

ECE 2210 / 00 homework # 3 Due: Thur, 1/24/19 You should finish p.1 by Mon.

Graded homework, labs and exams will be returned to a file cabinet in MEB 2101, filed by your alphabetically-assigned folder number.

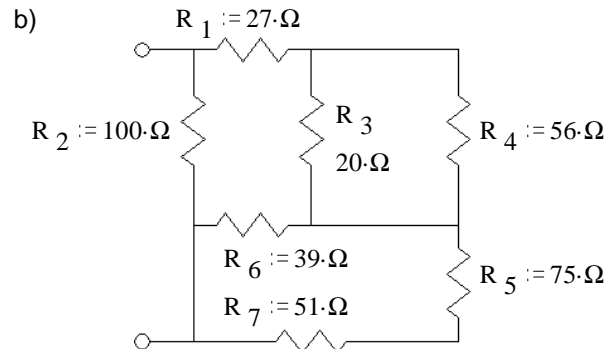
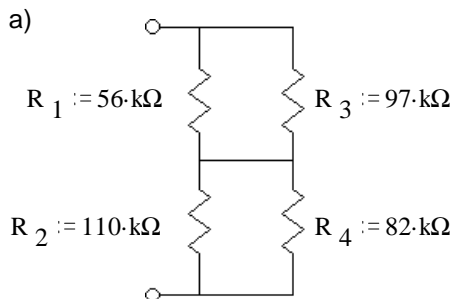
Answer the following problems on your own paper.

Since you have the answers, you must show the equations and work you used to arrive at the answer to get credit.

Don't forget: Write your folder number in the upper-left corner of your homework.

Equivalent resistance

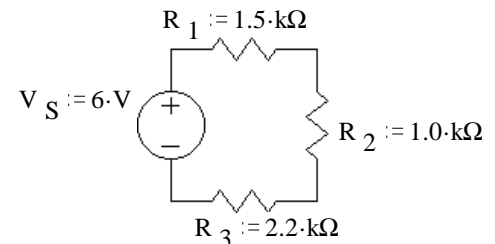
- Find the equivalent resistance of each of these networks, i.e. what would an ohmmeter read if hooked to the terminals. Work out and keep all your intermediate results -- they will help you in the problems to come.



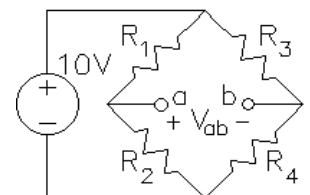
Note: the hard part of these problems is actually seeing which resistors are in parallel and which are in series. You may want to redraw the circuits a few times to help you figure it out.

Voltage dividers

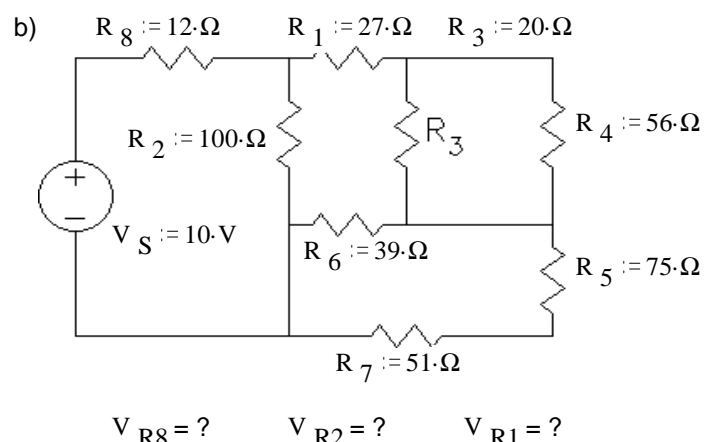
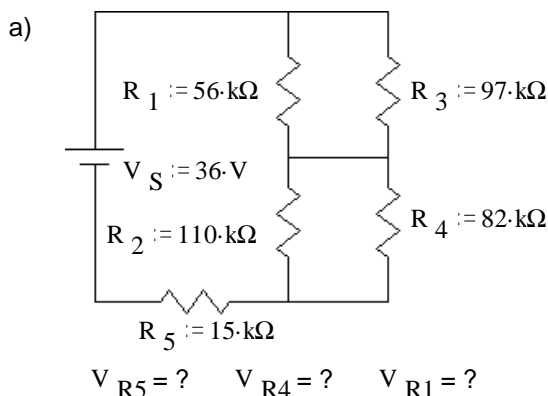
- Use the voltage divider concept to find the voltage across each of the resistors in the circuit at right. $V_{R1} = ?$ $V_{R2} = ?$ $V_{R3} = ?$
 - Confirm that the three resistor voltages add up to the source voltage, ie, confirm Kirchoff's voltage law.
 - Without recalculating anything, what would happen to all the resistor voltages if the source voltage were doubled? Tripled?



- The circuit at right is known as a *wheatstone bridge*, or simply a *bridge*. It is a very common measurement circuit, used with strain gauges, thermistors, and other devices whose resistance changes in response to something that you'd like to measure. Let's assume the resistors in this circuit are 100Ω strain gauges. The resistance of these gauges changes slightly when you stretch or compress them. They are glued to a material (often steel) and are used to measure deformations of the material (called strain).



- Due to deformation, R_1 and R_4 decrease by 1% and R_2 and R_3 increase by 1%. Find V_{ab} .
 - Due to a temperature change, the resistances of all the gages increase by 5%. Find the % change in V_{ab} .
 - Why do you think the bridge circuit is used in this case?
- Use voltage divider concepts to find the voltages indicated in the following circuits. You may want to use some of your results from problem 1. You may need to use the voltage divider equation more than once.



ECE 2210 / 00 homework # 3 p.2

Current Dividers

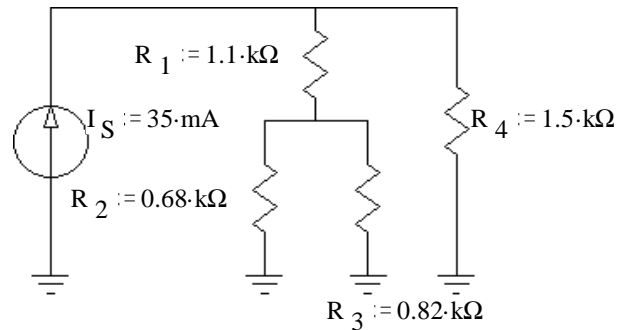
5. The circuit at right shows a current source hooked to a resistor network. Remember that the grounds are all connected together. You can draw lines between them if it helps you.

a) Use the current divider concept to find the current through each of the resistors in the circuit at right.

$$I_{R1} = ? \quad I_{R2} = ? \quad I_{R3} = ? \quad I_{R4} = ?$$

b) Confirm that $I_{R2} + I_{R3} = I_{R1}$ and that $I_{R1} + I_{R4} = I_S$, ie, confirm Kirchoff's current law twice.

c) Without recalculating anything, what would happen to all the currents if the source current were doubled? Tripled?



6. Refer back to the circuit of problem 4b.

a) Find the equivalent resistance as seen by the source (R_8 + your answer for problem 1b) and use that to find the source current (I_S or I_{R8}).

b) Find these currents by current divider methods. $I_{R2} = ? \quad I_{R1} = ? \quad I_{R4} = ?$

c) Using Ohm's law and the currents you found in this problem, confirm the voltages found in problem 4b.

Power

7. Refer to the circuit of problem 2.

a) How much power is dissipated by each resistor? $P_{R1} = ? \quad P_{R2} = ? \quad P_{R3} = ?$

b) Independently determine the power that the source is contributing to the circuit. $P_S = V_S \cdot I_S = ?$

c) Show that power is conserved (Σ answers to a = answer to b).

8. Refer to the circuit of problem 5.

a) How much power is dissipated by each resistor? $P_{R1} = ? \quad P_{R2} = ? \quad P_{R3} = ? \quad P_{R4} = ?$

b) Independently determine the power that the source is contributing to the circuit. $P_S = V_S \cdot I_S = ?$

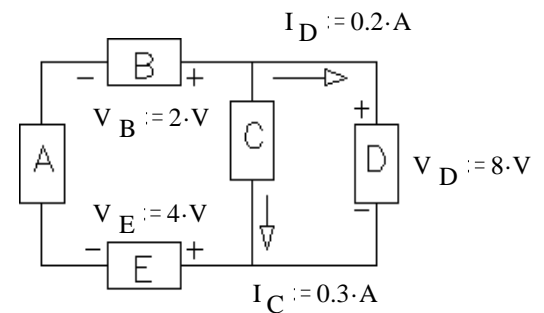
c) Show that power is conserved.

9. The circuit at right has five unknown components labeled A through E.

a) Which of the components are absorbing power from the circuit?

b) Which of the components are contributing power to the circuit?

c) Show that power is conserved.



Answers

1. a) $R_{eq} := 82.5 \cdot k\Omega$

b) $R_{eq} := 41.7 \cdot \Omega$

2. a) 1.91·V, 1.28·V, 2.81·V

b) 1.91·V + 1.28·V + 2.81·V = 6·V

c) double, triple

3. a) 100·mV b) 0% change

c) Reading won't be affected by temperature.

4. a) 5.54·V, 17.35·V, 13.11·V

b) 2.23·V, 7.77·V, 2.93·V

5. a) 17.67·mA, 9.66mA, 8.01·mA, 17.33·mA

b) both check

c) double, triple

6. a) 53.7·Ω, 0.186·A

b) 77.65·mA, 108.6·mA, 28.6·mA

c) all agree

7. a) 2.44·mW, 1.63·mW, 3.59·mW

b) 7.66·mW

c) $P_S = P_{R1} + P_{R2} + P_{R3}$

8. a) 0.343·W, 0.0634·W, 0.0526·W, 0.451·W

b) 0.910·W

c) $P_{R1} + P_{R2} + P_{R3} + P_{R4} = P_S$

9. a) C, D, E

b) A, B

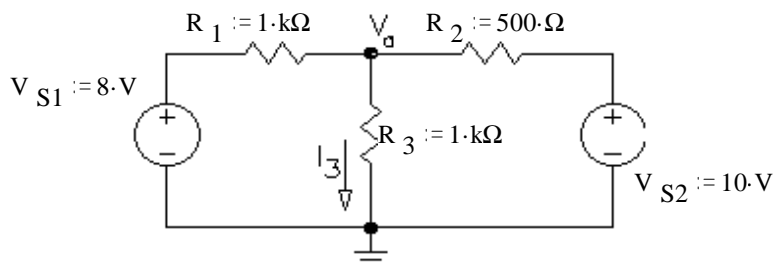
c) 6·W = 6·W

Answer the following problems on your own paper. Show your equations and work to get credit on this and all future homeworks.

