ECE 2210 Frequency Response, Filters & Bode Plots

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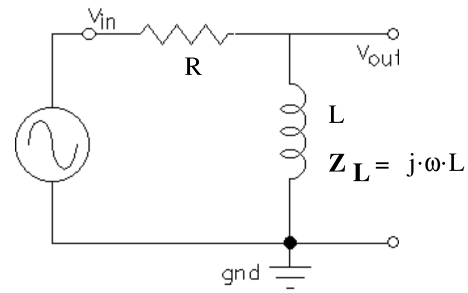
Frequency Response

In the Capacitors lab you made a "frequency dependent voltage divider" whose output was not the same for all frequencies of input. You made a graph of the output voltage as a function of the input frequency. That was a *frequency response* graph of the circuit. You made similar graphs in the Resonance lab. These graphs help show the relationship of the output to the input as a function of frequency. This relationship is known as the frequency response of the circuit. You may have heard the term used before in connection with speakers or microphones. All electrical and mechanical systems have frequency response characteristics. Sometimes the frequency response can be quite dramatic, like the Tacoma Narrows bridge.

Filter Circuits

A circuit which *passes* some frequencies and *filters out* other frequencies is called (surprise, surprise) a "filter" and this selection and rejection of frequencies is called "filtering". The tone or equalization controls on your stereo are frequency filters. So are the tuners in TVs and radios.

If a filter passes high frequencies and rejects low frequencies, then it is a high-pass filter. Conversely, if it passes low frequencies and rejects high ones, it is a low-pass filter. A filter that passes a range or band of frequencies and rejects frequencies lower or higher than that band, is a band-pass filter. The opposite of this is a band-rejection filter, or if the band is narrow, a notch filter or trap.



Look at the circuit at right. At low frequencies the impedance of the inductor is low and the output voltage is essentially shorted to ground. At high frequencies the impedance of the inductor is high and the output is about the same as the input. This is a high-pass filter. We can determine the relationship between the input and output:

$$\mathbf{V_{out}} = \frac{j\cdot\omega\cdot L}{R + j\cdot\omega\cdot L}\cdot\mathbf{V_{in}} \qquad \text{OR:} \qquad \frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = \frac{j\cdot\omega\cdot L}{R + j\cdot\omega\cdot L} = \mathbf{H}(\omega)$$

= The "Transfer Function"

A *transfer function* is a general term used for any linear system that has an input and an output. It is simply the ratio of output to input. The idea is that if you multiply the input by the transfer function, you get the output.

$$\mathbf{H}(\omega) = \frac{j\cdot\omega\cdot L}{R + j\cdot\omega\cdot L}$$

At low frequencies: $R >> j\cdot\omega L$ and $\mathbf{H}(\omega) \simeq \dfrac{j\cdot\omega\cdot L}{R}$ output is proportional to frequency

At high frequencies: $R << j\cdot\omega L$ and $\mathbf{H}(\omega) \simeq \dfrac{j\cdot\omega\cdot L}{j\cdot\omega\cdot L} = 1$ output is about the same as the input.

Naturally, a plot of the transfer function verses frequency would be a handy thing. You've already made similar plots in the lab. It turns out that these plots are best done on a log-log scale. Unfortunately, they are actually plotted on a semilog scale using a special unit in the vertical axis called the *decibel* (dB) and the log is built into this dB unit. The dB unit doesn't really simplify things, but it is widely used and you'll need to know about it, so here goes.

Decibels

Your ears respond to sound logarithmically, both in frequency and in intensity.
Musical octaves are in ratios of two. "A" in the middle octave is 220 Hz, in the next, 440 Hz, then 880 Hz, etc...
It takes about ten times as much power for you to sense one sound as twice as loud as another.

10x power $\simeq$ 2x loudness
A bel is such a 10x ratio of power.

Power ratio expressed in bels = $\log\left(\dfrac{P_2}{P_1}\right)$ bels

The bel is named for Alexander Graham Bell, who did original research in hearing.

It is a logarithmic expression of a unitless ratio (like the magnitude of H(ω) or gain of an amplifier).

The bel unit is never actually used, instead we use the decibel (dB, 1/10th of a bel).

Power ratio expressed in dB = $10\cdot\log\left(\dfrac{P_2}{P_1}\right)$ dB dB are also used to express voltage and current ratios, which is related to power when squared. $P = \dfrac{V^2}{R} = I^2 \cdot R$

$$\text{Voltage ratio expressed in dB} = 10\cdot\log\left(\frac{V_2{}^2}{V_1{}^2}\right)\ \text{dB} = 20\cdot\log\left(\frac{V_2}{V_1}\right)\ \text{dB}$$

$$\text{Current ratio expressed in dB} = 20\cdot\log\left(\frac{I_2}{I_1}\right)\ \text{dB}$$

These are the most common formulas used for dB

Some common ratios expressed as dB

$$20\cdot\log\left(\frac{1}{\sqrt{2}}\right) = -3.01\cdot\text{dB} \qquad 10^{-\frac{3}{20}} = 0.708 \qquad 20\cdot\log\left(\sqrt{2}\right) = 3.01\cdot\text{dB} \qquad 10^{\frac{3\cdot\text{dB}}{20}} = 1.413$$

$$20\cdot\log\left(\frac{1}{2}\right) = -6.021\cdot\text{dB} \qquad 10^{-\frac{6}{20}} = 0.501 \qquad 20\cdot\log(2) = 6.021\cdot\text{dB} \qquad 10^{\frac{6\cdot\text{dB}}{20}} = 1.995$$

$$20\cdot\log\left(\frac{1}{10}\right) = -20\cdot\text{dB} \qquad 10^{-\frac{20}{20}} = 0.1 \qquad 20\cdot\log(10) = 20\cdot\text{dB} \qquad 10^{\frac{20\cdot\text{dB}}{20}} = 10$$

$$20\cdot\log\left(\frac{1}{100}\right) = -40\cdot\text{dB} \qquad 10^{-\frac{40}{20}} = 0.01 \qquad 20\cdot\log(100) = 40\cdot\text{dB} \qquad 10^{\frac{40\cdot\text{dB}}{20}} = 100$$

Other dB-based units

You may have encountered dB as an absolute measure of sound intensity (Sound Pressure Level or SPL). In that case the RMS sound pressure is compared as a ratio to a reference of 2 x 10^{-5} Pascals.

dBm is another absolute power scale expressed in dB. Powers are referenced to 1mW.

Volume Units (VU) are dBm with the added spec that the load resistor is 600Ω.

Bode Plots

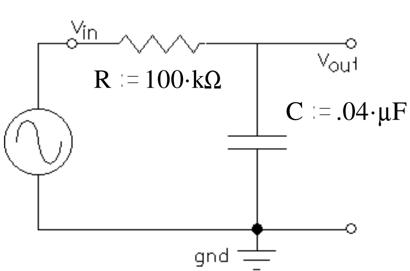
Named after Hendrik W. Bode (bo-dee), bode plots are just frequency response curves made on semilog paper where the horizontal axis is frequency on a $\log_{10}$ scale and the vertical axis is either dB or phase angle. The plots are nothing special, but the method that Bode came up with to make them quickly and easily *is* special. We aren' t going to bother with the phase-angle plots in this class, but since the bode method of making frequency plots is so simple it' s worth our time to see how it' s done.

Basically, these are the steps:

1. Find the transfer function.
2. Analyze the transfer function to find "corner frequencies" and use these to divide the frequency into ranges.
3. Simplify and approximate the magnitude of the transfer function in each of these ranges.
4. Draw a "straight-line approximation" of the frequency response curve.
5. Use a few memorized facts to draw the actual frequency response curve.

The best way to learn the method is by examples.

Ex. 1



$$\frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = \frac{\dfrac{1}{j\cdot\omega\cdot C}}{\dfrac{1}{j\cdot\omega\cdot C} + R} = \frac{1}{1 + R\cdot(j\cdot\omega\cdot C)} = \mathbf{H}(\omega) \quad \text{= The "Transfer Function"}$$

corner frequency is where real = imaginary (in denominator in this case)

$$1 = \omega_c\cdot R\cdot C \qquad \omega_c := \frac{1}{R\cdot C} \qquad \omega_c = 250\cdot\frac{\text{rad}}{\text{sec}} \qquad \text{So...}\quad \mathbf{H}(\omega) := \frac{1}{1 + j\cdot\dfrac{\omega}{250\cdot\frac{\text{rad}}{\text{sec}}}}$$

ω_c is also called a "pole" frequency

The transfer function is said to have one "pole" at ω_c

To make a straight-line approximation of the magnitude of $\mathbf{H}(\omega)$ we'll approximate $|\mathbf{H}(\omega)|$ in two regions, one below the corner frequency, and one above the corner frequency. Keep only the real or only the imaginary part of the denominator, depending on which is greater.

below the corner frequency: $\omega < \omega_c$ $\quad \mathbf{H}(\omega) \simeq \dfrac{1}{1} \quad |\mathbf{H}(\omega)| \simeq 1 \quad 20\cdot\log(1) = 0\cdot\text{dB}$

above the corner frequency: $\omega > \omega_c$ $\quad \mathbf{H}(\omega) \simeq \dfrac{1}{j\cdot\dfrac{\omega}{250\cdot\frac{\text{rad}}{\text{sec}}}} \quad |\mathbf{H}(\omega)| \simeq \dfrac{1}{\omega}\cdot\left(250\cdot\dfrac{\text{rad}}{\text{sec}}\right)$ inversely proportional to ω.

Inverse proportionality is a straight 1 to 1 down slope on a log-log plot, with dB it's a only slightly different. Since 10x corresponds to 20 dB, the line goes down 20 dB for every 10x increase in frequency (called a decade).

That's all you need to make the straight-line approximation shown in the plot below. (If you know the slope)

Try some values above the corner frequency:

$$20\cdot\log\left[\frac{1}{10\cdot\omega_c}\cdot\left(250\cdot\frac{\text{rad}}{\text{sec}}\right)\right] = -20\cdot\text{dB} \qquad 20\cdot\log\left[\frac{1}{100\cdot\omega_c}\cdot\left(250\cdot\frac{\text{rad}}{\text{sec}}\right)\right] = -40\cdot\text{dB}$$

The slope above the corner frequency is -20 dB per "decade".
A decade is a 10x increase in frequency.
This slope is also -6dB per "octave" (a 2x increase in frequency).

Let's find the actual magnitude of H(ω) right at the corner frequency ($H(\omega_c)$): $\omega = \omega_c$

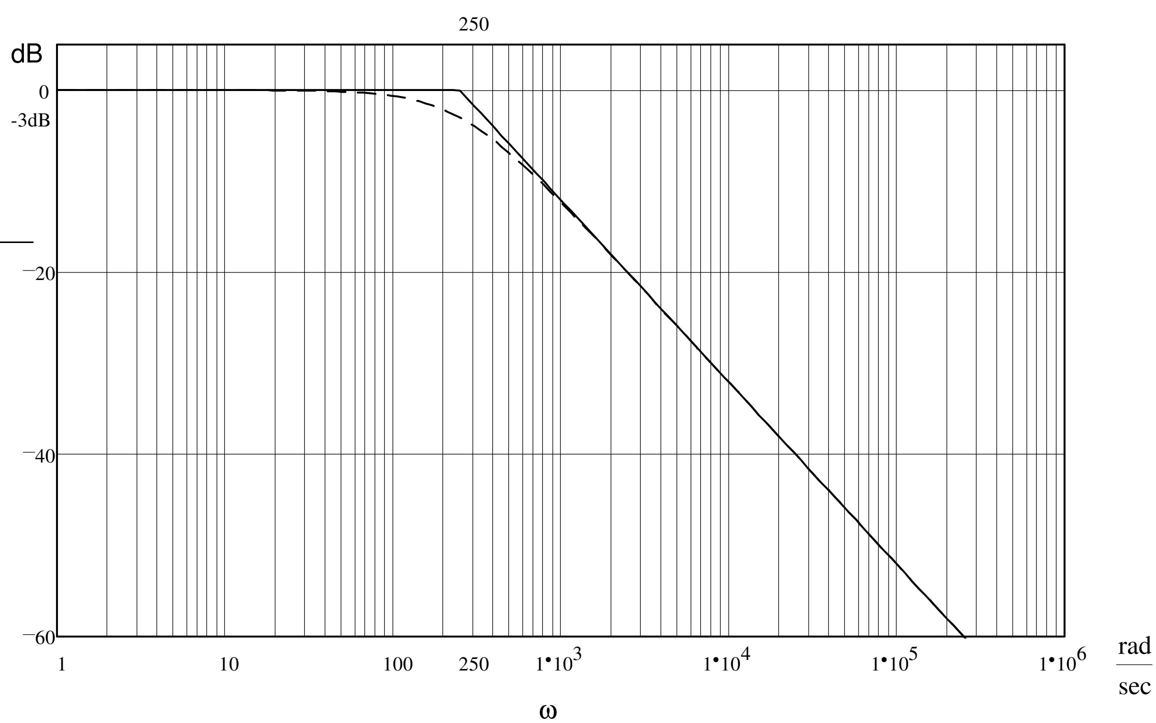
$$\mathbf{H}(\omega) = \frac{1}{1 + j\cdot\dfrac{\omega_c}{250\cdot\frac{\text{rad}}{\text{sec}}}} = \frac{1}{1 + j\cdot 1}$$

$$|\mathbf{H}(\omega)| = \frac{1}{\sqrt{2}} \qquad 20\cdot\log\left(\frac{1}{\sqrt{2}}\right) = -3.01\cdot\text{dB}$$

