There are a number of lockers on the 2nd floor of the MEB, in the center hallway. These lockers have slots cut in their doors so that homework and lab notebooks can be dropped through the slots. Turn in your homework in the locker marked "ECE 2210/00 Homework". (Sometimes lockers are separated as "ECE 2200 Homework" and "ECE 2210 Homework", look carefully the first time.) Homework is due by 5:00 p.m. on the due date.

The following problems are not meant to be hard. You should be able to do most of them in your head with no special formulas or calculations. In fact you should find them rather dumb and trivial. That's the point, I want to drill these concepts into your head so that you'll find them easy.

1. The figure at right shows a hydraulic system with a pump that converts rotational energy to fluid energy and two turbines which convert that energy back to rotational energy. Do NOT assume that the turbines are equal in size. This is a closed system containing an incompressible fluid with no places for that fluid to collect; i.e. flow in = flow out of any point or object. Kirchhoff's current law applies. The volumetric fluid flows are indicated by the arrows.

   \[ I_1 = 0.01 \text{ m}^3/\text{s}, \quad I_2 = 0.007 \text{ m}^3/\text{s} \]

   \[ I_3 = \quad I_4 = \quad I_5 = \quad I_6 = \]  

2. The figure at right shows an electrical circuit with a battery that converts chemical energy to electrical energy and two resistors which convert that electrical energy to heat energy. Do NOT assume that the resistors are equal in size. All electrical circuits are closed systems containing incompressible charges with no places for those charges to collect; i.e. flow in = flow out of any point or object. Kirchhoff's current law applies. The electrical currents are indicated by the arrows.

   \[ I_1 = 0.01 \text{ A}, \quad I_2 = 0.007 \text{ A} \]

   \[ I_3 = \quad I_4 = \quad I_5 = \quad I_6 = \]  

3. The figure at right shows a similar electrical circuit only now the electrical currents are indicated by the arrows next to the wires. This is a more common way to show the current flow because a little arrow in the wire is too easily confused with the electrical symbol for a diode. You'll learn about diodes later.

   \[ I_2 = 20 \text{ mA}, \quad I_5 = 14 \text{ mA} \]

   \[ I_6 = \quad I_1 = \quad I_3 = \quad I_4 = \]  

4. 

   \[ I_3 = 0.004 \text{ m}^3/\text{s}, \quad I_5 = 0.001 \text{ m}^3/\text{s}, \quad I_4 = \]  

   \[ I_2 = \quad I_1 = \quad I_6 = \]  

   \[ I_7 = \quad I_8 = \]
5. I_3 = 4.5 mA, I_5 = 1.2 mA
   I_4 = _______
   I_2 = _______
   I_1 = _______
   I_6 = _______
   I_7 = _______
   I_8 = _______

6. Again, a similar electrical circuit with the electrical current arrows in the more common position, next to the wires.

   I_6 = 0.03 A, I_7 = 0.08 A
   I_8 = _______
   I_1 = _______
   I_2 = _______
   I_3 = _______
   I_4 = _______
   I_5 = _______

7. I_9 = 0.04 m^3/s
   I_1 = _______
   I_2 = _______
   I_3 = _______
   I_4 = _______
   I_5 = _______
   I_6 = _______
   I_7 = _______
   I_8 = _______
   I_9 = _______

8. I_9 = 0.06 A
   I_1 = _______
   I_2 = _______
   I_3 = _______
   I_4 = _______
   I_5 = _______
   I_6 = _______
   I_7 = _______
   I_8 = _______
   I_9 = _______

9. I_4 = 0.05 m^3/s, I_5 = 0.014 m^3/s, I_6 = 0.03 m^3/s
   I_1 = _______
   I_2 = _______
   I_3 = _______
   I_4 = _______
10. \[ I_4 = 20\, \text{mA} \quad I_5 = 10\, \text{mA} \quad I_6 = 22\, \text{mA} \]
\[
I_1 = _______ \\
I_2 = _______ \\
I_3 = _______ \\
I_7 = _______
\]

11. Careful here, there are now two pumps. Also, given the flow arrows shown, one or more of the flows must come out negative.

\[ I_2 = 0.005\, \text{m}^3/\text{s} \quad I_6 = 0.03\, \text{m}^3/\text{s} \quad I_7 = 0.015\, \text{m}^3/\text{s} \]

\[
I_1 = _______ \\
I_3 = _______ \\
I_4 = _______ \\
I_5 = _______
\]

12. What does a negative fluid flow physically mean?

13. \[ I_1 = 0.01\, \text{A} \quad I_5 = -20\, \text{mA} \quad I_6 = 35\, \text{mA} \]
\[
I_2 = _______ \\
I_3 = _______ \\
I_4 = _______ \\
I_7 = _______
\]

14. What does a negative electrical current physically mean?

15. \[ I_4 = 0.05\, \text{m}^3/\text{s} \quad I_5 = 0.03\, \text{m}^3/\text{s} \quad I_7 = 0.045\, \text{m}^3/\text{s} \]
\[
I_9 = 0.06\, \text{m}^3/\text{s} \\
I_1 = _______ \\
I_2 = _______ \\
I_3 = _______ \\
I_6 = _______ \\
I_8 = _______ \\
I_{10} = _______ \\
I_{11} = _______
\]
16. \[ I_1 := 100 \text{ mA} \quad I_2 := 50 \text{ mA} \quad I_3 := 30 \text{ mA} \]

\[ I_6 := 66 \text{ mA} \]

\[ I_4 = \ldots \]

\[ I_5 = \ldots \]

\[ I_7 = \ldots \]

\[ I_8 = \ldots \]

\[ I_9 = \ldots \]

\[ I_{10} = \ldots \]

\[ I_{11} = \ldots \]

17. The figure at right shows the pressure differentials across elements in a hydraulic system. The side indicated by the + sign is the higher pressure side. Conversely, - indicates the lower pressure. $\Delta P_S$ is the pressure difference supplied by the pump (S for Source). $\Delta P_2$ is the pressure difference driving the left turbine and $\Delta P_4$ is the pressure difference driving the right turbine. Assume no pressure losses or discontinuities in the pipes, joints, or corners; i.e. all connected pipes are at exactly the same pressure. Finally, the fluid has no mass, so gravity and Bernoulli can go take a hike.

\[ \Delta P_S := 12 \text{ N/m}^2 = 12 \text{ Pa} \]

$\Delta P_2 = \ldots$

$\Delta P_4 = \ldots$

Yes, I know that these are ridiculously low pressures for a hydraulic system.

