Introduction to AC Power, RMS

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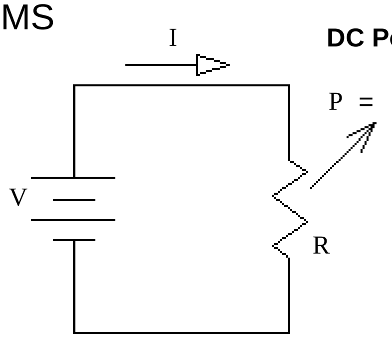
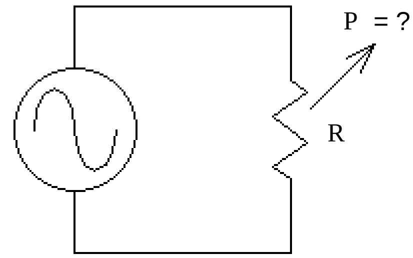
2/20/10
11/23/13
4/19/20

DC Power

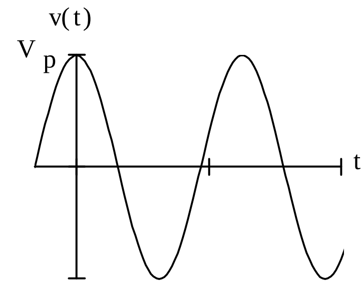


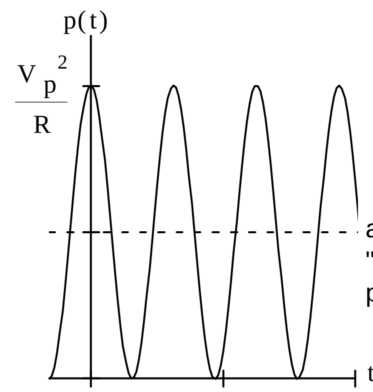
$$P = V \cdot I = \frac{V^2}{R} = I^2 \cdot R$$

AC Power

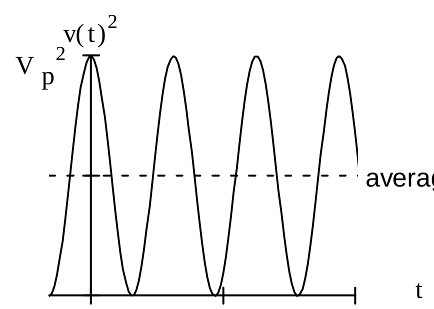


$$v(t) = V_p \cdot \cos(\omega \cdot t)$$





average or "effective" power = $\dfrac{\left(\dfrac{V_p^2}{R}\right)}{2}$



average = $\dfrac{V_p^2}{2}$

Couldn't we define an "effective" voltage that would allow us to use the same relationships for AC power as used for DC power?

$$P_{ave} = \frac{\left(\frac{V_p^2}{R}\right)}{2} = \frac{\left(\frac{V_p^2}{2}\right)}{R} = \frac{\left(\frac{V_p}{\sqrt{2}}\right)^2}{R}$$

$$V_{eff} = \sqrt{\left(\frac{V_p}{\sqrt{2}}\right)^2} = \frac{V_p}{\sqrt{2}} = V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T (v(t))^2\, dt}$$

Square

Mean (average)

Root

RMS **R**oot of the **M**ean of the **S**quare

Use RMS in power calculations

Sinusoids

$$V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T (v(t))^2\, dt} = \sqrt{\frac{1}{T} \cdot \int_0^T \left(V_p \cdot \cos(\omega \cdot t)\right)^2 dt} = \sqrt{\frac{1}{T} \cdot \int_0^T V_p^2 \cdot \left(\frac{1}{2} + \frac{1}{2} \cdot \cos(2 \cdot \omega \cdot t)\right) dt}$$

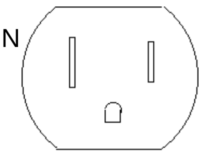
$$= \frac{V_p}{\sqrt{2}} \cdot \sqrt{\frac{1}{T} \cdot \int_0^T (1)\, dt + \frac{1}{T} \cdot \int_0^T \cos(2 \cdot \omega \cdot t)\, dt} = \frac{V_p}{\sqrt{2}} \cdot \sqrt{1 + 0}$$

Common household power

$$f = 60\cdot Hz$$

$$\omega = 377\cdot\frac{rad}{sec}$$

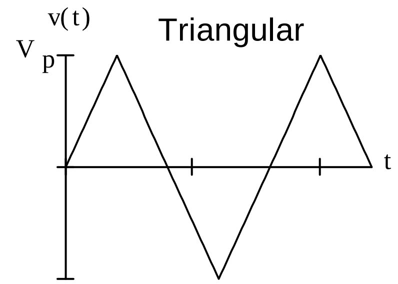
$$T = 16.67\cdot ms$$

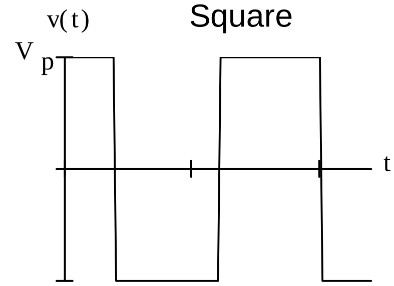


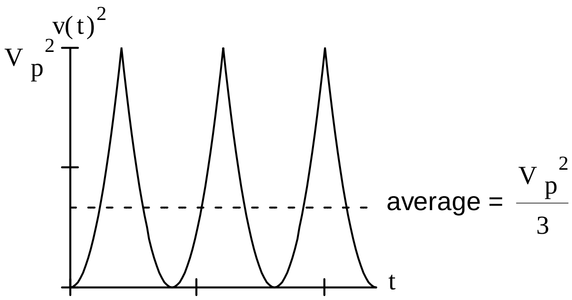
$$V_{rms} := 120\cdot V$$

$$V_p = V_{rms}\cdot\sqrt{2} = 170\cdot V$$

What about other wave shapes??

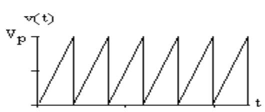


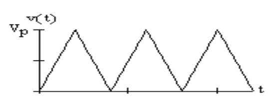


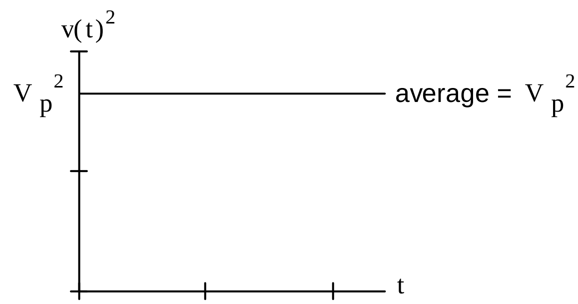


$$V_{rms} = \frac{V_p}{\sqrt{3}}$$

Works for all types of triangular and sawtooth waveforms



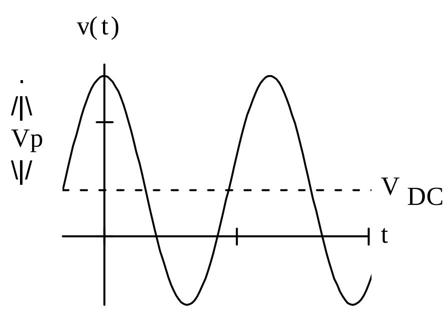




$$V_{rms} = V_p$$

Same for DC

How about AC + DC ?



