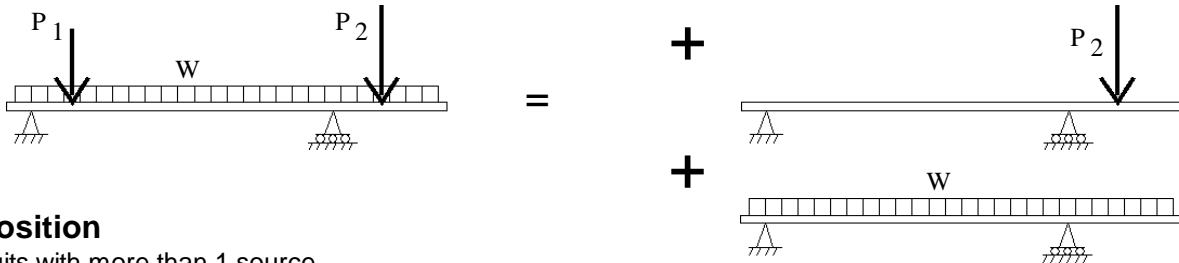


ECE 2210 Lecture notes Superposition

Circuits with more than one Source

A. Stolp
9/3/08,
7/31/09

Recall Statics. To find the reaction at each support, the reactions to each load on a beam (or anything else) can be found separately for each load. Simply add them up to find the total reactions.



Superposition

For circuits with more than 1 source.

1) Zero all but one source.

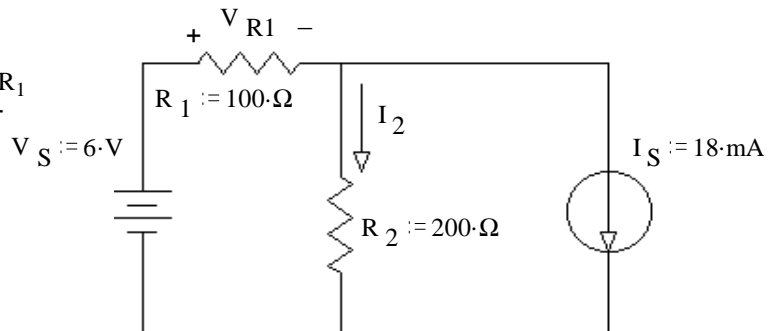
(To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)

2) Compute your wanted voltage or current due to the remaining source. Careful, some may be negative.

3) Repeat the first two steps for all the sources.

4) Sum all the contributions from all the sources to find the actual voltage or current. **Watch your signs!**

Ex1. Use the method of superposition to find the current I_2 (through R_2) and the voltage across R_1 (V_{R1}). Be sure to clearly show and **circle** your intermediate results.



superposition:

Eliminate current source

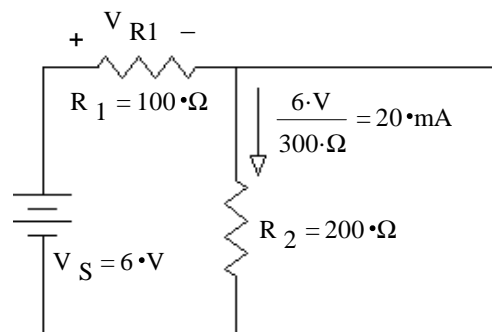
$$I_{2,V_S} := \frac{V_S}{R_1 + R_2}$$

$$I_{2,V_S} = 20 \text{ mA}$$

$$V_{R1,V_S} := \frac{R_1}{R_1 + R_2} \cdot V_S$$

$$V_{R1,V_S} = 2 \text{ V}$$

$$20 \text{ mA} \cdot 100 \Omega = 2 \text{ V}$$



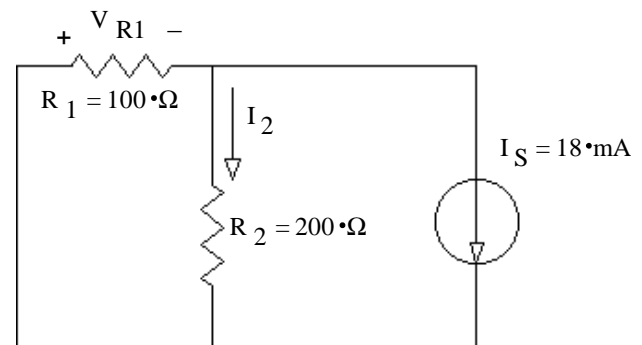
Eliminate voltage source

$$I_{2,I_S} := -\frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} \cdot I_S$$

$$I_{2,I_S} = -6 \text{ mA}$$

$$V_{R1,I_S} := -I_{2,I_S} \cdot R_2$$

$$V_{R1,I_S} = 1.2 \text{ V}$$



Add results

$$I_2 := I_{2,V_S} + I_{2,I_S}$$

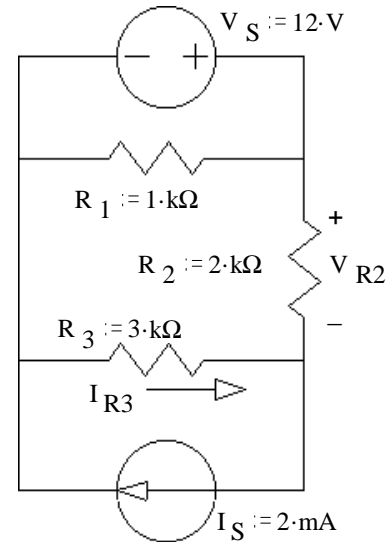
$$I_2 = 14 \text{ mA}$$

$$V_{R1} := V_{R1,V_S} + V_{R1,I_S}$$

$$V_{R1} = 3.2 \text{ V}$$

ECE 2210 Lecture notes p2

Ex2. Use the method of superposition to find the voltage across R_2 and the current through R_3 . Be sure to clearly show and **circle** your intermediate results.

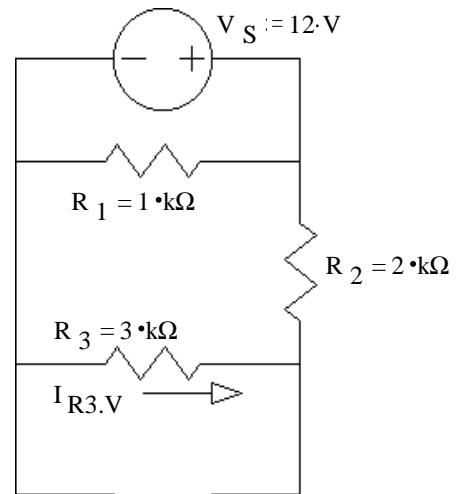


Eliminate current source

R_1 is a separate path and doesn't matter.

$$V_{R2.Vs} := \frac{R_2}{R_2 + R_3} \cdot V_S \quad V_{R2.Vs} = 4.8 \cdot V$$

$$I_{R3.Vs} := -\frac{V_S}{R_2 + R_3} \quad I_{R3.Vs} = -2.4 \cdot mA$$

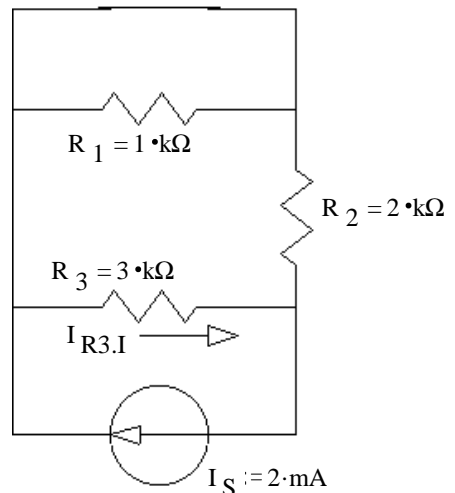


Eliminate voltage source

R_1 is shorted and doesn't matter.

$$V_{R2.Is} := I_S \cdot \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} \quad V_{R2.Is} = 2.4 \cdot V$$

$$I_{R3.Is} := \frac{\frac{1}{R_3}}{\frac{1}{R_2} + \frac{1}{R_3}} \cdot I_S \quad I_{R3.Is} = 0.8 \cdot mA$$