18. The figure at right shows the voltage differentials across elements in an electrical circuit. The side indicated by the + sign is the higher voltage side. Conversely, - indicates the lower voltage. $V_S$ is the voltage supplied by the battery. $V_2$ is the voltage across the left resistor and $V_4$ is the voltage across the right resistor. You may assume no voltage drops across any of the wires or connections in practically all electrical schematics; i.e. all connected wires are at exactly the same voltage (electrical potential).

\[ V_S := 12 \text{ V} \quad (V = \text{volts}) \]

$V_2 = \ldots$

$V_4 = \ldots$

19. \[ \Delta P_S := 400 \text{ kPa} \quad \Delta P_1 := 180 \text{ kPa} \quad \Delta P_3 := 100 \text{ kPa} \]

$\Delta P_5 = \ldots$

$\Delta P_7 = \ldots$

20. \[ V_1 := 10 \text{ V} \quad V_5 := 3 \text{ V} \quad V_7 := 2 \text{ V} \]

$V_S = \ldots$

$V_3 = \ldots$
21. \[ \Delta P_{S1} = 200 \text{kPa} \quad \Delta P_{S2} = 150 \text{kPa} \]
\[ \Delta P_2 = 50 \text{kPa} \quad \Delta P_6 = 60 \text{kPa} \]
\[ \Delta P_4 = \quad \Delta P_7 = \] 

22. \[ V_{S1} = 6 \text{V} \quad V_2 = 2 \text{V} \]
\[ V_6 = 2.4 \text{V} \quad V_7 = 3.2 \text{V} \]
\[ \Delta P_S = \quad \Delta P_2 = \]
\[ \Delta P_5 = \]

23. \[ \Delta P_3 = 120 \text{kPa} \quad \Delta P_4 = 80 \text{kPa} \quad \Delta P_6 = 110 \text{kPa} \]
\[ \Delta P_S = \quad \Delta P_2 = \]
\[ \Delta P_5 = \]

24. What does a negative pressure difference physically mean?

25. \[ V_3 = 2.3 \text{V} \quad V_5 = 0.5 \text{V} \quad V_6 = 3.2 \text{V} \]
\[ V_S = \quad V_2 = \]
\[ V_4 = \]

26. Watch your + and - signs very carefully now.

27. \[ V_{S1} = 14 \text{V} \quad V_{S2} = 3 \text{V} \quad V_2 = 6 \text{V} \quad V_3 = 4 \text{V} \]
\[ V_4 = \quad V_5 = \]
\[ V_6 = \quad \text{Think about the current through the 2nd battery. What is happening to that battery?} \]
ΔP₁ = 200-kPa  ΔP₂ = 1100-kPa
ΔP₃ = 600-kPa  ΔP₉ = 1800-kPa

ΔP₅ = 

ΔP₆ = 

ΔP₁₀ = 

V₅ = 10-V, V₆ = 2-V, V₇ = -5-V, battery is charging

V₁ = 4-V, V₂ = 8-V, V₆ = 6-V, V₉ = 14-V, V₁₀ = 0-V
ECE 2210 / 00  homework # 2  Due: Thur, 1/16/20

Don't forget: Write your folder number in the upper-left corner of ALL your homework.

Graded homework, labs and exams will be returned to a file cabinet in MEB 2101, filed by your alphabetically-assigned folder number.

You may do the following problems here or on your own paper. But, since you have the answers, you MUST show your work to get credit.

1. Ohm's law
   Consider the figure at right for each of the cases below, find the missing value.

   \[ I : 0.01 \text{ A} \quad V_R : 4 \text{ V} \quad R = ? \]

   \[ I : 50 \text{ mA} \quad R : 560 \Omega \quad V_R = ? \]

   \[ V_R : 12 \text{ V} \quad R : 1.5 \text{ k}\Omega \quad I = ? \]

2. Power and Ohm's law. Same circuit as above. For each of the cases below, find the missing values.

   \[ I : 5 \text{ mA} \quad R : 2 \text{ k}\Omega \quad V_R = \quad P_R = \]

   \[ V_R : 25 \text{ V} \quad R : 100 \Omega \quad I = \quad P_R = \]

   \[ V_R : 20 \text{ V} \quad I : 0.01 \text{ A} \quad R = \quad P_R = \]

   Ignore the fact that the following items run on AC

   \[ P_R : 900 \text{ W} \quad V_R : 120 \text{ V} \quad I = \quad R = \]

   \[ P_R : 1500 \text{ W} \quad R : 9.6 \Omega \quad I = \quad V_S = \]

   \[ P_R : 2500 \text{ W} \quad I : 10.5 \text{ A} \quad R = \quad V_S = \]

3. Find the equivalent resistance of each of these networks, i.e. what would an ohmmeter read if hooked to the terminals.

   a) \[ R_1 := 1.8 \text{ k}\Omega \quad R_2 := 1.0 \text{ k}\Omega \quad R_3 := 2.5 \text{ k}\Omega \quad R_4 := 5.6 \text{ k}\Omega \]
b) \( R_2 = 2.2 \, \text{k}\Omega \), \( R_1 = 1.5 \, \text{k}\Omega \), \( R_3 = 900 \, \Omega \), \( R_4 = 3 \, \text{k}\Omega \)

c) \( R_1 = 120 \, \Omega \), \( R_2 = 80 \, \Omega \), \( R_3 = 80 \, \Omega \)

d) \( R_1 = 110 \, \text{k}\Omega \), \( R_2 = 68 \, \text{k}\Omega \), \( R_3 = 82 \, \text{k}\Omega \)

e) \( R_1 = 150 \, \Omega \), \( R_2 = 56 \, \Omega \), \( R_3 = 22 \, \Omega \)

Don't forget: Write your folder number in the upper-left corner of your homework.

