Introduction to AC Power, RMS

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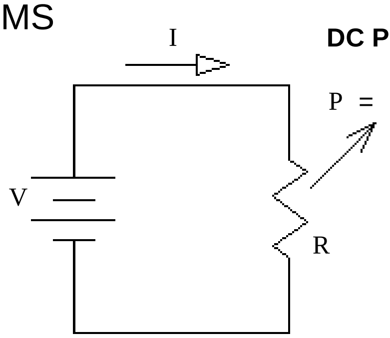
3/31/09,
2/20/10
11/23/13
4/15/20

RMS

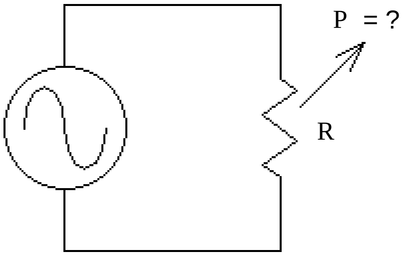
DC Power

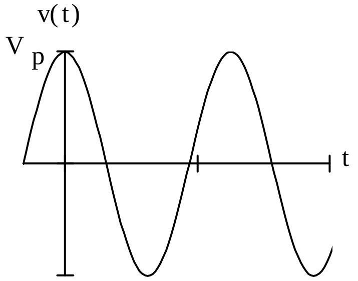


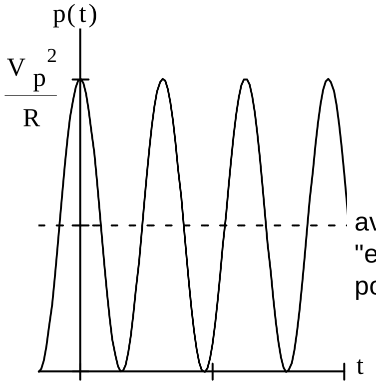
$$P = V\cdot I = \frac{V^2}{R} = I^2\cdot R$$

AC Power

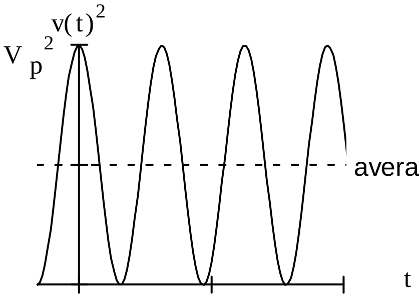
$$v(t) = V_p\cdot\cos(\omega\cdot t)$$







average or "effective" power = $\dfrac{\left(\dfrac{V_p^{\,2}}{R}\right)}{2}$



average = $\dfrac{V_p^{\,2}}{2}$

Couldn't we define an "effective" voltage that would allow us to use the same relationships for AC power as used for DC power?

$$P_{ave} = \frac{\left(\frac{V_p^{\,2}}{R}\right)}{2} = \frac{\left(\frac{V_p^{\,2}}{2}\right)}{R} = \frac{\left(\frac{V_p}{\sqrt{2}}\right)^2}{R}$$

$$V_{eff} = \sqrt{\left(\frac{V_p}{\sqrt{2}}\right)^2} = \frac{V_p}{\sqrt{2}} = V_{rms} = \sqrt{\frac{1}{T}\cdot\int_0^T (v(t))^2\,dt}$$

Square

Root

Mean (average)

RMS **R**oot of the **M**ean of the **S**quare

Use RMS in power calculations

Sinusoids

$$V_{rms} = \sqrt{\frac{1}{T}\cdot\int_0^T (v(t))^2\,dt} = \sqrt{\frac{1}{T}\cdot\int_0^T \left(V_p\cdot\cos(\omega\cdot t)\right)^2 dt} = \sqrt{\frac{1}{T}\cdot\int_0^T V_p^{\,2}\cdot\left(\frac{1}{2}+\frac{1}{2}\cdot\cos(2\cdot\omega\cdot t)\right) dt}$$

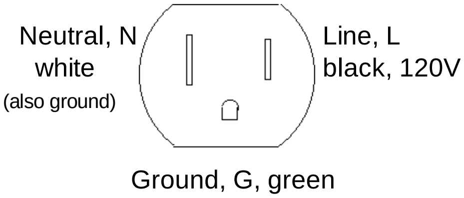
$$= \frac{V_p}{\sqrt{2}}\cdot\sqrt{\frac{1}{T}\cdot\int_0^T (1)\,dt + \frac{1}{T}\cdot\int_0^T \cos(2\cdot\omega\cdot t)\,dt} = \frac{V_p}{\sqrt{2}}\cdot\sqrt{1+0}$$

Common household power

$f = 60\cdot Hz$

$\omega = 377\cdot\frac{rad}{sec}$

$T = 16.67\cdot ms$

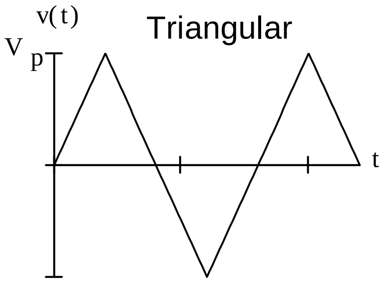


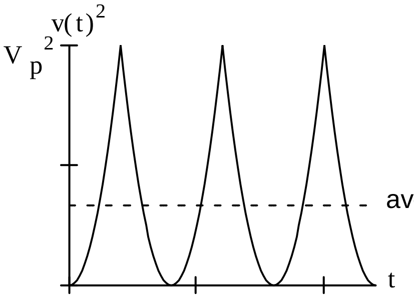
$V_{rms} := 120\cdot V$

$V_p = V_{rms}\cdot\sqrt{2} = 170\cdot V$

What about other wave shapes??

Triangular

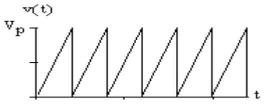




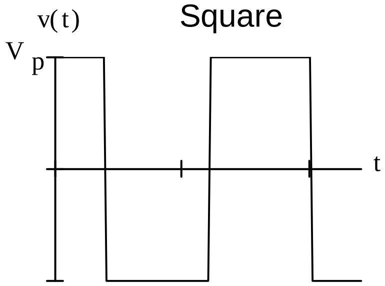
$\text{average} = \frac{V_p^{\,2}}{3}$

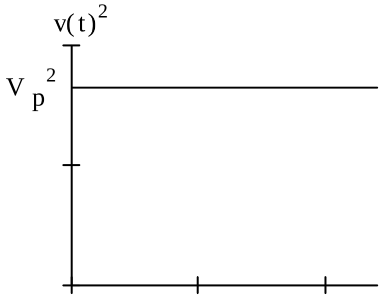
$V_{rms} = \frac{V_p}{\sqrt{3}}$

Works for all types of triangular and sawtooth waveforms



Square



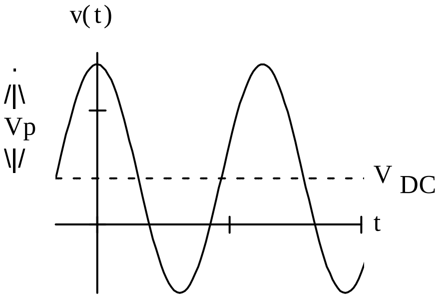


$\text{average} = V_p^{\,2}$

$V_{rms} = V_p$

Same for DC

How about AC + DC ?



$$V_{rms} = \sqrt{\frac{1}{T}\cdot\int_0^T (v(t))^2\,dt}$$

$$= \sqrt{\frac{1}{T}\cdot\int_0^T \left(V_p\cdot\cos(\omega\cdot t) + V_{DC}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T}\cdot\int_0^T \left[\left(V_p\cdot\cos(\omega\cdot t)\right)^2 + 2\cdot\left(V_p\cdot\cos(\omega\cdot t)\right)\cdot V_{DC} + V_{DC}^{\,2}\right] dt}$$

$$= \sqrt{\frac{1}{T}\cdot\int_0^T \left(V_p\cdot\cos(\omega\cdot t)\right)^2 dt + \frac{1}{T}\cdot\int_0^T 2\cdot\left(V_p\cdot\cos(\omega\cdot t)\right)\cdot V_{DC}\,dt + \frac{1}{T}\cdot\int_0^T V_{DC}^{\,2}\,dt}$$