Don't forget: Write your folder number in the upper-left corner of your homework.

Superposition

- Use superposition to find I_3 . Circle your intermediate solutions on your paper. Your intermediate solutions show how much of I_3 is due to V_{S1} , and how much is due to V_{S2} .

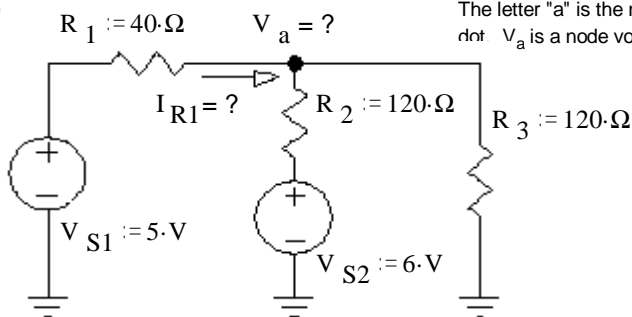


- Use superposition to solve following problems: Each problem asks for both a current and a voltage.

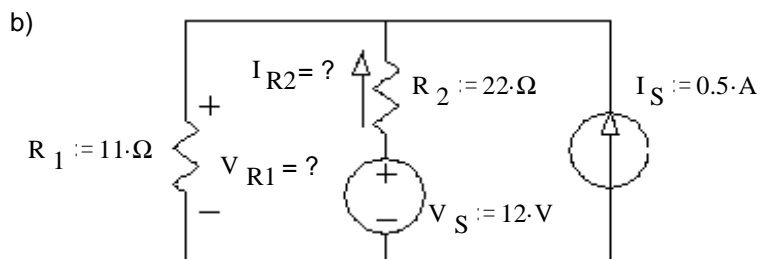
Clearly indicate your intermediate answers, the grader will look for those.

- The letter "a" is the name of the "node" at the black dot. V_a is a node voltage, referenced to ground.

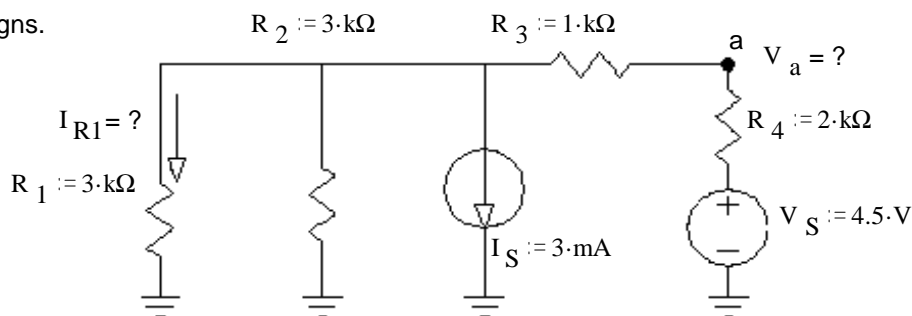
$V_a = V_{R3}$



These are ground symbols. They are all connected together, although that connection is not explicitly shown.



- Watch your signs.



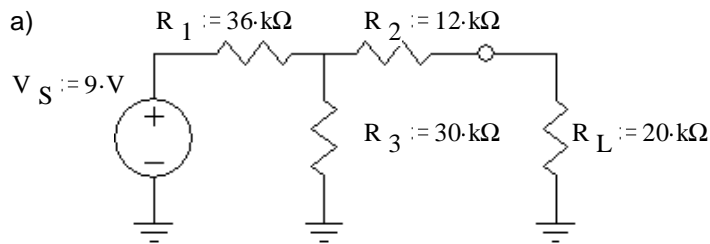
Answers

- $2\text{ mA} + 5\text{ mA} = 7\text{ mA}$
- a) 4.2 V , 20 mA b) 7.67 V , 197 mA
- c) 0.5 V , -0.5 mA

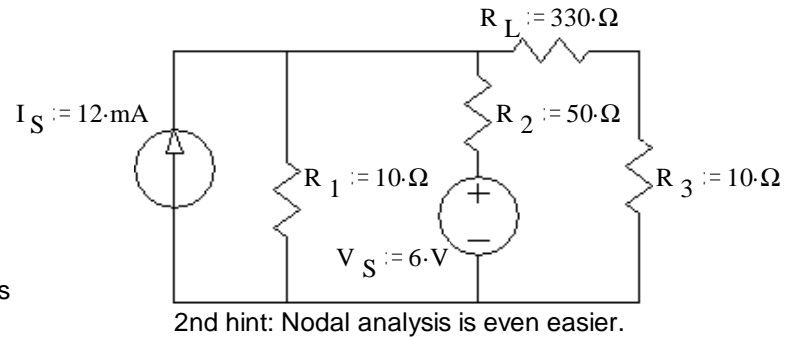
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Thevenin & Norton equivalent circuits

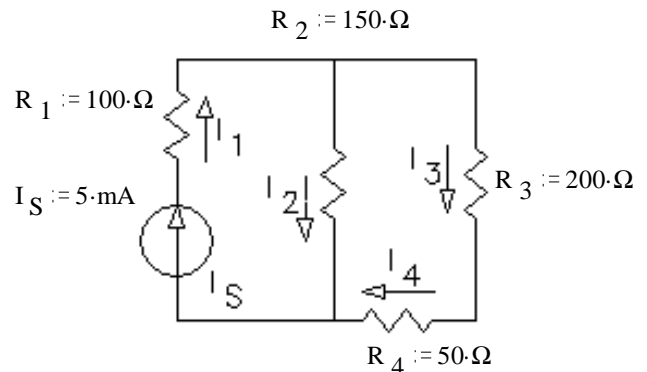
1. For each of the circuits below, find and draw the Thevenin equivalent circuit.



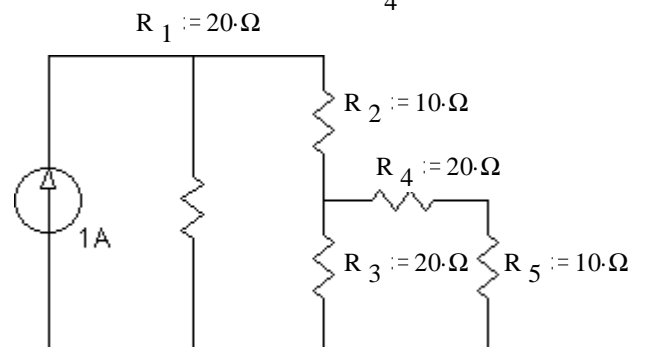
b) The load resistor is R_L , and is in a strange place in this circuit.
 Hint: use superposition to find V_{Th} .



- For the circuit of problem 1a, find the voltage across R_L (V_L) and the current through R_L (I_L) using your Thevenin equivalent circuit.
- For each of the circuits in problem 1, find and draw the Norton equivalent circuit.
- For the circuit of problem 1b, find V_L and I_L using your Norton equivalent circuit.
- For the circuit shown at right, use Thevenin's theorem to find the current through the $50\text{ }\Omega$ resistor R_4 .



- For the circuit shown, use Norton's theorem to find the value of the current in R_5 . Hint: You can find I_N either by calculation of the open circuit voltage (V_{OC}) and R_N or by direct calculation of the short-circuit current (I_{SC}), however, there is something about the values of the resistors which makes the second method easier than it would at first appear.

**Source resistance**

- The terminal voltage of a car's battery drops from 12.5 V to 8.5 volts when starting. The starter motor draws 60 A of current.

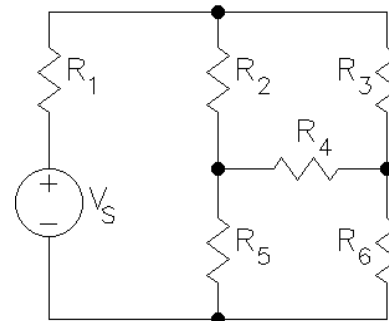
- Draw the voltage-source model (Thevenin equivalent) of this battery. Include the values of V_S and R_S .
- Draw the current-source model (Norton equivalent) of this battery. Include the values of I_S and R_S .
- Which of these two models is more appropriate for the car battery?
- What terminal voltage would you expect if this battery were being charged at 20 A?

Answers

1. a) 4.091 V , $28.4\text{ k}\Omega$ b) 1.1 V , $18.3\text{ }\Omega$ 2. 1.69 V , $84.6\text{ }\mu\text{A}$
3. a) 0.144 mA , $28.4\text{ k}\Omega$ b) 60 mA , $18.3\text{ }\Omega$ 4. 3.16 mA , 1.042 V 5. 1.88 mA
6. 0.19 A 7. a) $V_S = 12.5\text{ V}$ $R_S := 0.0667\text{ }\Omega$ b) $I_S = 187.5\text{ A}$ $R_S := 0.0667\text{ }\Omega$
- c) Thevenin d) 13.83 V

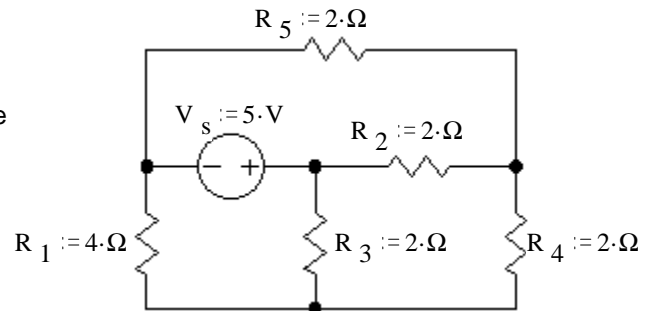
Nodal Analysis

1. a) If you select the bottom node as ground, how many unknown node voltages remain? (Assume V_S is a known quantity.) How many simultaneous equations would you need to solve to analyze this circuit?
- b) Use nodal analysis to find all the necessary simultaneous equations.