Answers

1. a) \( R = 400 \, \Omega \)  
   b) \( V_R = 28 \, \text{V} \)  
   c) \( I = 8 \, \text{mA} \)

2. a) \( V_R = 10 \, \text{V} \), \( P_R = 50 \, \text{mW} \)  
   b) \( I = 0.25 \, \text{A} \), \( P_R = 6.25 \, \text{W} \)  
   c) \( R = 2.0 \, \text{k}\Omega \)  
   d) \( I = 7.5 \, \text{A} \)  
   e) \( I = 12.5 \, \text{A} \)  
   f) \( R = 22.7 \, \text{\Omega} \)  

3. a) \( R_{eq} = 10.9 \, \text{k}\Omega \)  
   b) \( R_{eq} = 390 \, \Omega \)  
   c) \( R_{eq} = 51.3 \, \Omega \)  
   d) \( R_{eq} = 81 \, \text{k}\Omega \)  
   e) \( R_{eq} = 51.3 \, \Omega \)
Graded homework, labs and exams will be returned to a file cabinet in MEB 2101, filed by your alphabetically-assigned folder number.

Answer the following problems on your own paper. Since you have the answers, you must show the equations and work you used to arrive at the answer to get credit.

Don't forget: Write your folder number in the upper-left corner of your homework.

Equivalent resistance
1. Find the equivalent resistance of each of these networks, i.e. what would an ohmmeter read if hooked to the terminals. Work out and keep all your intermediate results -- they will help you in the problems to come.

a)

R_1 := 56-k\Omega  
R_2 := 110-k\Omega 
R_3 := 97-k\Omega  
R_4 := 82-k\Omega

b)

R_1 := 27-\Omega  
R_2 := 100-\Omega  
R_3 := 20-\Omega  
R_4 := 56-\Omega 
R_6 := 39-\Omega  
R_7 := 51-\Omega  
R_5 := 75-\Omega

Note: the hard part of these problems is actually seeing which resistors are in parallel and which are in series. You may want to redraw the circuits a few times to help you figure it out.

Voltage dividers
2. a) Use the voltage divider concept to find the voltage across each of the resistors in the circuit at right. V_{R1} = ?  V_{R2} = ?  V_{R3} = ?

b) Confirm that the three resistor voltages add up to the source voltage, i.e. confirm Kirchoff's voltage law.

c) Without recalculating anything, what would happen to all the resistor voltages if the source voltage were doubled? Tripled?

3. The circuit at right is known as a wheatstone bridge, or simply a bridge. It is a very common measurement circuit, used with strain gauges, thermistors, and other devices whose resistance changes in response to something that you'd like to measure. Let's assume the resistors in this circuit are 100\Omega strain gauges. The resistance of these gauges changes slightly when you stretch or compress them. They are glued to a material (often steel) and are used to measure deformations of the material (called strain).

a) Due to deformation, R_1 and R_4 decrease by 1% and R_2 and R_3 increase by 1%. Find V_{ab}.

b) Due to a temperature change, the resistances of all the gages increase by 5%. Find the % change in V_{ab}.

c) Why do you think the bridge circuit is used in this case?

4. Use voltage divider concepts to find the voltages indicated in the following circuits. You may want to use some of your results from problem 1. You may need to use the voltage divider equation more than once.

a)

R_1 := 56-k\Omega  
R_2 := 110-k\Omega  
R_3 := 97-k\Omega  
R_4 := 82-k\Omega 
V_S := 36-V  
V_{R5} = ?  
V_{R4} = ?  
V_{R1} = ?

b)

R_1 := 27-\Omega  
R_2 := 100-\Omega  
R_3 := 20-\Omega  
R_4 := 56-\Omega 
R_6 := 39-\Omega  
R_7 := 51-\Omega  
R_5 := 75-\Omega  
V_{R8} = ?  
V_{R2} = ?  
V_{R1} = ?
Current Dividers

5. The circuit at right shows a current source hooked to a resistor network. Remember that the grounds are all connected together. You can draw lines between them if it helps you.

   The circuit at right shows a current source hooked to a resistor network. Remember that the grounds are all connected together. You can draw lines between them if it helps you.

   a) Use the current divider concept to find the current through each of the resistors in the circuit at right.

   I_{R1} = ?  I_{R2} = ?  I_{R3} = ?  I_{R4} = ?

   b) Confirm that I_{R2} + I_{R3} = I_{R1} and that I_{R1} + I_{R4} = I_S, i.e., confirm Kirchoff's current law twice.

   c) Without recalculating anything, what would happen to all the currents if the source current were doubled? Tripled?

6. Refer back to the circuit of problem 4b.

   a) Find the equivalent resistance as seen by the source (R_8 + your answer for problem 1b) and use that to find the source current (I_S or I_{R8}).

   b) Find these currents by current divider methods.

   I_{R2} = ?  I_{R1} = ?  I_{R4} = ?

   c) Using Ohm's law and the currents you found in this problem, confirm the voltages found in problem 4b.

Power

7. Refer to the circuit of problem 2.

   a) How much power is dissipated by each resistor? P_{R1} = ?  P_{R2} = ?  P_{R3} = ?

   b) Independently determine the power that the source is contributing to the circuit. P_S = V_S I_S = ?

   c) Show that power is conserved (Σ answers to a = answer to b).

8. Refer to the circuit of problem 5.

   a) How much power is dissipated by each resistor? P_{R1} = ?  P_{R2} = ?  P_{R3} = ?  P_{R4} = ?

   b) Independently determine the power that the source is contributing to the circuit. P_S = V_S I_S = ?

   c) Show that power is conserved.

9. The circuit at right has five unknown components labeled A through E.

   a) Which of the components are absorbing power from the circuit? b) Which of the components are contributing power to the circuit? c) Show that power is conserved.