- - - zero over one period - - -

$$= \sqrt{V_{rmsAC}^{\,2} + 0 + V_{DC}^{\,2}} \qquad = \sqrt{V_{rmsAC}^{\,2} + V_{DC}^{\,2}}$$

sinusoid: $V_{rms} = \frac{V_p}{\sqrt{2}}$ $I_{rms} = \frac{I_p}{\sqrt{2}}$

triangular: $V_{rms} = \frac{V_p}{\sqrt{3}}$ $I_{rms} = \frac{I_p}{\sqrt{3}}$

square: $V_{rms} = V_p$ $I_{rms} = I_p$

waveform + DC $V_{rms} = \sqrt{V_{rmsAC}^2 + V_{DC}^2}$

rectified average $V_{ra} = \frac{1}{T}\cdot\int_0^T |v(t)|\,dt$

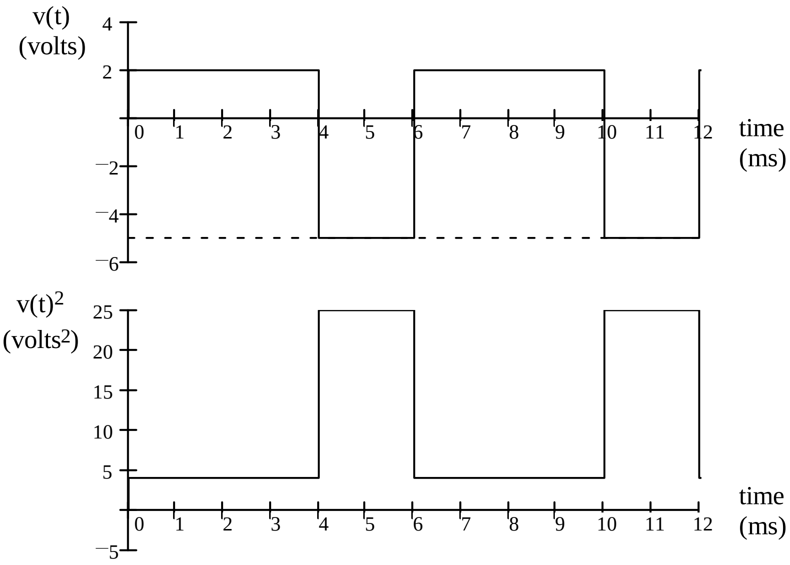
$V_{ra} = \frac{2}{\pi}\cdot V_p$ $I_{ra} = \frac{2}{\pi}\cdot I_p$

$V_{ra} = \frac{1}{2}\cdot V_p$ $I_{ra} = \frac{1}{2}\cdot I_p$

$V_{ra} = V_{rms} = V_p$ $I_{ra} = I_{rms} = I_p$

Most AC meters don't measure true RMS. Instead, they measure V_{ra} , display $1.11V_{ra}$, and call it RMS. That works for sine waves but not for any other waveform.

Some waveforms don't fall into these forms, then you have to perform the math from scratch



For waveform shown

The average DC (V_{DC}) value

$$\frac{2\cdot V\cdot(4\cdot ms) + (-5\cdot V)\cdot(2\cdot ms)}{6\cdot ms} = -0.333\cdot V$$

The RMS (effective) value

Graphical way

$$\frac{4\cdot V^2\cdot(4\cdot ms) + 25\cdot V^2\cdot(2\cdot ms)}{6\cdot ms} = 11\cdot V^2$$

$V_{RMS} := \sqrt{11\cdot V^2}$ $V_{RMS} = 3.32\cdot V$

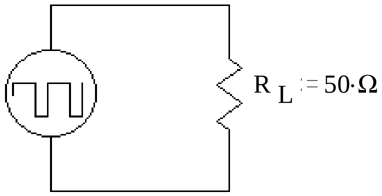
OR...

$$V_{RMS} = \sqrt{\frac{1}{T}\cdot\int_0^T (v(t))^2\,dt}$$

$$= \sqrt{\frac{1}{6\cdot ms}\cdot\left[\int_{0\cdot ms}^{4\cdot ms} (2\cdot V)^2\,dt + \int_{4\cdot ms}^{6\cdot ms} (-5\cdot V)^2\,dt\right]} = \sqrt{\frac{1}{6\cdot ms}\cdot\left[4\cdot ms\cdot(2\cdot V)^2 + 2\cdot ms\cdot(-5\cdot V)^2\right]} = 3.32\cdot V$$

The voltage is hooked to a resistor, as shown, for 6 seconds.

The energy is transferred to the resistor during that 6 seconds:



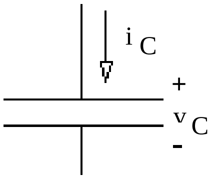
$P_L := \frac{V_{RMS}^2}{R_L}$ $P_L = 0.22\cdot W$

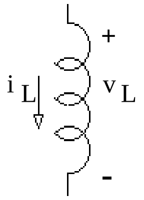
$W_L := P_L\cdot 6\cdot sec$ $W_L = 1.32\cdot joule$ All converted to heat

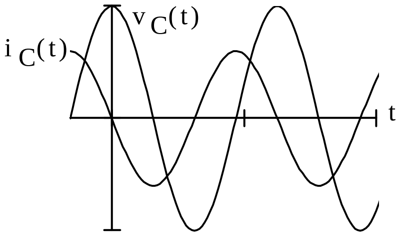
Use RMS in power calculations

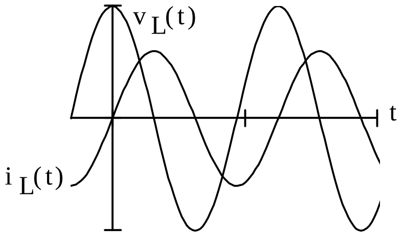
$$P = I_{Rrms}^{2} \cdot R = \frac{V_{Rrms}^{2}}{R}$$ for Resistors ONLY ! !

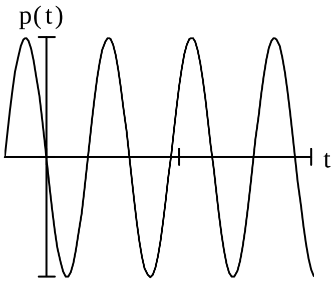
Capacitors and Inductors

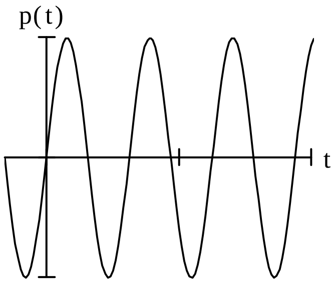












Average power is ZERO $P = 0$

Average power is ZERO $P = 0$

Capacitors and Inductors DO NOT dissipate (real) average power.

Reactive power is negative

$$Q_C = -I_{Crms} \cdot V_{Crms}$$

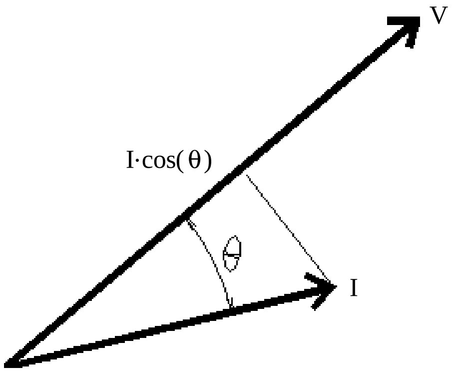
$$= -I_{Crms}^{2} \cdot \frac{1}{\omega \cdot C} = -V_{Crms}^{2} \cdot \omega \cdot C$$

Reactive power is positive

$$Q_L = I_{Lrms} \cdot V_{Lrms}$$

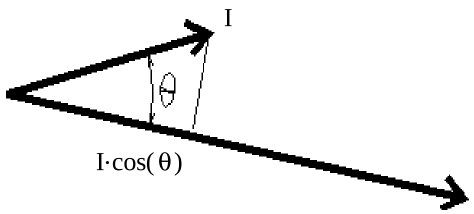
$$= I_{Lrms}^{2} \cdot \omega \cdot L = \frac{V_{Lrms}^{2}}{\omega \cdot L}$$

If current and voltage are not in phase, only the in-phase part of the current matters for the power-- DOT PRODUCT



"Lagging" power

Inductor dominates



"Leading" Power

Capacitor dominates

All voltages and currents shown are RMS

BOLD is a complex number

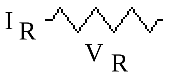
Real Power

$$P = V\cdot I\cdot\cos(\theta) = I^2\cdot|\mathbf{Z}|\cdot\cos(\theta) = \frac{V^2}{|\mathbf{Z}|}\cdot\cos(\theta)$$

P = "Real" Power (average) = $V\cdot I\cdot pf = I^2\cdot|\mathbf{Z}|\cdot pf = \frac{V^2}{|\mathbf{Z}|}\cdot pf$ units: watts, kW, MW, etc.