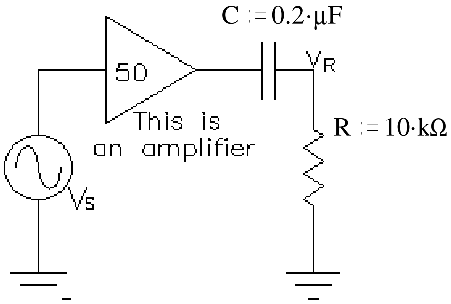
Magnitude plot $|\mathbf{H}(\omega)|$

Straight-line approximation ____

Actual _ _ _ _



Ex. 2



$$\frac{\mathbf{V_R}}{\mathbf{V_S}} = 50 \cdot \frac{R}{\frac{1}{j \cdot \omega \cdot C} + R} = \frac{50 \cdot (R \cdot (j \cdot \omega \cdot C))}{1 + R \cdot (j \cdot \omega \cdot C)} = \mathbf{H}(\omega)$$

Transfer function has one pole at ω_c

corner frequency is where real = imaginary

$$1 = \omega_c \cdot R \cdot C \qquad \omega_c := \frac{1}{R \cdot C} \qquad \omega_c = 500 \cdot \frac{rad}{sec}$$

So... $$\mathbf{H}(\omega) := \frac{50 \cdot j \cdot \frac{\omega}{500 \cdot \frac{rad}{sec}}}{1 + j \cdot \frac{\omega}{500 \cdot \frac{rad}{sec}}} = \frac{50 \cdot j \cdot \omega}{500 \cdot \frac{rad}{sec} + j \cdot \omega}$$ OR: $$\mathbf{H}(\omega) := \frac{50 \cdot j \cdot \frac{\omega}{\omega_c}}{1 + j \cdot \frac{\omega}{\omega_c}}$$

$$\omega < \omega_c \qquad \mathbf{H}(\omega) \simeq \frac{50 \cdot j \cdot \frac{\omega}{500 \cdot \frac{rad}{sec}}}{1} = \frac{0.1 \cdot \frac{sec}{rad} \cdot j \cdot \omega}{1} \qquad |\mathbf{H}(\omega)| \simeq 0.1 \cdot \frac{sec}{rad} \cdot \omega$$

Proportional to ω. That' s all we need to know here. This proportionality to ω will result in a +20 dB per decade slope for all frequencies below the corner frequency

$$\omega > \omega_c \qquad \mathbf{H}(\omega) \simeq \frac{50 \cdot j \cdot \frac{\omega}{500 \cdot \frac{rad}{sec}}}{j \cdot \frac{\omega}{500 \cdot \frac{rad}{sec}}} \qquad |\mathbf{H}(\omega)| \simeq 50 \qquad 20 \cdot \log(50) = 33.98 \cdot dB$$

The "pass band"

Actual value at the corner frequency

$$\omega = \omega_c \qquad \mathbf{H}(\omega) = \frac{50 \cdot j \cdot \omega}{500 \cdot \frac{rad}{sec} + j \cdot \omega} = \frac{50 \cdot j}{1 + j \cdot 1} = 25 + 25j \qquad |25 + 25 \cdot j| = 35.355 \qquad 20 \cdot \log(35.355) = 30.97 \cdot dB$$

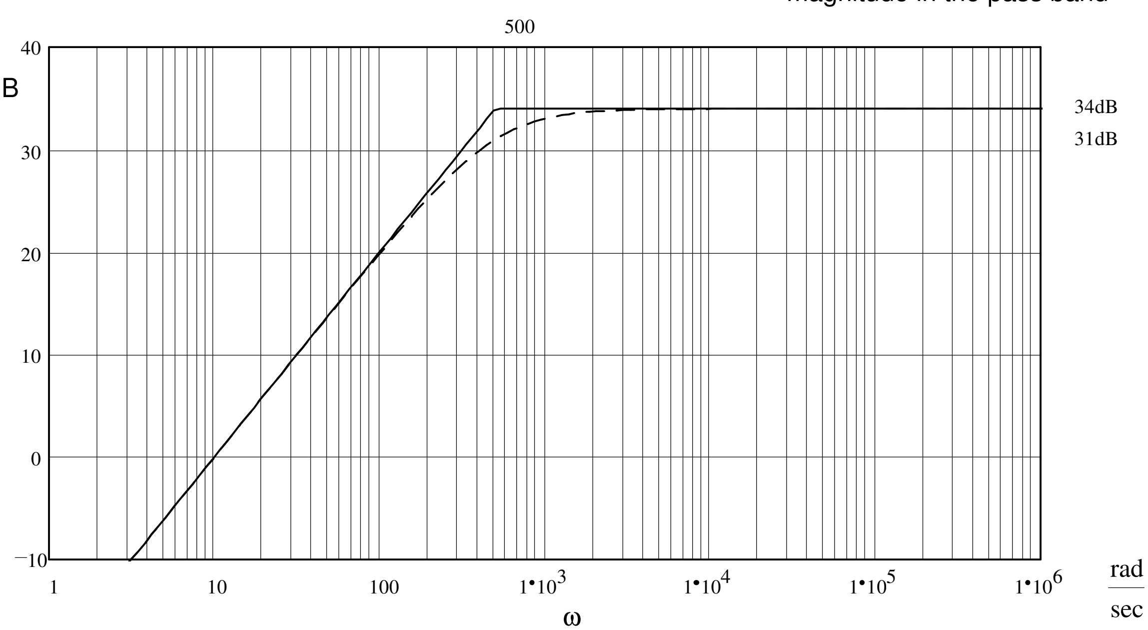
3 dB lower than the magnitude in the pass band

Magnitude plot

$|\mathbf{H}(\omega)|$ dB

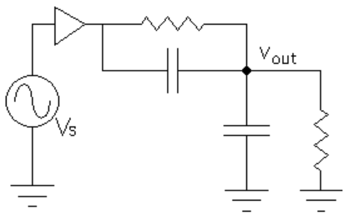
Straight-line approximation _____

Actual _ _ _ _



Ex. 3 The transfer function may already be worked out: $\mathbf{H}(f) := 10\cdot\dfrac{1+j\cdot\dfrac{f}{10\cdot Hz}}{1+j\cdot\dfrac{f}{500\cdot Hz}}$

Could come from
a circuit like this:



The real and imaginary parts of the numerator are equal at the one corner frequency (called a "zero") $\quad 1 = j\cdot\dfrac{f_c}{10\cdot Hz} \qquad f_{c1} := 10\cdot Hz$

The real and imaginary parts of the denominator are equal at the other corner frequency (pole) $\quad 1 = j\cdot\dfrac{f_c}{500\cdot Hz} \qquad f_{c2} := 500\cdot Hz$

There are now three regions to approximate |H(f)|

Below the first corner frequency: $\quad f < 10\cdot Hz \qquad |\mathbf{H}(f)| \simeq \left|10\cdot\dfrac{1}{1}\right| = 10 \qquad 20\cdot\log(10) = 20\cdot dB$

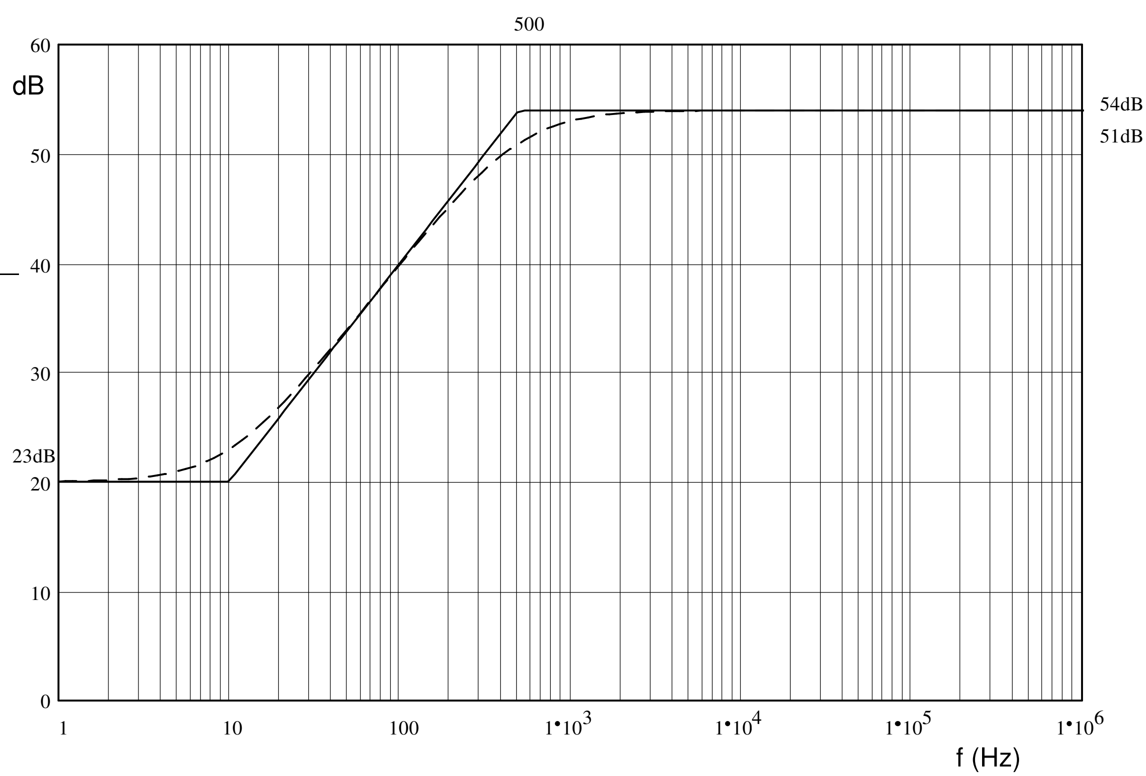
Between the corner frequencies: $\quad 10\cdot Hz < f < 500\cdot Hz \qquad |\mathbf{H}(f)| \simeq \left|10\cdot\dfrac{j\cdot\dfrac{f}{10}}{1}\right| = f \qquad$ proportional to f

Above the second corner frequency: $\quad 1000\cdot Hz < f \qquad |\mathbf{H}(f)| \simeq \left|10\cdot\dfrac{j\cdot\dfrac{f}{10}}{j\cdot\dfrac{f}{500}}\right| = 500 \qquad 20\cdot\log(500) = 53.98\cdot dB$

Magnitude plot $|\mathbf{H}(f)|$

Straight-line approximation _____

Actual _ _ _ _



Ex. 4

A Transfer function of a typical amplifier:
$$\mathbf{H}(\omega) := \frac{j\cdot\omega\cdot 0.182\cdot \text{sec}}{\left(1+\dfrac{j\cdot\omega}{6.875\cdot 10^4\cdot\dfrac{\text{rad}}{\text{sec}}}\right)\cdot\left(1+\dfrac{j\cdot\omega}{416.67\cdot\dfrac{\text{rad}}{\text{sec}}}\right)}$$

$$\omega_{C1} := 416.67\cdot\frac{\text{rad}}{\text{sec}}$$

$$\omega_{C2} := 6.875\cdot 10^4\cdot\frac{\text{rad}}{\text{sec}}$$

Between the two poles (passband): $\mathbf{H}(\omega) \simeq \dfrac{j\cdot\omega_i\cdot 0.182}{(1)\cdot\left(\dfrac{j\cdot\omega_i}{416.67}\right)} = 75.834$ $\quad 20\cdot\log(75.834) = 37.6$