Answers

1. a) R_{eq} = 82.5 kΩ  b) R_{eq} = 41.7 kΩ
2. a) 1.91 V, 1.28 V, 2.81 V  b) 1.91 V + 1.28 V + 2.81 V = 6 V  c) double, triple
3. a) 100 mV  b) 0% change  c) Reading won't be affected by temperature.
4. a) 5.54 V, 17.35 V, 13.11 V  b) 2.23 V, 7.77 V, 2.93 V
5. a) 17.67 mA, 9.66 mA, 8.01 mA, 17.33 mA  b) both check  c) double, triple
6. a) 53.7 kΩ, 0.186 A  b) 77.65 mA, 108.6 mA, 28.6 mA  c) all agree
7. a) 2.44 mJ, 1.63 mJ, 3.59 mJ  b) 7.66 mJ  c) P_S = P_{R1} + P_{R2} + P_{R3}
8. a) 0.343 W, 0.0634 W, 0.0526 W, 0.451 W  b) 0.910 W  c) P_{R1} + P_{R2} + P_{R3} + P_{R4} = P_S
9. a) C, D, E  b) A, B  c) 6 W = 6 W

Don't forget: Write your folder number in the upper-left corner of your homework.
Answer the following problems on your own paper. Show your equations and work to get credit on this and all future homeworks.

**Don't forget: Write your folder number in the upper-left corner of your homework.**

### Superposition

1. Use superposition to find $I_3$. Circle your intermediate solutions on your paper. Your intermediate solutions show how much of $I_3$ is due to $V_{S1}$, and how much is due to $V_{S2}$.

2. Use superposition to solve following problems: Each problem asks for both a current and a voltage. Clearly indicate your intermediate answers, the grader will look for those.

#### a)

\[
\begin{align*}
V_{S1} &= 8 \text{ V} \\
R_1 &= 1 \text{ k}\Omega \\
V_a &= ? \\
R_2 &= 500 \text{ } \Omega \\
R_3 &= 1 \text{ k}\Omega \\
V_{S2} &= 10 \text{ V}
\end{align*}
\]

#### b)

\[
\begin{align*}
R_1 &= 40 \text{ } \Omega \\
V_{S1} &= 5 \text{ V} \\
V_{S2} &= 6 \text{ V} \\
R_2 &= 120 \text{ } \Omega \\
R_3 &= 120 \text{ } \Omega \\
I_{R1} &= ?
\end{align*}
\]

These are ground symbols. They are all connected together, although that connection is not explicitly shown.

#### c)

\[
\begin{align*}
R_1 &= 11 \text{ } \Omega \\
V_{S} &= 12 \text{ V} \\
R_2 &= 22 \text{ } \Omega \\
I_{R2} &= ?
\end{align*}
\]

Watch your signs.

### Answers

1. $2 \text{ mA} + 5 \text{ mA} = 7 \text{ mA}$
2. a) $4.2 \text{ V}, 20 \text{ mA}$  
   b) $7.67 \text{ V}, 197 \text{ mA}$  
   c) $0.5 \text{ V}, -0.5 \text{ mA}$

**Don't forget: Write your folder number in the upper-left corner of your homework.**
Thevenin & Norton equivalent circuits
1. For each of the circuits below, find and draw the Thevenin equivalent circuit.

   a) R₁ = 36 kΩ  R₂ = 12 kΩ
      Vₛ := 9 V  R₃ = 30 kΩ  R₄ = 20 kΩ
      Iₛ := 12 mA

   b) The load resistor is R₄, and is in a strange place in this circuit.
      Hint: use superposition to find Vₚh.

   2. For the circuit of problem 1a, find the voltage across R₄ (V₄) and the current through R₄ (I₄) using your Thevenin equivalent circuit.

   3. For each of the circuits in problem 1, find and draw the Norton equivalent circuit.

   4. For the circuit of problem 1b, find V₄ and I₄ using your Norton equivalent circuit.

   5. For the circuit shown at right, use Thevenin's theorem to find the current through the 50 Ω resistor R₅.

   6. For the circuit shown, use Norton's theorem to find the value of the current in R₅. Hint: You can find I₅ either by calculation of the open circuit voltage (V_OC) and R₅ or by direct calculation of the short-circuit current (I_SC), however, there is something about the values of the resistors which makes the second method easier than it would at first appear.

Source resistance
7. The terminal voltage of a car's battery drops from 12.5 V to 8.5 volts when starting. The starter motor draws 60 A of current.
   a) Draw the voltage-source model (Thevenin equivalent) of this battery. Include the values of Vₛ and Rₛ.
   b) Draw the current-source model (Norton equivalent) of this battery. Include the values of Iₛ and Rₛ.
   c) Which of these two models is more appropriate for the car battery?
   d) What terminal voltage would you expect if this battery were being charged at 20 A?

Answers
1. a) 4.091 V, 28.4 kΩ  b) 1.1 V, 18.3 Ω
2. 1.69 V, 84.6 µA
3. a) 0.144 mA, 28.4 kΩ  b) 60 mA, 18.3 Ω
4. 3.16 mA, 1.042 V
5. 1.88 mA
6. 0.19 A  7. a) Vₛ = 12.5 V  Rₛ := 0.0667 Ω  b) Iₛ = 187.5 A  Rₛ := 0.0667 Ω
   c) Thevenin  d) 13.83 V

ECE 2210 / 00 homework # 5
Nodal Analysis
1. a) If you select the bottom node as ground, how many unknown node voltages remain? (Assume $V_S$ is a known quantity.) How many simultaneous equations would you need to solve to analyze this circuit?

b) Use nodal analysis to find all the necessary simultaneous equations.

2. a) Use nodal analysis to find all the node voltages.

b) Your node voltages will depend on your selection of a reference node (ground) as well as your arbitrary node labels, so the grader won't look at these specifically. Use your node voltages to find the potential (voltage) across each resistor. Report the magnitude and polarity of each.

3. Use Nodal analysis to find $V_a$ and use $V_a$ to find $I_3$.

4. Use Nodal analysis to solve following problems: Each problem asks for at least 1 voltage and a current. Use the voltage(s) to find the current.

a) $V_{S1} = 5\text{ V}$, $V_{S2} = 6\text{ V}$

b) $V_{S1} = 4.5\text{ V}$

Don't forget your folder number.