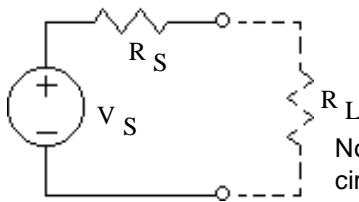
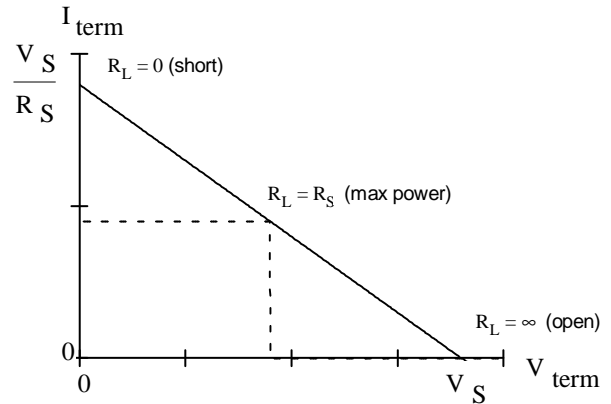
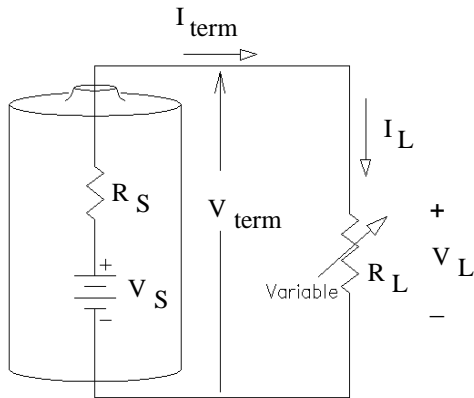
Add results

$$V_{R2} := V_{R2.Vs} + V_{R2.Is} \quad V_{R2} = 7.2 \cdot V$$

$$I_{R3} := I_{R3.Vs} + I_{R3.Is} \quad I_{R3} = -1.6 \cdot mA$$

Simple Model of a Real Source

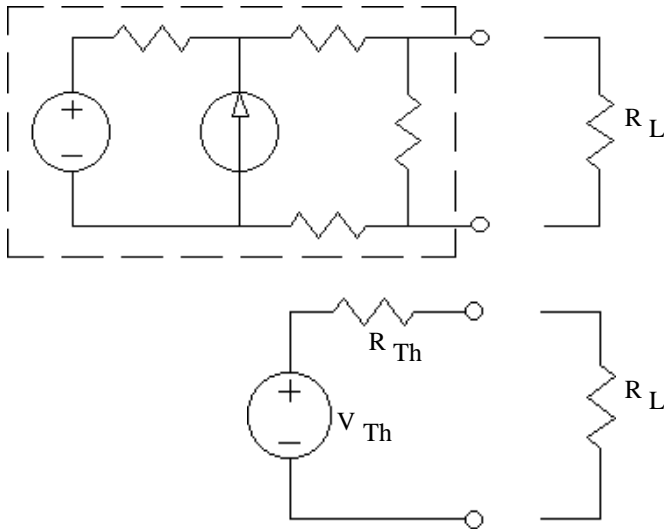
Real sources are not ideal, but we will model them with two ideal components.



Note: R_L is NOT part of the Thévenin equivalent circuit and does not need to be shown.

Thévenin Equivalent Circuit

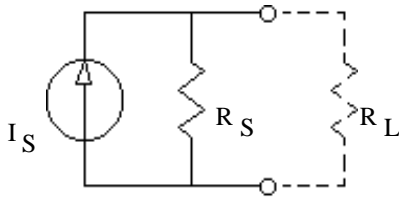
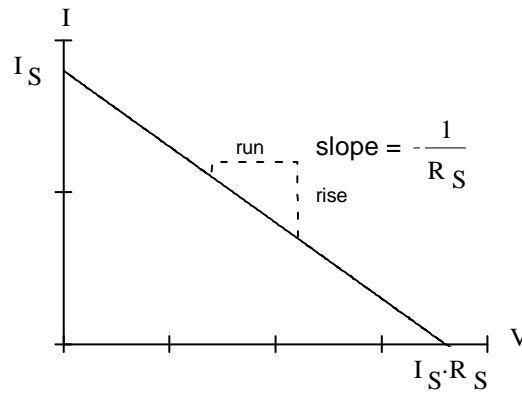
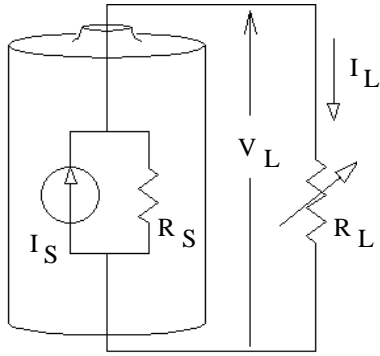
The same model can be used for any combination of sources and resistors.



Thévenin equivalent

To calculate a circuit's Thévenin equivalent:

- 1) Remove the load and calculate the open-circuit voltage where the load used to be.
This is the Thévenin voltage (V_{Th}).
- 2) Zero all the sources.
(To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
- 3) Compute the total resistance between the load terminals.
(DO NOT include the load in this resistance.) This is the Thévenin source resistance (R_{Th}).
- 4) Draw the Thévenin equivalent circuit and add your values.

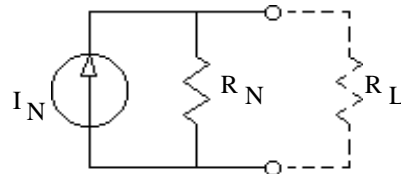


Note: R_L is not part of the Norton equivalent and does not need to be shown.

Norton equivalent

To calculate a circuit's Norton equivalent:

- 1) Replace the load with a short (a wire) and calculate the short-circuit current in this wire.
This is the Norton current (I_N). Remove the short.
- 2) Zero all the sources.
(To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
- 3) Compute the total resistance between the load terminals.
(DO NOT include the load in this resistance.) This is the Norton source resistance (R_N).
(Exactly the same as the Thévenin source resistance (R_{Th})).
- 4) Draw the Norton equivalent circuit and add your values.



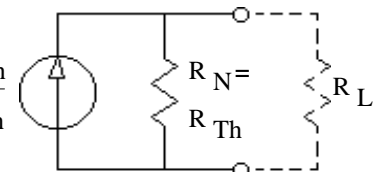
OR (the more common way)...

- 1) Find the Thévenin equivalent circuit.
- 2) Convert to Norton circuit, then >>>

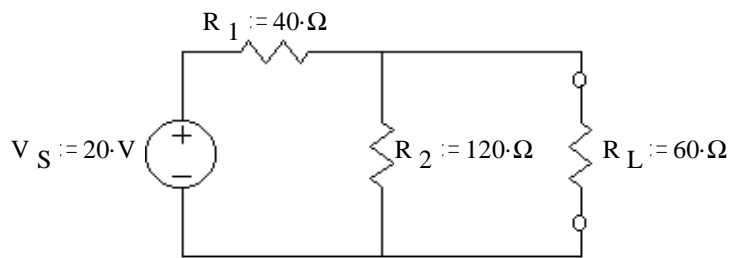
$$R_N = R_{Th}$$

and

$$I_N = \frac{V_{Th}}{R_{Th}}$$



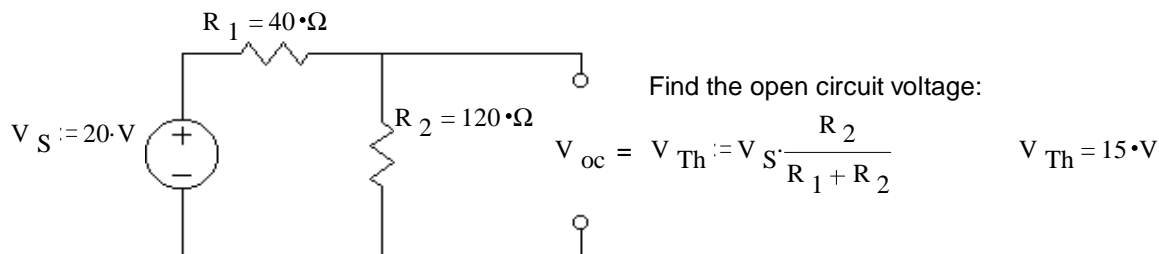
Ex 1 Find the Thévenin equivalent:



To calculate a circuit's Thévenin equivalent:

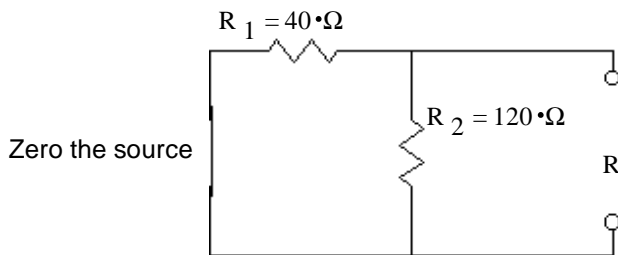
1) Remove the load and calculate the open-circuit voltage where the load used to be.