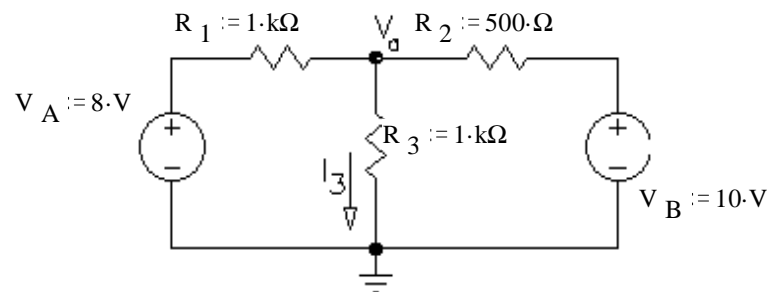


2. a) Use nodal analysis to find all the node voltages.

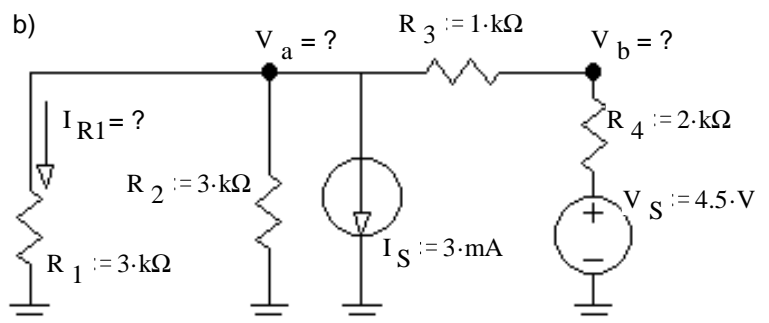
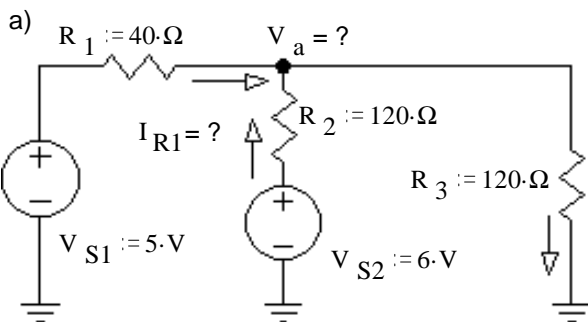
- b) Your node voltages will depend on your selection of a reference node (ground) as well as your arbitrary node labels, so the grader won't look at these specifically. Use your node voltages to find the potential (voltage) across each resistor. Report the magnitude and polarity of each.



3. Use Nodal analysis to find V_a and use V_a to find I_3 .



4. Use Nodal analysis to solve following problems: Each problem asks for at least 1 voltage and a current. Use the voltage(s) to find the current.



Don't forget your folder number.

Answers

$$1. \text{ a) } 3,3 \quad \text{ b) } V_a \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) - \frac{V_b}{R_2} - \frac{V_c}{R_3} = \frac{V_S}{R_1}, \quad \frac{V_a}{R_2} - V_b \cdot \left(\frac{1}{R_2} + \frac{1}{R_5} + \frac{1}{R_4} \right) + \frac{V_c}{R_4} = 0 \cdot A$$

$$\frac{V_a}{R_3} + \frac{V_b}{R_4} - V_c \cdot \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_6} \right) = 0 \cdot A$$

2. a) Answer will depend on your choice of ground, so check your answers to part b to see if you did part a right.

b) $3.077 \cdot V$, + bottom, $2.308 \cdot V$, + left, $1.923 \cdot V$, + top, $0.385 \cdot V$, + bottom, $2.692 \cdot V$, + right

3. $7 \cdot V$, $7 \cdot \text{mA}$ 4. a) $4.2 \cdot V$, $20 \cdot \text{mA}$ b) $V_a = -1.5 \cdot V$, $V_b = 0.5 \cdot V$, $I_{R1} = -0.5 \cdot \text{mA}$

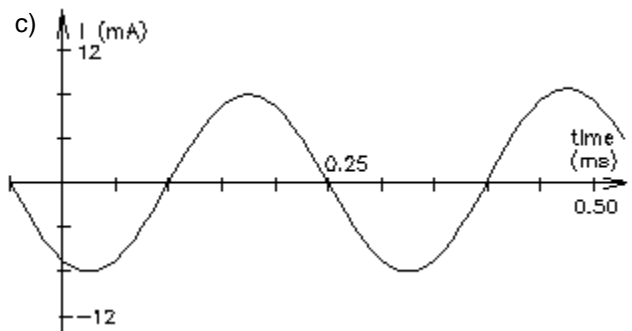
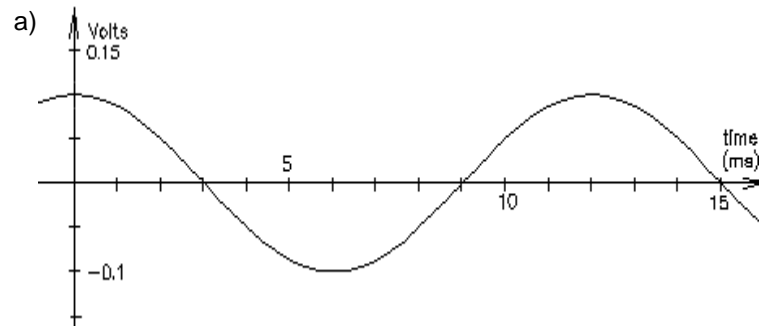
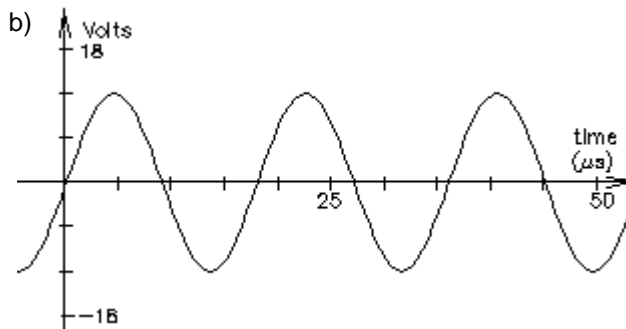
You may not get this homework back before the **1st exam**. Photocopy it if you want to be sure to have it.

1st exam on Fri. 2/8/19 will include this material

Answer the following problems on your own paper.

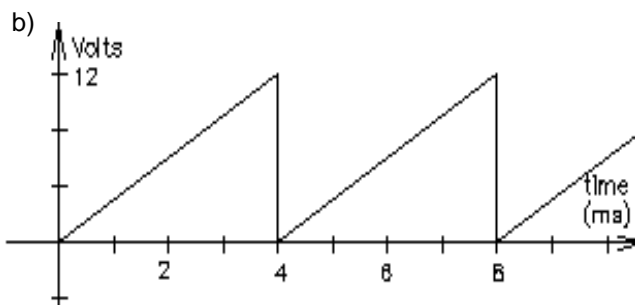
1. For each of the following sinusoidal waves, find:

- 1) peak-to-peak voltage or current, V_{pp} or I_{pp}
- 2) amplitude, A , V_p , or I_p
- 3) period, T
- 4) frequency f in cycles/sec or Hz
- 5) an expression for $v(t)$ or $i(t)$ in terms of $A\cos(\omega t + \phi)$
the frequency ω is in radians/sec
the phase angle ϕ is in rad/sec or degrees

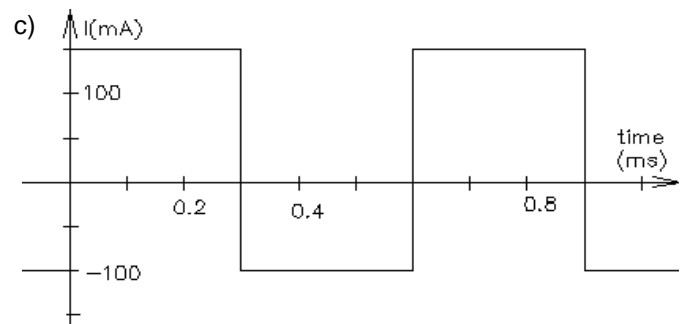
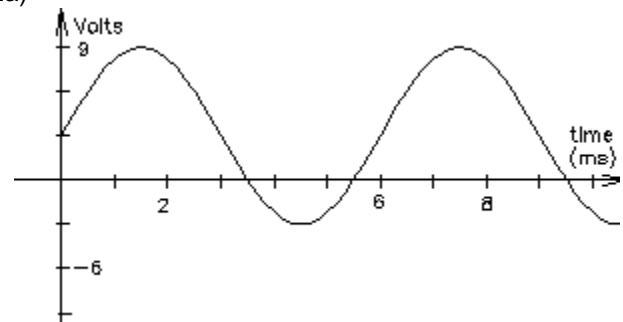


2. For each of the following waveforms, find:

- 1) Peak-to-peak voltage or current, V_{pp} or I_{pp}
- 2) Average, (V_{DC} , I_{DC} , V_{ave} , or I_{ave})
- 3) Period, T
- 4) Frequency f in cycles/sec or Hz



2a)



3. For problem 2a above, write a full expression for $v(t)$ in terms of $v(t) = A\cos(\omega t + \phi) + V_{DC}$

Answers

1. a) 0.2-V 0.1-V 12-ms 83.3-Hz $0.1 \cdot V \cdot \cos(523.6 \cdot t)$

b) 24-V 12-V 0.018-ms 55.6-kHz
 $v(t) := 12 \cdot V \cdot \cos(349100 \cdot t - 90 \cdot \text{deg})$

c) 16-mA 8-mA 0.3-ms 3333-Hz
 $8 \cdot \text{mA} \cdot \cos(20940 \cdot t + 150 \cdot \text{deg})$