Answers
1. a) 3, 3 
   b) $V_a = \frac{V_S}{R_1}, \quad V_b = \frac{V_a - V_c}{R_3 + \frac{1}{R_6}}$ = 0.25 A

2. a) Answer will depend on your choice of ground, so check your answers to part b to see if you did part a right.
   b) 3.077 V, + bottom, 2.308 V, + left, 1.923 V, + top, 0.385 V, + bottom, 2.692 V, + right

3. 7 V, 7 mA

4. a) 4.2 V, 20 mA 
   b) $V_a = -1.5 \text{ V}$, $V_b = 0.5 \text{ V}$, $I_{R1} = -0.5 \text{ mA}$

You may not get this homework back before the 1st exam. Photocopy it if you want to be sure to have it.
Answer the following problems on your own paper.

1. For each of the following sinusoidal waves, find:
   1) peak-to-peak voltage or current, $V_{pp}$ or $I_{pp}$
   2) amplitude, $A$, $V_p$, or $I_p$
   3) period, $T$
   4) frequency $f$ in cycles/sec or Hz
   5) an expression for $v(t)$ or $i(t)$ in terms of $A\cos(\omega t + \phi)$
      the frequency $\omega$ is in radians/sec
      the phase angle $\phi$ is in rad/sec or degrees

   ![Waveform 1](image1)

   ![Waveform 2](image2)

   ![Waveform 3](image3)

2. For each of the following waveforms, find:
   1) Peak-to-peak voltage or current, $V_{pp}$ or $I_{pp}$
   2) Average, $(V_{DC}, I_{DC}, V_{ave},$ or $I_{ave})$
   3) Period, $T$
   4) Frequency $f$ in cycles/sec or Hz

   ![Waveform 4](image4)

   ![Waveform 5](image5)

3. For problem 2a above, write a full expression for $v(t)$ in terms of $v(t) = A\cos(\omega t + \phi) + V_{DC}$

**Answers**

1. a) 0.2-V 0.1-V 12-ms 83.3-Hz 0.1-V-cos(523.6-t)
   b) 24-V 12-V 0.018-ms 55.6-kHz
      $v(t) := 12-V\cos(349100t - 90\text{-deg})$
   c) 16-mA 8-mA 0.3-ms 3333-Hz
      8-mA-cos(20940-t + 150-deg)

2. a) 12-V 3-V 6-ms 167-Hz
   b) 12-V 6-V 4-ms 250-Hz
   c) 250-mA 25-mA 0.6-ms 1.667-kHz

3. $v(t) := 6-V\cos(1047-t - 90\text{-deg}) + 3-V$
1) Find $C_{eq}$ in each case

a) \[ C_1 = 0.2\mu F \quad C_2 = 0.4\mu F \]

b) \[ C_1 = 0.02\mu F \quad C_2 = 0.06\mu F \]

c) \[ C_1 = 3\mu F \quad C_2 = 3\mu F \quad C_3 = 1.2\mu F \quad C_4 = 1.8\mu F \]

2. Each of the following circuits have been connected as shown for a long time. Find the voltage across each capacitor and the energy stored in each.

a) \[ R_1 = 22\Omega \quad C = 5\mu F \quad V_S = 5.5V \quad R_2 = 330\Omega \]

b) \[ R_1 = 2.4k\Omega \quad C = 0.47\mu F \quad I_S = 25mA \]

c) \[ R_2 = 4.8k\Omega \quad C_1 = 0.68\mu F \quad C_2 = 0.22\mu F \quad R_3 = 10k\Omega \quad V_S = 16V \]
3. The current waveform shown below flows through a 0.025 \( \mu \text{F} \) capacitor. Make an accurate drawing of the voltage across it. Label your graph. Assume the initial voltage across the capacitor is 0 V.

4. The voltage across a 2 \( \mu \text{F} \) capacitor is shown below. Make an accurate drawing of the capacitor current. Label your graph.

5. The voltage across a 0.68 \( \mu \text{F} \) capacitor is \( v_c = 6 \cdot \text{V} \cdot \cos \left( 200 \cdot t + \frac{\pi}{2} \right) \). Find \( i_c \).

6. The current through a 0.0047 \( \mu \text{F} \) capacitor is \( i_c = 18 \cdot \mu \text{A} \cdot \cos \left( 628 \cdot t - \frac{\pi}{4} \right) \). Find \( v_c \).

7. A capacitor voltage and current are shown at right. What value is the capacitor?

Answers
1. a) 0.6-\( \mu \text{F} \) b) 0.015-\( \mu \text{F} \) c) 4.5-\( \mu \text{F} \)
2. a) 3.3V 0.027-mJ b) 37.5-V 0.33-mJ c) 11-V 0.0411-mJ 5-V 2.75-mJ
3. 1.8-V 0.6-V 2.4-V
4. -6-mA 12-mA ramp to -8mA
5. \( i_c = 0.816 \cdot \mu \text{A} \cdot \cos(200 \cdot t + \pi) \)
6. \( v_c = 6.1 \cdot \text{V} \cdot \cos \left( 628 \cdot t - \frac{3 \cdot \pi}{4} \right) \)
7. 0.25-\( \mu \text{F} \)
1. Find $L_{eq}$ in each case
   a) $L_1 := 2 \text{ mH}$
      $L_2 := 3 \text{ mH}$
   b) $L_1 := 0.22 \text{ mH}$
      $L_2 := 0.4 \text{ mH}$

2. Find the stored energy in each capacitor and/or inductor under steady-state conditions. Note: Treat caps as opens and inductors as shorts to find DC voltages and currents.
   a) $R := 200 \Omega$
      $L := 40 \text{ mH}$
      $C_1 := 10 \mu \text{F}$
   b) $L_1 := 0.5 \text{ mH}$
      $R_1 := 30 \Omega$
      $V_S := 18 \text{ V}$
      $R_2 := 60 \Omega$
      $L_2 := 4 \text{ mH}$
      $R_3 := 30 \Omega$
      $C_2 := 2 \mu \text{F}$

3. The current waveform shown below flows through a $2 \text{ mH}$ inductor. Make an accurate drawing of the voltage across it. Label your graph.