This is the Thévenin voltage (V_{Th}).



2) Zero all the sources.

(To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)



3) Compute the total resistance between the load terminals.

(DO NOT include the load in this resistance.)

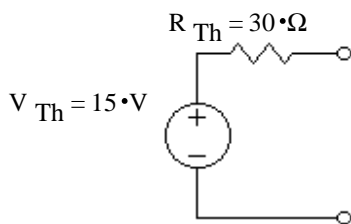
This is the Thévenin source resistance (R_{Th}).

Find the Thevenin resistance:

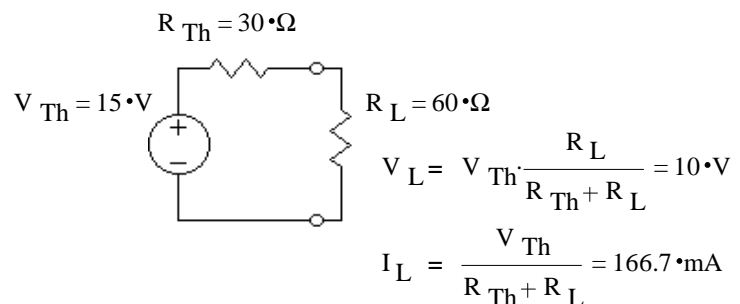
$$R_{Th} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \quad R_{Th} = 30 \cdot \Omega$$

4) Draw the Thévenin equivalent circuit and add your values.

Thevenin equivalent circuit:

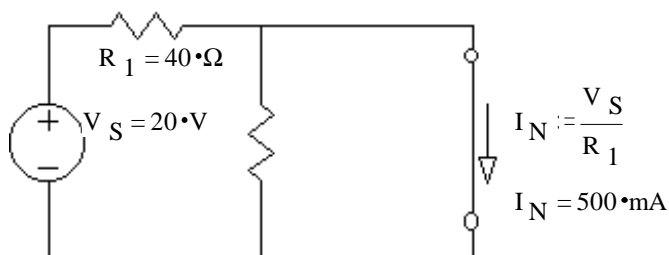


If the load were reconnected:

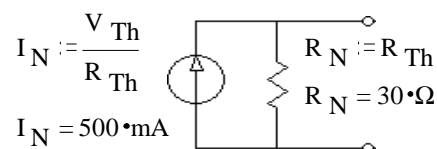


$$P_L = 10 \cdot V \cdot 166.7 \cdot mA = 1.667 \cdot W$$

b) Find the Norton equivalent circuit:



Norton equivalent circuit:



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c) Show that the Thévenin circuit is indeed equivalent to the original at several values of R_L .

	Original Circuit		Thévenin Circuit	
R_L	V_L	I_L	I_L	V_L
$R_L := 0 \cdot \Omega$	0·V	$\frac{V_S}{R_1} = 500 \cdot \text{mA}$	$\frac{V_{Th}}{R_{Th} + R_L} = 500 \cdot \text{mA}$	$500 \cdot \text{mA} \cdot 0 \cdot \Omega = 0 \cdot \text{V}$

Using either numbers: $P_L = V_L \cdot I_L = 0 \cdot \text{W}$

$R_L := 10 \cdot \Omega$	$R_o := \frac{1}{\frac{1}{R_2} + \frac{1}{R_L}}$	$R_o = 9.231 \cdot \Omega$	$I_L := \frac{V_{Th}}{R_{Th} + R_L}$	$V_L := I_L \cdot R_L$
			$I_L = 375 \cdot \text{mA}$	$V_L = 3.75 \cdot \text{V}$

$$V_L = V_S \cdot \frac{R_o}{R_1 + R_o} = 3.75 \cdot \text{V}$$

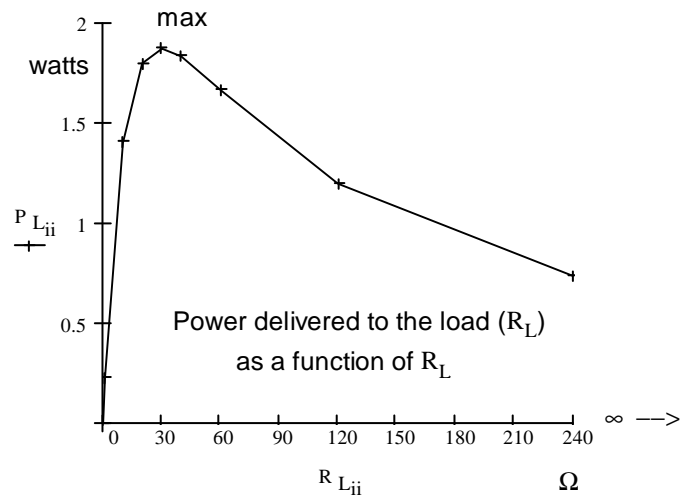
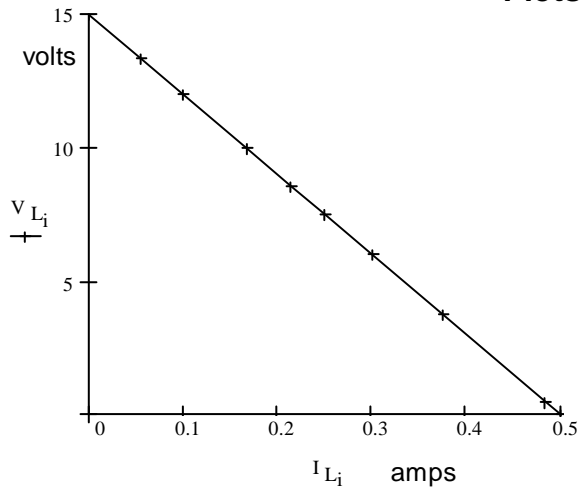
$$I_L = \frac{V_L}{R_L} = 375 \cdot \text{mA}$$

Using either numbers: $P_L = V_L \cdot I_L = 1.406 \cdot \text{W}$

Repeat these calculations for a number of load resistors

$R_{L_i} :=$	R_{o_i}	$V_L = V_S \cdot \frac{R_{o_i}}{R_1 + R_{o_i}}$	$I_L = \frac{V_{L_i}}{R_{L_i}}$	$I_L = \frac{V_{Th}}{R_{Th} + R_{L_i}}$	$V_L = I_{L_i} \cdot R_{L_i}$	P_{L_i}
	Ω	V	mA	mA	V	W
0·Ω	0	0	0	500	0	0
1·Ω	0.992	0.484	483.871	483.871	0.484	0.234
10·Ω	9.231	3.75	375	375	3.75	1.406
20·Ω	17.143	6	300	300	6	1.8
30·Ω	24	7.5	250	250	7.5	1.875 max
40·Ω	30	8.571	214.286	214.286	8.571	1.837
60·Ω	40	10	166.667	166.667	10	1.667
120·Ω	60	12	100	100	12	1.2
240·Ω	80	13.333	55.556	55.556	13.333	0.741
∞·Ω	120	15	0	0	15	0

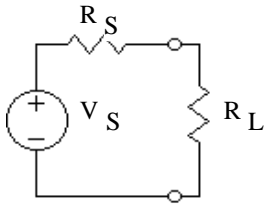
Plots



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Maximum power transfer

If I wanted to maximize the power dissipated by the load, what R_L would I choose?