2. a) 12-V 3-V 6-ms 167-Hz

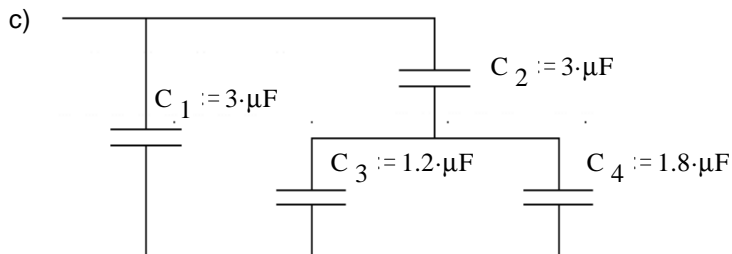
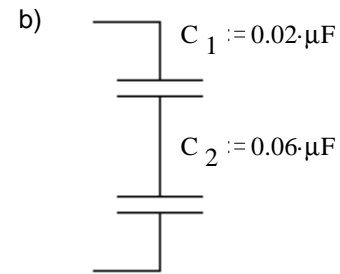
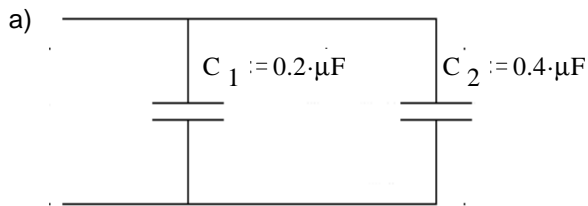
b) 12-V 6-V 4-ms 250-Hz

c) 250-mA 25-mA 0.6-ms 1.667-kHz

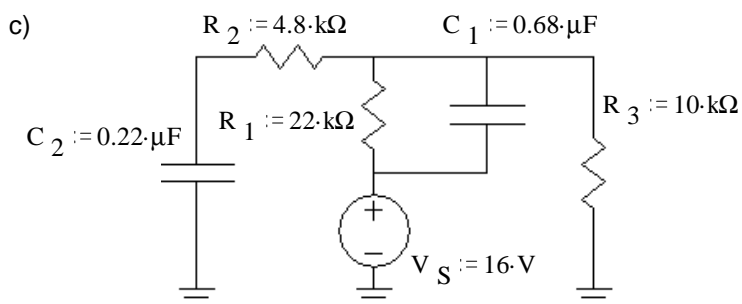
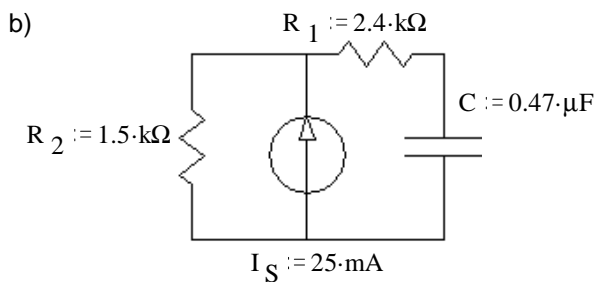
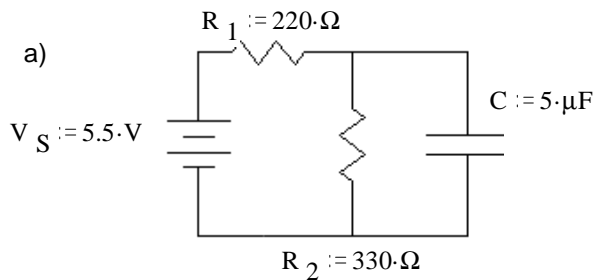
3. $v(t) := 6 \cdot V \cdot \cos(1047 \cdot t - 90 \cdot \text{deg}) + 3 \cdot V$

1st exam on Fri. 2/8 may include p.1 of this homework

(Listen for details in class)

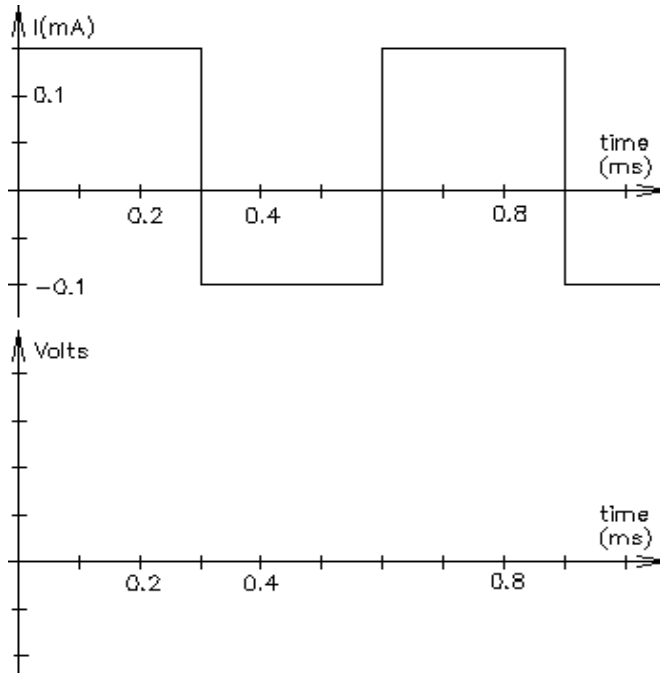
1) Find C_{eq} in each case

2. Each of the following circuits have been connected as shown for a long time. Find the voltage across each capacitor and the energy stored in each.

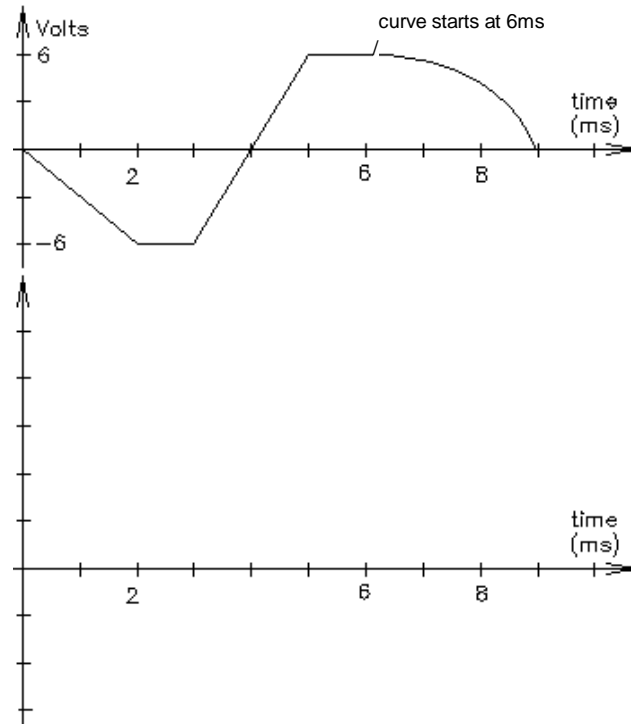


Name: _____ You may want to hand in this page with answers to problems 3 & 4.

3. The current waveform shown below flows through a $0.025 \mu\text{F}$ capacitor. Make an accurate drawing of the voltage across it. Label your graph. Assume the initial voltage across the capacitor is 0 V .



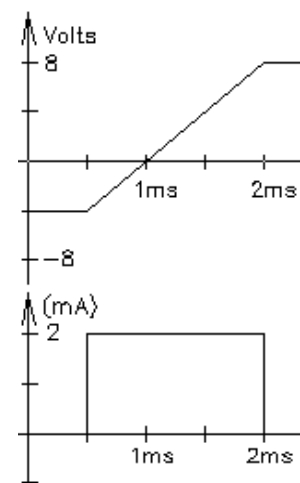
4. The voltage across a $2 \mu\text{F}$ capacitor is shown below. Make an accurate drawing of the capacitor current. Label your graph.



5. The voltage across a $0.68 \mu\text{F}$ capacitor is $v_c = 6 \cdot V \cdot \cos\left(200 \cdot t + \frac{\pi}{2}\right)$ find i_c .

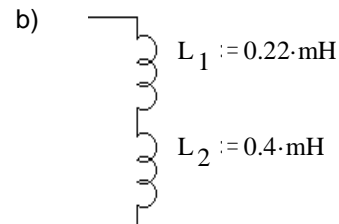
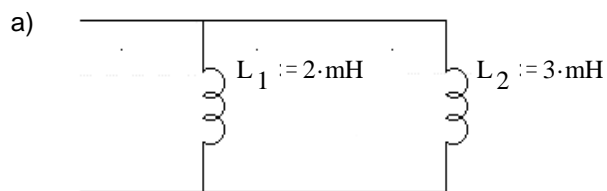
6. The current through a $0.0047 \mu\text{F}$ capacitor is $i_c = 18 \cdot \mu\text{A} \cdot \cos\left(628 \cdot t - \frac{\pi}{4}\right)$ find v_c .

7. A capacitor voltage and current are shown at right. What value is the capacitor?

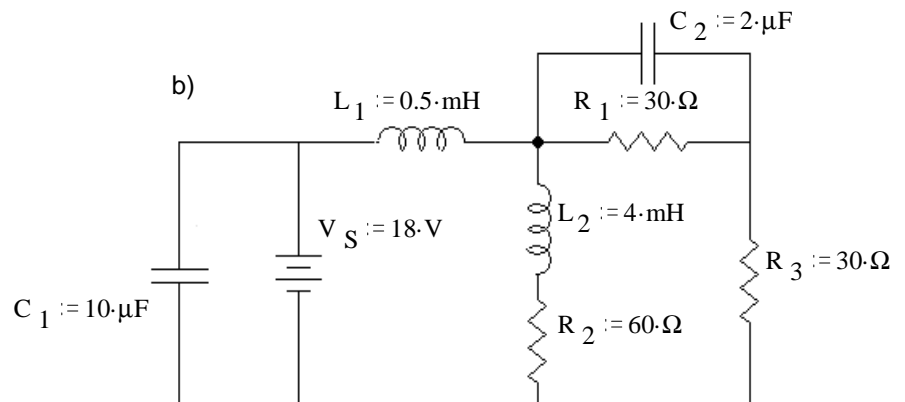
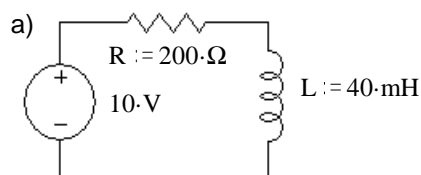
Answers1. a) $0.6 \mu\text{F}$ b) $0.015 \mu\text{F}$ c) $4.5 \mu\text{F}$ 2. a) 3.3 V 0.027 mJ b) 37.5 V 0.33 mJ c) 11 V 0.0411 mJ 5 V $2.75 \mu\text{J}$ 3. 1.8 V 0.6 V 2.4 V 4. -6 mA 12 mA ramp to -8 mA 5. $i_c = 0.816 \text{ mA} \cdot \cos(200 \cdot t + \pi)$ 6. $v_c = 6.1 \cdot \text{V} \cdot \cos\left(628 \cdot t - \frac{3 \cdot \pi}{4}\right)$ 7. $0.25 \mu\text{F}$ 

You will need another paper for your calculations, but you may want to hand this sheet in with your drawings.