$pf = \cos(\theta)$ = power factor

otherwise....

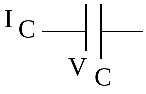


for resistors only part that uses real average power $P = I_R^2\cdot R = \frac{V_R^2}{R}$

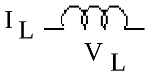
Reactive Power

Q = Reactive "power" = $V\cdot I\cdot\sin(\theta)$ units: VAR, kVAR, etc. "volt-amp-reactive"

otherwise....



capacitors -> **-** Q $Q_C = I_C^2\cdot X_C = \frac{V_C^2}{X_C}$ $X_C = -\frac{1}{\omega\cdot C}$ and is a negative number



inductors -> + Q $Q_L = I_L^2\cdot X_L = \frac{V_L^2}{X_L}$ $X_L = \omega\cdot L$ and is a positive number

Complex and Apparent Power

$\mathbf{S}$ = Complex "power" = $P + jQ = VI\underline{/\theta} = \mathbf{V}\cdot\overset{\text{complex conjugate}}{\overline{\mathbf{I}}} = I^2\cdot\mathbf{Z}$ units: VA, kVA, etc. "volt-amp"

NOT $\mathbf{V}\cdot\mathbf{I}$ **NOR** $\frac{\mathbf{V}^2}{\mathbf{Z}}$

S = Apparent "power" = $|\mathbf{S}| = \sqrt{P^2 + Q^2} = V\cdot I$ units: VA, kVA, etc. "volt-amp"

Power factor

$pf = \cos(\theta)$ = power factor (sometimes expressed in %) $0 \leq pf \leq 1$

θ is the **phase angle** between the voltage and the current or the phase angle of the impedance. $\theta = \theta_Z$

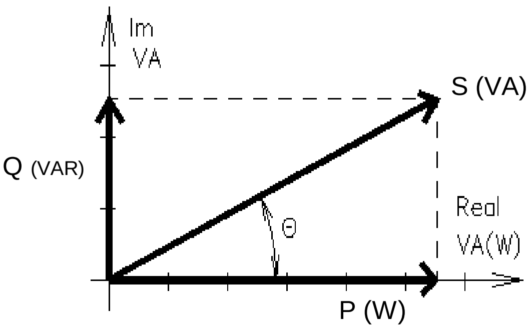
$\theta < 0$ Load is "Capacitive", power factor is "leading". This condition is very rare

$\theta > 0$ Load is "Inductive", power factor is "lagging". This condition is so common you can assume any power factor given is lagging unless specified otherwise. Transformers and motors make most loads inductive.

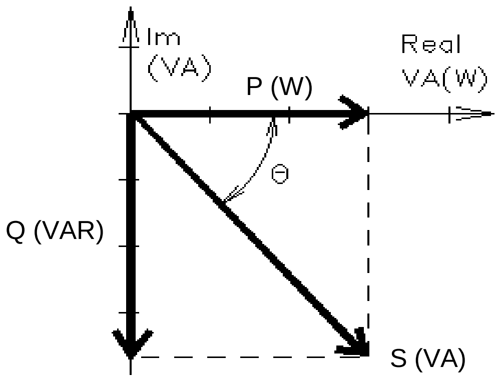
Industrial users are charged for the reactive power that they use, so power factor < 1 is a bad thing.

Power factor < 1 is also bad for the power company. To deliver the same power to the load, they have more line current (and thus more line losses).

Power factors are "corrected" by adding capacitors (or capacitve loads) in parallel with the inductive loads which cause the problems. (In the rare case that the load is capacitive, the pf would be corrected by an inductor.)



"Lagging" power



"Leading" Power

Transformer basics and ratings

A Transformer is two coils of wire that are magnetically coupled.

Transformers are only useful for AC, which is one of the big reasons electrical power is generated and distributed as AC.

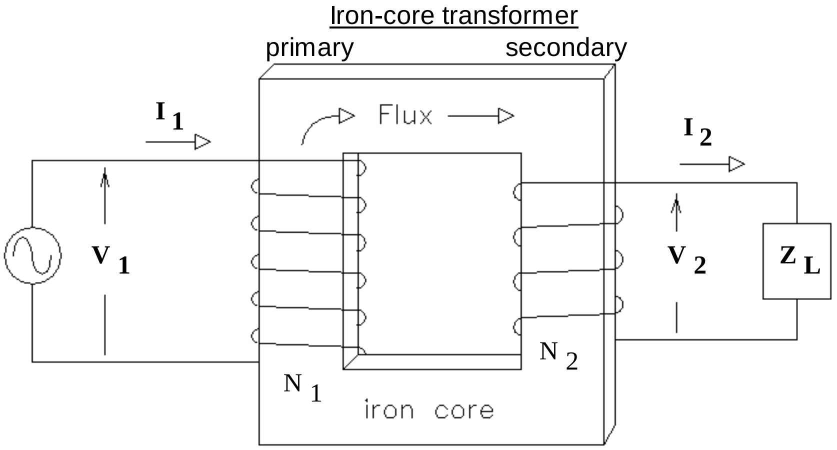
Transformer turns and turns ratios are rarely given, V_p/V_s is much more common where V_p/V_s is the rated primary over rated secondary voltages. You may take this to be the same as N_1/N_2 although in reality N_2 is usually a little bit bigger to make up for losses. Also common: $V_p : V_s$.

Both RMS

Transformers are rated in VA Transformer Rating (VA) = (rated V) x (rated I) , on either side.

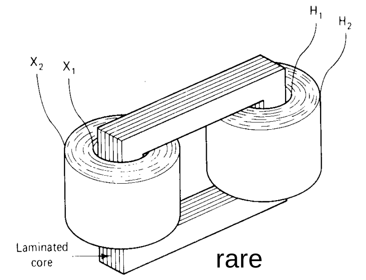
Don't allow voltages over the rated V , regardless of the actual current.
Don't allow currents over the rated I , regardless of the actual voltage.

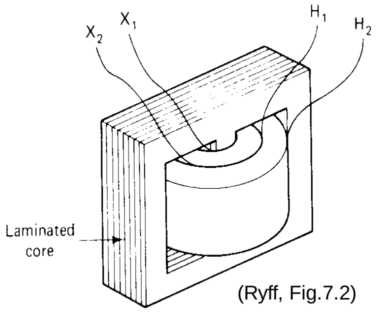
Ideal Transformers



Ideal: $P_1 = P_2$

power in = power out





(Ryff, Fig.7.2)

common

Transformation of voltage and current

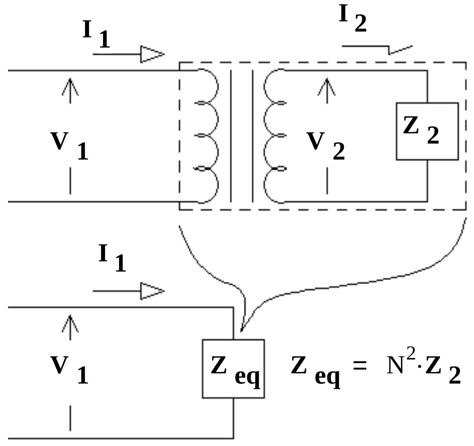
$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

Turns ratio

Turns ratio as defined in most books: $N = \frac{N_1}{N_2}$

Note: some other texts define the turns ratio as: $\frac{N_2}{N_1}$ Be careful how you and others use this term.

Transformation of impedance



$$Z_{eq} = N^2 \cdot Z_2 = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_2$$

You can replace the entire transformer and load with (Z_{eq}). This "impedance transformation" can be very handy.

Transformers can be used for "impedance matching"

This also works the opposite way, to move an impedance from the primary to the secondary, multiply by: $\left(\frac{N_2}{N_1}\right)^2$

Other Transformers

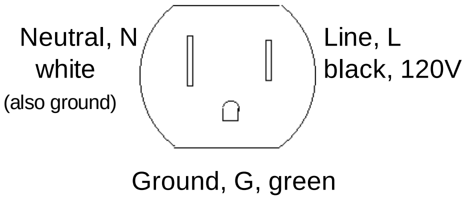
Multi-tap transformers:	Many transformers have more than two connections to primary and/or the secondary. The extra connections are called "taps" and may allow you to select from several different voltages or get more than one voltage at the same time.
Isolation Transformers:	Almost all transformers isolate the primary from the secondary. An Isolation transformer has a 1:1 turns ratio and is just for isolation.
Auto Transformers:	Auto transformers have only one winding with taps for various voltages. The primary and secondary are simply parts of the same winding. These parts may overlap. Any regular transformer can be wired as an auto transformer. Auto transformers DO NOT provide isolation.
Vari-AC:	A special form of auto transformer with an adjustable tap for an adjustable output voltage.
LVDT	A Linear-Variable-Differential-Transformers has moveable core which couples the primary winding to the secondary winding(s) in such a way the the secondary voltage is proportional to the position of the core. LVDTs are used as position sensors.