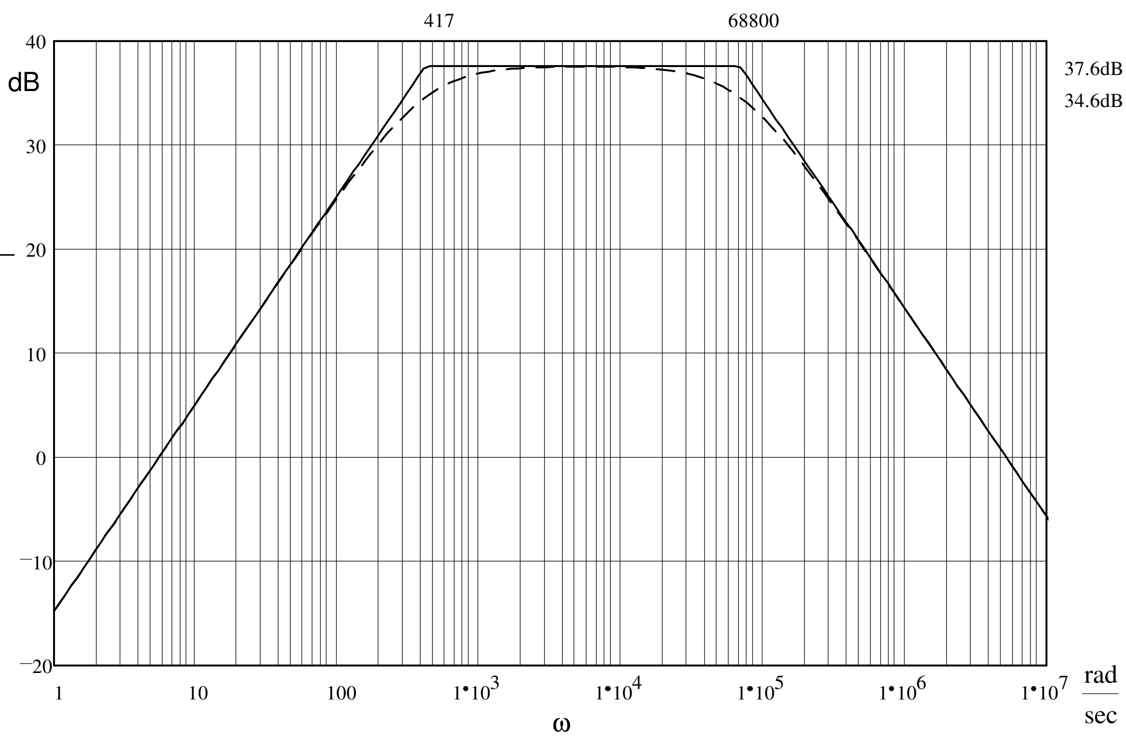
Below ω_{C1} $\quad \mathbf{H}(\omega) \simeq \dfrac{j\cdot\omega\cdot 0.182}{(1)\cdot(1)}$ $\quad$ proportional to ω

Above ω_{C2} $\quad \mathbf{H}(\omega) \simeq \dfrac{j\cdot\omega\cdot 0.182}{\left(\dfrac{j\cdot\omega}{6.875\cdot 10^4}\right)\cdot\left(\dfrac{j\cdot\omega}{416.67}\right)}$ $\quad$ inversely proportional to ω

Magnitude plot $|\mathbf{H}(\omega)|$

Straight-line approximation _____

Actual _ _ _ _



Warning

The Bode plots that we' ve covered here are the simplest types and only magnitude plots. This will do for an initial introduction to simple filters, but this coverage *is not* complete.
Complete Bode plots also include phase plots which we haven' t looked at at all. Also, if some poles and zeroes are too close to each other they can interact and even result in complex poles.
If asked in a future classes if you have "covered" Bode plots, do not make the mistake of saying "yes".

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1 10 100 $1 \cdot 10^3$ $1 \cdot 10^4$ $1 \cdot 10^5$ $1 \cdot 10^6$

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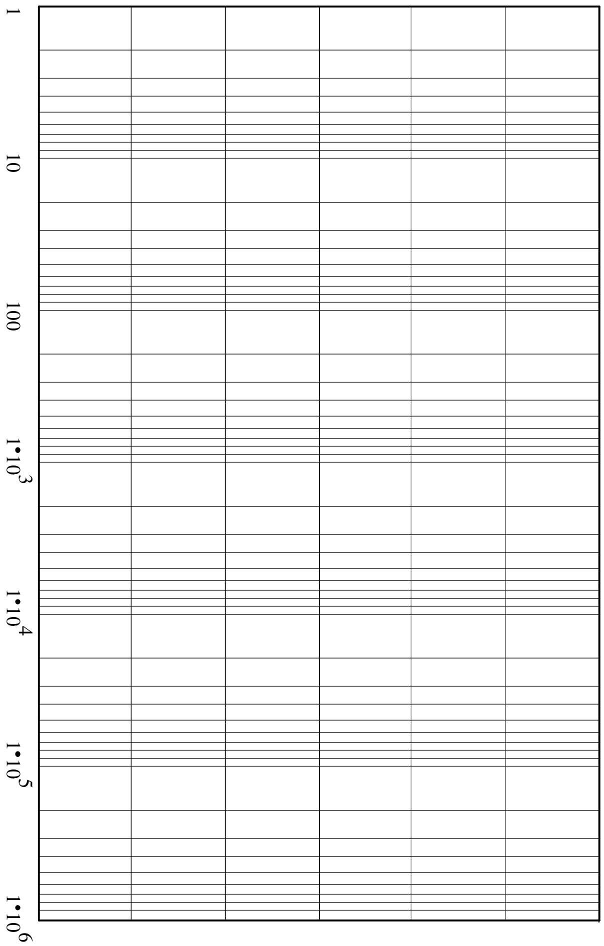
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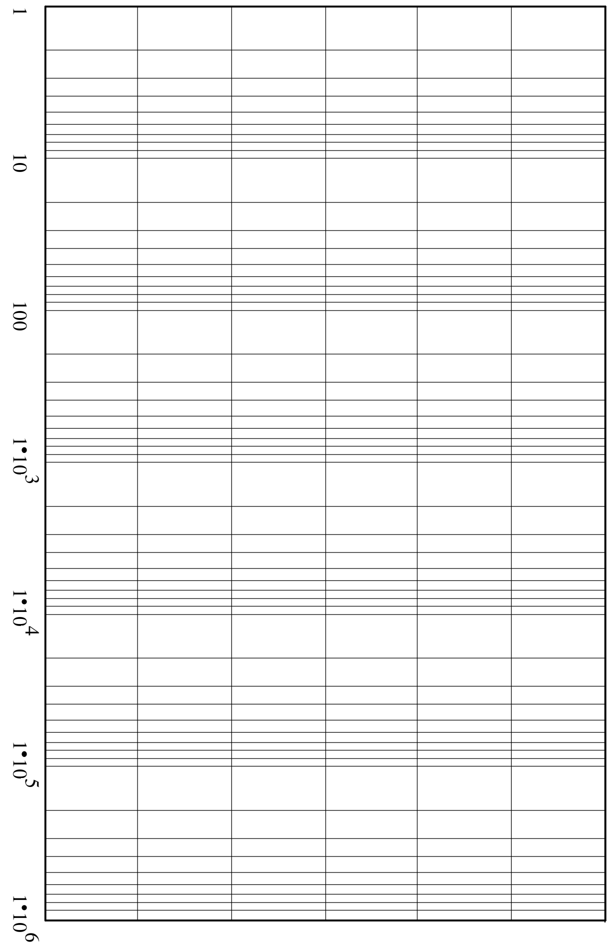
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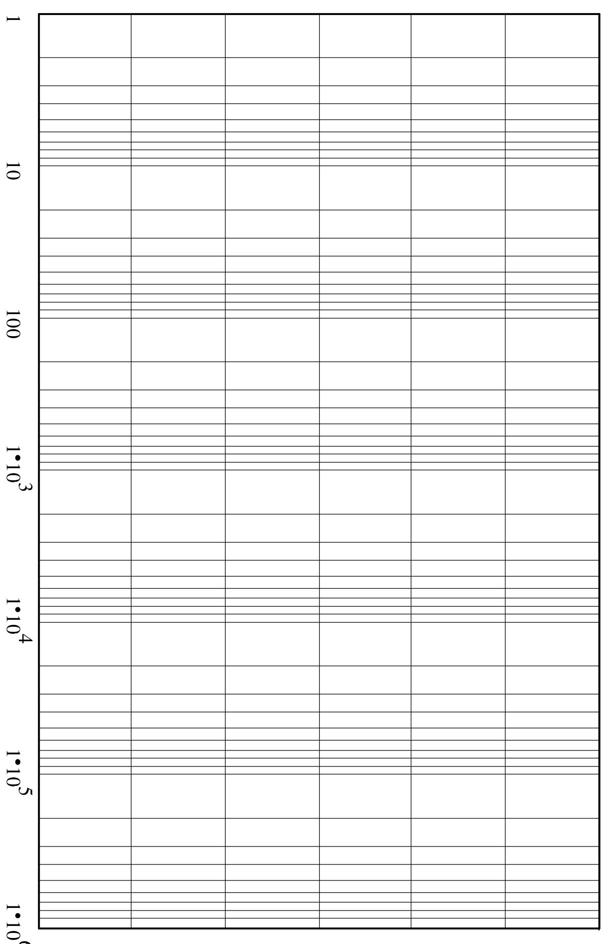
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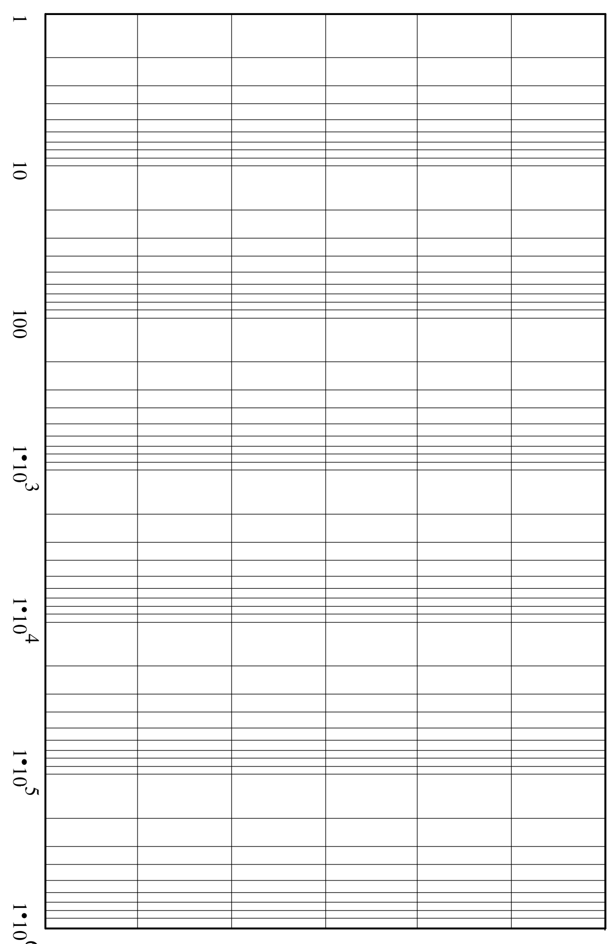
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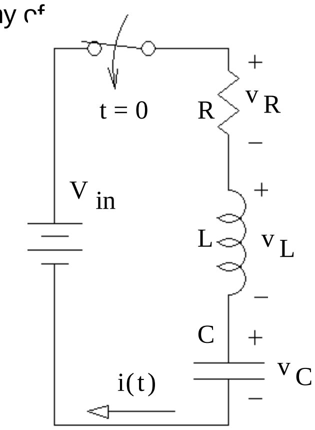
1.6 Second-Order Transients

A circuit with both a capacitor and an inductor is like a mechanical system with both a mass and a spring. When there are two different types of energy-storage elements, the transient responses can be much more interesting than the simple exponential curves that we've seen so far. Many of these systems can oscillate or "ring" when a transient is applied. When you analyze a circuit with a capacitor and an inductor you get a second-order differential equation, so the transient voltages and currents are called second-order transients.



Series RLC circuit, traditional way: Look at the circuit at right. The same current flows through all three elements ($i(t)$ or just i). That current will begin to flow after time $t = 0$, when the switch is closed. Using basic circuit laws:

$$V_{in} = v_R + v_L + v_C$$

$$= i\cdot R + L\cdot\frac{d}{dt}i + \frac{1}{C}\cdot\int_{-\infty}^{t} i_C\,dt \quad \text{Making the obvious substitutions.}$$

The next step here would be to differentiate both sides of the equation, but we've been through this before with the RC circuit. If you're a little more clever, there's an easier way.

Make this substitution instead $\quad i = i_C = C\cdot\frac{d}{dt}v_C$, $\quad$ to get $\quad V_{in} = R\cdot C\cdot\frac{d}{dt}v_C + L\cdot C\cdot\frac{d^2}{dt^2}v_C + v_C$

Rearrange this equation to get $\quad V_{in} = L\cdot C\cdot\frac{d^2}{dt^2}v_C + R\cdot C\cdot\frac{d}{dt}v_C + v_C \quad$ and $\quad \frac{V_{in}}{L\cdot C} = \frac{d^2}{dt^2}v_C + \frac{R\cdot C}{L\cdot C}\cdot\frac{d}{dt}v_C + \frac{1}{L\cdot C}\cdot v_C$

This is the classical second-order differential equation and it is solved just like the first-order differential equation, by guessing a solution of the right form and then finding the particulars of that solution.

Standard differential equation answer: $\quad v_C(t) = A + B\cdot e^{s\cdot t}$

Differentiate: $\quad \frac{d}{dt}v_C = B\cdot s\cdot e^{s\cdot t}$

And again: $\quad \frac{d^2}{dt^2}v_C = B\cdot s^2\cdot e^{s\cdot t}$

Note: It will turn out that there will be two s's (s_1 and s_2), and two B's (B and D) for the second-order solution. For now I'll leave out that added complexity.

Substitute these back into the original equation:

$$\frac{V_{in}}{L\cdot C} = \frac{d^2}{dt^2}v_C + \frac{R}{L}\cdot\frac{d}{dt}v_C + \frac{1}{L\cdot C}\cdot v_C$$

$$= B\cdot s^2\cdot e^{s\cdot t} + \frac{R}{L}\cdot B\cdot s\cdot e^{s\cdot t} + \frac{1}{L\cdot C}\cdot\left(A + B\cdot e^{s\cdot t}\right)$$

$$= B\cdot s^2\cdot e^{s\cdot t} + \frac{R}{L}\cdot B\cdot s\cdot e^{s\cdot t} + \frac{1}{L\cdot C}\cdot B\cdot e^{s\cdot t} + \frac{1}{L\cdot C}\cdot A$$

We can separate this equation into two parts, one which is time dependent and one which is not. Each part must still be an equation.