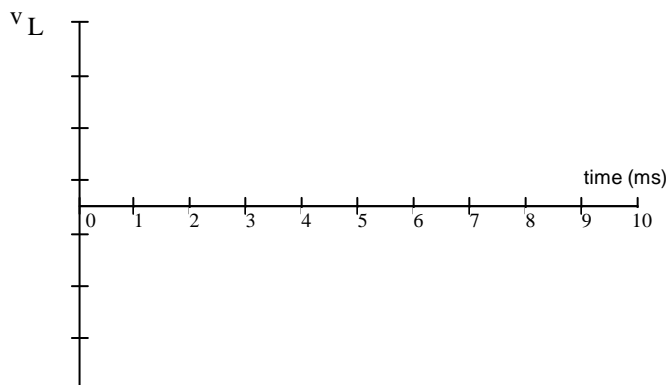
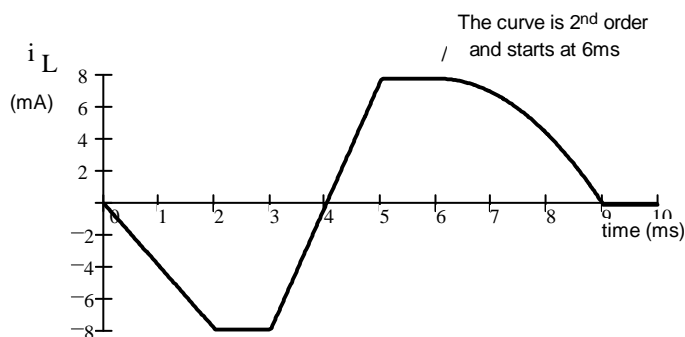
1. Find L_{eq} in each case



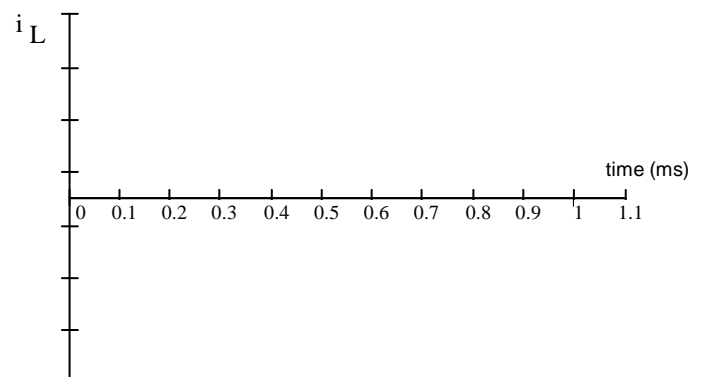
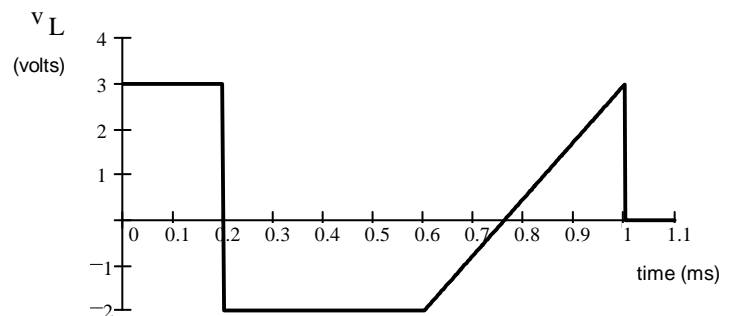
2. Find the stored energy in each capacitor and/or inductor under steady-state conditions. Note: Treat caps as opens and inductors as shorts to find DC voltages and currents.



3. The current waveform shown below flows through a 2 mH inductor. Make an accurate drawing of the voltage across it. Label your graph.



4. The voltage across a 0.5 mH inductor is shown below. Make an accurate drawing of the inductor current. Label your graph. Assume the initial current is 0 mA.



ECE 2210 / 00 homework # 9

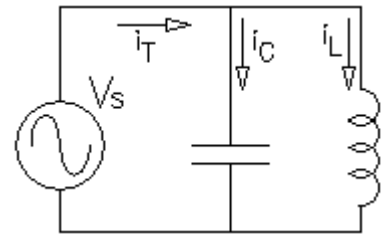
5. The voltage across a 1.2 mH inductor is $v_L = 4 \cdot \text{mV} \cdot \cos(300 \cdot t)$ find i_L .

6. The current through a 0.08 mH inductor is $i_L = 20 \cdot \text{mA} \cdot \cos\left(628 \cdot t - \frac{\pi}{4}\right)$ find v_L .

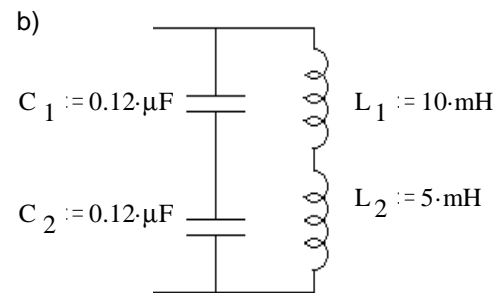
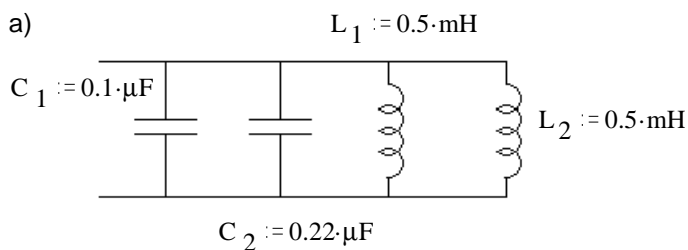
7. Refer to the circuit shown. Assume that V_s is a sinusoidal input voltage whose frequency can be adjusted. At some frequency of V_s this circuit can resonate. At that frequency $i_C(t) = -i_L(t)$. ($i_C(t)$ is 180 degrees out-of-phase with $i_L(t)$).

Show that resonance occurs at this frequency:

$$\omega_o = \frac{1}{\sqrt{L \cdot C}}, \quad f_o = \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}}$$



8. Find the resonant frequency, f_o in each case.



Answers

1. 1.2 mH 0.62 mH 2. a) 0.05 mJ b) 1.62 mJ 0.081 mJ 0.09 mJ 0.18 mJ
3. Straight lines between the following points: (0ms, -8mV), (2ms, -8mV), (2ms, 0mV), (3ms, 0mV), (3ms, 16mV), (5ms, 16mV), (5ms, 0mV), (6ms, 0mV), (9ms, -10.67mV), (9ms, 0mV), (10ms, 0mV)

4. Straight lines between the following points: (0ms, 0A), (0.2ms, 1.2A), (0.6ms, -0.4A), curves until it's flat at (0.76ms, -0.72A), continues to curve up to (1ms, 0A), (1.1ms, 0A)

$$5. \quad i_L = 11.1 \cdot \text{mA} \cdot \cos(300 \cdot t - 90 \cdot \text{deg}) \quad 6. \quad v_L = 1 \cdot \text{mV} \cdot \cos\left(628 \cdot t + \frac{1}{4} \cdot \pi\right)$$

7. Assume a sinusoidal voltage, find i_C and i_L by integration and differentiation, and show that they are equal and opposite at the resonant frequency.

8. a) 17.79 kHz b) 5305 Hz

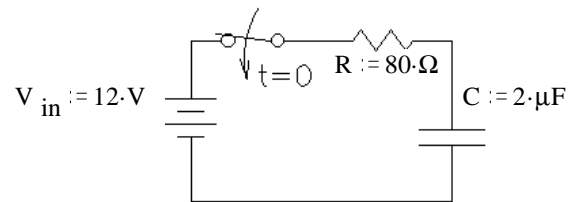
1. An FE style problem

A 10-microfarad capacitor has been charged to a potential of 150 volts. A resistor of $25\ \Omega$ is then connected across the capacitor through a switch. When the switch has been closed for 10 time constants the total energy dissipated by the resistor is most nearly

- (A) 1.0×10^{-7} joules
 (B) 1.1×10^{-1} joules
 (C) 9.0×10^1 joules
 (D) 9.0×10^3 joules

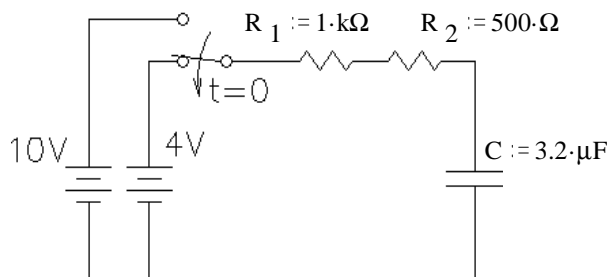
2. a) The switch is closed at time $t = 0$ and $v_C(0) = 0\text{V}$, find $v_C(t)$.

b) What is the value of the voltage across C at $t = 40\ \mu\text{s}$

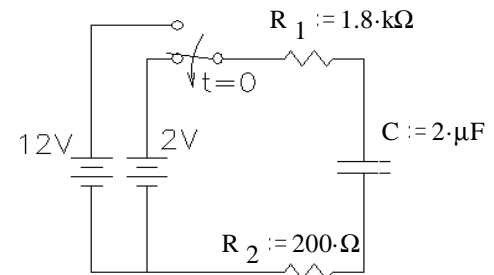


3. In the circuit below, the switch has been in the upper position for a long time and is switched down at time $t = 0$.

What is the capacitor voltage (V_C) at $t = 4\text{-ms}$



4. The switch below has been in the upper position for a long time and is switched down at time $t = 0$. At what time is $v_C = 4\text{ V}$?