4. The voltage across a $0.5 \text{ mH}$ inductor is shown below. Make an accurate drawing of the inductor current. Label your graph. Assume the initial current is 0 mA.
5. The voltage across a 1.2 mH inductor is \( v_L = 4 \text{mV} \cdot \cos(300t) \) find \( i_L \).

6. The current through a 0.08 mH inductor is \( i_L = 20 \text{mA} \cdot \cos\left(628t - \frac{\pi}{4}\right) \) find \( v_L \).

7. Refer to the circuit shown. Assume that \( V_s \) is a sinusoidal input voltage whose frequency can be adjusted. At some frequency of \( V_s \) this circuit can resonate. At that frequency \( i_C(t) = -i_L(t) \). \( i_C(t) \) is 180 degrees out-of-phase with \( i_L(t) \).

Show that resonance occurs at this frequency:

\[
\omega_0 = \frac{1}{\sqrt{L \cdot C}}, \quad f_0 = \frac{1}{2\pi \sqrt{L \cdot C}}
\]

8. Find the resonant frequency, \( f_0 \) in each case.

a) \( L_1 := 0.5 \text{mH} \)

\[ C_1 := 0.1 \mu\text{F} \]
\[ C_2 := 0.22 \mu\text{F} \]

b) \( L_1 := 10 \text{mH} \)

\[ C_1 := 0.12 \mu\text{F} \]
\[ C_2 := 0.12 \mu\text{F} \]

Answers
1. 1.2 mH 0.62 mH
2. a) 0.05 mJ b) 1.62 mJ 0.081 mJ 0.09 mJ 0.18 mJ
3. Straight lines between the following points: (0ms, -8mV), (2ms, -8mV), (2ms, 0mV), (3ms, 0mV), (3ms, 16mV), (5ms, 16mV), (5ms, 0mV), (6ms, 0mV), (9ms, -10.67mV), (9ms, 0mV), (10ms, 0mV)
4. Straight lines between the following points: (0ms, 0A), (0.2ms, 1.2A), (0.6ms, -0.4A), continues to curve up to (1ms, 0A), (1.1ms, 0A)
5. \( i_L = 11.1 \text{mA} \cdot \cos(300t - 90\text{deg}) \)
6. \( v_L = 1 \text{mV} \cdot \cos\left(628t + \frac{1}{4}\pi\right) \)
7. Assume a sinusoidal voltage, find \( i_C \) and \( i_L \) by integration and differentiation, and show that they are equal and opposite at the resonant frequency.
8. a) 17.79 kHz b) 5305 Hz
1. An FE style problem
A 10-microfarad capacitor has been charged to a potential of 150 volts. A resistor of 25 $\Omega$ is then connected across the capacitor through a switch. When the switch has been closed for 10 time constants the total energy dissipated by the resistor is most nearly

(A) $1.0 \times 10^{-7}$ joules
(B) $1.1 \times 10^{-1}$ joules
(C) $9.0 \times 10^{1}$ joules
(D) $9.0 \times 10^{3}$ joules

2. a) The switch is closed at time $t = 0$ and $v_C(0) = 0V$, find $v_C(t)$.

b) What is the value of the voltage across C at $t := 40\mu s$

3. In the circuit below, the switch has been in the upper position for a long time and is switched down at time $t = 0$.
What is the capacitor voltage ($V_C$) at $t := 4\text{ms}$

4. The switch below has been in the upper position for a long time and is switched down at time $t = 0$.
At what time is $v_C = 4V$?

5. a) What is the time constant of this circuit?
Hint: Use a Thevenin equivalent circuit.

b) What will be the final value of $v_C$?
(After the switch has been closed for a long time)

6. In a circuit with two capacitors, the left capacitor ($C_1$) has an initial charge and the right capacitor ($C_2$) does not. When the switch is closed at time $t = 0$, current $i(t)$ flows, discharging $C_1$ and charging $C_2$.

a) Derive the differential equation for $i(t)$. Hint: write an equation in terms of $i$ and integrals of $i$, then differentiate the whole equation.

Write your DE in this form:

$$\text{Constant} = x(t) + \tau \frac{dx(t)}{dt}$$

What is the time constant ($\tau$)?

b) Find $i(t)$ given $C_1 := 24\mu F$, $C_2 := 12\mu F$, $R := 400\Omega$, $v_{C1}(0) = 18V$, $v_{C2}(0) = 0V$

c) Find $v_{C2}(t)$ for the same values. Hint: The trick here will be finding the final condition. Realize that charge will be conserved. If $C_1$ discharges $x$ coulombs, then $C_2$ will charge $x$ coulombs. Charges will stop flowing when $v_{C1} = v_{C2}$. It may help to think of two water tanks, one with half the cross-sectional area of the other.

d) Find the initial and final stored energy of the system ($W_{C1} + W_{C2}$) to find the total "loss". What happened to that energy?

Answers

1. B
2.a) $12V - 12V e^{-0.16ms}$
2.b) $2.65V$
3. $6.61V$ 4. $6.44V$ 5. a) $5.87ms$ 5. b) $5V$
6.a) $\tau = \frac{R}{C}$
6.b) $i(t) = 45mA e^{-0.16ms}$
6.c) $12V - 12V e^{-3.2ms}$
6.d) $1.3mJ$
1. A 12 V car ignition coil has an inductance of 10 mH and resistance of 2 Ω (so its equivalent circuit is a 10 mH inductor in series with a 2 Ω resistor). Calculate how long it takes the current to build up to 95% of its maximum value after a 12 V battery is connected to the coil.

2. A constant voltage is applied to a series RL circuit by closing a switch. The voltage across L is 30 volts at t = 0 and drops to 6 volts at t = .0025 sec. If L = 0.2 H, what must be the value of R?