$$P_L = \frac{V_L^2}{R_L} = \left(\frac{R_L}{R_S + R_L} \cdot V_S \right)^2 \cdot \frac{1}{R_L} = \frac{R_L^2}{(R_S + R_L)^2} \cdot V_S^2 \cdot \frac{1}{R_L}$$

$$= \frac{R_L^2}{R_S^2 + 2 \cdot R_S \cdot R_L + R_L^2} \cdot V_S^2 \cdot \frac{1}{R_L} = \frac{R_L}{R_S^2 + 2 \cdot R_S \cdot R_L + R_L^2} \cdot V_S^2$$

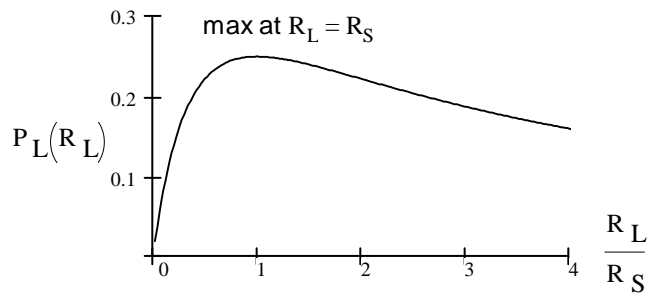
$$= \frac{1}{\frac{R_S^2}{R_L} + 2 \cdot R_S + R_L} \cdot V_S^2$$

Next step would be to differentiate $\frac{d}{dR_L} P_L(R_L)$,

set this equal to 0 and solve for R_L to find the maximum

Unfortunately this function is a pain to differentiate. What if we just differentiate the denominator and find its minimum, wouldn't that work just as well?

$$\frac{d}{dR_L} \left(\frac{R_S^2}{R_L} + 2 \cdot R_S + R_L \right) = -1 \cdot \frac{R_S^2}{R_L^2} + 0 + 1 = 0$$



Maximum power transfer happens when: $R_L = R_S$

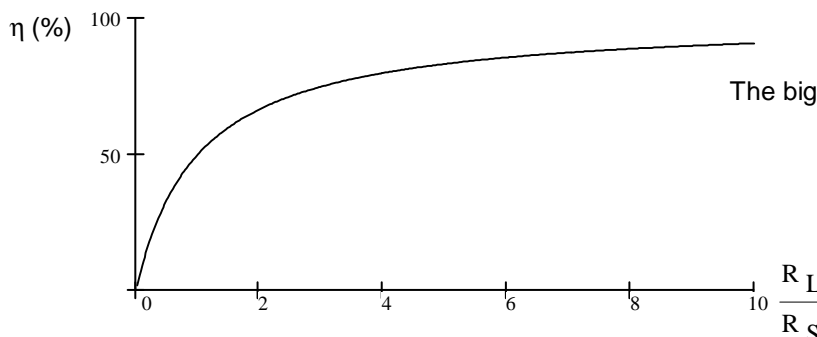
Just what we saw in Example 1

This is rarely important in power circuitry, where there should be plenty of power and R_S should be small. It is much more likely to be important in signal circuitry where the voltages can be very small and the source resistance may be significant -- say a microphone or a radio antenna.

All you need to remember is: $R_L = R_S$ to maximize the power dissipation in R_L

What about efficiency?

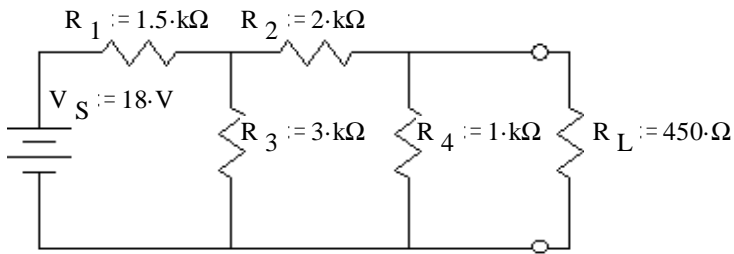
$$\frac{P_L(R_L)}{P_S(R_L)} = \frac{I^2 \cdot R_L}{I^2 \cdot (R_S + R_L)} = \frac{R_L}{R_S + R_L}$$



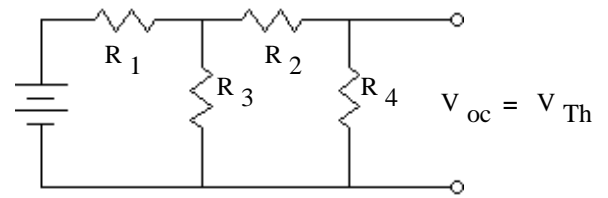
The bigger R_L is, the higher the efficiency.

Ex 2 a) Find and draw the Thévenin equivalent circuit.

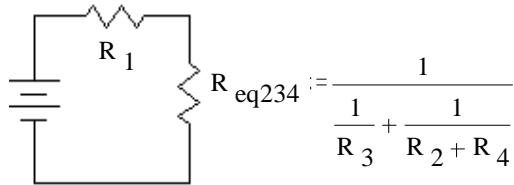
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Find the open circuit voltage:



First do some simplification:

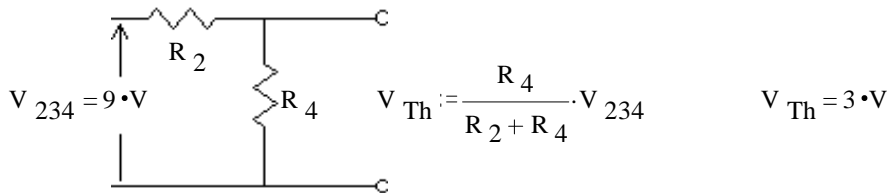


$$R_{eq234} = 1.5 \cdot k\Omega$$

$$V_{234} := \frac{R_{eq234}}{R_1 + R_{eq234}} \cdot V_S$$

$$V_{234} = 9 \cdot V$$

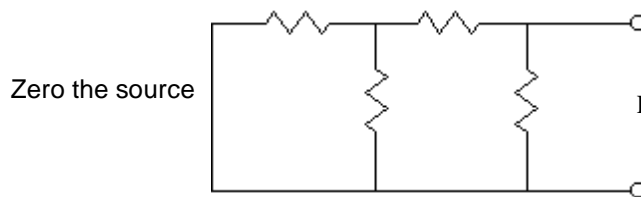
Divide this voltage between R_2 and R_4 :



$$V_{Th} := \frac{R_4}{R_2 + R_4} \cdot V_{234}$$

$$V_{Th} = 3 \cdot V$$

Find the Thévenin resistance:

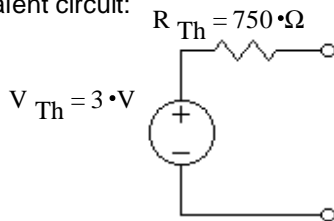


Zero the source

$$R_{Th} := \frac{1}{\frac{1}{R_4} + \frac{1}{R_2 + \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_3}}\right)}}$$

$$R_{Th} = 750 \cdot \Omega$$

Thévenin equivalent circuit:



If the load were reconnected:

$$V_L := V_{Th} \cdot \frac{R_L}{R_{Th} + R_L}$$

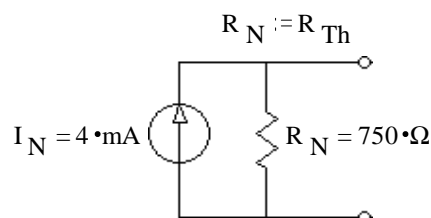
$$V_L = 1.125 \cdot V$$

$$I_L := \frac{V_{Th}}{R_{Th} + R_L}$$

$$I_L = 2.5 \cdot mA$$

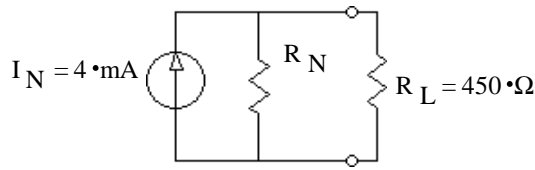
b) Find and draw the Norton equivalent circuit.

$$I_N := \frac{V_{Th}}{R_{Th}}$$



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c) Use your Norton equivalent circuit to find the current through the load.



$$I_L := \frac{\frac{1}{R_L}}{\left(\frac{1}{R_N} + \frac{1}{R_L}\right)} \cdot I_N \quad I_L = 2.5 \text{ mA}$$

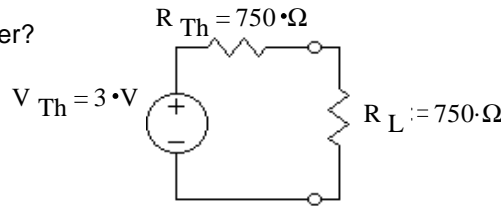
$$V_L := I_L \cdot R_L \quad V_L = 1.125 \text{ V}$$

same as above

d) What value of R_L would result in the maximum power delivery to R_L ?