$$V_{rms} = \sqrt{\frac{1}{T}\cdot\int_0^T (v(t))^2\,dt}$$

$$= \sqrt{\frac{1}{T}\cdot\int_0^T \left(V_p\cdot\cos(\omega\cdot t) + V_{DC}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T}\cdot\int_0^T \left[\left(V_p\cdot\cos(\omega\cdot t)\right)^2 + 2\cdot\left(V_p\cdot\cos(\omega\cdot t)\right)\cdot V_{DC} + V_{DC}^2\right] dt}$$

$$= \sqrt{\frac{1}{T}\cdot\int_0^T \left(V_p\cdot\cos(\omega\cdot t)\right)^2 dt + \frac{1}{T}\cdot\int_0^T 2\cdot\left(V_p\cdot\cos(\omega\cdot t)\right)\cdot V_{DC}\,dt + \frac{1}{T}\cdot\int_0^T V_{DC}^2\,dt}$$

- - - zero over one period - - -

$$= \sqrt{V_{rmsAC}^2 + 0 + V_{DC}^2} \qquad = \sqrt{V_{rmsAC}^2 + V_{DC}^2}$$

rectified average $V_{ra} = \frac{1}{T} \cdot \int_0^T |v(t)|\, dt$

sinusoid:	$V_{rms} = \frac{V_p}{\sqrt{2}}$	$I_{rms} = \frac{I_p}{\sqrt{2}}$	$V_{ra} = \frac{2}{\pi} \cdot V_p$	$I_{ra} = \frac{2}{\pi} \cdot I_p$
triangular:	$V_{rms} = \frac{V_p}{\sqrt{3}}$	$I_{rms} = \frac{I_p}{\sqrt{3}}$	$V_{ra} = \frac{1}{2} \cdot V_p$	$I_{ra} = \frac{1}{2} \cdot I_p$
square:	$V_{rms} = V_p$	$I_{rms} = I_p$	$V_{ra} = V_{rms} = V_p$	$I_{ra} = I_{rms} = I_p$

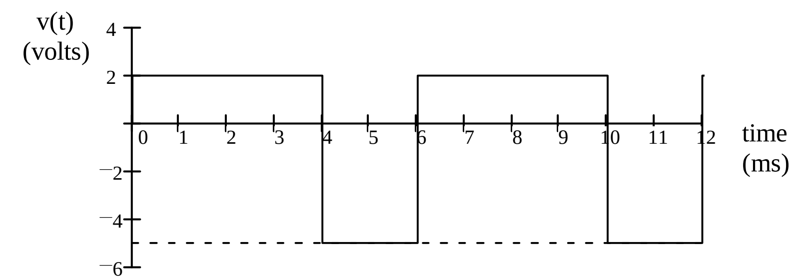
waveform + DC $V_{rms} = \sqrt{V_{rmsAC}^2 + V_{DC}^2}$

Most AC meters don't measure true RMS. Instead, they measure V_{ra} , display $1.11 V_{ra}$, and call it RMS. That works for sine waves but not for any other waveform.

Use RMS in power calculations

Some waveforms don't fall into these forms, then you have to perform the math from scratch

For waveform shown

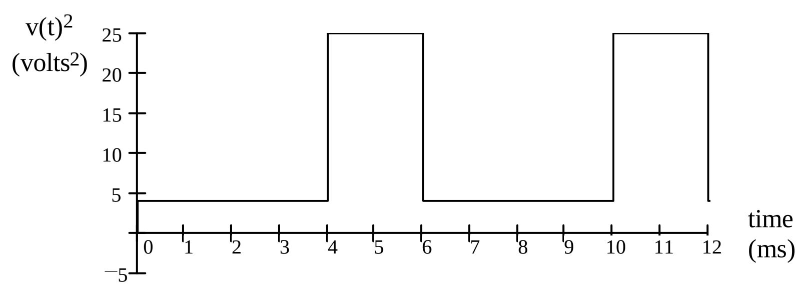


The average DC (V_{DC}) value

$$\frac{2 \cdot V \cdot (4 \cdot ms) + (-5 \cdot V) \cdot (2 \cdot ms)}{6 \cdot ms} = -0.333 \cdot V$$

The RMS (effective) value

Graphical way



$$\frac{4 \cdot V^2 \cdot (4 \cdot ms) + 25 \cdot V^2 \cdot (2 \cdot ms)}{6 \cdot ms} = 11 \cdot V^2$$

$V_{RMS} := \sqrt{11 \cdot V^2}$ $\qquad V_{RMS} = 3.32 \cdot V$

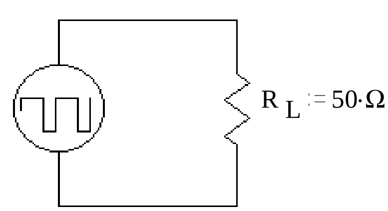
OR...

$$V_{RMS} = \sqrt{\frac{1}{T} \cdot \int_0^T (v(t))^2\, dt}$$

$$= \sqrt{\frac{1}{6 \cdot ms} \cdot \left[\int_{0 \cdot ms}^{4 \cdot ms} (2 \cdot V)^2\, dt + \int_{4 \cdot ms}^{6 \cdot ms} (-5 \cdot V)^2\, dt \right]} \quad = \sqrt{\frac{1}{6 \cdot ms} \cdot \left[4 \cdot ms \cdot (2 \cdot V)^2 + 2 \cdot ms \cdot (-5 \cdot V)^2 \right]} = 3.32 \cdot V$$

The voltage is hooked to a resistor, as shown, for 6 seconds.

The energy is transferred to the resistor during that 6 seconds:



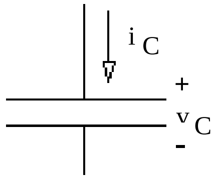
$P_L := \frac{V_{RMS}^2}{R_L}$ $\qquad P_L = 0.22 \cdot W$

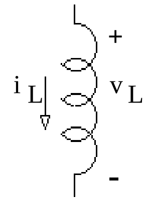
$W_L := P_L \cdot 6 \cdot sec$ $\qquad W_L = 1.32 \cdot joule$ All converted to heat

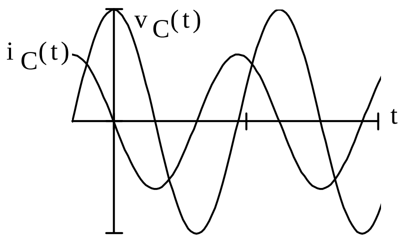
Use RMS in power calculations

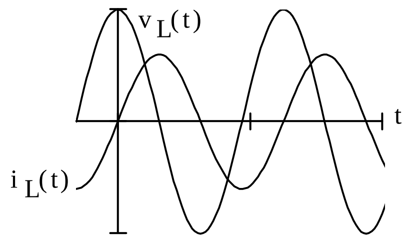
$$P = I_{Rrms}^{2} \cdot R = \frac{V_{Rrms}^{2}}{R}$$ for Resistors ONLY ! !