Home power

Standard 120 V outlet connections are shown at right.

The 3 lines coming into your house are **NOT** 3-phase.
They are +120 V, Gnd, -120 V

(The two 120s are 180^o out-of-phase, allowing for 240 V connections)

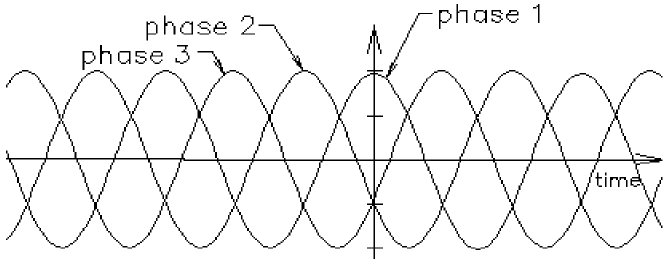


3-Phase Power (FYI ONLY)

Single phase power pulses at 120 Hz. This is not good for motors or generators over 5 hp.

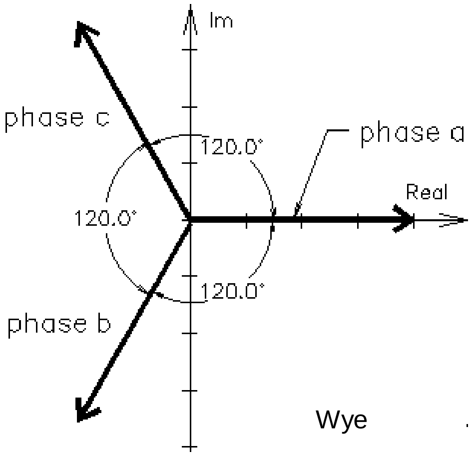
Three phase power is constant as long as the three loads are balanced.

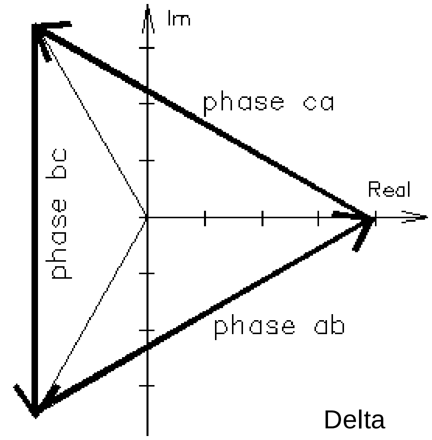
Three lines are needed to transmit 3-phase power.
If loads are balanced, ground return current will be zero.



Wye connection:
Connect each load or generator phase between a line and ground.

Delta connection:
Connect each load or generator phase between two lines.



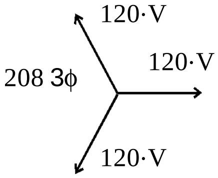


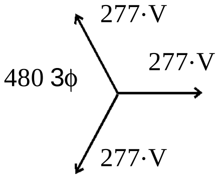
$$V_{LN} = \frac{V_{LL}}{\sqrt{3}} \qquad I_L = \sqrt{3}\cdot I_{LL}$$

$$V_{LL} = \sqrt{3}\cdot V_{LN} \qquad I_{LL} = \frac{I_L}{\sqrt{3}}$$

3-Phase Power (FYI ONLY)

Common 3-phase voltages:

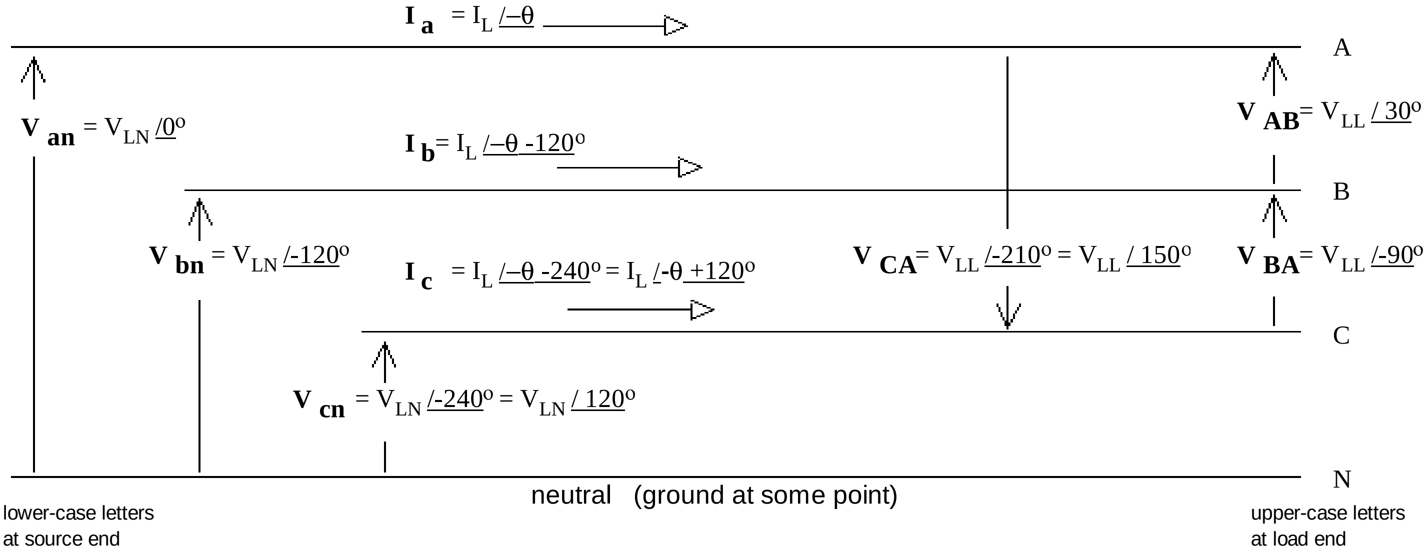


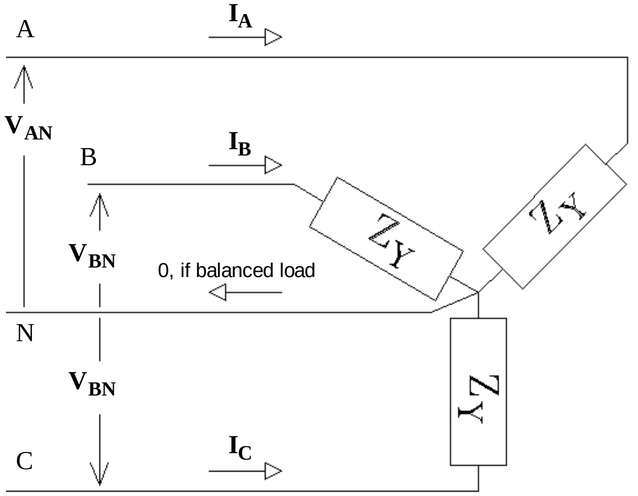


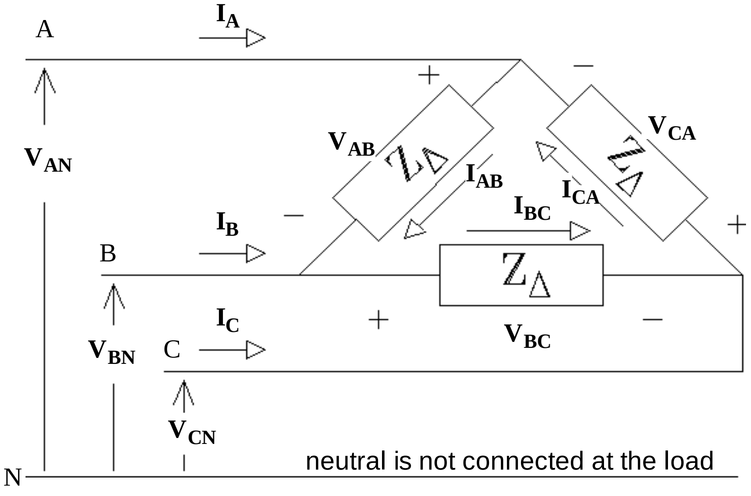
Apparent Power: $\left|S_{3\phi}\right| = 3\cdot V_{LN}\cdot I_L = 3\cdot V_{LL}\cdot I_{LL} = \sqrt{3}\cdot V_{LL}\cdot I_L$