Time independent (forced) part: $\quad V_{in} = A$, $\quad A = V_{in} =$ final condition $= v_C(\infty)$ $\quad$ just like before

Time dependent (transient) part: $\quad 0 = B\cdot s^2\cdot e^{s\cdot t} + \frac{R}{L}\cdot B\cdot s\cdot e^{s\cdot t} + \frac{1}{L\cdot C}\cdot B\cdot e^{s\cdot t}$

Divide both sides by $\quad B\cdot e^{s\cdot t}$ $\quad$ to get: $\quad 0 = s^2 + \frac{R}{L}\cdot s + \frac{1}{L\cdot C}$ $\quad$ **= characteristic equation**

This equation is important. It is called the characteristic equation and we'll need to find one like it for every second-order circuit that we analyze. Luckily, there's a much easier way to get it, using impedances similar to those we used in phasor analysis. I'll talk about that in the next section, in the meantime, let's continue with this problem.

Once you have the characteristic equation

characteristic equation: $s^2 + \frac{R}{L} \cdot s + \frac{1}{L \cdot C} = 0$

Solutions to the characteristic equation:

$$s_1 = -\frac{R}{2 \cdot L} + \frac{1}{2} \cdot \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{L \cdot C}} \qquad s_2 = -\frac{R}{2 \cdot L} - \frac{1}{2} \cdot \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{L \cdot C}}$$

This results in three possible types of solutions, depending on what's under the radical, +, -, or 0.

The characteristic equation is solved using the quadratic equation, recall:

if $a \cdot x^2 + b \cdot x + c = 0$

there are two solutions

$$x_1 = \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

and

$$x_2 = \frac{-b - \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

Notice also that there are two s values (s_1 and s_2) and that leads to two two B's (we'll call them B and D)

Overdamped The part under the radical is +

if $\left(\frac{R}{L}\right)^2 - \frac{4}{L \cdot C} > 0$ then s_1 and s_2 are both real and $s_1 \neq s_2$ and our guessed solution $v_C(t) = A + B \cdot e^{s \cdot t}$

will become $v_C(t) = v_C(\infty) + B \cdot e^{s_1 \cdot t} + D \cdot e^{s_2 \cdot t}$ and is simply the combination of two exponentials.

Also both s_1 and s_2 will always be negative (unless you find a negative R, C, or L), meaning the exponential parts will decay with time and are thus transient.

This is the overdamped case, like a class of students on a Monday morning. Pretty dull and soon to be asleep.

Underdamped The part under the radical is -

if $\left(\frac{R}{L}\right)^2 - \frac{4}{L \cdot C} < 0$ then s_1 and s_2 are both complex and and can be expressed as

$s_1 = \alpha + j \cdot \omega$ and $s_2 = \alpha - j \cdot \omega$

Well, if you start putting complex numbers in exponentials, what do you get? Euler's equations show that you'll get sines and cosines. In this case its much easier to rephrase the guessed solution like this.

$$v_C(t) = v_C(\infty) + e^{\alpha \cdot t} \cdot \left(B_2 \cdot \cos(\omega \cdot t) + D_2 \cdot \sin(\omega \cdot t)\right)$$

This form can be derived directly from $v_C(t) = A + B \cdot e^{s_1 \cdot t} + D \cdot e^{s_2 \cdot t}$

using Euler's equation, $e^{j \cdot \theta} = \cos(\theta) + j \cdot \sin(\theta)$, but we won't bother to here.

In fact, although B_2 and D_2 are <u>not</u> the same as B and D, I'll drop the "2" subscripts because we'll never actually need to convert between these two forms and the extra subscripts just become annoying.

So: $v_C(t) = v_C(\infty) + e^{\alpha \cdot t} \cdot (B \cdot \cos(\omega \cdot t) + D \cdot \sin(\omega \cdot t))$

α and ω come from the s_1 and s_2 solutions to the characteristic equation. ω is frequency at which the underdamped circuit will "ring" or "oscillate" in response to a transient. α sets the decay rate of that oscillation. Because α will always be negative the e^{at} term insures that the transient ringing dies out in time.

This is the underdamped case, like students on spring break in Fort Lauderdale.

Natural Frequency and the Damping Ratio

These are commonly used terms to describe the underdamped response in a normalized way, similar to the τ used to decribe first-order transient responses.

The "natural frequency" is defined as: $\omega_n = \sqrt{\alpha^2 - \omega^2}$

It is the frequency that the system would oscillate at if there were no damping ($R = 0$ in our case)

for this case: $\omega_n = \frac{1}{\sqrt{L \cdot C}}$

The damping ratio is defined as: $\zeta = \frac{\alpha}{\omega_n}$ (ζ is zeta)

Critically damped The part under the radical is 0

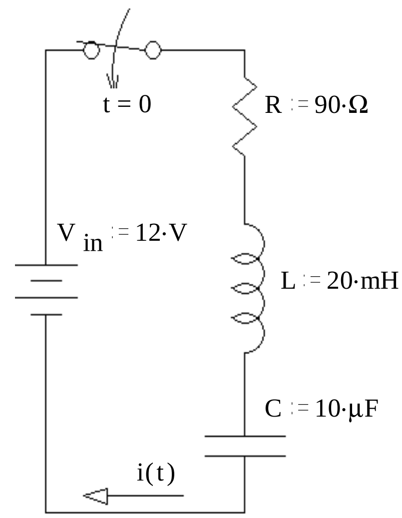
if $\left(\frac{R}{L}\right)^2 - \frac{4}{L \cdot C} = 0$ then s_1 and s_2 are both real and exactly the same. Now our guessed solution must be

modified to $v_C(t) = v_C(\infty) + B \cdot e^{s_1 \cdot t} + D \cdot t \cdot e^{s_2 \cdot t}$ and can result in a single overshoot.

This is actually a trivial case since it relies on an exact equality which will never happen in reality. The best use of the critically damped case is as a conceptual border between the over- and under-damped cases.

RLC examples

Let's use some component values in the RLC circuit and see what happens.



Overdamped Example

$\left(\frac{R}{L}\right)^2 - \frac{4}{L \cdot C} > 0$ s_1 and s_2 are real and negative, overdamped.

$$s_1 := -\frac{R}{2 \cdot L} + \frac{1}{2} \cdot \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{L \cdot C}} \qquad s_1 = -2000 \cdot \sec^{-1}$$

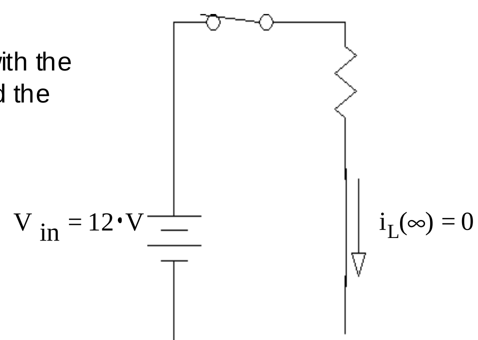
$$s_2 := -\frac{R}{2 \cdot L} - \frac{1}{2} \cdot \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{L \cdot C}} \qquad s_2 = -2500 \cdot \sec^{-1}$$

$$v_C(t) = v_C(\infty) + B \cdot e^{s_1 \cdot t} + D \cdot e^{s_2 \cdot t}$$

(As an example, the form is the same for all variables in this circuit)

Final conditions

REDRAW the circuit with the inductor as a short and the capacitor as an open.

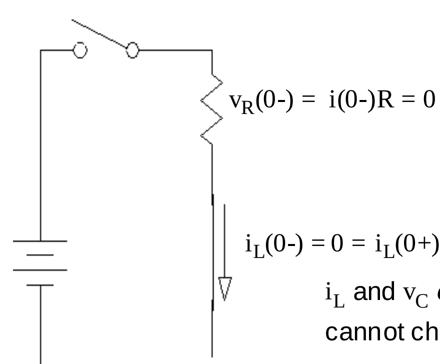


$v_C(\infty)$ = final condition = $12 \cdot V$

The capacitor will eventually charge up to V_{in}.

Initial conditions

REDRAW the circuit before the switch changes. Find two initial conditions, $i_L(0-)$ and $v_C(0-)$.



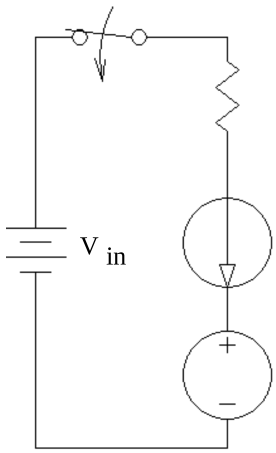
i_L and v_C ***cannot change instantly***, so cannot change the instant the switch changes.

$v_C(0-) = 0 = v_C(0+)$

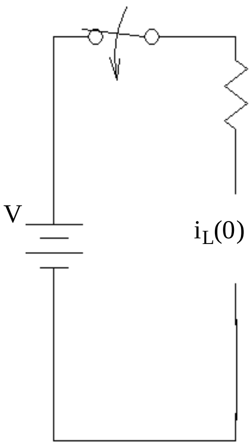
Pretty easy in this case

(assuming no initial charge)

REDRAW the circuit again just after the switch changes. Show the inductor as a current source of $i_L(0)$ (same as $i_L(0\text{-})$) and the capacitor as a voltage source of $v_C(0)$ (same as $v_C(0\text{-})$). Find two more initial conditions, $v_L(0)$ and $i_C(0)$. Both $v_L(0)$ or $i_C(0)$ ***can change instantly***, so you ***must*** find them from $i_L(0)$ and $v_C(0)$.



In this particular case, the circuit can be redrawn again for clarity:



Again, pretty easy in this case

Rearrange the basic equations for inductors and capacitors to find the initial slopes from $v_L(0)$ or $i_C(0)$.

Rearrange $\quad v_L = L \cdot \frac{d}{dt} i_L \quad$ to $\quad \frac{d}{dt} i_L(0) = \frac{v_L(0)}{L} \quad = \frac{12 \cdot V}{L} = 600 \cdot \frac{A}{sec}$

In this case

or, $\quad i_C = C \cdot \frac{d}{dt} v_C \quad$ to $\quad \frac{d}{dt} v_C(0) = \frac{i_C(0)}{C} \quad = \frac{0 \cdot A}{C} = 0 \cdot \frac{V}{sec}$

Note: You will need only the first one if you are looking for $i_L(t)$.

You will need only the second one if you are looking for $v_C(t)$.

You may need both if you are looking for any other variable in the circuit.
Other variables can usually be found most easily from $i_L(t)$ and/or $v_C(t)$.

To Find $v_C(t)$

At time $t = 0$ $\quad v_C(0) = v_C(\infty) + B + D = 0$

$0 = 12 \cdot V + B + D$ $\qquad$ Rearranging: $D = -12 \cdot V - B$

This equation has two unknowns. The initial slope will give us the needed second equation.

Differentiate the solution: $\quad v_C(t) = v_C(\infty) + B \cdot e^{s_1 \cdot t} + D \cdot e^{s_2 \cdot t}$

to get: $\quad \frac{d}{dt} v_C(t) = 0 + B \cdot s_1 \cdot e^{s_1 \cdot t} + D \cdot s_2 \cdot e^{s_2 \cdot t}$

At time $t = 0$: $\quad \frac{d}{dt} v_C(0) = B \cdot s_1 + D \cdot s_2$

From initial conditions, above: $\quad \frac{d}{dt} v_C(0) = \frac{i_C(0)}{C} = 0 \cdot \frac{V}{sec}$

Combining: $\quad 0 \cdot \frac{V}{sec} = B \cdot s_1 + D \cdot s_2$ $\qquad$ The second equation!

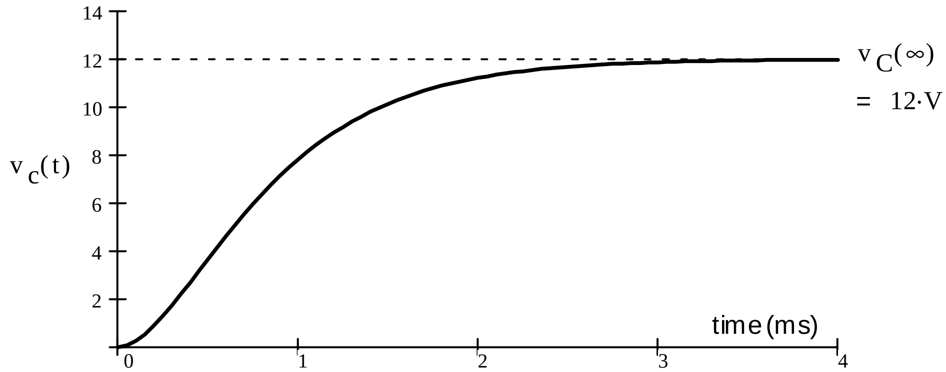
Solve simultaneously for B and D: $\quad 0 \cdot \frac{V}{sec} = B \cdot s_1 + (-12 \cdot V - B) \cdot s_2$ $\qquad B = s_2 \cdot \frac{12 \cdot V}{(s_1 - s_2)} = -60 \cdot V$

$D = -12 \cdot V - B = -12 \cdot V - -60 \cdot V = 48 \cdot V$

recall the solution: $v_C(t) = v_C(\infty) + B\cdot e^{s_1\cdot t} + D\cdot e^{s_2\cdot t}$

Substitute everything back in back in: $v_C(t) := 12\cdot V - 60\cdot V\cdot e^{-\frac{2000}{sec}\cdot t} + 48\cdot V\cdot e^{-\frac{2500}{sec}\cdot t}$

Notice that this is not a simple exponential curve, although admittedly it's not much more interesting.