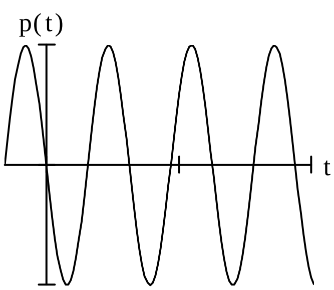
Capacitors and Inductors

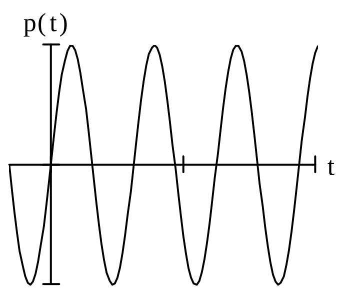












Average power is ZERO $P = 0$

Average power is ZERO $P = 0$

Capacitors and Inductors DO NOT dissipate (real) average power.

Reactive power is negative

$$Q_C = -I_{Crms} \cdot V_{Crms}$$

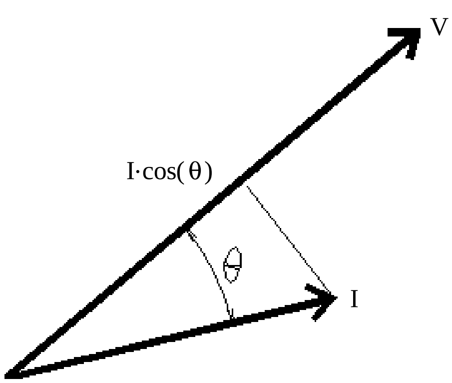
$$= -I_{Crms}^{2} \cdot \frac{1}{\omega \cdot C} = -V_{Crms}^{2} \cdot \omega \cdot C$$

Reactive power is positive

$$Q_L = I_{Lrms} \cdot V_{Lrms}$$

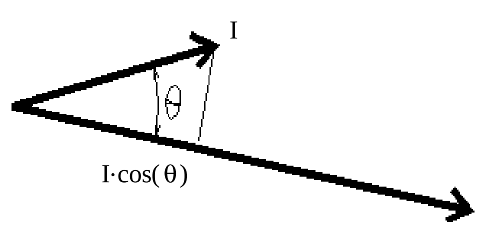
$$= I_{Lrms}^{2} \cdot \omega \cdot L = \frac{V_{Lrms}^{2}}{\omega \cdot L}$$

If current and voltage are not in phase, only the in-phase part of the current matters for the power-- DOT PRODUCT



"Lagging" power

Inductor dominates



"Leading" Power

Capacitor dominates

All voltages and currents shown are RMS

Real Power

$$P = V \cdot I \cdot \cos(\theta) = I^2 \cdot |\mathbf{Z}| \cdot \cos(\theta) = \frac{V^2}{|\mathbf{Z}|} \cdot \cos(\theta)$$

BOLD is a complex number

P = "Real" Power (average) = $V \cdot I \cdot pf = I^2 \cdot |\mathbf{Z}| \cdot pf = \dfrac{V^2}{|\mathbf{Z}|} \cdot pf$ units: watts, kW, MW, etc.

$pf = \cos(\theta)$ = power factor

otherwise....

I_R V_R for resistors only part that uses real average power $P = I_R{}^2 \cdot R = \dfrac{V_R{}^2}{R}$

Reactive Power

Q = Reactive "power" = $V \cdot I \cdot \sin(\theta)$ units: VAR, kVAR, etc. "volt-amp-reactive"

otherwise....

I_C V_C capacitors -> - Q $Q_C = I_C{}^2 \cdot X_C = \dfrac{V_C{}^2}{X_C}$ $X_C = -\dfrac{1}{\omega \cdot C}$ and is a negative number

I_L V_L inductors -> + Q $Q_L = I_L{}^2 \cdot X_L = \dfrac{V_L{}^2}{X_L}$ $X_L = \omega \cdot L$ and is a positive number

Complex and Apparent Power

$\mathbf{S}$ = Complex "power" = $P + jQ$ = $VI\,\underline{/\theta}$ = $\mathbf{V} \cdot \overline{\mathbf{I}}$ = $I^2 \cdot \mathbf{Z}$ units: VA, kVA, etc. "volt-amp"

complex conjugate

NOT $\mathbf{V} \cdot \mathbf{I}$ **NOR** $\dfrac{V^2}{\mathbf{Z}}$

S = Apparent "power" = $|\mathbf{S}| = \sqrt{P^2 + Q^2} = V \cdot I$ units: VA, kVA, etc. "volt-amp"

Power factor

$pf = \cos(\theta)$ = power factor (sometimes expressed in %) $0 \leq pf \leq 1$

θ is the **phase angle** between the voltage and the current or the phase angle of the impedance. $\theta = \theta_Z$

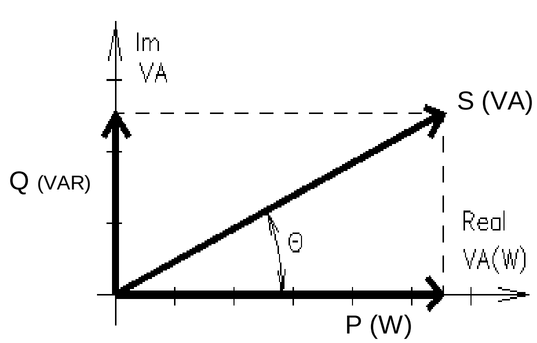
$\theta < 0$ Load is "Capacitive", power factor is "leading". This condition is very rare

$\theta > 0$ Load is "Inductive", power factor is "lagging". This condition is so common you can assume any power factor given is lagging unless specified otherwise. Transformers and motors make most loads inductive.

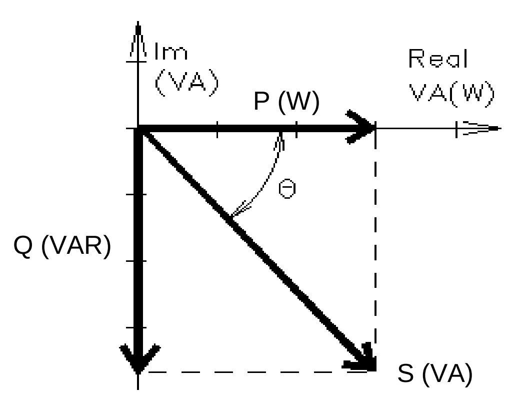
Industrial users are charged for the reactive power that they use, so power factor < 1 is a bad thing.

Power factor < 1 is also bad for the power company. To deliver the same power to the load, they have more line current (and thus more line losses).

Power factors are "corrected" by adding capacitors (or capacitve loads) in parallel with the inductive loads which cause the problems. (In the rare case that the load is capacitive, the pf would be corrected by an inductor.)



"Lagging" power



"Leading" Power

Transformer basics and ratings

A Transformer is two coils of wire that are magnetically coupled.