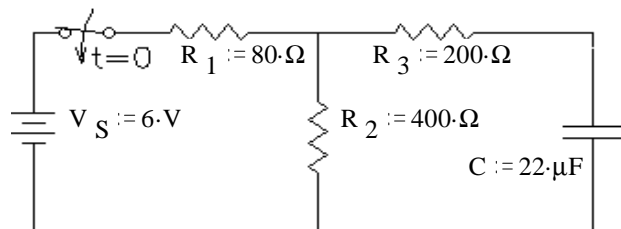


5. a) What is the time constant of this circuit?

Hint: Use a Thevenin equivalent circuit.

b) What will be the final value of v_C ?

(After the switch has been closed for a long time)

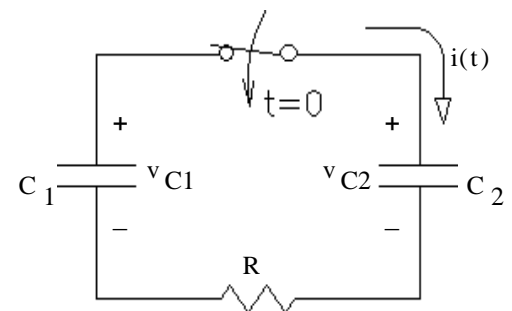


6. In a circuit with two capacitors, the left capacitor (C_1) has an initial charge and the right capacitor (C_2) does not. When the switch is closed at time $t = 0$, current $i(t)$ flows, discharging C_1 and charging C_2 .

a) Derive the differential equation for $i(t)$. Hint: write an equation in terms of i and integrals of i , then differentiate the whole equation.

Write your DE in this form: $\text{Constant} = x(t) + \tau \frac{d}{dt}x(t)$

What is the time constant (τ)?



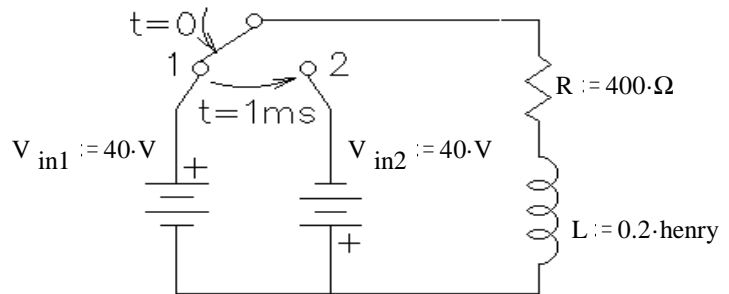
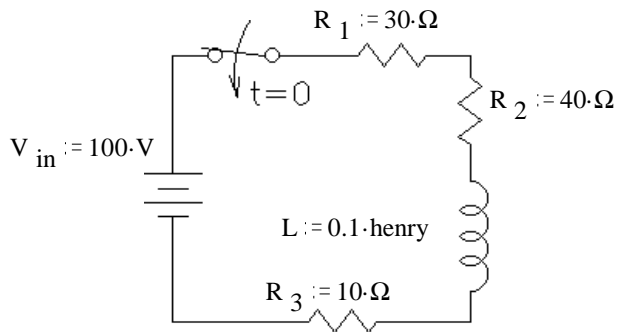
b) Find $i(t)$ given $C_1 = 24\ \mu\text{F}$, $C_2 = 12\ \mu\text{F}$, $R = 400\ \Omega$, $v_{C1}(0) = 18\text{-V}$, $v_{C2}(0) = 0\text{-V}$

c) Find $v_{C2}(t)$ for the same values. Hint: The trick here will be finding the final condition. Realize that charge will be conserved. If C_1 discharges x coulombs, then C_2 will charge x coulombs. Charges will stop flowing when $v_{C1} = v_{C2}$. It may help to think of two water tanks, one with half the cross-sectional area of the other. $V = \frac{Q}{C}$

d) Find the initial and final stored energy of the system ($W_{C1} + W_{C2}$) to find the total "loss". What happened to that energy?

Answers 1. B 2.a) $12\text{-V} - 12\text{-V} \cdot e^{-\frac{t}{0.16\text{-ms}}}$ 3. 6.61-V 4. 6.44-ms 5. a) 5.87-ms 6.a) $\tau = R \cdot \frac{1}{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}$
 b) 5-V 6.b) $i(t) = 45\text{-mA} \cdot e^{-\frac{t}{3.2\text{-ms}}}$ c) $12\text{-V} - 12\text{-V} \cdot e^{-\frac{t}{3.2\text{-ms}}}$ d) 1.3-mJ dissipated in resistor

1. A 12 V car ignition coil has an inductance of 10 mH and resistance of $2\ \Omega$ (so its equivalent circuit is a 10 mH inductor in series with a $2\ \Omega$ resistor). Calculate how long it takes the current to build up to 95% of its maximum value after a 12 V battery is connected to the coil.
2. A constant voltage is applied to a series RL circuit by closing a switch. The voltage across L is 30 volts at $t = 0$ and drops to 6 volts at $t = .0025$ sec. If $L = 0.2$ H, what must be the value of R?
3. In the circuit shown, the switch is closed at $t = 0$. Find the transient current expression.
4. In the circuit shown, the switch is closed on position 1 at $t = 0$, and then instantly moved to position 2 after 1 millisecond. Find the time at which the current is zero and reversing its direction.



note the different battery directions

If you learn to use the complex math feature of your calculator, you may use that to work the following problems. In that case you may report the answers without showing any work.

5. Convert the following complex numbers to polar form ($m\angle\theta$ or $me^{j\theta}$).
a) $1 + j$ b) $2.6 + 8.7j$ c) $3 + 4j$ d) $3 - 4j$ e) $-3 + 4j$ f) $-3 - 4j$
6. Convert the following complex numbers to rectangular form ($a + bj$).
a) $10 \cdot e^{j \cdot 60 \cdot \text{deg}}$ b) $0.4 \cdot e^{j \cdot 12 \cdot \text{deg}}$ c) $1500 \cdot e^{j \cdot \frac{\pi}{2} \cdot \text{rad}}$ d) $10 \cdot e^{-j \cdot 45 \cdot \text{deg}}$ e) $20 \cdot e^{j \cdot 120 \cdot \text{deg}}$ f) $30 \cdot e^{j \cdot 210 \cdot \text{deg}}$
7. Perform the following additions and subtractions of complex numbers.
a) $(3 + 2j) + (6 + 9j)$ b) $(9 - 10j) - (9 + 10j)$ c) $(-2 - 2j) + (-6 + 9j)$ d) $(3 + 0j) - (0 + 9j)$
e) $(5 + 6j) + 5 \cdot e^{j \cdot 53 \cdot \text{deg}}$ f) $(-2 + 3j) - 8 \cdot e^{-j \cdot 37 \cdot \text{deg}}$
8. Perform the following multiplications of complex numbers.
a) $(8 + j) \cdot 3$ b) $(3 + 2j) \cdot j$ c) $(20 \cdot e^{j \cdot 40 \cdot \text{deg}}) \cdot (10 \cdot e^{j \cdot 60 \cdot \text{deg}})$ d) $(-6 + 9j) \cdot (10 \cdot e^{j \cdot 60 \cdot \text{deg}})$ e) $(-2 - j) \cdot (-6 - 9j)$
9. Perform the following divisions of complex numbers.
a) $\frac{20 \cdot e^{j \cdot 40 \cdot \text{deg}}}{10 \cdot e^{j \cdot 60 \cdot \text{deg}}}$ b) $\frac{9 - 10j}{3 \cdot e^{-j \cdot 20 \cdot \text{deg}}}$ c) $\frac{3 + 0j}{0 + 9j}$ d) $\frac{-2 - 2j}{-6 + 9j}$

Answers

1. 15 ms 2. $129\ \Omega$ 3. $1.25 \cdot A \cdot \left(1 - e^{-\frac{t}{1.25 \cdot \text{ms}}}\right)$ 4. 1.312 ms
5. a) $1.414 \cdot e^{j45 \cdot \text{deg}}$ b) $9.08 \cdot e^{j73.4 \cdot \text{deg}}$ c) $5 \cdot e^{j53.1 \cdot \text{deg}}$ d) $5 \cdot e^{-j53.1 \cdot \text{deg}}$ e) $5 \cdot e^{j126.9 \cdot \text{deg}}$ f) $5 \cdot e^{-j126.9 \cdot \text{deg}}$
6. a) $5 + 8.66j$ b) $0.391 + 0.083j$ c) $1500j$ d) $7.071 - 7.071j$ e) $-10 + 17.321j$ f) $-25.981 - 15j$
7. a) $9 + 11j$ b) $-20j$ c) $-8 + 7j$ d) $3 - 9j$ e) $8.009 + 9.993j$ f) $-8.389 + 7.815j$
8. a) $24 + 3j$ b) $-2 + 3j$ c) $200 \cdot e^{j100 \cdot \text{deg}}$ d) $108 \cdot e^{-j176 \cdot \text{deg}}$ e) $24.2 \cdot e^{j82.9 \cdot \text{deg}}$
9. a) $2 \cdot e^{-j20 \cdot \text{deg}}$ b) $4.485 \cdot e^{-j28.01 \cdot \text{deg}}$
c) $0.333 \cdot e^{-j90 \cdot \text{deg}}$ d) $-0.051 + 0.256j$

Read about complex numbers and phasors in your textbook (sections 2.26 & 2.27, starting on p.159).

1. For the complex numbers $\mathbf{z}_1 := -4 + 5j$ and $\mathbf{z}_2 := 2 + 4j$ Determine the following

a) Does $|\mathbf{z}_1 \cdot \mathbf{z}_2|$ equal $|\mathbf{z}_1| \cdot |\mathbf{z}_2|$?

b) Does $\left| \frac{\mathbf{z}_1}{\mathbf{z}_2} \right|$ equal $\frac{|\mathbf{z}_1|}{|\mathbf{z}_2|}$?

c) Does $|\mathbf{z}_1 + \mathbf{z}_2|$ equal $|\mathbf{z}_1| + |\mathbf{z}_2|$?