Power: $P_{3\phi} = 3\cdot V_{LN}\cdot I_L\cdot pf = 3\cdot V_{LL}\cdot I_{LL}\cdot pf = \sqrt{3}\cdot V_{LL}\cdot I_L\cdot pf = S_{3\phi}\cdot pf$ $\qquad pf = \cos(\theta)$

Reactive power: $Q_{3\phi} = 3\cdot V_{LN}\cdot I_L\cdot \sin(\theta)$ etc... $= \sqrt{\left(\left|S_{3\phi}\right|\right)^2 - P_{3\phi}^{\;2}}$







$$\left|V_{AN}\right| = \left|V_{BN}\right| = \left|V_{CN}\right| = V_{LN} = \frac{V_{LL}}{\sqrt{3}} \qquad \left|V_{AB}\right| = \left|V_{BC}\right| = \left|V_{CA}\right| = V_{LL} = \sqrt{3}\cdot V_{LN}$$

$$\left|I_A\right| = \left|I_B\right| = \left|I_C\right| = I_L = \sqrt{3}\cdot I_{LL} \qquad \left|I_{AB}\right| = \left|I_{BC}\right| = \left|I_{CA}\right| = I_{LL} = \frac{I_L}{\sqrt{3}}$$

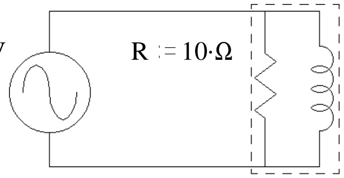
To get equivalent line currents with equivalent voltages

$$Z_Y = \frac{Z_\Delta}{3} \qquad Z_\Delta = 3\cdot Z_y$$

ECE 2210 AC Power Examples

A.Stolp
11/06/02
rev 2/27/07

Ex. 1 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.



$V_{in} := 110\cdot V$ $\quad \omega := 377\cdot\frac{rad}{sec}$ $\quad R := 10\cdot\Omega$ $\quad L := 25\cdot mH$

$$\mathbf{Z} := \frac{1}{\left(\frac{1}{R}+\frac{1}{j\cdot\omega\cdot L}\right)} = \frac{1}{0.1458\cdot\frac{1}{\Omega}\cdot e^{-j\cdot 46.7\cdot deg}}$$

$\mathbf{Z} = 4.704 + 4.991j \;\bullet\Omega$ $\quad |\mathbf{Z}| = 6.859\bullet\Omega$ $\quad \theta := \arg(\mathbf{Z})$ $\quad \theta = 46.7\bullet deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.686$

$\mathbf{I} := \frac{\mathbf{V}_{in}}{\mathbf{Z}}$ $\quad \mathbf{I} = 11 - 11.671j \;\bullet A$ $\quad |\mathbf{I}| = 16.038\bullet A$ $\quad \arg(\mathbf{I}) = -46.7\bullet deg$

$P := |\mathbf{V}_{in}|\cdot|\mathbf{I}|\cdot pf$ $\quad P = 1.21\bullet kW$

$Q := |\mathbf{V}_{in}|\cdot|\mathbf{I}|\cdot\sin(\theta)$ $\quad Q = 1.284\bullet kVAR$ OR... $Q := |\mathbf{V}_{in}|\cdot|\mathbf{I}|\cdot\sqrt{1^2 - pf^2}$ $\quad Q = 1.284\bullet kVAR$

$\mathbf{S} := \mathbf{V}_{in}\cdot\overline{\mathbf{I}}$ OR.. $\mathbf{S} := P + j\cdot Q$ $\quad \mathbf{S} = 1.21 + 1.284j \;\bullet kVA$ $\quad S := \sqrt{Re(\mathbf{S})^2 + Im(\mathbf{S})^2} = |\mathbf{S}| = 1.764\bullet kVA$

$\operatorname{atan}\left(\frac{Im(\mathbf{S})}{Re(\mathbf{S})}\right) = 46.696\bullet deg$

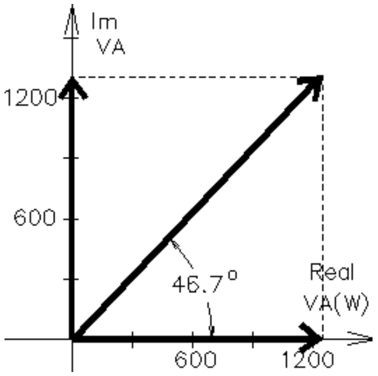
$\mathbf{S}$ = 1.764kVA $\underline{/46.7^{o}}$

OR, since we know that the voltage across each element of the load is V_{in} ...
Real power is dissipated only by resistors

$P := \frac{(|\mathbf{V}_{in}|)^2}{R}$ $\quad P = 1.21\bullet kW$ $\quad Q := \frac{(|\mathbf{V}_{in}|)^2}{\omega\cdot L}$ $\quad Q = 1.284\bullet kVAR$

$\mathbf{S} := P + j\cdot Q$

$S = |\mathbf{S}| = \sqrt{P^2 + Q^2} = 1.764\bullet kVA$ $\quad pf = \frac{P}{|\mathbf{S}|} = 0.686$



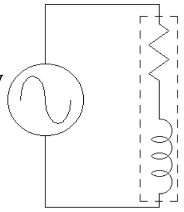
What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

$Im(\mathbf{I}) = -11.671\bullet A$ $\quad X_C := \frac{\mathbf{V}_{in}}{Im(\mathbf{I})}$ $\quad X_C = -9.425\bullet\Omega = \frac{-1}{\omega\cdot C}$

$\frac{1}{|X_C|\cdot\omega} = 281\bullet\mu F$ OR.. $\omega = \frac{1}{\sqrt{L\cdot C}}$ $\quad C := \frac{1}{L\cdot\omega^2}$ $\quad C = 281\bullet\mu F$

Ex. 2 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

Series R & L



$V_{in} := 110\cdot V$ $\quad \omega := 377\cdot\frac{rad}{sec}$ $\quad R := 10\cdot\Omega$ $\quad L := 25\cdot mH$

$\mathbf{Z} := R + j\cdot\omega\cdot L$

$\mathbf{Z} = 10 + 9.425j \;\bullet\Omega$ $\quad |\mathbf{Z}| = 13.742\bullet\Omega$

$\theta := \arg(\mathbf{Z})$ $\quad \theta = 43.304\bullet deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.728$

$\mathbf{I} := \frac{\mathbf{V}_{in}}{\mathbf{Z}}$ $\quad \mathbf{I} = 5.825 - 5.49j \;\bullet A$

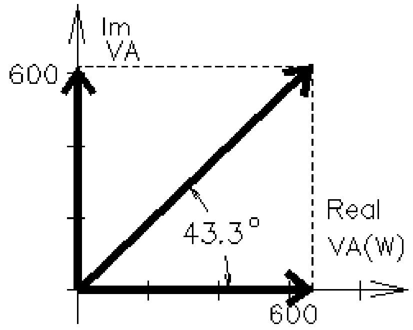
$|\mathbf{I}| = 8.005\bullet A$ $\quad \arg(\mathbf{I}) = -43.304\bullet deg$

$P := |\mathbf{V}_{in}|\cdot|\mathbf{I}|\cdot pf$ $\quad P = 0.641\bullet kW$

$Q := |\mathbf{V}_{in}|\cdot|\mathbf{I}|\cdot\sin(\theta)$ $\quad Q = 0.604\bullet kVAR$

$\mathbf{S} := \mathbf{V}_{in}\cdot\overline{\mathbf{I}}$ $\quad \mathbf{S} = 0.641 + 0.604j \;\bullet kVA$

$|\mathbf{S}| = 0.881\bullet kVA$ $\quad \arg(\mathbf{S}) = 43.304\bullet deg$ $\quad \mathbf{S}$ = 881VA$\underline{/43.3^{o}}$



OR, if we first find the magnitude of the current which flows through each element of the load...

$$|\mathbf{I}| = \frac{\mathbf{V}_{\mathbf{in}}}{\sqrt{R^2 + (\omega \cdot L)^2}} = 8.005 \bullet A$$

$P := (|\mathbf{I}|)^2 \cdot R$ $\quad P = 0.641 \bullet kW$ $\quad Q := (|\mathbf{I}|)^2 \cdot (\omega \cdot L)$ $\quad Q = 0.604 \bullet kVAR$

$\mathbf{S} := P + j \cdot Q$ $\quad |\mathbf{S}| = \sqrt{P^2 + Q^2} = 0.881 \bullet kVA$ $\quad pf = \frac{P}{|\mathbf{S}|} = 0.728$