To Find $i_L(t)$ or $i_R(t)$ or $i_C(t)$ which all the same $i(t)$.

$i(t) = i(\infty) + B\cdot e^{s_1\cdot t} + D\cdot e^{s_2\cdot t}$

From final and initial conditions

$i(0) = i(\infty) + B + D \quad = \quad 0 \quad = \quad 0 + B + D$ $\qquad\qquad$ $D = -B$

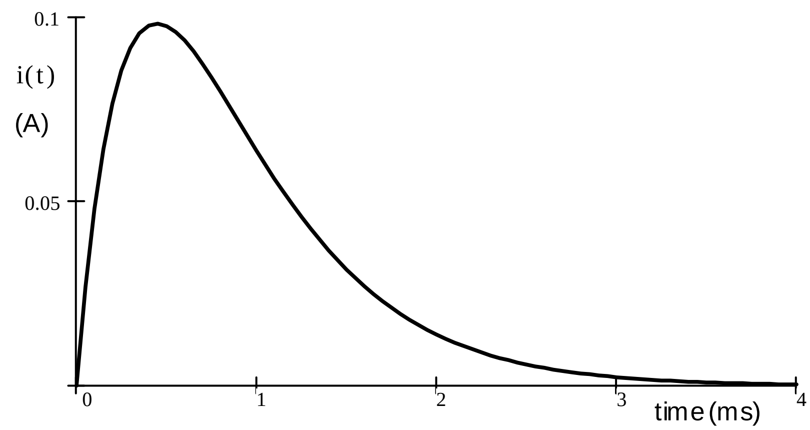
$\frac{d}{dt}i(0) = B\cdot s_1 + D\cdot s_2 \quad = \quad \frac{12\cdot V}{L} = 600\cdot\frac{A}{sec}$

Solve simultaneously for B and D $\qquad \frac{12\cdot V}{L} = B\cdot s_1 - B\cdot s_2 \qquad B = \frac{\left(\frac{12\cdot V}{L}\right)}{\left(s_1 - s_2\right)} = 1.2\cdot A$

$D = -B = -1.2\cdot A$

Substitute back in: $i(t) := 1.2\cdot e^{-\frac{2000}{sec}\cdot t} - 1.2\cdot e^{-\frac{2500}{sec}\cdot t} \quad A$

However you get to it, at least this curve is slightly more interesting than the $v_C(t)$.



We could have found the same result from $v_C(t)$, using that to find $i_L(t)$:

$$i_C(t) = C\cdot\frac{d}{dt}v_C(t) = C\cdot\frac{d}{dt}\left(12\cdot V - 60\cdot V\cdot e^{-\frac{2000}{sec}\cdot t} + 48\cdot V\cdot e^{-\frac{2500}{sec}\cdot t}\right)$$

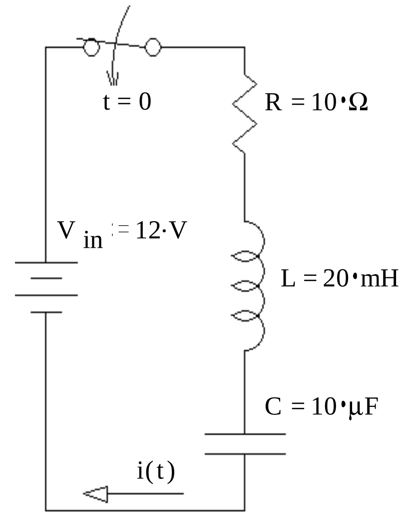
$$= C\cdot(-60\cdot V)\cdot\left(-\frac{2000}{sec}\right)\cdot e^{-\frac{2000}{sec}\cdot t} + C\cdot 48\cdot V\cdot\left(-\frac{25}{sec}\right)\cdot e^{-\frac{2500}{sec}\cdot t}$$

$C\cdot(-60\cdot V)\cdot\left(-\frac{2000}{sec}\right) = 1.2\cdot A \qquad C\cdot 48\cdot V\cdot\left(-\frac{2500}{sec}\right) = -1.2\cdot A$ $\quad$ and $\quad i(t) := 1.2\cdot e^{-\frac{2000}{sec}\cdot t} - 1.2\cdot e^{-\frac{2500}{sec}\cdot t}$

same

Underdamped Example

$R := 10\cdot\Omega \qquad L := 20\cdot mH \qquad C := 10\cdot\mu F$



$$s_1 := -\frac{R}{2\cdot L} + \frac{1}{2}\cdot\sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{L\cdot C}} \qquad s_1 = -250 + 2.222\cdot 10^3 j \quad \cdot sec^{-1}$$

$$s_2 := -\frac{R}{2\cdot L} - \frac{1}{2}\cdot\sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{L\cdot C}} \qquad s_2 = -250 - 2.222\cdot 10^3 j \quad \cdot sec^{-1}$$

$$\alpha := -250\cdot\frac{1}{sec} \qquad \omega := \mathrm{Im}(s_1) \qquad \omega = 2222\cdot\frac{rad}{sec}$$

The final and initial conditions are the same as before, since they did not depend on R and R is the only component that is different.

Let's find the current again this time.

$i(t) = \; i(\infty) + e^{\alpha\cdot t}\cdot(B\cdot\cos(\omega\cdot t) + D\cdot\sin(\omega\cdot t))$ (underdamped this time)

$i(0) = \; i(\infty) + B,$

$0 \; = \; 0 + B \qquad B := 0\cdot A$

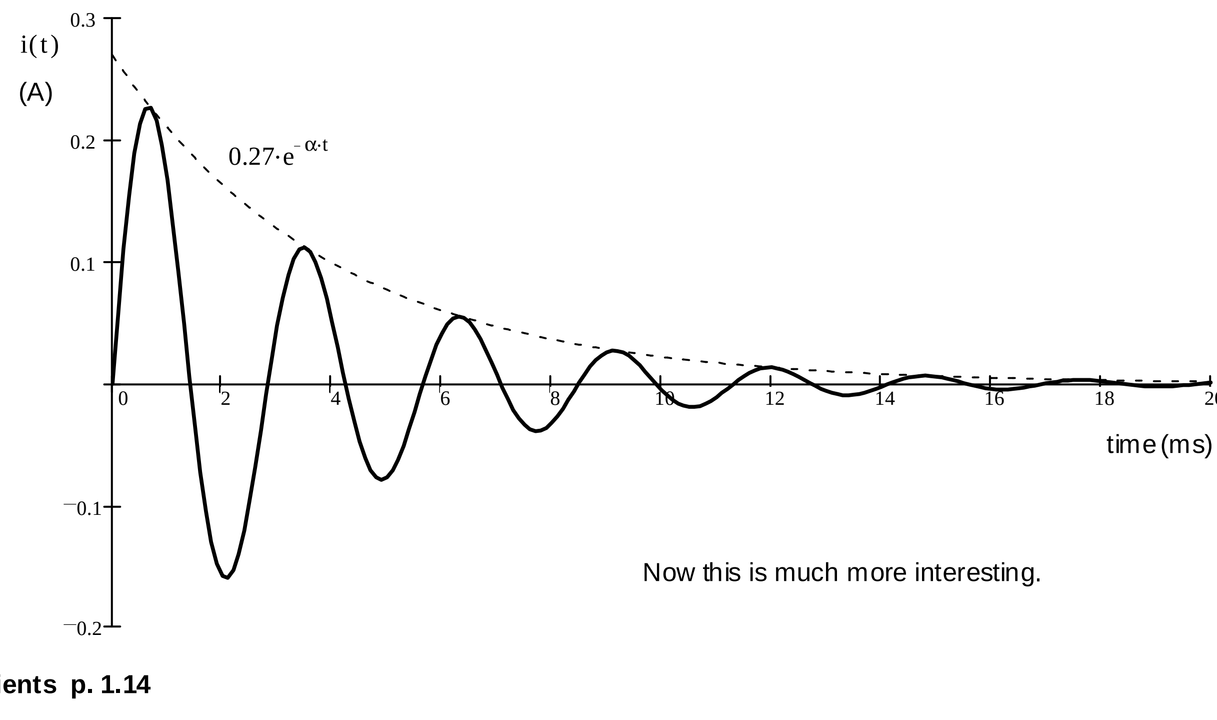
Differentiate the solution: $i(t) = \; i(\infty) + e^{\alpha\cdot t}\cdot(B\cdot\cos(\omega\cdot t) + D\cdot\sin(\omega\cdot t))$

to get: $\frac{d}{dt}i(t) \; = \; \alpha\cdot e^{\alpha\cdot t}\cdot(B\cdot\cos(\omega\cdot t) + D\cdot\sin(\omega\cdot t)) + e^{\alpha\cdot t}\cdot(-B\cdot\sin(\omega\cdot t)\cdot\omega + D\cdot\cos(\omega\cdot t)\cdot\omega)$

At time t = 0: $\frac{d}{dt}i(0) = \; B\cdot\alpha + D\cdot\omega$ Solve for D: $D \; = \; \frac{\frac{d}{dt}i(0) - B\cdot\alpha}{\omega}$

$$\frac{d}{dt}i(0) = \; \frac{12\cdot V}{L} \qquad D \; = \; \frac{\frac{12\cdot V}{L} - B\cdot\alpha}{\omega} = 0.27\cdot A$$

Substitute back in: $i(t) := e^{\alpha\cdot t}\cdot(0.27\cdot\sin(\omega\cdot t)) \quad A$



Now this is much more interesting.

Critically Damped Example

First we have to figure out how to get this case

Change R's value to create critical damping: $\left(\frac{R}{L}\right)^2 - \frac{4}{L \cdot C} = 0$ $\quad R := 2 \cdot \sqrt{\frac{L}{C}}$ $\quad R = 89.44271909999159 \cdot \Omega$

(exactly)

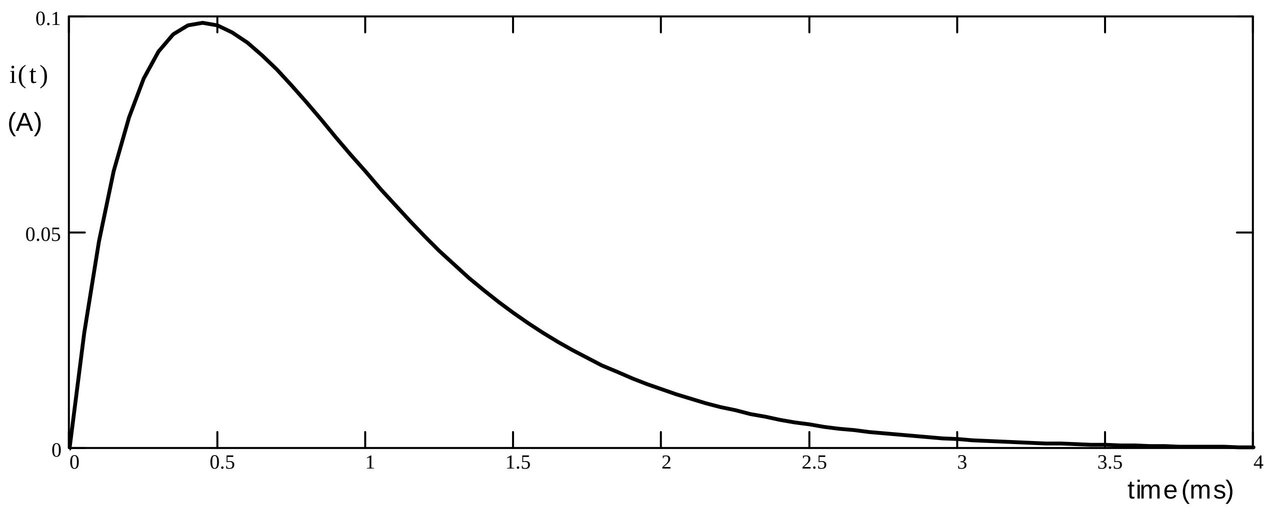
$s_1 := -\frac{R}{2 \cdot L} + \frac{1}{2} \cdot \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{L \cdot C}}$ $\quad s_1 = -2236 \cdot \mathrm{sec}^{-1}$ $\qquad s_2 := -\frac{R}{2 \cdot L} - \frac{1}{2} \cdot \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{L \cdot C}}$ $\quad s_2 = -2236 \cdot \mathrm{sec}^{-1}$

$i(t) = i(\infty) + B \cdot e^{s_1 \cdot t} + D \cdot t \cdot e^{s_2 \cdot t}$ $\quad i(\infty)$ = final condition $= 0 \cdot A$ Capacitor will charge up and current will stop.

