = P_{R1} + P_{R2} + P_{R3}$
8. a) $0.343\cdot W$, $0.0634\cdot W$, $0.0526\cdot W$, $0.451\cdot W$ b) $0.910\cdot W$ c) $P_{R1} + P_{R2} + P_{R3} + P_{R4} = P_S$
9. a) C, D, E b) A, B c) $6\cdot W = 6\cdot W$

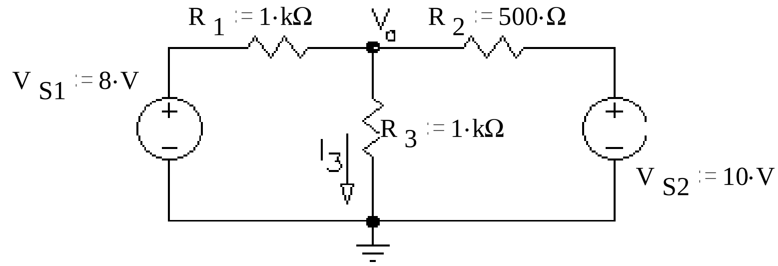
# ECE 2210 / 00 homework # 4 Due: Tue, 2/2/21

a

Answer the following problems on your own paper. Show your equations and work to get credit on this and all future homeworks.

**Superposition**

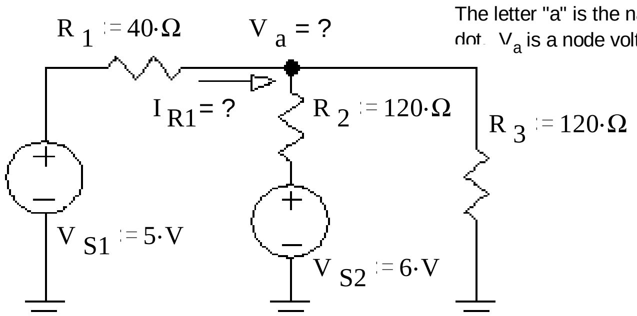
1. Use superposition to find $I_3$. Circle your intermediate solutions on your paper. Your intermediate solutions show how much of $I_3$ is due to $V_{S1}$, and how much is due to $V_{S2}$.




2. Use superposition to solve following problems: Each problem asks for both a current and a voltage.

   Clearly indicate your intermediate answers, the grader will look for those.

   a)

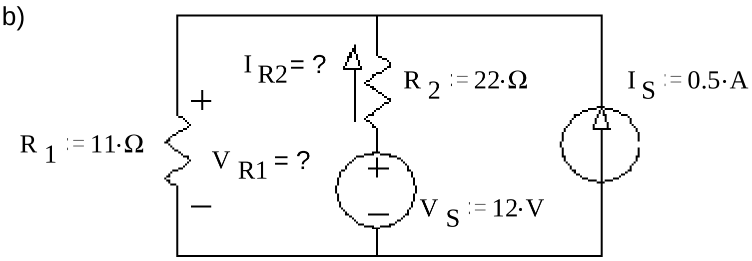



The letter "a" is the name of the "node" at the black dot. $V_a$ is a node voltage, referenced to ground.

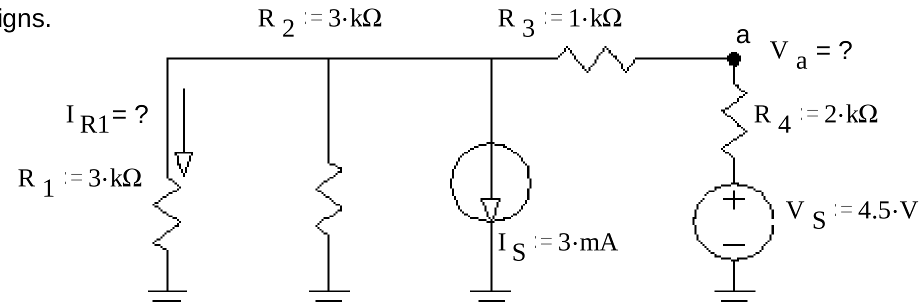
$V_a = V_{R3}$

These are ground symbols. They are all connected together, although that connection is not explicitly shown.

   b)




   c) Watch your signs.




**<u>Answers</u>**

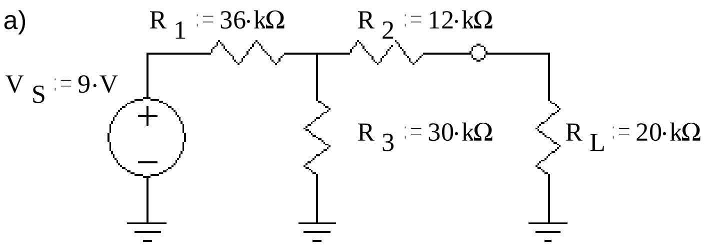
1. $2\cdot mA + 5\cdot mA = 7\cdot mA$
2. a) $4.2\cdot V$ , $20\cdot mA$ b) $7.67\cdot V$ , $197\cdot mA$
   c) $0.5\cdot V$ , $-0.5\cdot mA$