Transformers are only useful for AC, which is one of the big reasons electrical power is generated and distributed as AC.

Transformer turns and turns ratios are rarely given, V_p/V_s is much more common where V_p/V_s is the rated primary over rated secondary voltages. You may take this to be the same as N_1/N_2 although in reality N_2 is usually a little bit bigger to make up for losses. Also common: $V_p : V_s$.

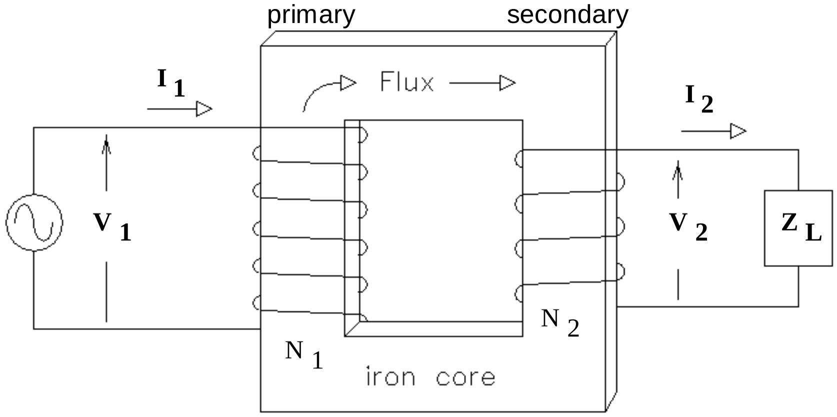
Transformers are rated in VA   Transformer Rating (VA) = (rated V) x (rated I) , on either side. (Both RMS)

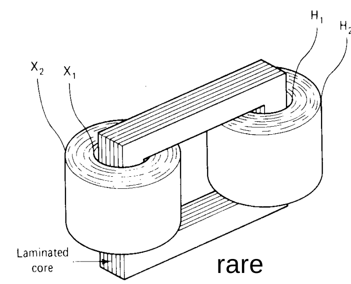
Don't allow voltages over the rated V , regardless of the actual current.
Don't allow currents over the rated I , regardless of the actual voltage.

Ideal Transformers

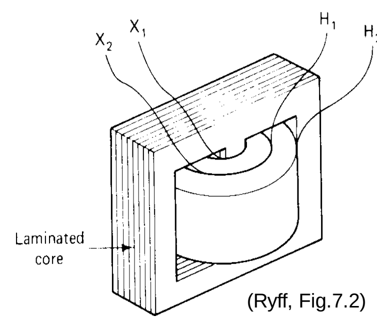
Iron-core transformer

primary   secondary





rare



(Ryff, Fig.7.2)

common

Ideal: $P_1 = P_2$

power in = power out

Transformation of voltage and current

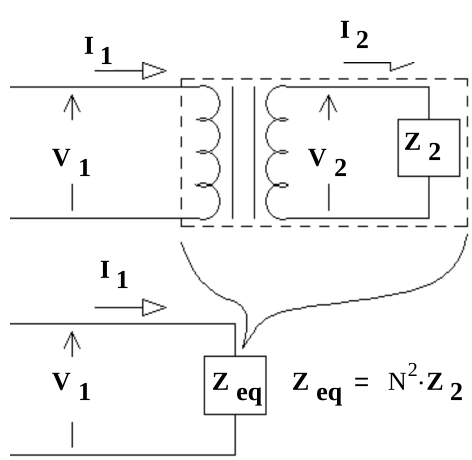
$$\frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

Turns ratio

Turns ratio as defined in most books: $N = \dfrac{N_1}{N_2}$

Note: some other texts define the turns ratio as: $\dfrac{N_2}{N_1}$ Be careful how you and others use this term.

Transformation of impedance



$$Z_{eq} = N^2 \cdot Z_2 = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_2$$

You can replace the entire transformer and load with (Z_{eq}). This "impedance transformation" can be very handy.

Transformers can be used for "impedance matching"

This also works the opposite way, to move an impedance from the primary to the secondary, multiply by: $\left(\dfrac{N_2}{N_1}\right)^2$

Other Transformers

Multi-tap transformers: Many transformers have more than two connections to primary and/or the secondary. The extra connections are called "taps" and may allow you to select from several different voltages or get more than one voltage at the same time.

Isolation Transformers: Almost all transformers isolate the primary from the secondary. An Isolation transformer has a 1:1 turns ratio and is just for isolation.

Auto Transformers: Auto transformers have only one winding with taps for various voltages. The primary and secondary are simply parts of the same winding. These parts may overlap. Any regular transformer can be wired as an auto transformer. Auto transformers DO NOT provide isolation.

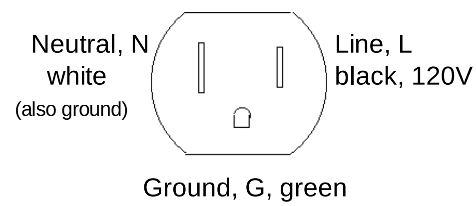
Vari-AC: A special form of auto transformer with an adjustable tap for an adjustable output voltage.

LVDT A Linear-Variable-Differential-Transformers has moveable core which couples the primary winding to the secondary winding(s) in such a way the the secondary voltage is proportional to the position of the core. LVDTs are used as position sensors.

Home power

Standard 120 V outlet connections are shown at right.

The 3 lines coming into your house are **NOT** 3-phase.
They are +120 V, Gnd, -120 V
(The two 120s are 180^o out-of-phase, allowing for 240 V connections)

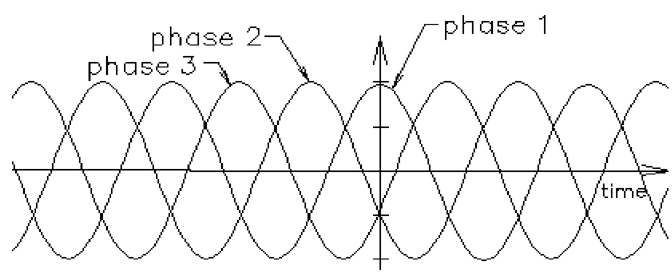


3-Phase Power (FYI ONLY)

Single phase power pulses at 120 Hz. This is not good for motors or generators over 5 hp.

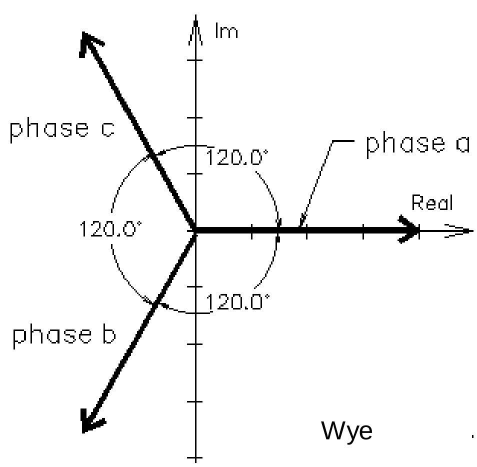
Three phase power is constant as long as the three loads are balanced.

Three lines are needed to transmit 3-phase power.
If loads are balanced, ground return current will be zero.