For maximum power transfer $R_L = R_{Th} = 750 \text{ ohms}$

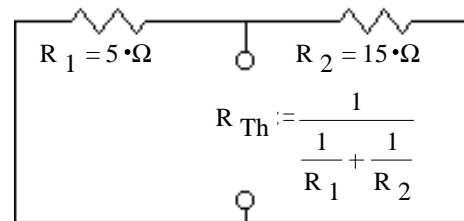
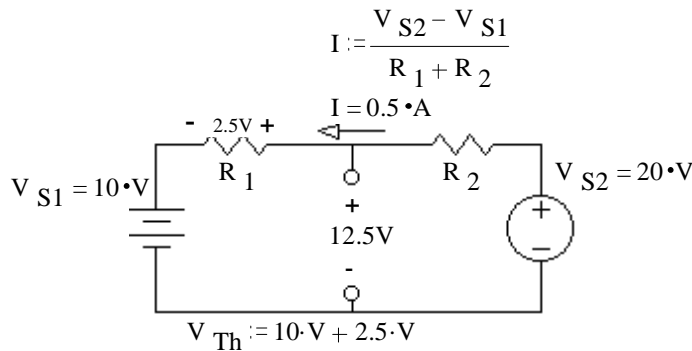
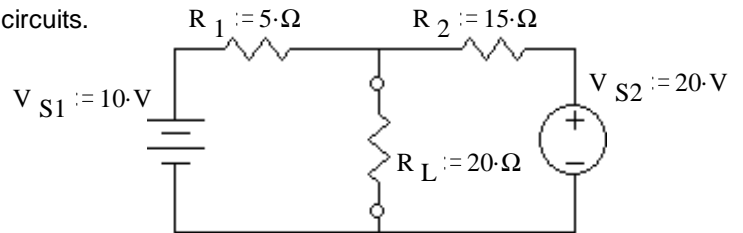
e) What is the maximum power transfer?



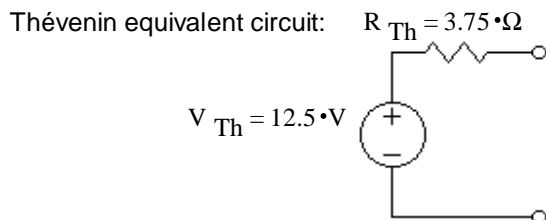
$$V_L := \frac{V_{Th}}{2}$$

$$P_L = \frac{V_L^2}{R_L} = 3 \text{ mW}$$

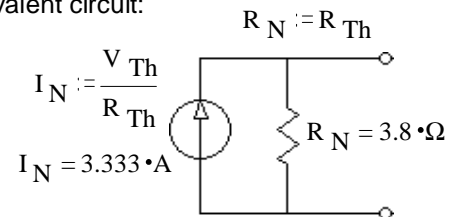
Ex 3 a) Find and draw the Thévenin & Norton equivalent circuits.



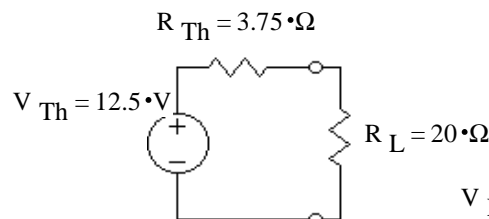
$$R_{Th} = 3.75 \text{ ohms}$$



Norton equivalent circuit:



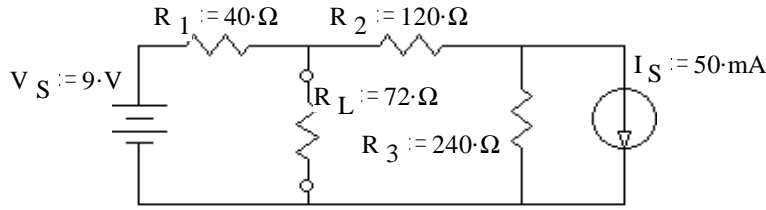
b) Use your Thévenin equivalent circuit to find the voltage across the load.



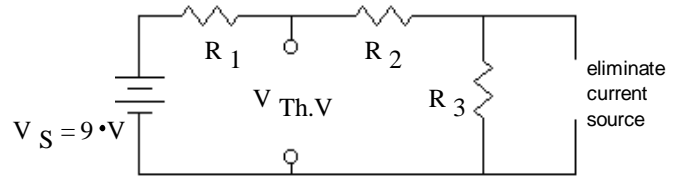
$$V_L = \frac{R_L}{R_{Th} + R_L} \cdot V_{Th} = 10.526 \text{ V}$$

Ex 4 a) Find and draw the Thévenin & Norton equivalent circuits.

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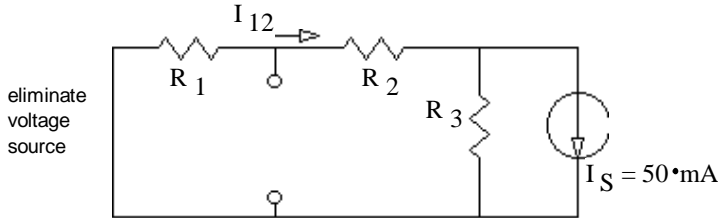


Use superposition to find V_{Th} .



$$V_{Th.V} := \frac{R_2 + R_3}{R_1 + R_2 + R_3} \cdot V_S$$

$$V_{Th.V} = 8.1 \cdot V$$



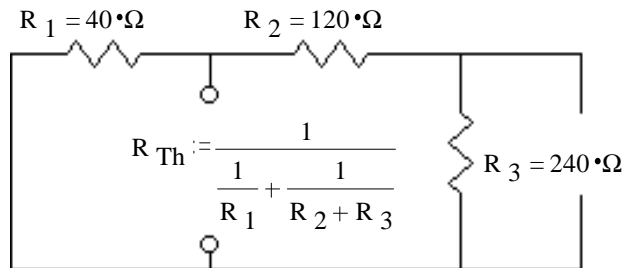
eliminate voltage source

$$\text{current divider: } I_{12} := \frac{\frac{1}{R_1 + R_2}}{\frac{1}{R_1 + R_2} + \frac{1}{R_3}} \cdot I_S \quad I_{12} = 30 \cdot \text{mA}$$

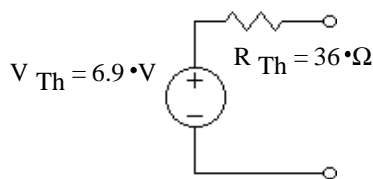
$$V_{Th.I} := -I_{12} \cdot R_1 \quad V_{Th.I} = -1.2 \cdot V$$

$$V_{Th} := V_{Th.V} + V_{Th.I} \quad V_{Th} = 6.9 \cdot V$$

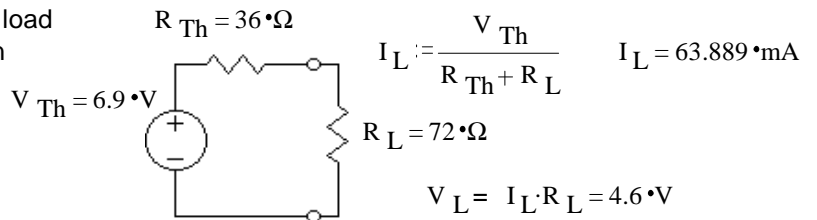
Find the Thévenin resistance



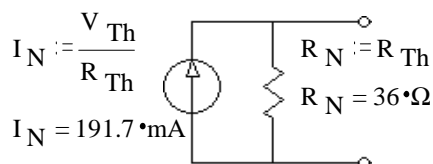
Thévenin equivalent circuit:



Put the load back on



Norton equivalent circuit:

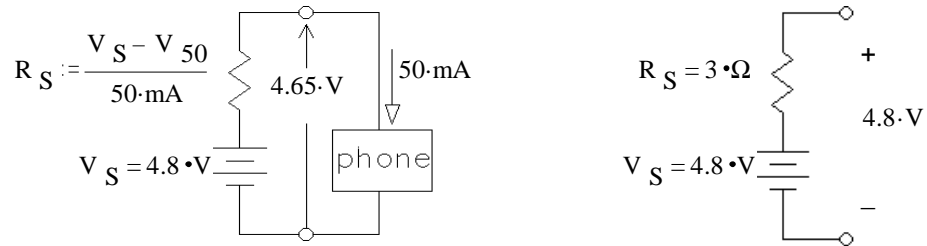


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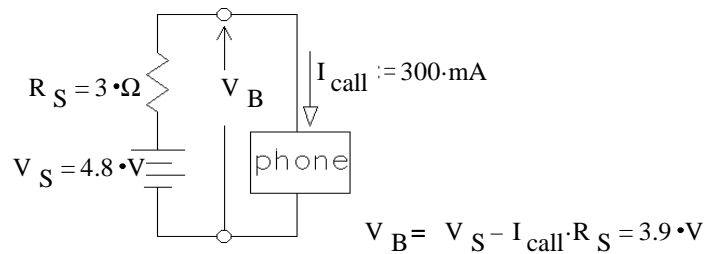
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Ex 5 A NiCad Battery pack is used to power a cell phone. When the phone is switched on the battery pack voltage drops from 4.80 V to 4.65 V and the cell phone draws 50 mA. $V_S := 4.80 \cdot V$ $V_{50} := 4.65 \cdot V$

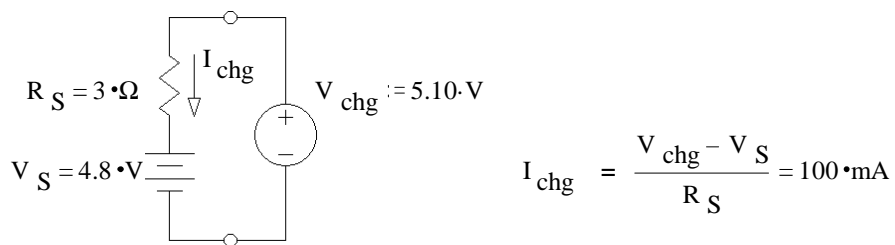
- a) Draw a simple, reasonable model of the battery pack using ideal parts. Find the value of each part.



- b) The cell phone is used to make a call. Now it draws 300 mA. What is the battery pack voltage now?



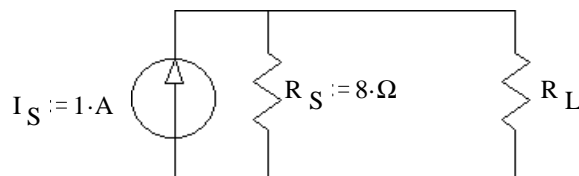
- c) The battery pack is placed in a charger. The charger supplies 5.10 V. How much current flows into the battery pack?



Ex 6 Consider the circuit at right.

- a) What value of load resistor (R_L) would you choose if you wanted to maximize the power dissipation in that load resistor.

$$R_L := R_S \quad R_L = 8 \cdot \Omega$$

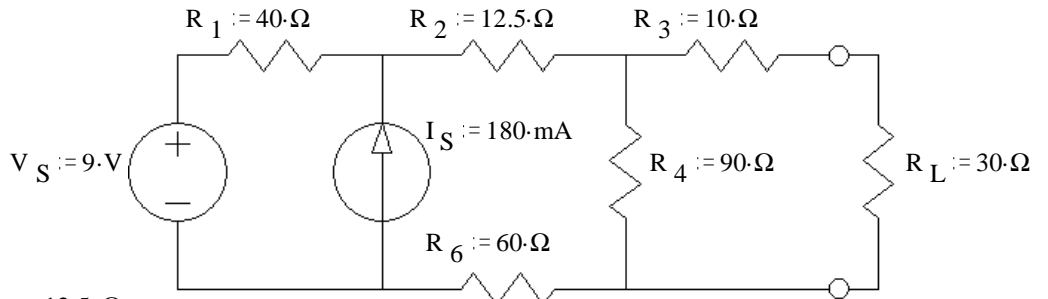


- b) With that load resistor (R_L) find the power dissipation in the load.

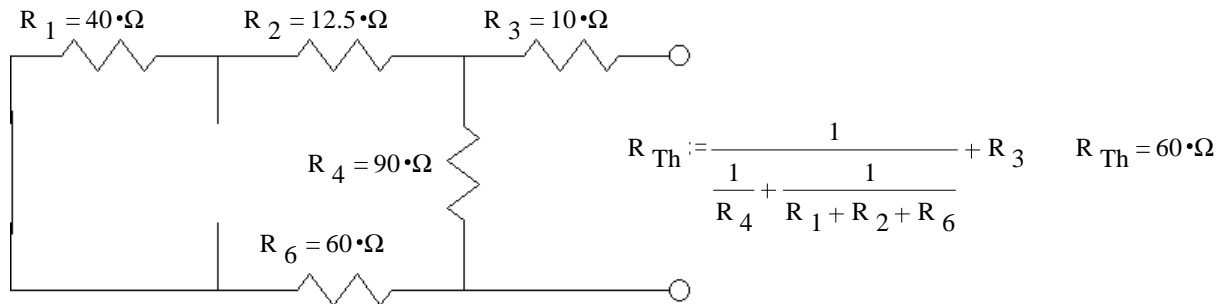
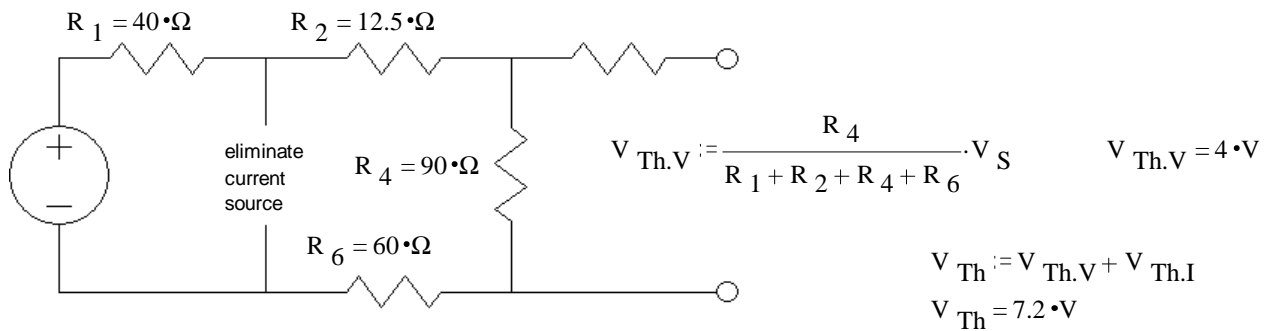
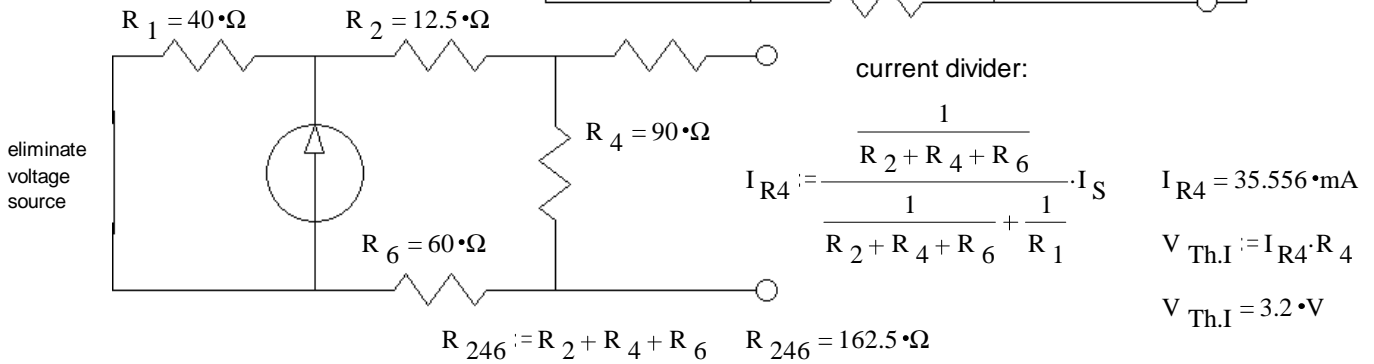
$$I_L := \frac{I_S}{2} \quad P_L = I_L^2 \cdot R_L = 2 \cdot W$$