3. In the circuit shown, the switch is closed at t = 0. Find the transient current expression.

4. In the circuit shown, the switch is closed on position 1 at t = 0, and then instantly moved to position 2 after 1 millisecond. Find the time at which the current is zero and reversing its direction.

If you learn to use the complex math feature of your calculator, you may use that to work the following problems. In that case you may report the answers without showing any work.

5. Convert the following complex numbers to polar form (rθ or rejθ).
   a) 1 + j    b) 2.6 + 8.7j    c) 3 + 4j   d) 3 - 4j  e) -3 + 4j  f) -3 - 4j

6. Convert the following complex numbers to rectangular form (a + bj).
   a) 10e^j60°    b) 0.4e^j12°    c) 1500e^j\frac{π}{2} rad    d) 10e^-j45°    e) 20e^j120°    f) 30e^-j210°

7. Perform the following additions and subtractions of complex numbers.
   a) (3 + 2j) + (6 + 9j)  b) (9 - 10j) - (9 + 10j)  c) (-2 - 2j) + (-6 + 9j)  d) (3 + 0j) - (0 + 9j)
   e) (5 + 6j) + 5e^j53°    f) (-2 + 3j) - 8e^-j37°

8. Perform the following multiplications of complex numbers.
   a) (8 + j)3    b) (3 + 2j)j    c) (20e^j40°)(10e^-j60°)  d) (-6 + 9j)(10e^-j60°)  e) (-2 - j)(-6 - 9j)

9. Perform the following divisions of complex numbers.
   a) \frac{20e^j40°}{10e^-j60°}    b) \frac{9 - 10j}{3e^-j20°}    c) \frac{3 + 0j}{0 + 9j}    d) \frac{2 - 2j}{6 + 9j}

**Answers**

1. 15 ms  2. 129 Ω  3. 1.25 · A(1 - e^-t/1.25 ms)  4. 1.312 ms

5. a) 1.414e^-j45°  b) 9.08e^-j734°  c) 5e^-j53.1°  d) 5e^j53.1°  e) 5e^j126.9°  f) 5e^-j126.9°

6. a) 5 + 8.66j  b) 0.391 + 0.083j  c) 1500j  d) 7.071 - 7.071j  e) -10 + 17.321j  f) -25.981 - 15j

7. a) 9 + 11j  b) -20j  c) -8 + 7j  d) 3 - 9j  e) 8.009 + 9.993j  f) -8.389 + 7.815j

8. a) 24 + 3j  b) -2 + 3j  c) 200e^j100°  d) 108e^-j176°  e) 24.2e^-j82.9°

9. a) 2e^-j20°  b) 4.485e^-j28.01°  c) 0.333e^-j90°  d) -0.051 + 0.256j
ECE 2210/00 homework #12

ECE 2210 Due: Wed, 2/26/20

ECE 2200 Due: Tue, 2/25/20, may be handed in Wed, 2/26 for full credit.

Read about complex numbers and phasors in your textbook (sections 2.26 & 2.27, starting on p.159).

1. For the complex numbers \( z_1 := -4 + 5j \) and \( z_2 := 2 + 4j \) Determine the following
   a) Does \( z_1 \cdot z_2 \) equal \( 1 + 2j \)?
   b) Does \( \frac{z_1}{z_2} \) equal \( 1 + 2j \)?
   c) Does \( z_1 + z_2 \) equal \( 1 + 2j \)?

2. a) Find the phasor for \( v(t) = 8.4 \cdot \cos(100 \cdot t - 90\deg) \) Express in both forms, polar and rectangular.
   b) The phasor representation of a current is \( i := (5 + j \cdot 12) \cdot \mu A \) Find the time-domain representation, \( i(t) \).

3. Add or subtract the sinusoidal voltages using phasors. Draw a phasor diagram which shows all 3 phasors, and give your final answer in time domain form.
   a) \( v_1(t) = 1.5 \cdot V \cdot \cos(\omega \cdot t + 10\deg) \)
   b) \( v_1(t) = 1.5 \cdot V \cdot \cos(\omega \cdot t + 10\deg) \) \( v_2(t) = 3.2 \cdot V \cdot \cos(\omega \cdot t + 25\deg) \) \( v_3(t) = v_1(t) + v_2(t) \) you may add \( V_4 \) to the phasor diagram you've already drawn for part a).
   c) \( v_1(t) = 50 \cdot V \cdot \cos(\omega \cdot t - 60\deg) \)
   d) \( v_1(t) = 0.9 \cdot V \cdot \cos(\omega \cdot t + 72\deg) \)
   e) \( v_1(t) = 0.9 \cdot V \cdot \cos(\omega \cdot t + 72\deg) \) \( v_2(t) = 1.2 \cdot V \cdot \cos(\omega \cdot t - 20\deg) \) \( v_3(t) = v_1(t) + v_2(t) \) you may add \( V_4 \) to the phasor diagram you've already drawn for part d).

4. Express the impedance of a 5.2mH inductor at 60 Hz in polar form.

5. a) A capacitor impedance has a magnitude of 240Ω at a frequency of 1.8kHz. What is the value of capacitor?
   b) What value inductor has the same impedance magnitude at the same frequency?
   c) Find the reactance (magnitude of the impedance with + or - sign) of this capacitor and this inductor at 3.6kHz?
   d) What would be the total impedance of this inductance and this capacitance connected in series at 2.7kHz?

6. Find \( Z_{eq} \) in each case.
   a) \[ V(j\omega) \]
   \[ 10 \cdot V \cdot e^{j0} \]
   \[ f := 1 \cdot kHz \]
   \[ R := 330 \cdot \Omega \]
   \[ L := 100 \cdot mH \]

   b) \[ V(j\omega) \]
   \[ 10 \cdot V \cdot e^{j0} \]
   \[ f := 2 \cdot kHz \]

   c) \[ V(j\omega) \]
   \[ 10 \cdot V \cdot e^{j0} \]
   \[ f := 4 \cdot kHz \]

   d) \[ V(j\omega) \]
   \[ 8 \cdot V \cdot e^{j0} \]
   \[ f := 1.5 \cdot kHz \]

7. Find the current \( I(j\omega) \) in each case above.
8. a) Find \( Z \). Hint: Find the total impedance \((R+Z)\) first.
   
   b) Which leads, current or voltage?
   
   c) By how much?
   I.E. what is the phase angle between the voltage and current?