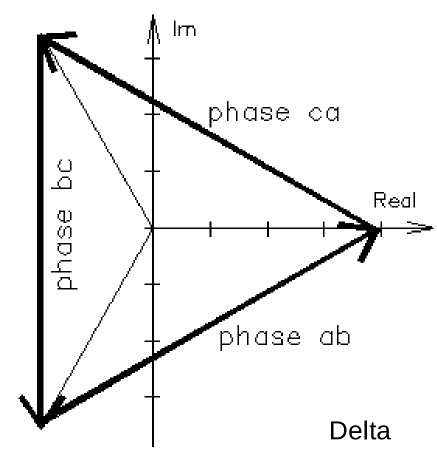


<u>Wye connection</u>:
Connect each load or generator phase between a line and ground.

<u>Delta connection</u>:
Connect each load or generator phase between two lines.



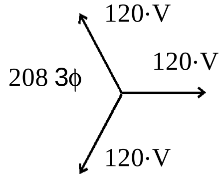
$$V_{LN} = \frac{V_{LL}}{\sqrt{3}} \qquad I_L = \sqrt{3}\cdot I_{LL}$$

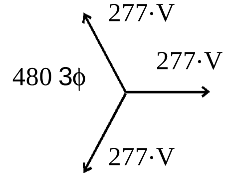


$$V_{LL} = \sqrt{3}\cdot V_{LN} \qquad I_{LL} = \frac{I_L}{\sqrt{3}}$$

3-Phase Power (FYI ONLY)

Common 3-phase voltages:

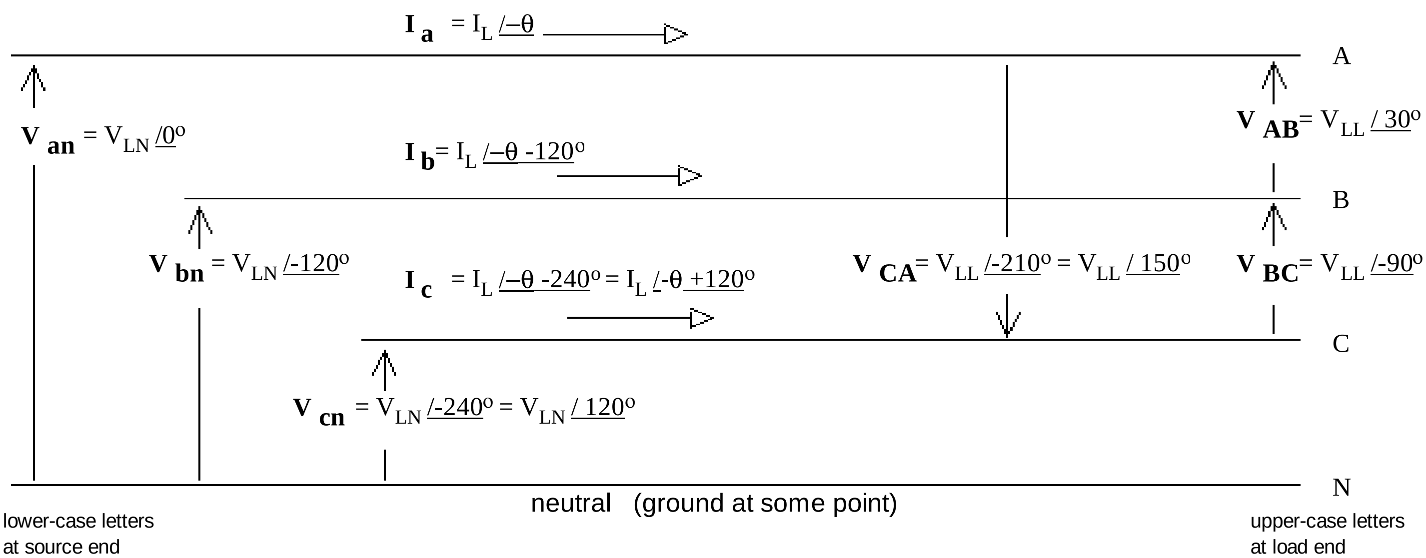


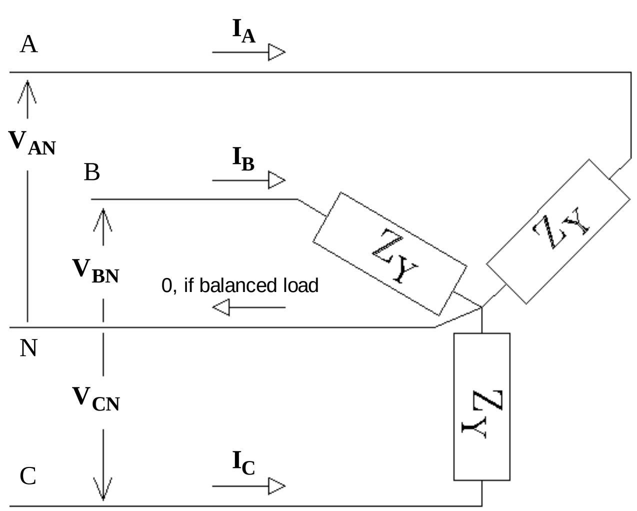


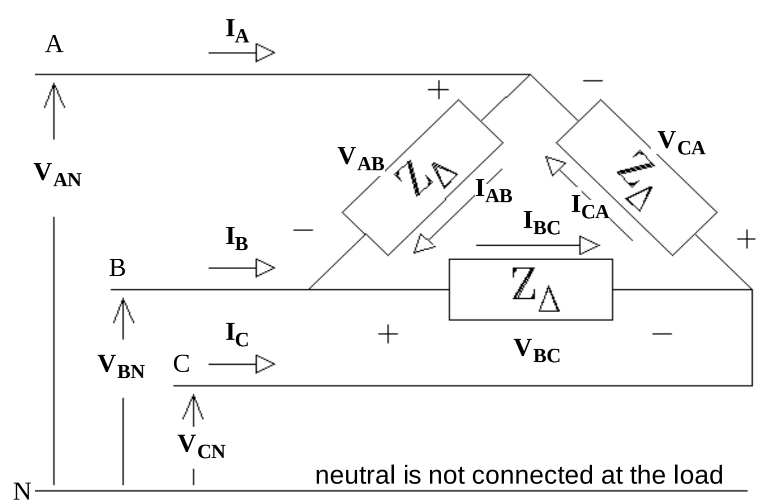
Apparent Power: $\left|S_{3\phi}\right| = 3 \cdot V_{LN} \cdot I_L = 3 \cdot V_{LL} \cdot I_{LL} = \sqrt{3} \cdot V_{LL} \cdot I_L$

Power: $P_{3\phi} = 3 \cdot V_{LN} \cdot I_L \cdot pf = 3 \cdot V_{LL} \cdot I_{LL} \cdot pf = \sqrt{3} \cdot V_{LL} \cdot I_L \cdot pf = S_{3\phi} \cdot pf$ $\quad pf = \cos(\theta)$