$i(0) = i(\infty) + B = 0 \ , \quad B = 0$

$\frac{d}{dt} i(0) = B \cdot s_1 \cdot e^{s_1 \cdot t} + D \cdot e^{s_2 \cdot t} + D \cdot t \cdot s_2 \cdot e^{s_2 \cdot t} = B \cdot s_1 + D = \frac{12 \cdot V}{L}$ Since all initial voltage will be across inductor.

Solve for D: $D = \frac{12 \cdot V}{L} = 600 \cdot \frac{A}{\mathrm{sec}}$ Substitute back in: $i(t) := 600 \cdot \frac{A}{\mathrm{sec}} \cdot t \cdot e^{-\frac{2236}{\mathrm{sec}} \cdot t}$



if you notice a remarkable similarity with the overdamped case, that's common for critical damping.

1.7 The Easy Way to get the Characteristic Equation

Recall from your Ordinary Differential Equations class, the Laplace transform method of solving differential equations. The Laplace transform allowed you to change time-domain functions to frequency-domain functions. We've already done this for steady-state AC circuits. We changed functions of t into functions of $j\omega$. That was the frequency domain. Laplace let's us do the same sort of thing for transients. The general procedure is as follows.

1) Transform your forcing functions into the frequency domain with the Laplace transform.

2) Solve your differential equations with plain old algebra, where:

$\frac{d}{dt}$ operation can be replaced with s, and $\int \blacksquare \, dt$ can be replaced by $\frac{1}{s}$

3) Transform your result back to the time domain with the inverse Laplace transform.

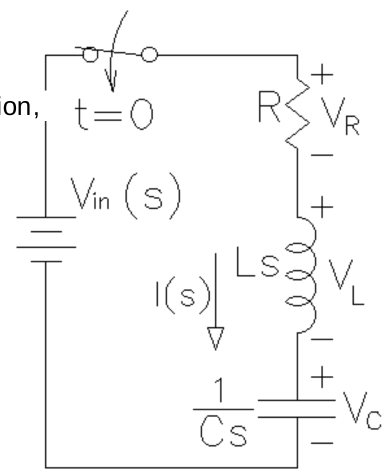
Step 1 isn't too bad, but step 3 can be a total pain without a good computer program to do the job. However, step 2 sounds great. It turns out that we can use step 2 alone and still learn a great deal about our circuits and other systems without ever bothering with steps 1 and 3.

First remember from your study of Laplace that differentiation in the time domain was the same as multiplication by s in the frequency domain. That's really all we need and we're off and running.

$$v_L(t) = L\cdot\frac{d}{dt}i_L(t) \dashrightarrow V_L(s) = L\cdot s\cdot I_L(s) \quad \text{and} \quad i_C(t) = C\cdot\frac{d}{dt}v_C(t) \dashrightarrow I_C(s) = C\cdot s\cdot V_C(s)$$

Leading to the Laplace impedances: Ls for an inductor and $\frac{1}{Cs}$ for a capacitor.

That's it, now we can use these impedances just like the $j\omega$ impedances, and we can use all the tools developed for DC. And with Laplace we don't even have to mess with complex numbers.



Look what happens to the RLC circuit now.

Pick any dependent variable ($I(s)$, $V_R(s)$, $V_L(s)$, or $V_C(s)$) and write a transfer function, which is a ratio of the dependent variable to the input ($V_{in}(s)$), like this:

$$V_{in}(s) = I(s)\cdot\left(\frac{1}{C\cdot s} + R + L\cdot s\right)$$

Transfer function = $H(s) = \dfrac{I(s)}{V_{in}(s)} = \dfrac{1}{\left(\frac{1}{C\cdot s} + R + L\cdot s\right)}$

Manipulate this transfer function into this form: $\dfrac{a_1\cdot s^2 + b_1\cdot s + k_1}{s^2 + b\cdot s + k}$

One polynomial divided by another.

$$\frac{I(s)}{V_{in}(s)} = \frac{1\cdot(C\cdot s)}{(1 + R + L\cdot s\cdot(C\cdot s))} = \frac{\frac{1}{L}\cdot s}{\left(s^2 + \frac{R}{L}\cdot s + \frac{1}{L\cdot C}\right)} \quad \text{in the correct form.}$$

Set the denominator to 0 and you get the characteristic equation: $s^2 + \frac{R}{L}\cdot s + \frac{1}{L\cdot C} = 0$

At this point you just proceed with the solution like you did before; Solve the characteristic equation to find s_1 and s_2. Decide which case you have (over-, under-, or critically damped). Use the two initial conditions, $i_L(0)$ and $v_C(0)$ to find the initial condition and the initial slope of your variable of interest, then use those to find the constants B and D.

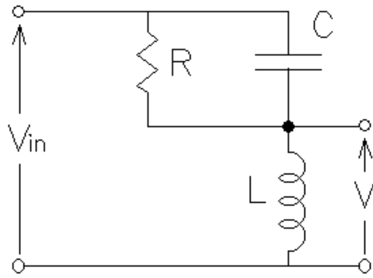
<u>Differential equation from the transfer function</u>

You can also use the transfer function to go back and find the differential equation, just replace each s with a $\frac{d}{dt}\blacksquare$ and go back to functions of t. $\frac{1}{L}\cdot\frac{d}{dt}V_{in}(t) = \left(\frac{d^2}{dt^2}i(t) + \frac{R}{L}\cdot\frac{d}{dt}i(t) + \frac{1}{L\cdot C}\cdot i(t)\right)$ Actually this is a pretty useless thing to do.

Second-Order Transient Examples

A.Stolp
10/29/02
rev 2/27/07

Ex. 1 a) Find the transfer function of the circuit shown. Write your equation in the form of one simple polynomial divided by another

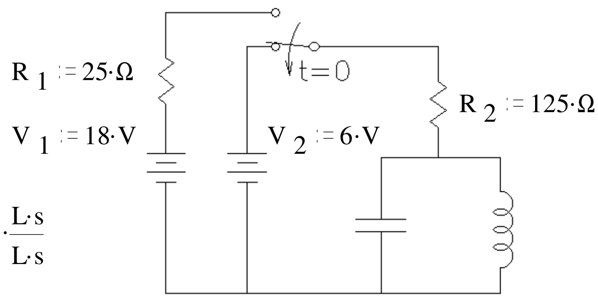


$$\mathbf{H}(s) = \frac{\mathbf{V}_o(s)}{\mathbf{V}_{in}(s)} = \frac{L\cdot s}{\dfrac{1}{\dfrac{1}{R}+C\cdot s}+L\cdot s}\cdot\frac{\left(\dfrac{1}{R}+C\cdot s\right)}{\left(\dfrac{1}{R}+C\cdot s\right)}$$

$$= \frac{\dfrac{L\cdot s}{R}+L\cdot C\cdot s^2}{\left[1+L\cdot s\cdot\left(\dfrac{1}{R}+C\cdot s\right)\right]} = \frac{\dfrac{L\cdot s}{R}+L\cdot C\cdot s^2}{1+\dfrac{L\cdot s}{R}+L\cdot C\cdot s^2} = \frac{\dfrac{L\cdot s}{R}+L\cdot C\cdot s^2}{L\cdot C\cdot s^2+\dfrac{L\cdot s}{R}+1}\cdot\frac{\dfrac{1}{L\cdot C}}{\dfrac{1}{L\cdot C}} = \frac{s^2+\dfrac{1}{C\cdot R}\cdot s}{s^2+\left(\dfrac{1}{C\cdot R}\right)\cdot s+\dfrac{1}{L\cdot C}}$$

b) Find the characteristic equation $\quad 0 = s^2+\dfrac{1}{C\cdot R}\cdot s+\dfrac{1}{L\cdot C}$

Ex. 2 a) Find the characteristic equation of the circuit shown (after the switch moves to the lower position at $t = 0$).



$C := 0.08\cdot\mu F \qquad L := 5\cdot mH$

$$\frac{\mathbf{V}_C(s)}{\mathbf{V}_{in}(s)} = \mathbf{H}(s) = \frac{\dfrac{1}{\dfrac{1}{L\cdot s}+C\cdot s}}{\dfrac{1}{\dfrac{1}{L\cdot s}+C\cdot s}+R_2}\cdot\frac{\left(\dfrac{1}{L\cdot s}+C\cdot s\right)}{\left(\dfrac{1}{L\cdot s}+C\cdot s\right)} = \frac{1}{1+\dfrac{R_2}{L\cdot s}+R_2\cdot C\cdot s}\cdot\frac{L\cdot s}{L\cdot s}$$

$$= \frac{L\cdot s}{L\cdot s+R_2+R_2\cdot C\cdot L\cdot s^2}\cdot\frac{\dfrac{1}{R_2\cdot C\cdot L}}{\dfrac{1}{R_2\cdot C\cdot L}} = \frac{\dfrac{1}{R_2\cdot C}\cdot s}{\dfrac{1}{R_2\cdot C}\cdot s+\dfrac{1}{L\cdot C}+s^2}$$

Characteristic equation:

$$0 = s^2+\frac{1}{R_2\cdot C}\cdot s+\frac{1}{L\cdot C}$$

b) Find the solutions of the characteristic equation. $\quad \dfrac{1}{R_2\cdot C} = 1\cdot 10^5\ \cdot sec^{-1} \qquad \dfrac{4}{L\cdot C} = 1\cdot 10^{10}\ \cdot sec^{-2}$

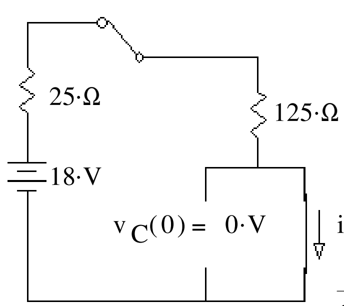
$$s_1 := \frac{-\dfrac{1}{R_2\cdot C}+\sqrt{\left(\dfrac{1}{R_2\cdot C}\right)^2-\dfrac{4}{L\cdot C}}}{2} \qquad s_1 = -5\cdot 10^4\ \cdot\frac{1}{sec}$$

$$s_2 := \frac{-\dfrac{1}{R_2\cdot C}-\sqrt{\left(\dfrac{1}{R_2\cdot C}\right)^2-\dfrac{4}{L\cdot C}}}{2} \qquad s_2 = -5\cdot 10^4\ \cdot\frac{1}{sec}$$

$s_1 = s_2$ so... critically damped

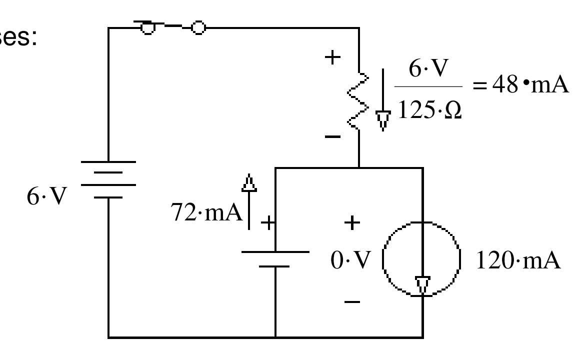
c) Find initial and final conditions for $v_C(t)$

before switch is moved:



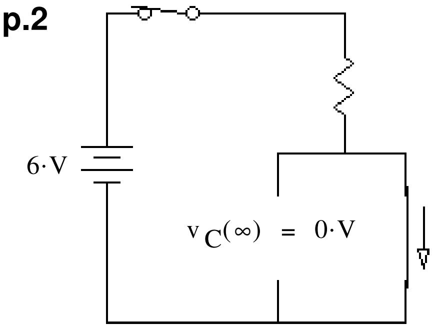
$$\frac{18\cdot V}{25\cdot\Omega+125\cdot\Omega} = 120\cdot mA$$

just after switch closes:



$$\frac{d}{dt}v_C(0) = \frac{-72\cdot mA}{C} = -9\cdot 10^5\ \cdot\frac{V}{sec}$$

Final conditions:



$v_C(\infty) = 0\cdot V$

$i_L(\infty) = \dfrac{6\cdot V}{125\cdot\Omega} = 48\cdot mA$

d) Find the full expression of $v_C(t)$.