9. a) A resistor and a capacitor are connected in series to create an impedance of
   at a frequency \( f' = 3 \text{ kHz} \). Find \( R \) and \( C \).

   b) A resistor and a capacitor are connected in parallel to create an impedance of
   at a frequency \( f' = 3 \text{ kHz} \). Find \( R \) and \( C \).

   Hint: invert \( Z_{\text{eq}} \). Instead of solving this:
   \[ 50 / 66^\circ = \frac{1}{R + j \omega C} \]
   solve this:
   \[ \frac{1}{Z_{\text{eq}}} = 0.02/66^\circ = \frac{1}{R + j \omega C} \]

10. The phasor diagram at right shows the voltage and current in the
circuit below

   Assume the load consists of a resistor in series with a reactive component and
the frequency is 60 Hz.

   a) What is the magnitude of the impedance?
   b) What is the value of the resistor?
   c) What is the reactive component (type and value)?

11. For the circuit shown, draw a phasor diagram showing \( V_s, I, V_R \), and \( V_C \). Draw the voltages to
scale so that you can show that they obey KVL.

Answers

1. a) 28.636 = 28.636 Yes    b) 1.432 = 1.432 Yes     c) 9.22 not = 10.875 No

2. a) 8.4 /-90° = 8.4\cdot e^{-90^\circ} = -8.4 \quad b) 13\cdot \mu A\cdot \cos(3770\cdot t + 67.4^\circ)

3. a) \begin{align*} v_1(t) + v_2(t) &= 4.67\cdot \cos(\omega t + 20.2^\circ) \cdot V \\ v_1(t) - v_2(t) &= 1.794\cdot \cos(\omega t - 142.5^\circ) \cdot V \\ v_1(t) + v_2(t) &= 1.48\cdot \cos(\omega t + 17.6^\circ) \cdot V \end{align*}

4. 1.96 \Omega /90^\circ    a) 0.368\mu F     b) 21.2\cdot \mu H    c) -120\Omega 480\Omega    d) 200j\cdot \Omega

5. \begin{align*} R \cdot \left( \frac{\omega L - \frac{1}{\omega C}}{R^2 + \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)^2} \right) \end{align*} angle: - \tan\left( \frac{\omega L - \frac{1}{\omega C}}{R} \right)

6. a) (330 + 628.3j) \cdot \Omega = 709.7\Omega /62.29^\circ   \quad c) R + \left(\frac{\omega L - \frac{1}{\omega C}}{j}\right) \quad d) 1.82k\Omega /15.2^\circ

7. a) (6.6 - 12.5j) \cdot mA = 14.1mA /-62.29^\circ \quad b) (13.8 + 15.1j) \cdot mA = 20.4mA /47.63^\circ \quad d) 4.4mA /15.2^\circ

8. a) 259 - 160j  b) The current leads the voltage  \quad c) 20^\circ

9. a) 20.34 \Omega 1.16\mu F \quad b) 123 \Omega 0.969\mu F

10. a) 19.5 \Omega \quad b) 11.2 \Omega \quad c) inductor 42.3\cdot \mu H

ECE 2210/00 homework # 12 p.2
Warning: This homework is longer than normal -- DO NOT put it off until the last minute.

1. For the circuit shown, find the following:
   a) At what frequency would the magnitude of the total impedance be 240Ω?
   b) At this frequency, what is the phase angle of the impedance?
   c) At this frequency, you want to add a capacitor in series to make the circuit appear purely resistive (the impedance has no imaginary component). Find the value of the capacitor.

2. You need to design a circuit in which the current \( i(t) \) leads the voltage \( v_S(t) \) by 36° of phase.
   a) What should go in the box: R, L, C?
   b) Find its value.

3. The phasor diagram at right shows the source voltage and two branch currents of a parallel circuit. Find the impedance of each of the two branches.

4. a) Find all the currents, \( I_1, I_2, \) and \( I_T \).
   b) Draw a phasor diagram showing \( I_1, I_2, \) and \( I_T \) to scale so that you can show that they obey KCL.

5. a) Find the AC current source, \( I_{in} \) in polar form.
   b) Find \( V_T \).
   c) Choose one:
      i) The source current leads the source voltage.
      ii) The source current lags the source voltage.
6. a) Find $Z_1$.

b) To make $Z_1$ in the simplest way, what part(s) would you need? Just determine the needed part(s) from the list below and state why you made that choice, don't find the values.

- resistor
- capacitor
- inductor
- power supply
- current source
- Thevenin resistor
- ideal transformer
- voltmeter
- ammeter
- scope

c) Choose one: i) $I_2$ leads the source voltage ($V_{in}$) ii) $I_2$ lags the source voltage ($V_{in}$)

d) Choose one: i) $I_1$ leads $I_2$ ii) $I_1$ lags $I_2$

7. Find $Z$.

8. a) Find the total impedance of the circuit.

b) Find $I_T$.

9. Find $Z_{eq}$ in simple polar form.

Answers
1. a) 11 kHz b) 60° c) 0.0694 μF
2. a) C b) 6.12 μF
3. $Z_1 = (19.2 - 33.3) \cdot \Omega$ $Z_2 = (46.0 + 19.6) \cdot \Omega$
4. a) $(0.197 + 0.138) - A + 0.096 - A = 0.293 + 0.138j \cdot A$
5. a) $60 / 36.87^\circ$ mA b) $11.54 / 21^\circ$ V c) i)
6. a) $172 / 53.4^\circ$ Ω b) phase angle > 0, resistor and inductor
   c) i) d) ii)
7. $657 \Omega / 67.4^\circ$ 8. a) $21.86 \Omega / -20.38^\circ$ b) $0.457A / 20.38^\circ$
9. $382 \Omega / -40.2^\circ$