Reactive power: $Q_{3\phi} = 3 \cdot V_{LN} \cdot I_L \cdot \sin(\theta)$ etc... $= \sqrt{\left(\left|S_{3\phi}\right|\right)^2 - P_{3\phi}{}^2}$







$$\left|V_{AN}\right| = \left|V_{BN}\right| = \left|V_{CN}\right| = V_{LN} = \frac{V_{LL}}{\sqrt{3}} \qquad \left|V_{AB}\right| = \left|V_{BC}\right| = \left|V_{CA}\right| = V_{LL} = \sqrt{3} \cdot V_{LN}$$

$$\left|I_A\right| = \left|I_B\right| = \left|I_C\right| = I_L = \sqrt{3} \cdot I_{LL} \qquad \left|I_{AB}\right| = \left|I_{BC}\right| = \left|I_{CA}\right| = I_{LL} = \frac{I_L}{\sqrt{3}}$$

To get equivalent line currents with equivalent voltages

$$Z_Y = \frac{Z_\Delta}{3} \qquad Z_\Delta = 3 \cdot Z_y$$

AC Power Examples

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11/06/02
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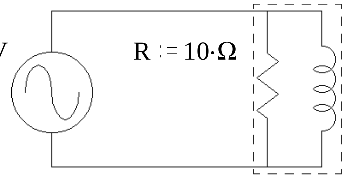
rev 12/4/23

Ex. 1 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

$\mathbf{V_{in}} := 110 \cdot V$ $\quad R := 10 \cdot \Omega$ $\quad L := 25 \cdot mH$

$\omega := 377 \cdot \frac{rad}{sec}$



$$\mathbf{Z} := \frac{1}{\left(\frac{1}{R} + \frac{1}{j \cdot \omega \cdot L}\right)} = \frac{1}{0.1458 \cdot \frac{1}{\Omega} \cdot e^{-j \cdot 46.7 \cdot deg}}$$

$\mathbf{Z} = 4.704 + 4.991j \cdot \Omega$ $\quad |\mathbf{Z}| = 6.859 \cdot \Omega$ $\quad \theta := \arg(\mathbf{Z})$ $\quad \theta = 46.7 \cdot deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.686$

$\mathbf{I} := \frac{\mathbf{V_{in}}}{\mathbf{Z}}$ $\quad \mathbf{I} = 11 - 11.671j \cdot A$ $\quad |\mathbf{I}| = 16.038 \cdot A$ $\quad \arg(\mathbf{I}) = -46.7 \cdot deg$

$P := |\mathbf{V_{in}}| \cdot |\mathbf{I}| \cdot pf$ $\quad P = 1.21 \cdot kW$

$Q := |\mathbf{V_{in}}| \cdot |\mathbf{I}| \cdot \sin(\theta)$ $\quad Q = 1.284 \cdot kVAR$ $\quad$ OR... $\quad Q := |\mathbf{V_{in}}| \cdot |\mathbf{I}| \cdot \sqrt{1^2 - pf^2}$ $\quad Q = 1.284 \cdot kVAR$

$\mathbf{S} := \mathbf{V_{in}} \cdot \overline{\mathbf{I}}$ OR.. $\mathbf{S} := P + j \cdot Q$ $\quad \mathbf{S} = 1.21 + 1.284j \cdot kVA = 1.764kVA\ \underline{/46.7^o}$ $\quad \text{atan}\left(\frac{\text{Im}(\mathbf{S})}{\text{Re}(\mathbf{S})}\right) = 46.696 \cdot deg$ same angle as **Z**

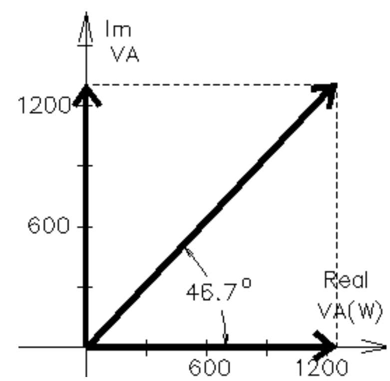
$S := \sqrt{P^2 + Q^2} = |\mathbf{S}| = 1.764 \cdot kVA = |\mathbf{V_{in}}| \cdot |\mathbf{I}| = 1.764 \cdot kVA$

OR, since we know that the voltage across each element of the load is V_{in} ...
Real power is dissipated only by resistors

$P := \frac{(|\mathbf{V_{in}}|)^2}{R}$ $\quad P = 1.21 \cdot kW$ $\quad Q := \frac{(|\mathbf{V_{in}}|)^2}{\omega \cdot L}$ $\quad Q = 1.284 \cdot kVAR$

$\mathbf{S} := P + j \cdot Q$

$S = |\mathbf{S}| = \sqrt{P^2 + Q^2} = 1.764 \cdot kVA$ $\quad pf = \frac{P}{|\mathbf{S}|} = 0.686$



What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

$Q_C := -Q = \frac{(110 \cdot V)^2}{X_C}$ $\quad X_C := \frac{(110 \cdot V)^2}{-Q}$ $\quad X_C = -9.425 \cdot \Omega$

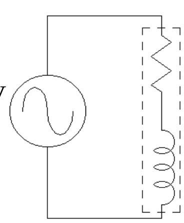
$\frac{1}{|X_C| \cdot \omega} = 281 \cdot \mu F$ $\quad$ OR.. $\quad Q_C := -Q = (110 \cdot V)^2 \cdot \omega \cdot C$ $\quad C = \frac{-Q}{(110 \cdot V)^2 \cdot \omega} = -281.434 \cdot \mu F$

Ex. 2 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

Series R & L

$\mathbf{V_{in}} := 110 \cdot V$

$\omega := 377 \cdot \frac{rad}{sec}$



$R := 10 \cdot \Omega$ $\quad \mathbf{Z} := R + j \cdot \omega \cdot L$

$\mathbf{Z} = 10 + 9.425j \cdot \Omega$ $\quad |\mathbf{Z}| = 13.742 \cdot \Omega$

$L := 25 \cdot mH$ $\quad \theta := \arg(\mathbf{Z})$ $\quad \theta = 43.304 \cdot deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.728$

$\mathbf{I} := \frac{\mathbf{V_{in}}}{\mathbf{Z}}$ $\quad \mathbf{I} = 5.825 - 5.49j \cdot A$

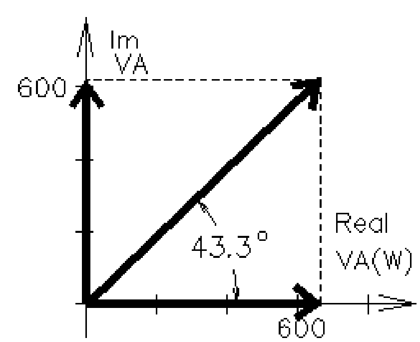
$|\mathbf{I}| = 8.005 \cdot A$ $\quad \arg(\mathbf{I}) = -43.304 \cdot deg$

$P := |\mathbf{V_{in}}| \cdot |\mathbf{I}| \cdot pf$ $\quad P = 0.641 \cdot kW$

$Q := |\mathbf{V_{in}}| \cdot |\mathbf{I}| \cdot \sin(\theta)$ $\quad Q = 0.604 \cdot kVAR$