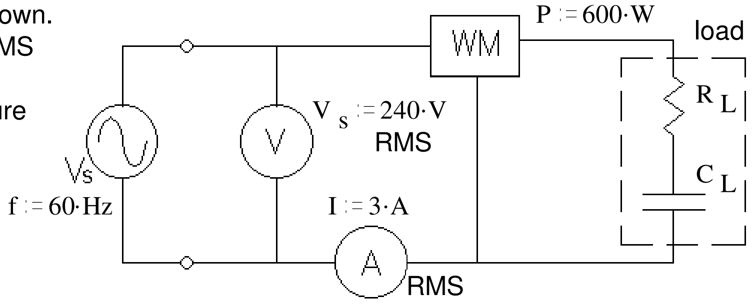
What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

$Q = 603.9 \bullet VAR$ so we need: $Q_C := -Q$ $\quad Q_C = -603.9 \bullet VAR = \frac{V_{in}^2}{X_C}$

$X_C := \frac{\mathbf{V}_{\mathbf{in}}^2}{Q_C}$ $\quad X_C = -20.035 \bullet \Omega = \frac{-1}{\omega \cdot C}$ $\quad C := \frac{1}{|X_C| \cdot \omega}$ $\quad C = 132 \bullet \mu F$

Check: $\frac{1}{\frac{1}{R + j \cdot \omega \cdot L} + j \cdot \omega \cdot C} = 18.883 \bullet \Omega$ No j term, so $\theta = 0^o$

Ex. 3 R, & C together are the load in the circuit shown. The RMS voltmeter measures 240 V, the RMS ammeter measures 3 A, and the wattmeter measures 600 W. Find the following: Be sure to show the correct units for each value.



a) The value of the load resistor. R_L = ?

$P = I^2 \cdot R_L$

$R_L := \frac{P}{I^2}$ $\quad R_L = 66.7 \bullet \Omega$

b) The apparent power. |**S**| = ? $\quad S := V_s \cdot I$ $\quad S = 720 \bullet VA$

c) The reactive power. Q = ? $\quad Q := -\sqrt{S^2 - P^2}$ $\quad Q = -398 \bullet VAR$

d) The complex power. **S** = ? $\quad \mathbf{S} := P + j \cdot Q$ $\quad \mathbf{S} = 600 - 398i \bullet VA$

e) The power factor. pf = ? $\quad pf := \frac{P}{V_s \cdot I}$ $\quad pf = 0.833$

f) The power factor is leading or lagging? leading (load is capacitive, Q is negative)

g) The two components of the load are in a box which cannot be opened. Add (draw it) another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and <u>find its value</u>. This component should not affect the real power consumption of the load.

$f = 60 \bullet Hz$ $\quad \omega := 377 \cdot \frac{rad}{sec}$

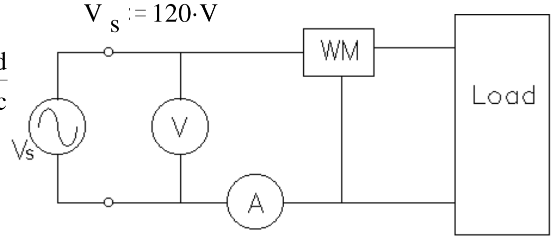
Add an inductor in parallel with load

$Q = -398 \bullet VAR$ so we need: $Q_L := -Q$ $\quad Q_L = 398 \bullet VAR = \frac{V_s^2}{X_L}$

$X_L := \frac{V_s^2}{Q_L}$ $\quad X_L = 144.725 \bullet \Omega = \omega \cdot L$ $\quad L := \frac{|X_L|}{\omega}$ $\quad L = 384 \bullet mH$

Ex. 4 For the 60 Hz load shown in the figure, the RMS voltmeter measures 120 V. The phasor diagram for the power is also shown. Find the following:

$V_s := 120 \cdot V$



$\omega := 377 \cdot \frac{rad}{sec}$

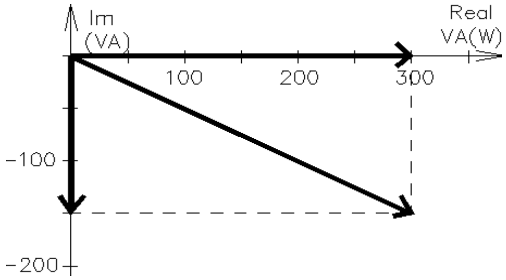
a) The complex power. $\mathbf{S}$ = ?

$P := 300 \cdot W$ $\quad$ $Q := -150 \cdot VA$

$\mathbf{S} := P + j \cdot Q$ $\quad$ $\mathbf{S} = (300 - 150j) \cdot VA$

b) The apparent power. |$\mathbf{S}$| = ? $\quad |\mathbf{S}| = \sqrt{P^2 + Q^2} = 335.4 \cdot VA$

c) The power factor. pf = ? $\quad pf := \frac{P}{|\mathbf{S}|}$ $\quad pf = 0.894$



d) The item marked "WM" in the figure is a wattmeter, what does it read? (give a number) $\quad P = 300 \cdot W$

e) The item marked "A" in the figure is an RMS ammeter, what does it read? (give a number)

$I := \frac{|\mathbf{S}|}{V_s}$ $\quad I = 2.795 \cdot A$ $\quad I = 2.8 \cdot A$

f) The power factor is leading or lagging? leading (Q is negative)

g) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and <u>find its value</u>. This component should not affect the real power consumption of the load.

Add an inductor in parallel with load

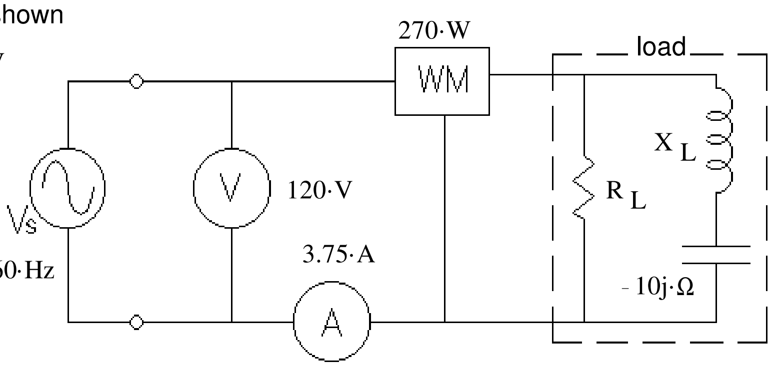
$Q = -150 \cdot VAR$ $\quad$ need: $Q_L := -Q$ $\quad Q_L = 150 \cdot VAR$ $\quad = \frac{V_s^2}{\omega \cdot L}$ $\quad L := \frac{V_s^2}{\omega \cdot Q_L}$ $\quad L = 255 \cdot mH$

Ex. 5 R, L, & C together are the load in the circuit shown

The RMS voltmeter measures 120 V. $V_s := 120 \cdot V$

The wattmeter measures 270 W. $P := 270 \cdot W$

The RMS ammeter measures 3.75 A. $I := 3.75 \cdot A$



$f := 60 \cdot Hz$

Find the following: Be sure to show the correct units for each value.

a) The value of the load resistor. R_L = ?

$P = \frac{V_s^2}{R_L}$ $\quad R_L := \frac{V_s^2}{P}$ $\quad R_L = 53.3 \cdot \Omega$

b) The magnitude of the impedance of the load inductor (reactance) . |$\mathbf{Z}_L$| = X_L = ?

$I_R := \frac{V_s}{R_L}$ $\quad I_R = 2.25 \cdot A$ $\quad I_L := \sqrt{I^2 - I_R^2}$ $\quad I_L = 3 \cdot A$ $\quad X := \frac{V_s}{I_L}$ $\quad X = 40 \cdot \Omega$

$X_C := -10 \cdot \Omega$ $\quad X_L := X - X_C$ $\quad X_L = 50 \cdot \Omega$

c) The reactive power. Q = ? $\quad Q := \sqrt{(V_s \cdot I)^2 - P^2}$ $\quad Q = 360 \cdot VAR$ $\quad$ positive, because the load is primarily inductive

d) The power factor is leading or lagging? lagging (load is inductive, Q is positive)

e) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and <u>find its value</u>. This component should not affect the real power consumption of the load.