2. a) Find the phasor for $v(t) = 8.4 \cdot \cos(100t - 90^\circ)$ Express in both forms, polar and rectangular.

b) The phasor representation of a current is $\mathbf{I} := (5 + j \cdot 12) \cdot \mu\text{A}$ Find the time-domain representation, $i(t)$. $f := 600 \cdot \text{Hz}$

3. Add or subtract the sinusoidal voltages using phasors. Draw a phasor diagram which shows all 3 phasors, and give your final answer in time domain form.

a) $v_1(t) = 1.5 \cdot V \cdot \cos(\omega \cdot t + 10^\circ)$ $v_2(t) = 3.2 \cdot V \cdot \cos(\omega \cdot t + 25^\circ)$ Find $v_3(t) = v_1(t) + v_2(t)$

b) $v_1(t) = 1.5 \cdot V \cdot \cos(\omega \cdot t + 10^\circ)$ $v_2(t) = 3.2 \cdot V \cdot \cos(\omega \cdot t + 25^\circ)$ Find $v_4(t) = v_1(t) - v_2(t)$

you may add \mathbf{V}_4 to the phasor diagram you've already drawn for part a).

c) $v_1(t) = 50 \cdot V \cdot \cos(\omega \cdot t - 60^\circ)$ $v_2(t) = 24 \cdot V \cdot \cos(\omega \cdot t + 15^\circ)$ Find $v_3(t) = v_1(t) + v_2(t)$

d) $v_1(t) = 0.9 \cdot V \cdot \cos(\omega \cdot t + 72^\circ)$ $v_2(t) = 1.2 \cdot V \cdot \cos(\omega \cdot t - 20^\circ)$ Find $v_3(t) = v_1(t) + v_2(t)$

e) $v_1(t) = 0.9 \cdot V \cdot \cos(\omega \cdot t + 72^\circ)$ $v_2(t) = 1.2 \cdot V \cdot \cos(\omega \cdot t - 20^\circ)$ Find $v_4(t) = v_2(t) - v_1(t)$

you may add \mathbf{V}_4 to the phasor diagram you've already drawn for part d).

4. Express the impedance of a 5.2mH inductor at 60 Hz in polar form.

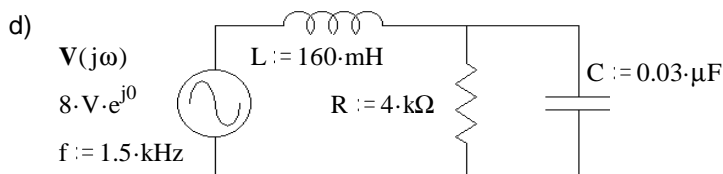
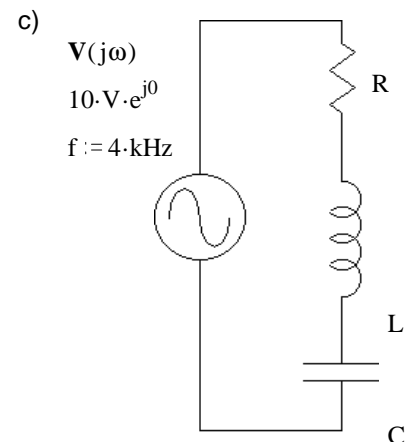
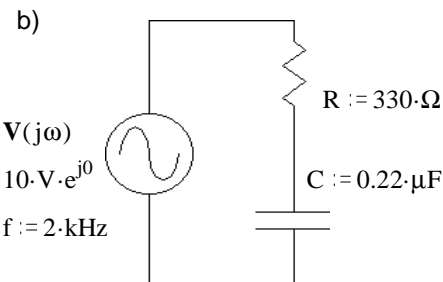
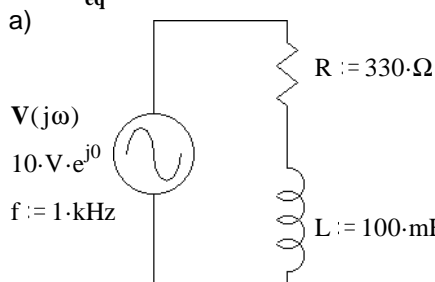
5. a) A capacitor impedance has a magnitude of 240Ω at a frequency of 1.8kHz. What is the value of capacitor?

b) What value inductor has the same impedance magnitude at the same frequency?

c) Find the reactance (magnitude of the impedance with + or - sign) of this capacitor and this inductor at 3.6kHz?

d) What would be the total impedance of this inductance and this capacitance connected in series at 2.7kHz?

6. Find \mathbf{Z}_{eq} in each case.



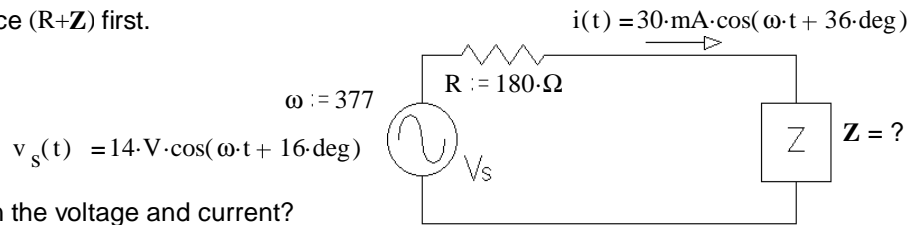
7. Find the current $\mathbf{I}(j\omega)$ in each case above.

8. a) Find Z . Hint: Find the total impedance ($R+Z$) first.

b) Which leads, current or voltage?

c) By how much?

I.E. what is the phase angle between the voltage and current?



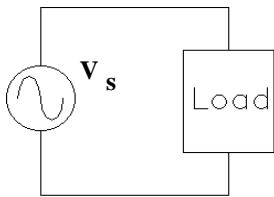
9. a) A resistor and a capacitor are connected in series to create an impedance of $Z := 50 \cdot \Omega \cdot e^{-j \cdot 66 \cdot \text{deg}} = 50 \Omega / -66^\circ$ at a frequency $f := 3 \cdot \text{kHz}$. Find R and C.

b) A resistor and a capacitor are connected in parallel to create an impedance of $Z := 50 \cdot \Omega \cdot e^{-j \cdot 66 \cdot \text{deg}} = 50 \Omega / -66^\circ$ at a frequency $f := 3 \cdot \text{kHz}$. Find R and C.

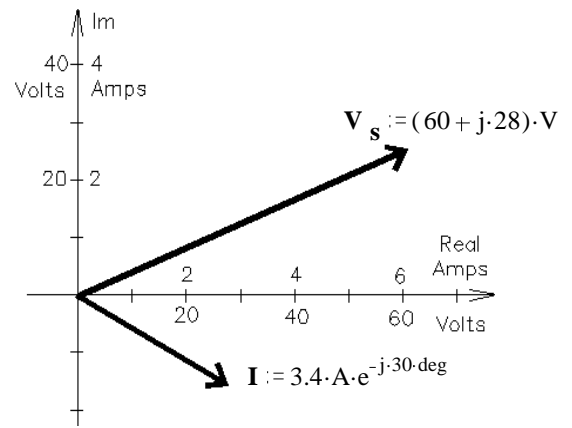
Hint: invert Z_{eq} . Instead of solving this: $50 / -66 \text{deg} = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C}$

solve this: $\frac{1}{Z_{eq}} = 0.02 / 66 \text{deg} = \frac{1}{R} + j \cdot \omega \cdot C$

10. The phasor diagram at right shows the voltage and current in the circuit below



Assume the load consists of a resistor in series with a reactive component and the frequency is 60 Hz.

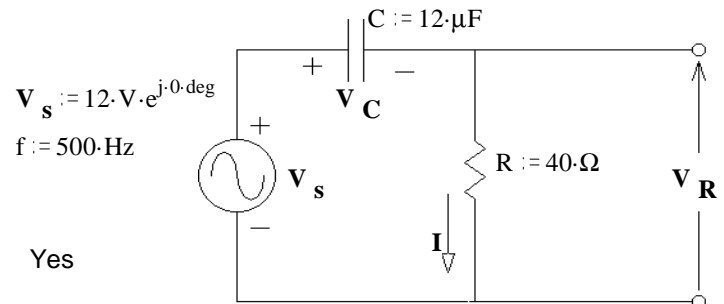


a) What is the magnitude of the impedance?

b) What is the value of the resistor?

c) What is the reactive component (type and value)?

11. For the circuit shown, draw a phasor diagram showing V_s , I , V_R , and V_C . Draw the voltages to scale so that you can show that they obey KVL.



Answers

1. a) $28.636 = 28.636$ Yes b) $1.432 = 1.432$ Yes
c) $9.22 \text{ not} = 10.875$ No

2. a) $8.4 / -90^\circ = 8.4 \cdot e^{-j \cdot 90 \cdot \text{deg}} = -8.4j$ b) $13 \cdot \mu\text{A} \cdot \cos(3770 \cdot t + 67.4 \cdot \text{deg})$

3. a) $v_1(t) + v_2(t) = 4.67 \cdot \cos(\omega \cdot t + 20.2 \cdot \text{deg}) \cdot \text{V}$

b) $v_1(t) - v_2(t) = 1.794 \cdot \cos(\omega \cdot t - 142.5 \cdot \text{deg}) \cdot \text{V}$

d) $v_1(t) + v_2(t) = 1.48 \cdot \cos(\omega \cdot t + 17.6 \cdot \text{deg}) \cdot \text{V}$

c) $v_1(t) + v_2(t) = 60.8 \cdot \cos(\omega \cdot t - 37.6 \cdot \text{deg}) \cdot \text{V}$

e) $v_2(t) - v_1(t) = 1.525 \cdot \cos(\omega \cdot t - 56.15 \cdot \text{deg}) \cdot \text{V}$

4. $1.96 \Omega / 90^\circ$ 5. a) $0.368 \cdot \mu\text{F}$ b) $21.2 \cdot \text{mH}$

c) $-120 \cdot \Omega$ $480 \cdot \Omega$ d) $200 \cdot j \cdot \Omega$

6. a) $(330 + 628.3 \cdot j) \cdot \Omega = 709.7 \Omega / 62.29^\circ$

b) $(330 - 361.7 \cdot j) \cdot \Omega = 489.6 \Omega / -47.63^\circ$

7. a) $(6.6 - 12.5 \cdot j) \cdot \text{mA} = 14.1 \text{mA} / -62.29^\circ$

b) $(13.8 + 15.1 \cdot j) \cdot \text{mA} = 20.4 \text{mA} / 47.63^\circ$

d) $4.4 \text{mA} / 15.2^\circ$

c) $R + \left(\omega \cdot L - \frac{1}{\omega \cdot C} \right) \cdot j$ d) $1.82 \text{k}\Omega / -15.2^\circ$