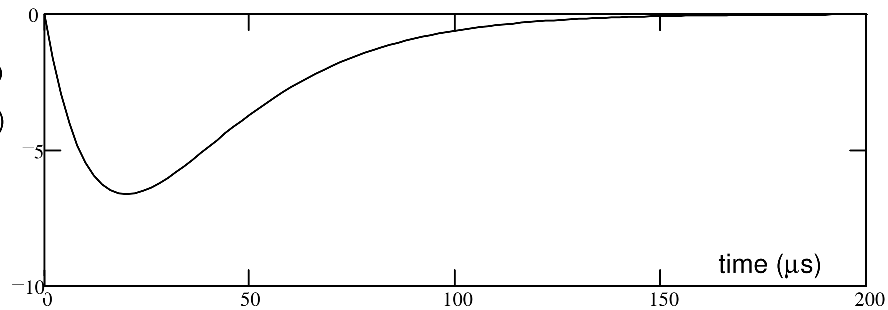
Critically damped $\quad v_C(t) = v_C(\infty) + B\cdot e^{s_1\cdot t} + D\cdot t\cdot e^{s_2\cdot t}$

$v_C(0) = v_C(\infty) + B$

$0 = 0 + B \qquad B = 0 \qquad \dfrac{d}{dt}v_C(0) = B\cdot s + D$

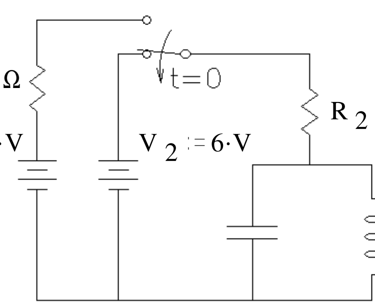
$-9\cdot 10^5\cdot\dfrac{V}{sec} = 0\cdot\left(-5\cdot 10^4\cdot\dfrac{1}{sec}\right) + D \qquad D := -9\cdot 10^5\cdot\dfrac{V}{sec}$

$v_C(t) := -9\cdot 10^5\cdot\dfrac{V}{sec}\cdot t\cdot e^{s_1\cdot t}$



Ex. 2 with bigger R_2

a) Find the characteristic equation of the circuit shown (after the switch moves to the lower position at t = 0).
as before:



$R_1 := 25\cdot\Omega \qquad V_1 := 18\cdot V \qquad V_2 := 6\cdot V \qquad R_2 := 1\cdot k\Omega$

$C := 0.08\cdot\mu F \qquad L := 5\cdot mH$

$$\frac{\mathbf{V}_C(s)}{\mathbf{V}_{in}(s)} = \mathbf{H}(s) = \frac{\dfrac{1}{R_2\cdot C}\cdot s}{\dfrac{1}{R_2\cdot C}\cdot s + \dfrac{1}{L\cdot C} + s^2}$$

Characteristic equation:

$$0 = s^2 + \frac{1}{R_2\cdot C}\cdot s + \frac{1}{L\cdot C}$$

b) Find the solutions of the characteristic equation.

$\dfrac{1}{R_2\cdot C} = 1.25\cdot 10^4 \cdot sec^{-1} \qquad \dfrac{4}{L\cdot C} = 1\cdot 10^{10} \cdot sec^{-2}$

$$s_1 := \frac{-\dfrac{1}{R_2\cdot C} + \sqrt{\left(\dfrac{1}{R_2\cdot C}\right)^2 - \dfrac{4}{L\cdot C}}}{2}$$

$s_1 = -6.25\cdot 10^3 + 4.961\cdot 10^4 j \cdot\dfrac{1}{sec}$

$s_2 = \overline{s_1} = -6.25\cdot 10^3 - 4.961\cdot 10^4 j \cdot\dfrac{1}{sec}$

s_1 & s_2 complex, so underdamped

$\alpha := \mathrm{Re}(s_1) \qquad \alpha = -6.25\cdot 10^3 \cdot sec^{-1}$

$\omega := \mathrm{Im}(s_1) \qquad \omega = 4.961\cdot 10^4 \cdot sec^{-1}$

c) Find initial and final conditions for $v_C(t)$
See drawings above

$$v_C(0) = 0 \cdot V$$

$$i_L(0) = \frac{18 \cdot V}{25 \cdot \Omega + 1 \cdot k\Omega} = 17.561 \cdot mA$$

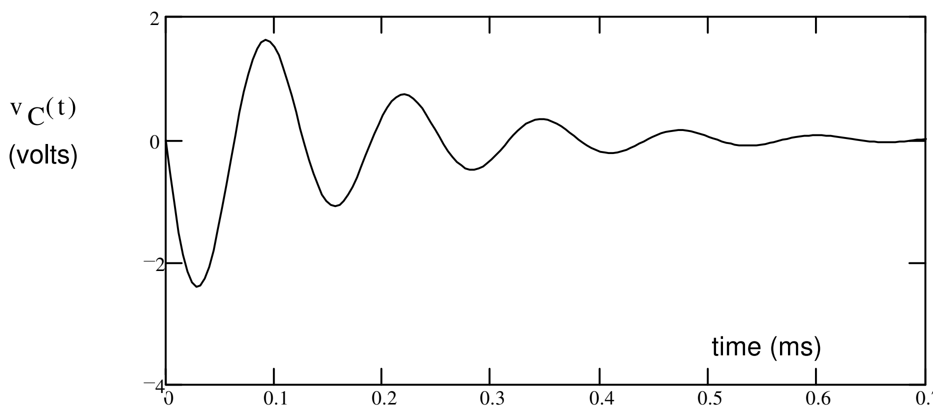
$$i_C(0) = \frac{6 \cdot V}{1 \cdot k\Omega} - 17.561 \cdot mA = -11.561 \cdot mA \qquad \frac{d}{dt} v_C(0) = \frac{-11.561 \cdot mA}{C} = -1.445 \cdot 10^5 \cdot \frac{V}{sec}$$

$$v_C(\infty) = 0 \cdot V$$

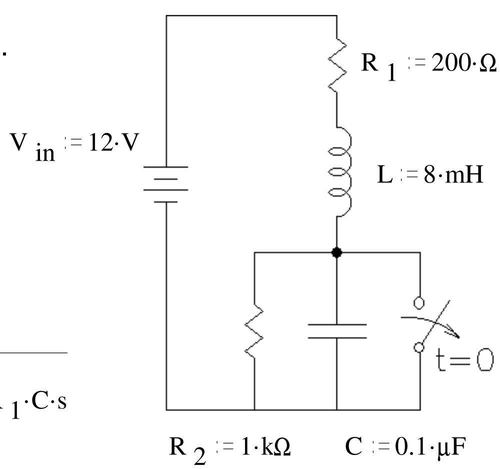
d) Find the full expression of $v_C(t)$. <u>Underdamped</u> $v_C(t) = v_C(\infty) + e^{\alpha t} \cdot (B \cdot \cos(\omega \cdot t) + D \cdot \sin(\omega \cdot t))$

$$B = v_C(0) - v_C(\infty) = 0 \cdot V - 0 \cdot V = 0 \cdot V$$

$$D = \frac{\frac{d}{dt} v_C(0) - B \cdot \alpha}{\omega} = \frac{-1.445 \cdot 10^5 \cdot \frac{V}{sec} - 0 \cdot V \cdot \alpha}{\omega} = -2.913 \cdot V \qquad v_C(t) := -2.913 \cdot V \cdot e^{-\frac{6250}{sec} \cdot t} \cdot \sin\left(\frac{49610}{sec} \cdot t\right)$$



Ex. 3 a) Find the characteristic equation of the circuit shown. (after the switch opens at $t = 0$). Write your equation in the form of a simple polynomials.



$$\mathbf{H}(s) = \frac{\mathbf{I_T}(s)}{\mathbf{V_{in}}(s)} = \frac{1}{\mathbf{Z}(s)} = \frac{1}{\frac{1}{\frac{1}{R_2} + C \cdot s} + L \cdot s + R_1} \cdot \frac{\left(\frac{1}{R_2} + C \cdot s\right)}{\left(\frac{1}{R_2} + C \cdot s\right)}$$

$$= \frac{\frac{1}{R_2} + C \cdot s}{1 + L \cdot s \cdot \left(\frac{1}{R_2} + C \cdot s\right) + R_1 \cdot \left(\frac{1}{R_2} + C \cdot s\right)} = \frac{\frac{1}{R_2} + C \cdot s}{1 + \frac{L \cdot s}{R_2} + L \cdot C \cdot s^2 + \frac{R_1}{R_2} + R_1 \cdot C \cdot s}$$

$$= \frac{\frac{1}{R_2} + C \cdot s}{L \cdot C \cdot s^2 + \frac{L \cdot s}{R_2} + R_1 \cdot C \cdot s + 1 + \frac{R_1}{R_2}} \cdot \frac{\frac{1}{L \cdot C}}{\frac{1}{L \cdot C}} = \frac{\frac{1}{L \cdot C \cdot R_2} + \frac{C}{L \cdot C} \cdot s}{s^2 + \left(\frac{L}{L \cdot C \cdot R_2} + \frac{R_1 \cdot C}{L \cdot C}\right) \cdot s + \left(1 + \frac{R_1}{R_2}\right) \cdot \frac{1}{L \cdot C}}$$

$$= \frac{1}{L} \cdot \frac{s + \frac{1}{C \cdot R_2}}{s^2 + \left(\frac{1}{C \cdot R_2} + \frac{R_1}{L}\right) \cdot s + \left(1 + \frac{R_1}{R_2}\right) \cdot \frac{1}{L \cdot C}}$$

Characteristic eq.: $0 = s^2 + \left(\frac{1}{C \cdot R_2} + \frac{R_1}{L}\right) \cdot s + \left(1 + \frac{R_1}{R_2}\right) \cdot \frac{1}{L \cdot C}$

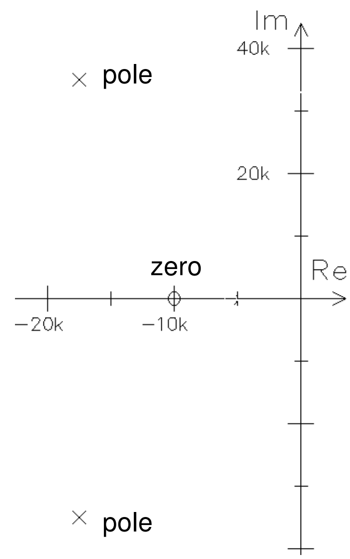
b) Find the solutions (numbers) of the characteristic equation:

$b := \dfrac{1}{C\cdot R_2} + \dfrac{R_1}{L}$ $\qquad b = 3.5\cdot 10^4 \cdot sec^{-1}$ $\qquad k := \left(1 + \dfrac{R_1}{R_2}\right)\cdot\dfrac{1}{L\cdot C}$ $\qquad k = 1.5\cdot 10^9 \cdot sec^{-2}$

$s_1 := \dfrac{-b + \sqrt{b^2 - 4\cdot k}}{2}$ $\qquad s_1 = -1.75\cdot 10^4 + 3.455\cdot 10^4 j \cdot\dfrac{1}{sec}$ $\qquad \alpha := -\dfrac{b}{2}$ $\qquad \alpha = -1.75\cdot 10^4 \cdot sec^{-1}$

$s_2 := \dfrac{-b - \sqrt{b^2 - 4\cdot k}}{2}$ $\qquad s_2 = -1.75\cdot 10^4 - 3.455\cdot 10^4 j \cdot\dfrac{1}{sec}$ $\qquad \omega := \dfrac{1}{2}\cdot\sqrt{4\cdot k - b^2}$ $\qquad \omega = 3.455\cdot 10^4 \cdot sec^{-1}$

Underdamped



c) Plot the poles and zeroes of the transfer function.

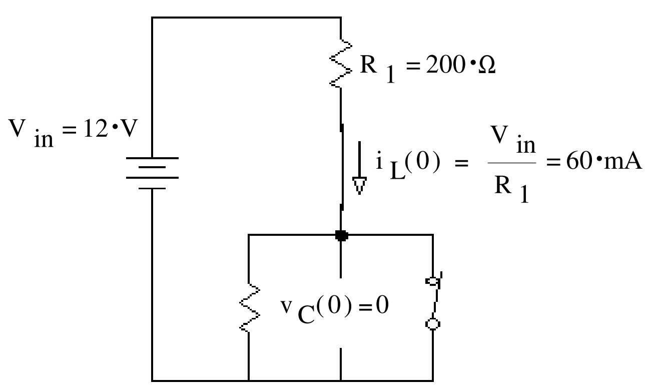
The poles are the s's where the denominator is zero, that is, the s_1 & s_2 solutions to the characteristic equation.

The zero is the s where the numerator is zero: $0 = \dfrac{1}{L\cdot C\cdot R_2} + \dfrac{C}{L\cdot C}\cdot s$

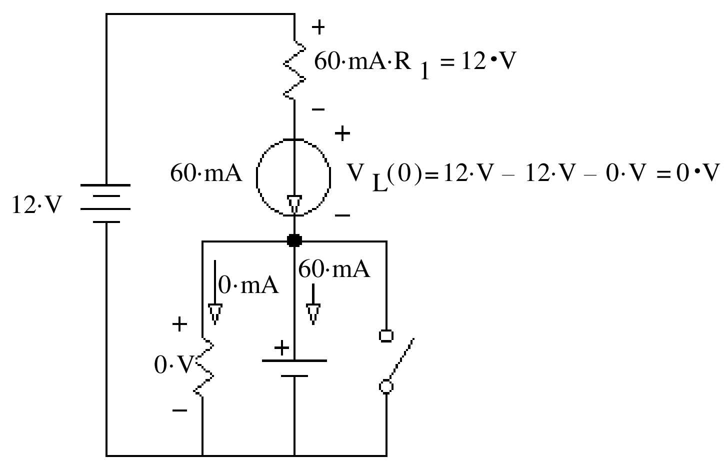
$$s = -\frac{1}{C\cdot R_2} = -1\cdot 10^4 \cdot sec^{-1}$$

d) Initial and final conditions for $i_L(t)$ and $v_C(t)$.