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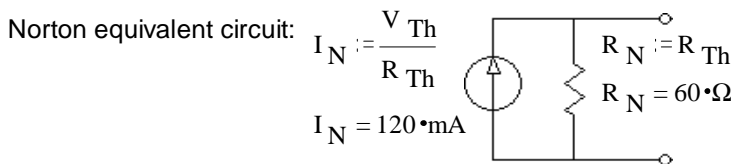
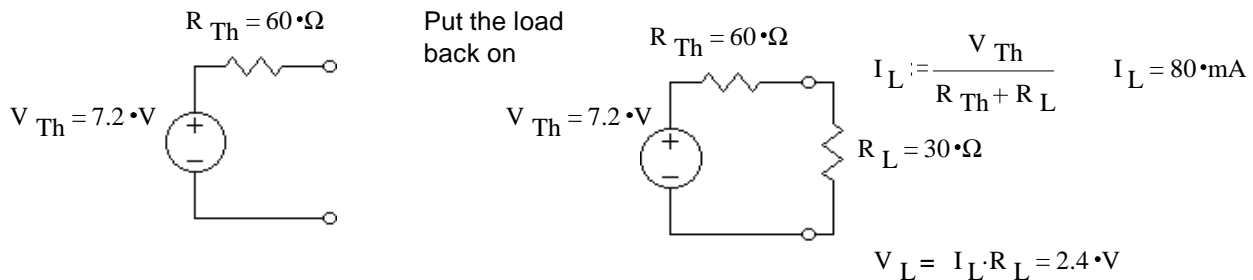
Ex 7



Use superposition to find V_{Th} .



Thévenin equivalent circuit:



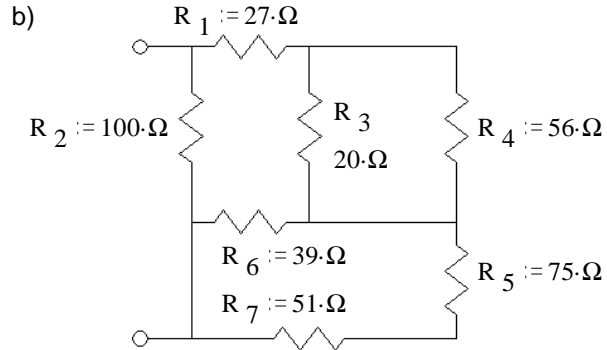
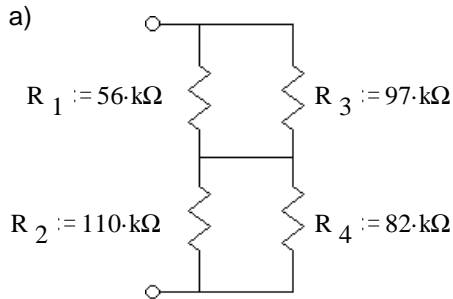
ECE 2210 / 00 homework # 3 Due: Fri, 9/4/20 You should finish p.1 by Wed.

Answer the following problems on your own paper or tablet.

Since you have the answers, you **must show** the equations and work you used to arrive at the answer to get credit.

Equivalent resistance

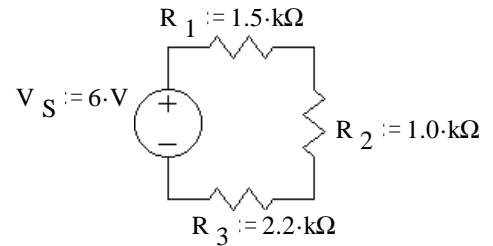
- Find the equivalent resistance of each of these networks, i.e. what would an ohmmeter read if hooked to the terminals. Work out and keep all your intermediate results -- they will help you in the problems to come.



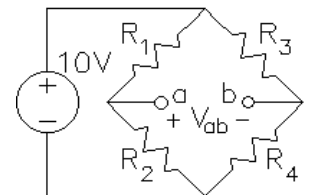
Note: the hard part of these problems is actually seeing which resistors are in parallel and which are in series. You may want to redraw the circuits a few times to help you figure it out.

Voltage dividers

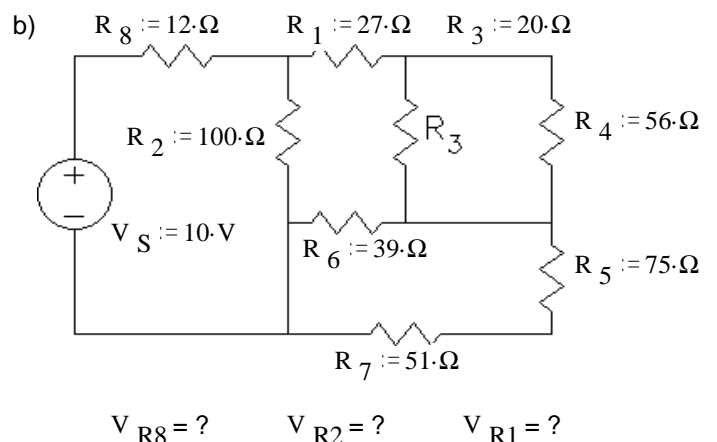
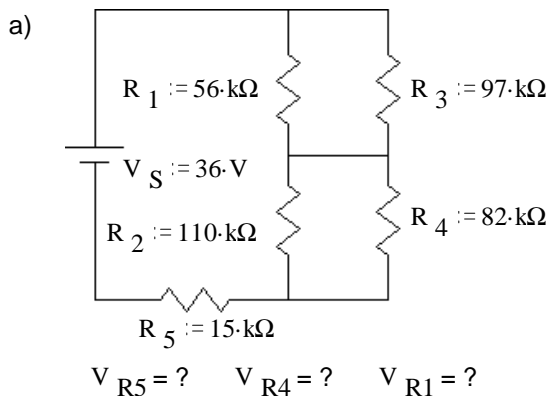
- Use the voltage divider concept to find the voltage across each of the resistors in the circuit at right. $V_{R1} = ?$ $V_{R2} = ?$ $V_{R3} = ?$
 - Confirm that the three resistor voltages add up to the source voltage, ie, confirm Kirchoff's voltage law.
 - Without recalculating anything, what would happen to all the resistor voltages if the source voltage were doubled? Tripled?



- The circuit at right is known as a *wheatstone bridge*, or simply a *bridge*. It is a very common measurement circuit, used with strain gauges, thermistors, and other devices whose resistance changes in response to something that you'd like to measure. Let's assume the resistors in this circuit are 100Ω strain gauges. The resistance of these gauges changes slightly when you stretch or compress them. They are glued to a material (often steel) and are used to measure deformations of the material (called strain).



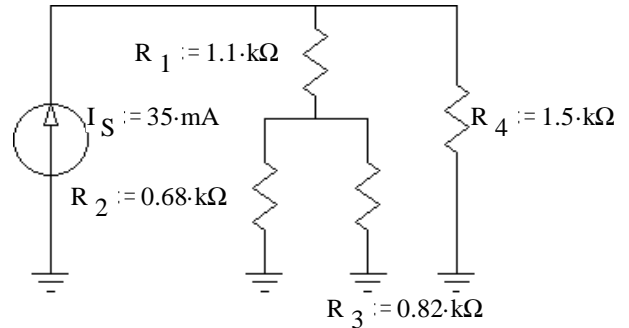
- Due to deformation, R_1 and R_4 decrease by 1% and R_2 and R_3 increase by 1%. Find V_{ab} .
 - Due to a temperature change, the resistances of all the gages increase by 5%. Find the % change in V_{ab} .
 - Why do you think the bridge circuit is used in this case?
- Use voltage divider concepts to find the voltages indicated in the following circuits. You may want to use some of your results from problem 1. You may need to use the voltage divider equation more than once.