Add a capacitor in parallel with load

$f = 60 \cdot \text{Hz} \qquad \omega := 377 \cdot \frac{\text{rad}}{\text{sec}}$

$Q = 360 \cdot \text{VAR}$ so we need: $Q_C := -Q \qquad Q_C = -360 \cdot \text{VAR} = -\frac{V_s^{\,2}}{\frac{1}{\omega \cdot C}} = -\omega \cdot C \cdot V_s^{\,2}$

$C := \frac{Q_C}{-\omega \cdot V_s^{\,2}} \qquad C = 66.3 \cdot \mu\text{F}$

Ex. 6 A step-down transformer has an output voltage of 220 V (rms) when the primary is connected across a 560 V (rms) source.

a) If there are 280 turns on the primary winding, how many turns are required on the secondary?

$280 \cdot \frac{220 \cdot \text{volt}}{560 \cdot \text{volt}} = 110$ turns

b) If the current in the primary is 2.4 A, what current flows in the load connected to the secondary?

$2.4 \cdot \text{amp} \cdot \frac{280}{110} = 6.11 \cdot \text{A}$

c) If the transformer is rated at 700/275 V, 2.1 kVA, what are the rated primary and secondary currents?

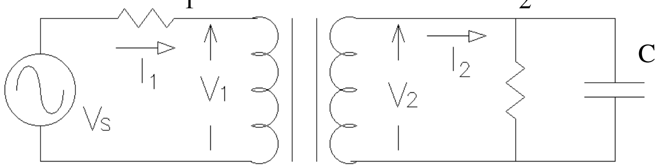
pri: $\frac{2.1 \cdot \text{kVA}}{700 \cdot \text{V}} = 3 \cdot \text{A}$ sec: $\frac{2.1 \cdot \text{kVA}}{275 \cdot \text{V}} = 7.636 \cdot \text{A}$

Ex. 7 The transformer shown in the circuit below is ideal. Find the following:

a) $|\mathbf{I_1}|$ = ?

$R_1 := 20 \cdot \Omega \qquad R_2 := 15 \cdot \Omega$

$V_s := 120 \cdot \text{V} \qquad \omega := 377 \cdot \frac{\text{rad}}{\text{sec}} \qquad C := 40 \cdot \mu\text{F}$

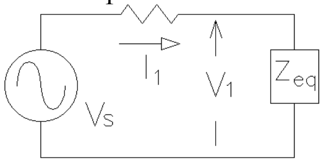


$N_1 := 150 \cdot \text{turns} \qquad N_2 := 50 \cdot \text{turns}$

$\mathbf{Z_L} := \frac{1}{\frac{1}{R_2} + j \cdot \omega \cdot C}$

Make an equivalent circuit:

$R_1 := 20 \cdot \Omega$



$\mathbf{Z_L} = 14.27 - 3.228j \cdot \Omega$

$\mathbf{Z_{eq}} := \left(\frac{N_1}{N_2}\right)^2 \cdot \mathbf{Z_L} \qquad \mathbf{Z_{eq}} = 128.429 - 29.051j \cdot \Omega$

$R_1 + \mathbf{Z_{eq}} = 148.429 - 29.051j \cdot \Omega \qquad \sqrt{148.429^2 + 29.051^2} = 151.245$

$|\mathbf{I_1}| = \frac{V_s}{|R_1 + \mathbf{Z_{eq}}|} = \frac{V_s}{151.245 \cdot \Omega} = 0.793 \cdot \text{A}$

b) $|\mathbf{I_2}|$ = ? $= \left(\frac{N_1}{N_2}\right) \cdot |\mathbf{I_1}| = \frac{150}{50} \cdot .793 \cdot \text{A} = 2.379 \cdot \text{A}$

c) $|\mathbf{V_1}|$ = ? $= V_s \cdot \left|\frac{\mathbf{Z_{eq}}}{R_1 + \mathbf{Z_{eq}}}\right|$ OR.. $|\mathbf{V_1}| = |\mathbf{I_1}| \cdot |\mathbf{Z_{eq}}| = .793 \cdot \text{A} \cdot \sqrt{128.429^2 + 29.051^2} \cdot \Omega = 104.417 \cdot \text{V}$

ECE 2210 homework PA1 Due: Sat, 4/24

c

Note: In the following problems, you may assume voltages and currents are RMS unless stated otherwise or given as a function of time.

1. Read the AC power notes and examples.

2. Compute the power factor for an inductive load consisting of $L := 20\cdot mH$ and $R := 6\cdot\Omega$ in series. $\omega := 377\cdot\frac{rad}{s}$

3. The complex power consumed by a load is $620 \underline{/29^o}$ VA. Find:

   a) Apparent power (as always, give the correct units). b) Real power. c) Reactive power.

   d) Power factor. e) Is the power factor leading or lagging? f) Draw a phasor diagram.

4. In the circuit shown, the voltmeter measures 120V, the ammeter measures 6.3A and the wattmeter measures 560W. The load consists of a resistor and an inductor. The frequency is 60Hz. Find the following:

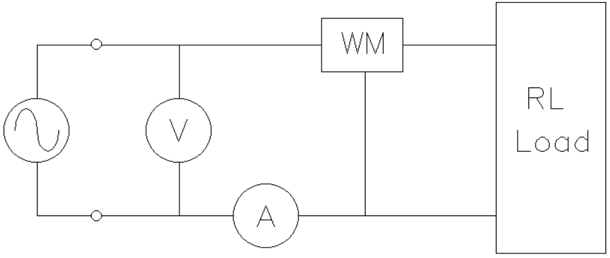
   a) Power factor b) Leading or lagging?

   c) Real power.

   d) Apparent power.

   e) Reactive power.

   f) Draw a phasor diagram.



   g) The load is in a box which cannot be opened. Add another component to the circuit above to correct the power factor (make pf = 1). Draw the correct component in the correct place and find its value. This component should not affect the real power consumption of the load.

5. For the circuit shown, find the following: (as always, give the correct units)

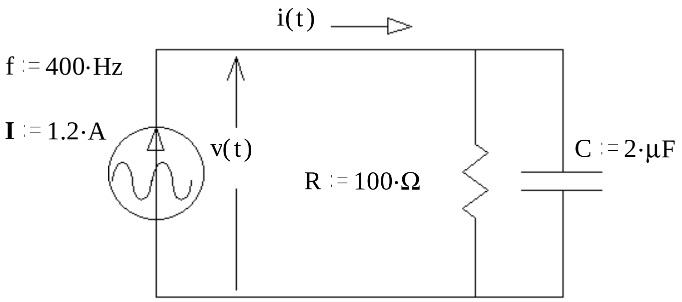
   a) The complex power.

   b) Real power.

   c) Reactive power.

   d) Apparent power.

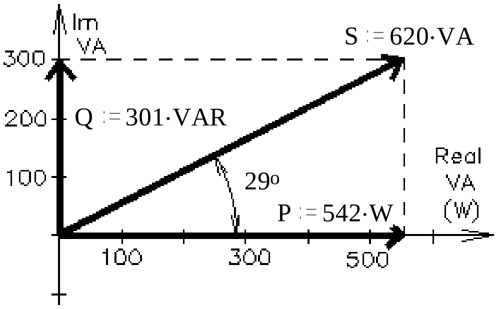
   e) Draw a power phasor diagram.



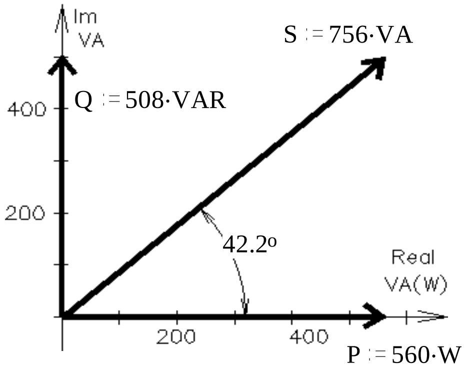
Answers

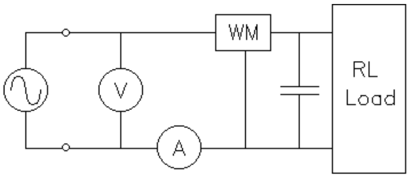
2. $pf := 0.623$

3. a) $620\cdot VA$
   b) $542\cdot W$
   c) $301\cdot VAR$
   d) 0.875
   e) lagging
   f) ------->

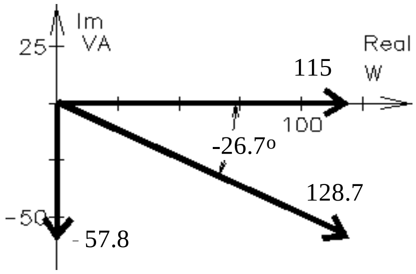


4. a) 0.741
   b) lagging
   c) $560\cdot W$
   d) $756\cdot VA$
   e) $508\cdot VAR$
   f) ------->
   g) $93.6\cdot\mu F$ capacitor in parallel with load





5. a) $(115 - 57.8\cdot j)\cdot VA$
   b) $115\cdot W$
   c) $-57.8\cdot VAR$
   d) $128.7\cdot VA$
   e) ------->



ECE 2210 homework PA2 Due: Tue, 4/27

Note: In the following problems, you may assume voltages and currents are RMS unless stated otherwise or given as a function of time. Transformers are ideal unless stated otherwise.