$\mathbf{S} := \mathbf{V_{in}} \cdot \overline{\mathbf{I}}$ $\quad \mathbf{S} = 0.641 + 0.604j \cdot kVA$

$|\mathbf{S}| = 0.881 \cdot kVA$ $\quad \arg(\mathbf{S}) = 43.304 \cdot deg$ $\quad \mathbf{S} = 881VA\underline{/43.3^o}$



OR, if we first find the magnitude of the current which flows through each element of the load...

$$|\mathbf{I}| \;=\; \frac{\mathbf{V_{in}}}{\sqrt{R^2+(\omega\cdot L)^2}} = 8.005\cdot A$$

$P := (|\mathbf{I}|)^2\cdot R$ $\qquad P = 0.641\cdot kW$ $\qquad Q := (|\mathbf{I}|)^2\cdot(\omega\cdot L)$ $\qquad Q = 0.604\cdot kVAR$

$\mathbf{S} := P + j\cdot Q$ $\qquad |\mathbf{S}| \;=\; \sqrt{P^2+Q^2} = 0.881\cdot kVA$ $\qquad pf \;=\; \dfrac{P}{|\mathbf{S}|} = 0.728$

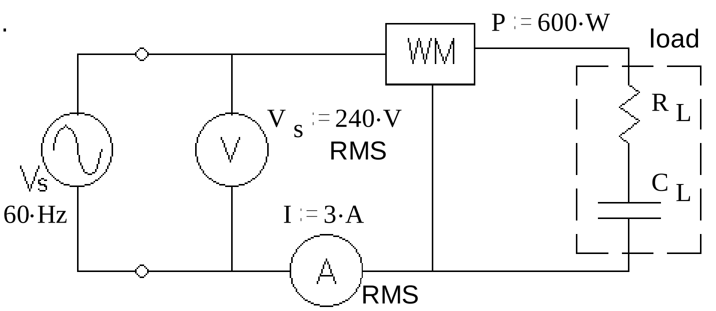
What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

$Q = 603.9\cdot VAR$ $\qquad$ so we need: $\quad Q_C := -Q$ $\qquad Q_C = -603.9\cdot VAR \;=\; \dfrac{V_{in}^{\;2}}{X_C}$

$X_C := \dfrac{\mathbf{V_{in}}^2}{Q_C}$ $\qquad X_C = -20.035\cdot\Omega \;=\; \dfrac{-1}{\omega\cdot C}$ $\qquad C := \dfrac{1}{|X_C|\cdot\omega}$ $\qquad C = 132\cdot\mu F$

Check: $\dfrac{1}{\dfrac{1}{R+j\cdot\omega\cdot L}+j\cdot\omega\cdot C} = 18.883\cdot\Omega$ $\qquad$ No j term, so $\theta = 0^o$

Ex. 3 R, & C together are the load in the circuit shown. The RMS voltmeter measures 240 V, the RMS ammeter measures 3 A, and the wattmeter measures 600 W. Find the following: Be sure to show the correct units for each value.



a) The value of the load resistor. R_L = ?

$$P \;=\; I^2\cdot R_L$$

$R_L := \dfrac{P}{I^2}$ $\qquad R_L = 66.7\cdot\Omega$

b) The apparent power. |**S**| = ? $\qquad S := V_s\cdot I$ $\qquad S = 720\cdot VA$

c) The reactive power. Q = ? $\qquad Q := -\sqrt{S^2-P^2}$ $\qquad Q = -398\cdot VAR$

d) The complex power. **S** = ? $\qquad \mathbf{S} := P + j\cdot Q$ $\qquad \mathbf{S} = 600 - 398i \;\cdot VA$

e) The power factor. pf = ? $\qquad pf := \dfrac{P}{V_s\cdot I}$ $\qquad pf = 0.833$

f) The power factor is leading or lagging? $\qquad$ leading (load is capacitive, Q is negative)

g) The two components of the load are in a box which cannot be opened. Add (draw it) another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and find its value. This component should not affect the real power consumption of the load.

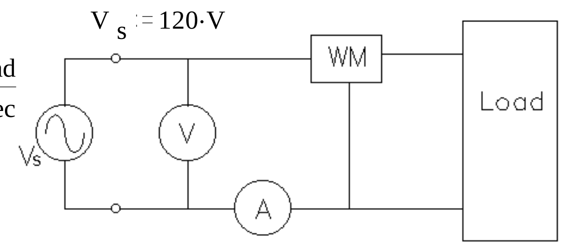
$f = 60\cdot Hz$ $\qquad \omega := 2\cdot\pi\cdot f$ $\qquad \omega = 376.991\cdot\dfrac{rad}{sec}$

Add an inductor in parallel with load

$Q = -398\cdot VAR$ $\qquad$ so we need: $\quad Q_L := -Q$ $\qquad Q_L = 398\cdot VAR \;=\; \dfrac{V_s^{\;2}}{X_L}$

$X_L := \dfrac{V_s^{\;2}}{Q_L}$ $\qquad X_L = 144.725\cdot\Omega \;=\; \omega\cdot L$ $\qquad L := \dfrac{|X_L|}{\omega}$ $\qquad L = 384\cdot mH$

Ex. 4 For the 60 Hz load shown in the figure, the RMS voltmeter measures 120 V. The phasor diagram for the power is also shown. Find the following:



$\omega := 377 \cdot \frac{rad}{sec}$

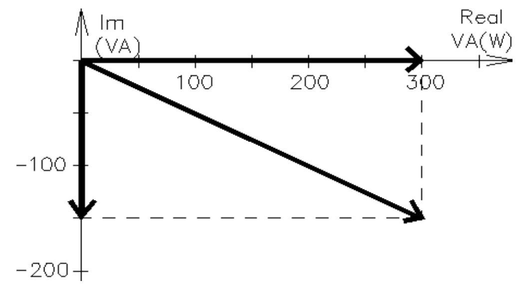
a) The complex power. **S** = ?

$P := 300 \cdot W$ $\qquad Q := -150 \cdot VA$

$\mathbf{S} := P + j \cdot Q$ $\qquad \mathbf{S} = 300 - 150j \cdot VA$

b) The apparent power. |**S**| = ? $\qquad |\mathbf{S}| = \sqrt{P^2 + Q^2} = 335.4 \cdot VA$

c) The power factor. pf = ? $\qquad pf := \frac{P}{|\mathbf{S}|}$ $\qquad pf = 0.894$



d) The item marked "WM" in the figure is a wattmeter, what does it read? (give a number) $\qquad P = 300 \cdot W$

e) The item marked "A" in the figure is an RMS ammeter, what does it read? (give a number)

$I := \frac{|\mathbf{S}|}{V_s}$ $\qquad I = 2.795 \cdot A$ $\qquad I = 2.8 \cdot A$

f) The power factor is leading or lagging? $\qquad$ leading (Q is negative)

g) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and <u>find its value</u>. This component should not affect the real power consumption of the load.