# ECE 2210 / 00 homework # 5 Due: Sat, 2/6/21 a

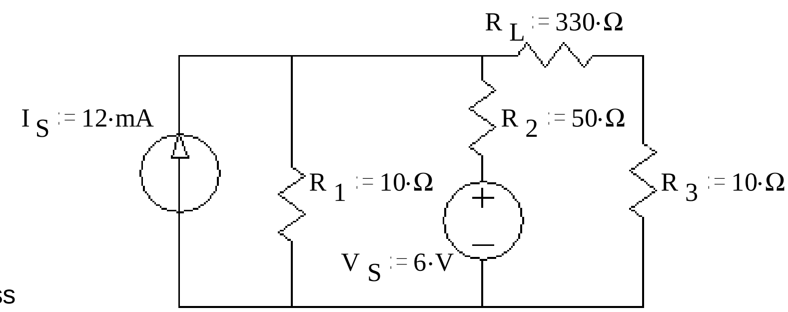
**Thevenin & Norton equivalent circuits**

1. For each of the circuits below, find and draw the Thevenin equivalent circuit.

a)




b) The load resistor is $R_L$, and is in a strange place in this circuit.
Hint: use superposition to find $V_{Th}$.



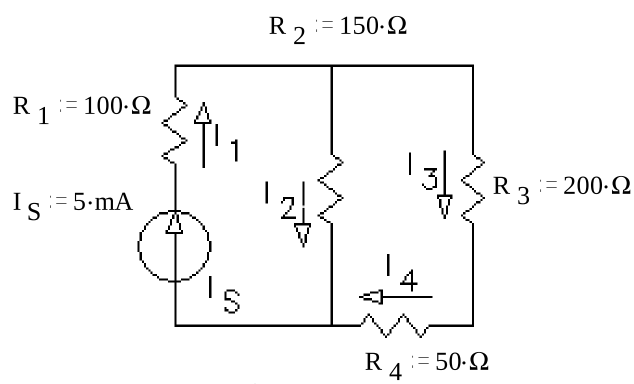

2nd hint: Nodal analysis is even easier.

2. For the circuit of problem 1a, find the voltage across $R_L$ ( $V_L$) and the current through $R_L$ ($I_L$) using your Thevenin equivalent circuit.

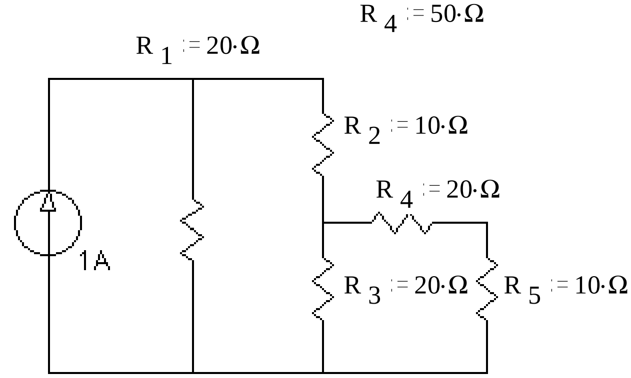
3. For each of the circuits in problem 1, find and draw the Norton equivalent circuit.

4. For the circuit of problem 1b, find $V_L$ and $I_L$ using your Norton equivalent circuit.

5. For the circuit shown at right, use Thevenin's theorem to find the current through the $50\ \Omega$ resistor $R_4$.




6. For the circuit shown, use Norton's theorem to find the value of the current in $R_5$. Hint: You can find $I_N$ either by calculation of the open circuit voltage ($V_{OC}$) and $R_N$ or by direct calculation of the short-circuit current ($I_{SC}$), however, there is something about the values of the resistors which makes the second method easier than it would at first appear.




**Source resistance**

7. The terminal voltage of a car's battery drops from 12.5 V to 8.5 volts when starting. The starter motor draws 60 A of current.

   a) Draw the voltage-source model (Thevenin equivalent) of this battery. Include the values of $V_S$ and $R_S$.

   b) Draw the current-source model (Norton equivalent) of this battery. Include the values of $I_S$ and $R_S$.

   c) Which of these two models is more appropriate for the car battery?

   d) What terminal voltage would you expect if this battery were being charged at 20 A?

## Answers

1. a) $4.091 \cdot V$ , $28.4 \cdot k\Omega$ b) $1.1 \cdot V$ , $18.3 \cdot \Omega$ 2. $1.69 \cdot V$ , $84.6 \cdot \mu A$

3. a) $0.144 \cdot mA$ , $28.4 \cdot k\Omega$ b) $60 \cdot mA$ , $18.3 \cdot \Omega$ 4. $3.16 \cdot mA$, $1.042 \cdot V$ 5. $1.88 \cdot mA$

6. $0.19 \cdot A$ 7. a) $V_S = 12.5 \cdot V$ $R_S := 0.0667 \cdot \Omega$ b) $I_S = 187.5 \cdot A$ $R_S := 0.0667 \cdot \Omega$

c) Thevenin d) $13.83 \cdot V$