1. A load draws 12kVA at 0.8 pf, lagging when hooked to 480V. A capacitance is hooked in parallel with the load and the power factor is corrected to 0.9, lagging.

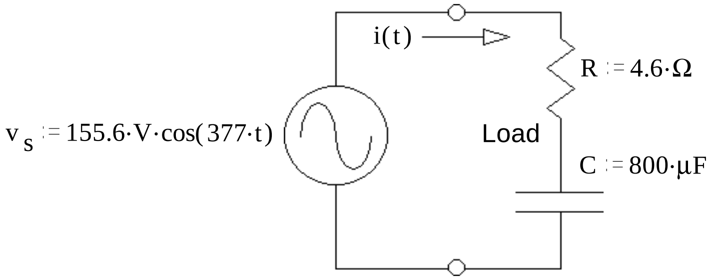
   a) Find the reactive power (VAR) of the capacitor. Draw a phasor diagram as part of the solution.

   b) Find the value of the capacitor assuming f = 60Hz.

2. Consider the circuit at right.
   The resistor and capacitor together make up the load.

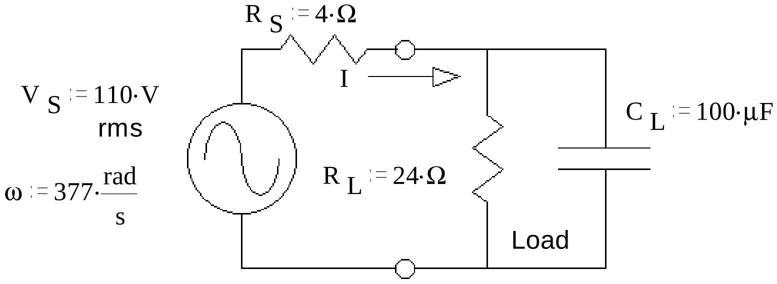
   a) Find the load impedance of the circuit.

   b) Compute the average power dissipated by the load.



3. a) Compute the average power dissipated by the load (R_L and C_L taken together).

   b) Compute the power dissipated by the internal source resistance (R_S) in this circuit.



4. Read sections 2.28, 3.8, & 7.1 in your textbook.

5. An ideal transformer has 330 turns on the primary winding and 36 turns on the secondary. If the primary is connected across a 110 V (rms) generator, what is the rms output voltage?

6. A transformer has $N_1 = 320$ turns and $N_2 = 1000$ turns. If the input voltage is v(t) = (255 V)cos(ωt), what rms voltage is developed across the secondary coil?

7. A step-up transformer is designed to have an output voltage of 2200 V (rms) when the primary is connected across a 240 V (rms) source.

   a) If there are 150 turns on the primary winding, how many turns are required on the secondary?

   b) If a load resistor across the secondary draws a current of 1.2 A, what is the current in the primary, assuming ideal conditions?

8. The primary current of an ideal transformer is 8.5 A and the primary voltage is 80 V. 1.0 A is delivered to a load resistor connected to the secondary. Calculate the voltage across the secondary.

9. An ideal transformer has a turns ratio ($N = N_1/N_2$) of 1.5 . It is desired to operate a 200 Ω resistive load at 150 V (rms).

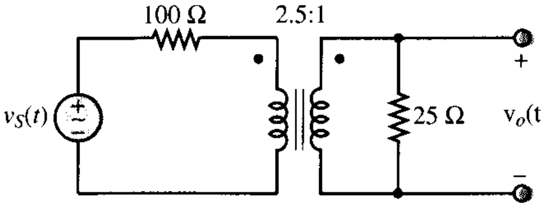
   a) Find the secondary and primary currents.

   b) Find the source voltage ($\mathbf{V_1}$).

   c) Find the power dissipated in the load resistor and the power delivered to the primary from the source.

   d) Find the impedance the source sees looking into the primary winding by calculating $\mathbf{Z_{eq}} = N^2\, \mathbf{Z_L}$ and again by calculating $\mathbf{V_1} / \mathbf{I_1}$.

10. For the ideal transformer shown in the figure, find $v_o(t)$ if $v_s(t)$ is 320Vcos(377t).



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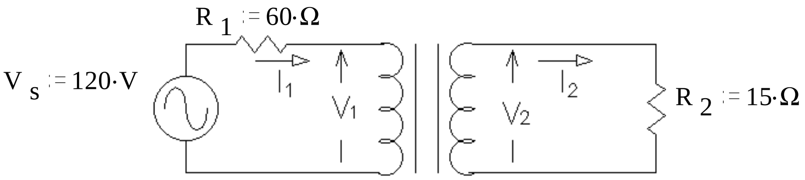
11. The transformer shown in the circuit below is ideal. It is rated at 120/30 V, 80 VA, 60 Hz
Find the following:

a) $I_1 = ?$

b) $V_2 = ?$



12. A transformer is rated at 13,800/480 V, 60 kVA, 60 Hz. (Note: kVA stands for kilo-Volt-Amp, in this case it is the transformer's voltage rating times its current rating.) Find the allowable primary and secondary currents at a supply voltage of 12,000 V at 100% power factor. Repeat for a power factor of 50%.

13. An ideal transformer has a rating of 500/125 V, 10 kVA, 60 Hz. It is loaded with an impedance of 5Ω at 80% pf (0.80). The source voltage applied to the primary winding is 440 V (rms). Find:
a) the load voltage
b) the load current
c) the kVA delivered to load
d) the power delivered to load
e) the primary current
f) the power factor of primary
g) the impedance the source sees looking into primary.

14. An ideal transformer is rated to deliver 400 kVA at 460 V to a customer.

a) How much current can the transformer supply to the customer?

b) If the customer's load is purely resistive (i.e. if the pf = 1), what is the maximum power the customer can receive?

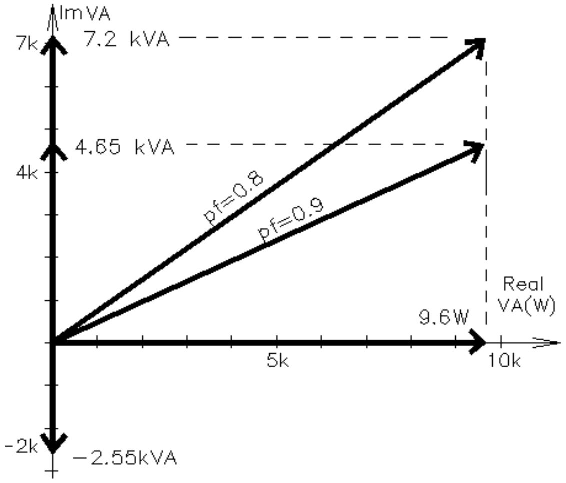
c) If the customer's power factor is 0.8 (lagging), what is the maximum usable power the customer can receive?

d) What is the maximum power if the power factor is 0.7 (lagging)?

e) If the customer requires 300 kW to operate, what is the minimum allowable power factor given the rating of this transformer?

Answers

1.



a) $-2.55\cdot kVA$ b) $29.4\cdot\mu F$

2. a) $\mathbf{Z} := 5.67\cdot\Omega\cdot e^{-j\cdot 35.8\cdot deg}$ b) $P_{av} := 1.73\cdot kW$

3. a) $P_{av} := 364\cdot W$ b) $110\cdot W$

5. 12 V

6. 563 V

7. a) 1375 turns b) 11 A

8. 680 V

9. a) 0.75 A, 0.50 A b) 225 V c) 112.5 W d) 450Ω

10. 78Vcos(377t)

11. a) $0.4\cdot A$ b) 24V

12. 4.35 A, 125 A any pf, (Using the transformer at a lower voltage does not increase its current rating.)

13. a) 110 V b) 22 A c) 2.42 kVA d) 1.94 kW e) 5.5 A f) 0.80 g) $80\Omega\ \underline{/36.9^{o}}\ \Omega$

14. a) $870\cdot A$ b) $400\cdot kW$ c) $320\cdot kW$ d) $280\cdot kW$ e) 0.75