Add an inductor in parallel with load

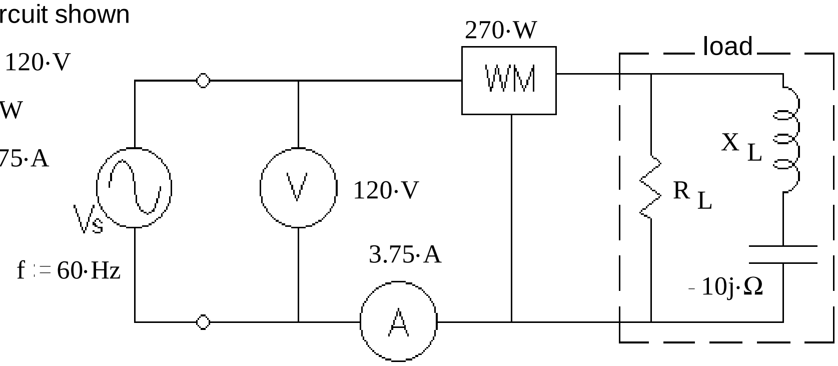
$Q = -150 \cdot VAR$ $\qquad$ need: $Q_L := -Q$ $\qquad Q_L = 150 \cdot VAR = \frac{V_s^2}{\omega \cdot L}$ $\qquad L := \frac{V_s^2}{\omega \cdot Q_L}$ $\qquad L = 255 \cdot mH$

Ex. 5 R, L, & C together are the load in the circuit shown

The RMS voltmeter measures 120 V. $\quad V_s := 120 \cdot V$

The wattmeter measures 270 W. $\quad P := 270 \cdot W$

The RMS ammeter measures 3.75 A. $\quad I := 3.75 \cdot A$



Find the following: Be sure to show the correct units for each value.

a) The value of the load resistor. R_L = ?

$P = \frac{V_s^2}{R_L}$ $\qquad R_L := \frac{V_s^2}{P}$ $\qquad R_L = 53.3 \cdot \Omega$

b) The magnitude of the impedance of the load inductor (reactance) . $|\mathbf{Z_L}| = X_L$ = ?

$I_R := \frac{V_s}{R_L}$ $\qquad I_R = 2.25 \cdot A$ $\qquad I_L := \sqrt{I^2 - I_R^2}$ $\qquad I_L = 3 \cdot A$ $\qquad X := \frac{V_s}{I_L}$ $\qquad X = 40 \cdot \Omega$

OR: $S := V_s \cdot I$ $\qquad S = 450 \cdot VA$ $\qquad Q := \sqrt{S^2 - P^2}$ $\qquad Q = 360 \cdot VAR = \frac{V_s^2}{X}$ $\qquad X = \frac{V_s^2}{Q} = 40 \cdot \Omega$

either way: $\quad X_C := -10 \cdot \Omega$ $\qquad X_L := X - X_C$ $\qquad X_L = 50 \cdot \Omega$

c) The reactive power. Q = ? $\qquad Q := \sqrt{(V_s \cdot I)^2 - P^2}$ $\qquad Q = 360 \cdot VAR$ $\qquad$ positive, because the load is primarily inductive

d) The power factor is leading or lagging? $\qquad$ lagging (load is inductive, Q is positive)

e) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and find its value. This component should not affect the real power consumption of the load.

$f = 60 \cdot Hz \qquad \omega := 377 \cdot \frac{rad}{sec}$

Add a capacitor in parallel with load

$Q = 360 \cdot VAR$ so we need: $Q_C := -Q \qquad Q_C = -360 \cdot VAR = -\frac{V_s^2}{\frac{1}{\omega \cdot C}} = -\omega \cdot C \cdot V_s^2$

$$C := \frac{Q_C}{-\omega \cdot V_s^2} \qquad C = 66.3 \cdot \mu F$$

Ex. 6 A step-down transformer has an output voltage of 220 V (rms) when the primary is connected across a 560 V (rms) source.

a) If there are 280 turns on the primary winding, how many turns are required on the secondary?

$280 \cdot \frac{220 \cdot volt}{560 \cdot volt} = 110$ turns

b) If the current in the primary is 2.4 A, what current flows in the load connected to the secondary?

$2.4 \cdot amp \cdot \frac{280}{110} = 6.11 \cdot A$

c) If the transformer is rated at 700/275 V, 2.1 kVA, what are the rated primary and secondary currents?

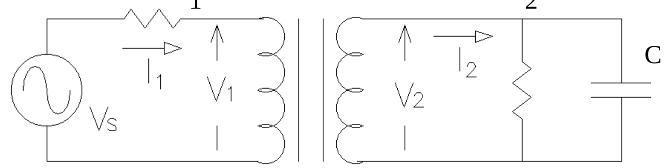
pri: $\frac{2.1 \cdot kVA}{700 \cdot V} = 3 \cdot A$ sec: $\frac{2.1 \cdot kVA}{275 \cdot V} = 7.636 \cdot A$

Ex. 7 The transformer shown in the circuit below is ideal. Find the following:

a) $|\mathbf{I_1}|$ = ?

$R_1 := 20 \cdot \Omega \qquad R_2 := 15 \cdot \Omega$

$V_s := 120 \cdot V \qquad \omega := 377 \cdot \frac{rad}{sec} \qquad C := 40 \cdot \mu F$



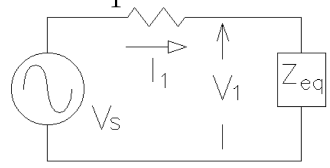
$N_1 := 150 \cdot turns \qquad N_2 := 50 \cdot turns$

$$\mathbf{Z_L} := \frac{1}{\frac{1}{R_2} + j \cdot \omega \cdot C}$$

$$\mathbf{Z_L} = 14.27 - 3.228j \cdot \Omega$$

Make an equivalent circuit:

$R_1 := 20 \cdot \Omega$



$$\mathbf{Z_{eq}} := \left(\frac{N_1}{N_2}\right)^2 \cdot \mathbf{Z_L} \qquad \mathbf{Z_{eq}} = 128.429 - 29.051j \cdot \Omega$$

$$R_1 + \mathbf{Z_{eq}} = 148.429 - 29.051j \cdot \Omega \qquad \sqrt{148.429^2 + 29.051^2} = 151.245$$

$$|\mathbf{I_1}| = \frac{V_s}{|R_1 + \mathbf{Z_{eq}}|} = \frac{V_s}{151.245 \cdot \Omega} = 0.793 \cdot A$$

b) $|\mathbf{I_2}|$ = ? $= \left(\frac{N_1}{N_2}\right) \cdot |\mathbf{I_1}| = \frac{150}{50} \cdot .793 \cdot A = 2.379 \cdot A$

c) $|\mathbf{V_1}|$ = ? $= V_s \cdot \left|\frac{\mathbf{Z_{eq}}}{R_1 + \mathbf{Z_{eq}}}\right|$ OR.. $|\mathbf{V_1}| = |\mathbf{I_1}| \cdot |\mathbf{Z_{eq}}| = .793 \cdot A \cdot \sqrt{128.429^2 + 29.051^2} \cdot \Omega = 104.417 \cdot V$