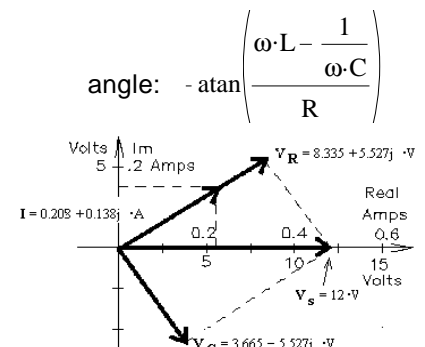
c) $\frac{10 \cdot \text{V}}{\sqrt{R^2 + \left(\omega \cdot L - \frac{1}{\omega \cdot C} \right)^2}}$

8. a) $259 - 160 \cdot j$ b) The current leads the voltage c) 20°

9. a) $20.34 \cdot \Omega$ $1.16 \cdot \mu\text{F}$ b) $123 \cdot \Omega$ $0.969 \cdot \mu\text{F}$

10. a) $19.5 \cdot \Omega$ b) $11.2 \cdot \Omega$ c) inductor $42.3 \cdot \text{mH}$

ECE 2210/00 homework # 12 p.2

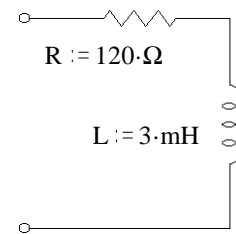


ECE 2200 Due: Tue, 2/26, may be handed in Mon, 3/4 for full credit

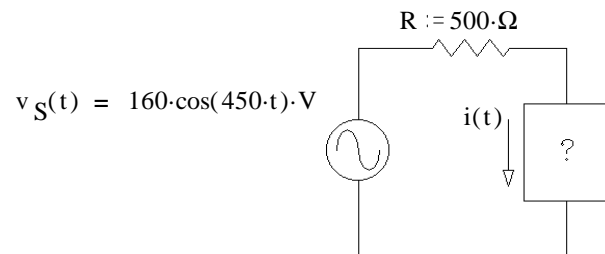
The 2nd exam will include this material**Warning: This homework is longer than normal -- DO NOT put it off until the last minute.**

1. For the circuit shown, find the following:

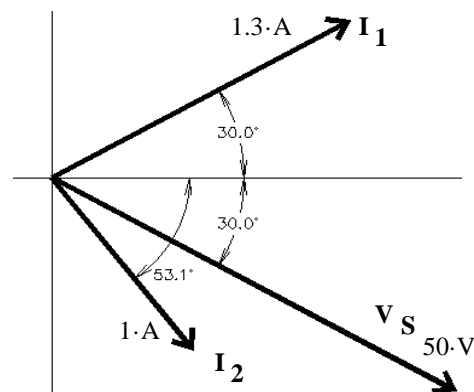
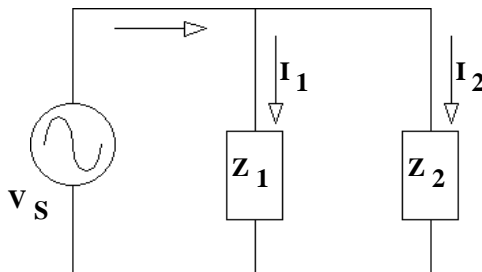
- At what frequency would the magnitude of the total impedance be 240Ω ?
- At this frequency, what is the phase angle of the impedance?
- At this frequency, you want to add a capacitor in series to make the circuit appear purely resistive (the impedance has no imaginary component). Find the value of the capacitor.

2. You need to design a circuit in which the current ($i(t)$) leads the voltage ($v_S(t)$) by 36° of phase.

- What should go in the box: R, L, C?
- Find its value.

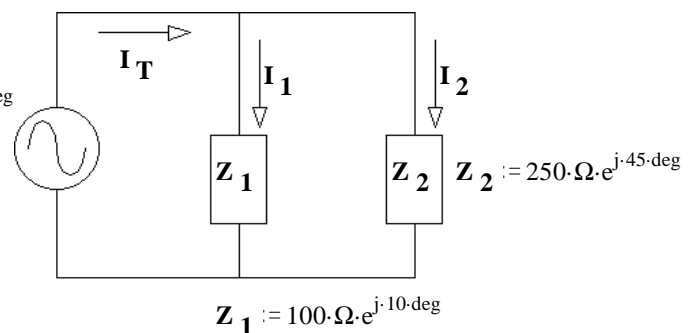


3. The phasor diagram at right shows the source voltage and two branch currents of a parallel circuit. Find the impedance of each of the two branches.

4. a) Find all the currents, I_1 , I_2 , and I_T .

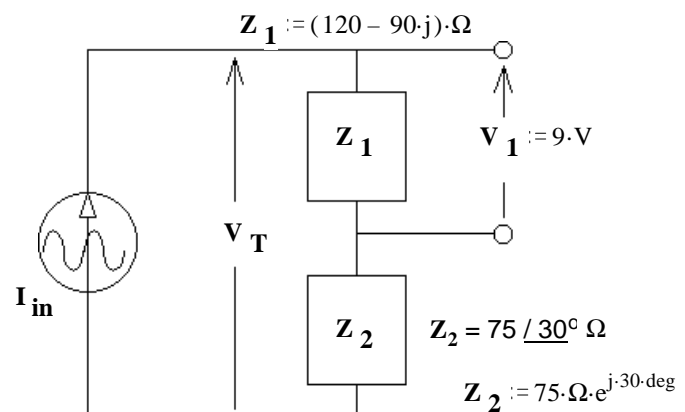
$$V_S := 24 \text{ V} \cdot e^{j \cdot 45 \cdot \text{deg}}$$

- Draw a phasor diagram showing I_1 , I_2 , and I_T to scale so that you can show that they obey KCL.

5. a) Find the AC current source, I_{in} in polar form.b) Find V_T .

c) Choose one:

- The source current leads the source voltage.
- The source current lags the source voltage.



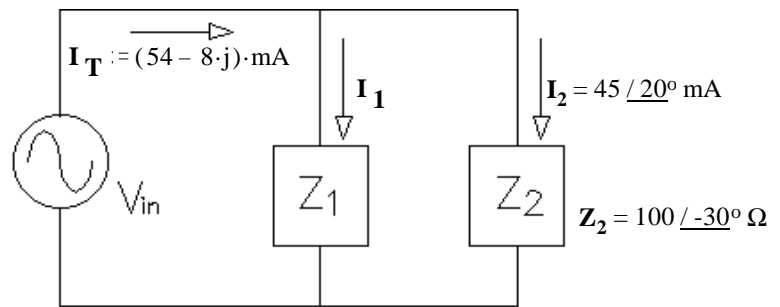
ECE 2210 homework # 13 p.2

6. a) Find Z_1 .

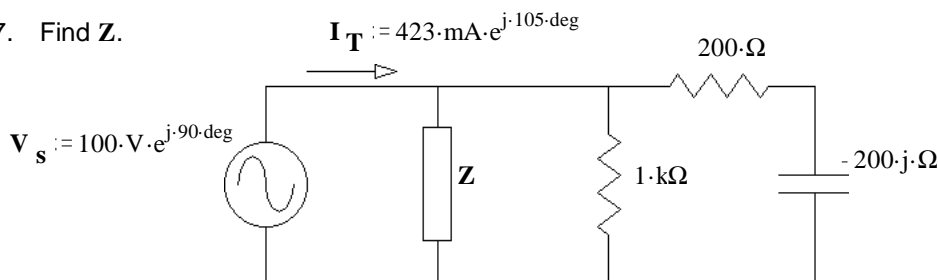
b) To make Z_1 in the simplest way, what part(s) would you need? Just determine the needed part(s) from the list below and state why you made that choice, don't find the values.

resistor capacitor inductor power supply current source
Thevenin resistor Ideal transformer voltmeter ammeter scope

c) Choose one: i) I_2 leads the source voltage (V_{in}) ii) I_2 lags the source voltage (V_{in})
d) Choose one: i) I_1 leads I_2 ii) I_1 lags I_2

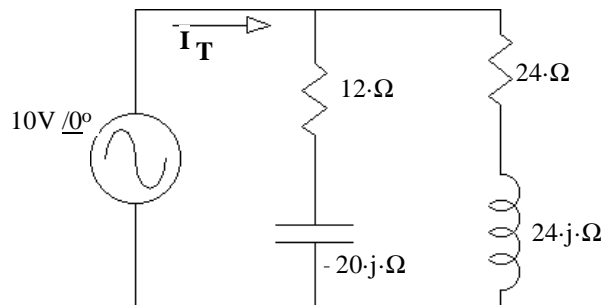


7. Find Z .



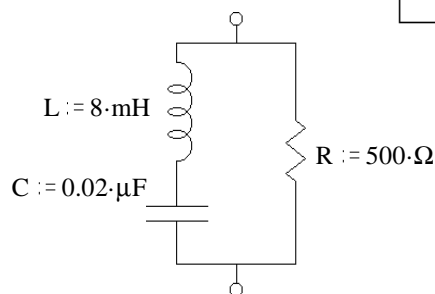
8. a) Find the total impedance of the circuit.

b) Find I_T .



9. Find Z_{eq} in simple polar form.

$f := 8000 \cdot \text{Hz}$



Answers

- a) 11 kHz b) 60° c) 0.0694 μF
- a) C b) 6.12 μF
- $Z_1 = (19.2 - 33.3j) \cdot \Omega$ $Z_2 = (46.0 + 19.6j) \cdot \Omega$
- a) $(0.197 + 0.138j) \cdot A + 0.096 \cdot A = 0.293 + 0.138j \cdot A$
- a) $60 / 36.87^\circ \text{ mA}$ b) $11.54 / 21^\circ \text{ V}$ c) i)
- a) $172 / 53.4^\circ \Omega$ b) phase angle > 0, resistor and inductor
c) i) d) ii)
- $657 \Omega / 67.4^\circ$ 8. a) $21.86 \Omega / -20.38^\circ$ b) $0.457 A / 20.38^\circ$
- $382 \Omega / -40.2^\circ$