ECE 2210 / 00 homework # 3 p.2

Current Dividers

5. The circuit at right shows a current source hooked to a resistor network. Remember that the grounds are all connected together. You can draw lines between them if it helps you.



a) Use the current divider concept to find the current through each of the resistors in the circuit at right.

$$I_{R1} = ? \quad I_{R2} = ? \quad I_{R3} = ? \quad I_{R4} = ?$$

b) Confirm that $I_{R2} + I_{R3} = I_{R1}$ and that $I_{R1} + I_{R4} = I_S$, ie, confirm Kirchoff's current law twice.

c) Without recalculating anything, what would happen to all the currents if the source current were doubled? Tripled?

6. Refer back to the circuit of problem 4b.

a) Find the equivalent resistance as seen by the source (R_8 + your answer for problem 1b) and use that to find the source current (I_S or I_{R8}).

b) Find these currents by current divider methods. $I_{R2} = ? \quad I_{R1} = ? \quad I_{R4} = ?$

c) Using Ohm's law and the currents you found in this problem, confirm the voltages found in problem 4b.

Power

7. Refer to the circuit of problem 2.

a) How much power is dissipated by each resistor? $P_{R1} = ? \quad P_{R2} = ? \quad P_{R3} = ?$

b) Independently determine the power that the source is contributing to the circuit. $P_S = V_S \cdot I_S = ?$

c) Show that power is conserved (Σ answers to a = answer to b).

8. Refer to the circuit of problem 5.

a) How much power is dissipated by each resistor? $P_{R1} = ? \quad P_{R2} = ? \quad P_{R3} = ? \quad P_{R4} = ?$

b) Independently determine the power that the source is contributing to the circuit. $P_S = V_S \cdot I_S = ?$

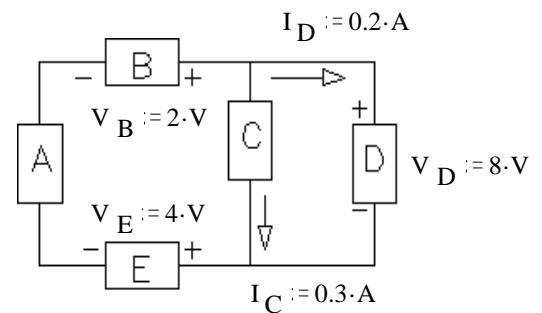
c) Show that power is conserved.

9. The circuit at right has five unknown components labeled A through E.

a) Which of the components are absorbing power from the circuit?

b) Which of the components are contributing power to the circuit?

c) Show that power is conserved.



Answers

1. a) $R_{eq} := 82.5 \cdot k\Omega$

b) $R_{eq} := 41.7 \cdot \Omega$

2. a) 1.91·V, 1.28·V, 2.81·V

b) 1.91·V + 1.28·V + 2.81·V = 6·V

c) double, triple

3. a) 100·mV b) 0% change

c) Reading won't be affected by temperature.

4. a) 5.54·V, 17.35·V, 13.11·V

b) 2.23·V, 7.77·V, 2.93·V

5. a) 17.67·mA, 9.66mA, 8.01·mA, 17.33·mA

b) both check

c) double, triple

6. a) 53.7·Ω, 0.186·A

b) 77.65·mA, 108.6·mA, 28.6·mA

c) all agree

7. a) 2.44·mW, 1.63·mW, 3.59·mW

b) 7.66·mW

c) $P_S = P_{R1} + P_{R2} + P_{R3}$

8. a) 0.343·W, 0.0634·W, 0.0526·W, 0.451·W

b) 0.910·W

c) $P_{R1} + P_{R2} + P_{R3} + P_{R4} = P_S$

9. a) C, D, E

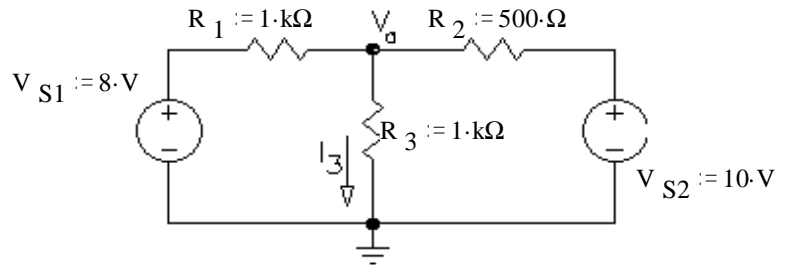
b) A, B

c) 6·W = 6·W

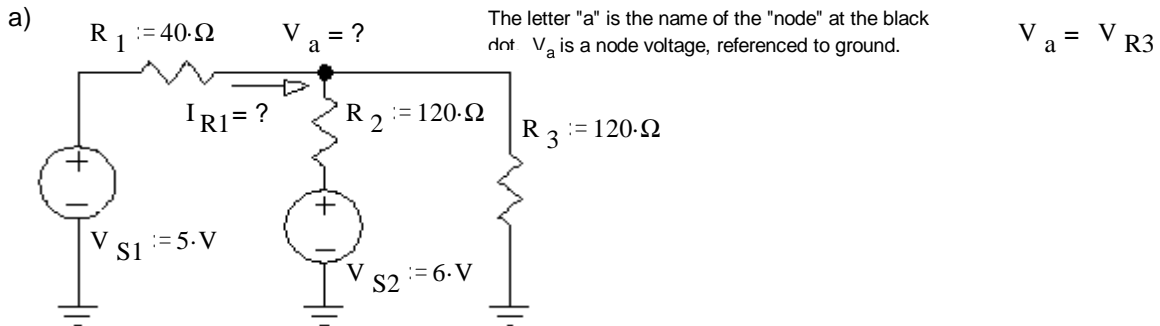
Answer the following problems on your own paper. Show your equations and work to get credit on this and all future homeworks.

Superposition

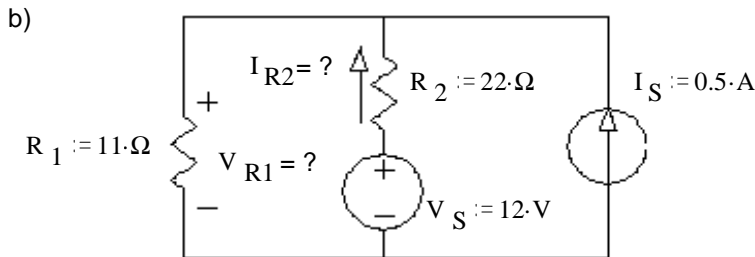
- Use superposition to find I_3 . Circle your intermediate solutions on your paper. Your intermediate solutions show how much of I_3 is due to V_{S1} , and how much is due to V_{S2} .



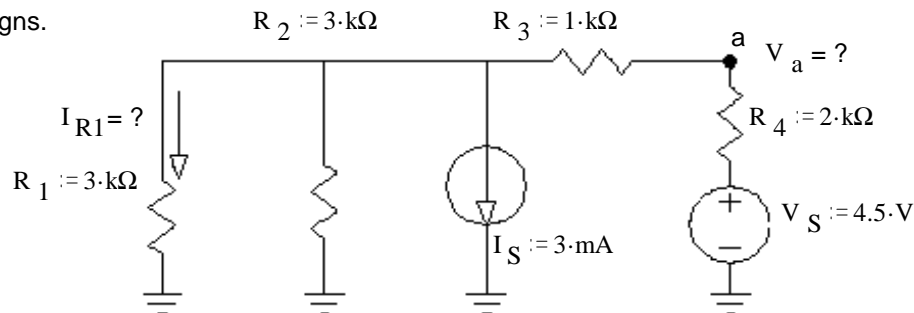
- Use superposition to solve following problems: Each problem asks for both a current and a voltage. Clearly indicate your intermediate answers, the grader will look for those.



These are ground symbols. They are all connected together, although that connection is not explicitly shown.



- c) Watch your signs.



Answers

- $2 \cdot \text{mA} + 5 \cdot \text{mA} = 7 \cdot \text{mA}$
- a) $4.2 \cdot \text{V}$, $20 \cdot \text{mA}$ b) $7.67 \cdot \text{V}$, $197 \cdot \text{mA}$
- $0.5 \cdot \text{V}$, $-0.5 \cdot \text{mA}$