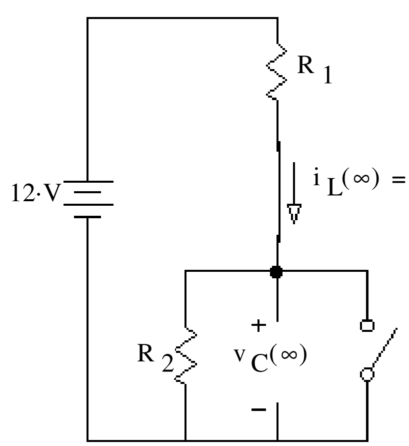
before the switch opens



just after the switch opens



Final condition:



$i_L(\infty) = \dfrac{V_{in}}{R_1 + R_2} = 10\cdot mA$

$\dfrac{d}{dt} i_L(0) = \dfrac{V_L(0)}{L} = \dfrac{0\cdot V}{L} = 0\cdot\dfrac{A}{sec}$

$\dfrac{d}{dt} v_C(0) = \dfrac{i(0)}{C} = \dfrac{60\cdot mA}{C} = 6\cdot 10^5 \cdot\dfrac{V}{sec}$

$v_C(\infty) = \dfrac{R_2}{R_1 + R_2}\cdot V_{in} = \dfrac{1\cdot k\Omega}{200\cdot\Omega + 1\cdot k\Omega}\cdot 12\cdot V = 10\cdot V$

e) Find the full expression of $i_L(t)$.

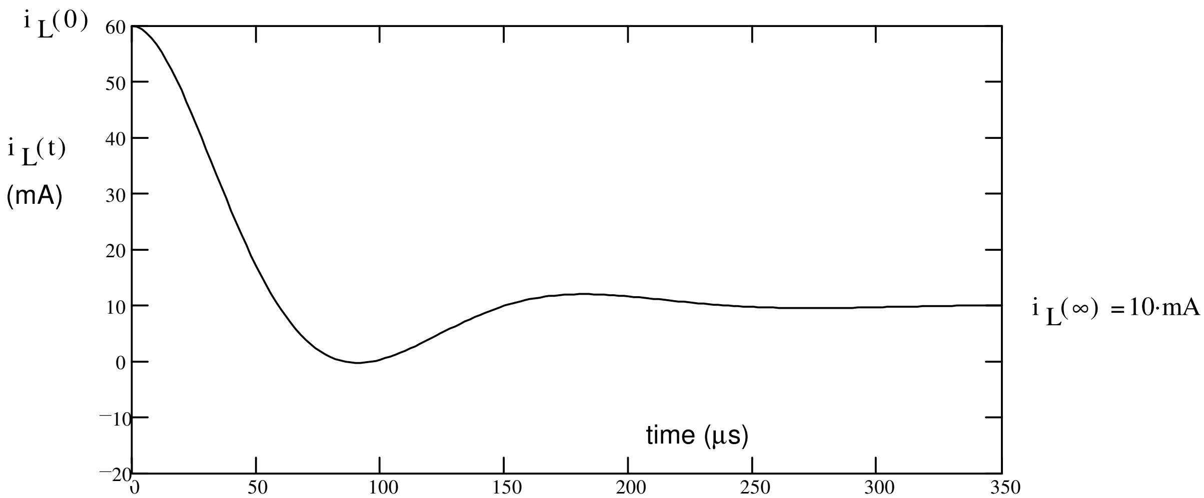
Underdamped $\quad X(t) = X(\infty) + e^{\alpha \cdot t} \cdot (B \cdot \cos(\omega \cdot t) + D \cdot \sin(\omega \cdot t))$

$$i_L(t) = i_L(\infty) + e^{\alpha \cdot t} \cdot (B \cdot \cos(\omega \cdot t) + D \cdot \sin(\omega \cdot t))$$

$i_L(0) = i_L(\infty) + B \quad$ so.. $\quad B = i_L(0) - i_L(\infty) \qquad B := 60 \cdot mA - 10 \cdot mA \qquad B = 50 \cdot mA$

$\frac{d}{dt} i_L(0) = B \cdot \alpha + D \cdot \omega \quad$ so.. $\quad D = \dfrac{\frac{d}{dt} i_L(0) - B \cdot \alpha}{\omega} \qquad D := \dfrac{0 \cdot \frac{A}{sec} - B \cdot \alpha}{\omega} \qquad D = 25.325 \cdot mA$

$$i_L(t) := 10 \cdot mA + e^{-\frac{17500}{sec} \cdot t} \cdot \left(50 \cdot mA \cdot \cos\left(\frac{34550}{sec} \cdot t\right) + 25.325 \cdot mA \cdot \sin\left(\frac{34550}{sec} \cdot t\right)\right)$$

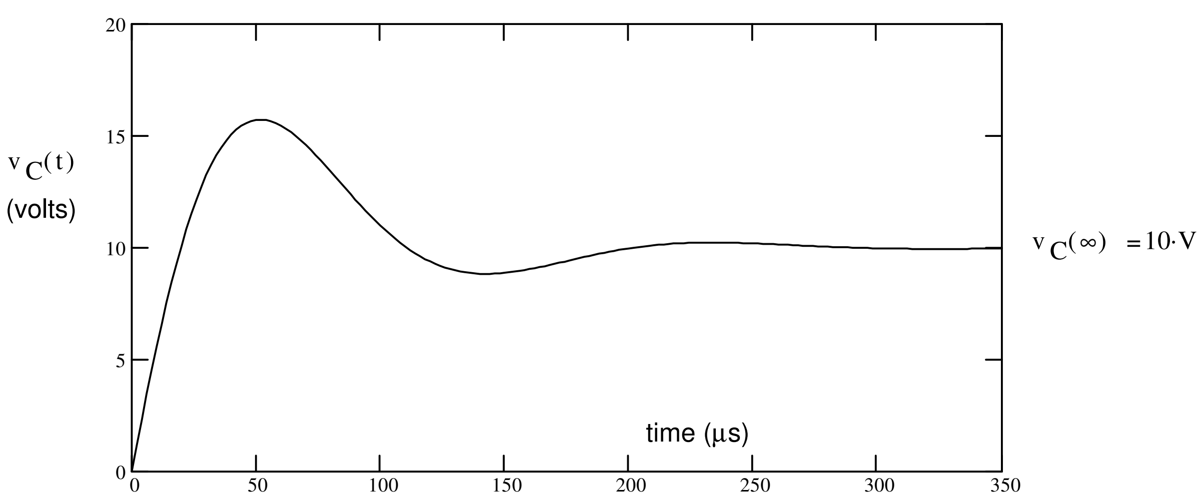


f) Find the full expression of $v_C(t)$.

$B := 0 \cdot V - 10 \cdot V \qquad B = -10 \cdot V \qquad D := \dfrac{6 \cdot 10^5 \cdot \frac{V}{sec} - B \cdot \alpha}{\omega} \qquad D = 12.301 \cdot V$

$$v_C(t) = v_C(\infty) + e^{\alpha \cdot t} \cdot (B \cdot \cos(\omega \cdot t) + D \cdot \sin(\omega \cdot t))$$

$$v_C(t) := 10 \cdot V + e^{-\frac{17500}{sec} \cdot t} \cdot \left(-10 \cdot V \cdot \cos\left(\frac{34550}{sec} \cdot t\right) + 12.301 \cdot V \cdot \sin\left(\frac{34550}{sec} \cdot t\right)\right)$$



h) What value of R_1 would make this system critically damped?

$$\left(\frac{1}{C\cdot R_2}+\frac{R_1}{L}\right)^2 = 4\cdot\left(1+\frac{R_1}{R_2}\right)\cdot\frac{1}{L\cdot C}$$

$$\frac{1}{C^2\cdot R_2^{\,2}}+\frac{2}{C\cdot R_2}\cdot\frac{R_1}{L}+\frac{R_1^{\,2}}{L^2} = \frac{4}{L\cdot C}+\frac{4}{C\cdot R_2}\cdot\frac{R_1}{L}$$

$$0 = \left(\frac{1}{L^2}\cdot R_1^{\,2}-\frac{2}{L\cdot C\cdot R_2}\cdot R_1\right)+\left(\frac{1}{C^2\cdot R_2^{\,2}}-\frac{4}{L\cdot C}\right)$$

$$0 = \left(R_1^{\,2}-\frac{2\cdot L}{C\cdot R_2}\cdot R_1\right)+\left(\frac{L^2}{C^2\cdot R_2^{\,2}}-\frac{4\cdot L}{C}\right)$$

Solve for R_1 with quadradic equation:

$$R_1 = \frac{\dfrac{2\cdot L}{C\cdot R_2}+\sqrt{\left(\dfrac{2\cdot L}{C\cdot R_2}\right)^2-4\cdot\left(\dfrac{L^2}{C^2\cdot R_2^{\,2}}-\dfrac{4\cdot L}{C}\right)}}{2}$$

Quadradic equation can be reduced to:

$$= \frac{L}{C\cdot R_2}-\frac{4}{2}\cdot\sqrt{\frac{L}{C}} = -485.7\,\Omega$$ this solution can't be

$$= \frac{L}{C\cdot R_2}+\frac{4}{2}\cdot\sqrt{\frac{L}{C}} = 645.7\,\Omega$$ this must be the solution

Ex. 2 with bigger R_1

$R_1 := 1\cdot k\Omega$ This should make the system overdamped

$$b := \frac{1}{C\cdot R_2}+\frac{R_1}{L} \qquad b = 1.35\cdot 10^5\ \cdot sec^{-1} \qquad k := \left(1+\frac{R_1}{R_2}\right)\cdot\frac{1}{L\cdot C} \qquad k = 2.5\cdot 10^9\ \cdot sec^{-2}$$

$$s_1 := \frac{-b+\sqrt{b^2-4\cdot k}}{2} \qquad s_1 = -2.215\cdot 10^4\ \cdot\frac{1}{sec} \qquad s_2 := \frac{-b-\sqrt{b^2-4\cdot k}}{2} \qquad s_2 = -1.128\cdot 10^5\ \cdot\frac{1}{sec}$$

Overdamped

$$v_C(0) = 0 \qquad i_L(0) = \frac{V_{in}}{R_1} = 12\cdot mA = i_C(0) \qquad \frac{d}{dt}v_C(0) = \frac{i(0)}{C} = \frac{12\cdot mA}{C} = 1.2\cdot 10^5\ \cdot\frac{V}{sec}$$

$$v_C(\infty) = \frac{R_2}{R_1+R_2}\cdot V_{in} = 6\cdot V \qquad i_L(\infty) = \frac{V_{in}}{R_1+R_2} = 6\cdot mA$$

$$1.2\cdot 10^5\cdot\frac{1}{sec} =$$

$$v_C(0) = v_C(\infty)+B+D$$

$$0\cdot V = 6\cdot V+B+D \qquad B = -(6\cdot V+D)$$

$$\frac{d}{dt}v_C(0) = B\cdot s_1+D\cdot s_2 = -6\cdot V\cdot s_1-D\cdot s_1+D\cdot s_2 \qquad D := \frac{1.2\cdot 10^5\cdot\dfrac{V}{sec}+6\cdot V\cdot s_1}{s_2-s_1}$$

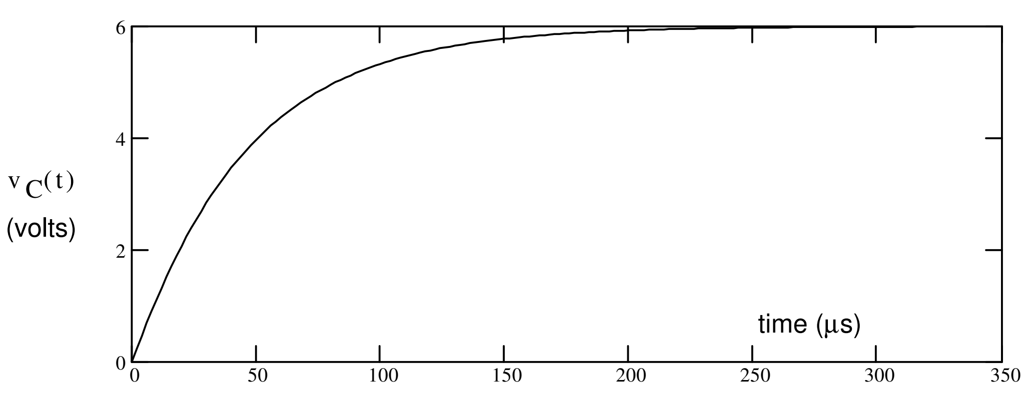
$$D = 0.143\cdot V$$

$$B = -(6\cdot V+D) = -6.143\cdot V$$

$$v_C(t) = v_C(\infty)+B\cdot e^{s_1\cdot t}+D\cdot e^{s_2\cdot t}$$

$$v_C(t) := 6\cdot V-6.143\cdot V\cdot e^{-\frac{2.215\cdot 10^4}{sec}\cdot t}+0.143\cdot V\cdot e^{-\frac{1.128\cdot 10^5}{sec}\cdot t}$$

$$v_C(\infty) = 6\cdot V$$

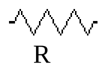
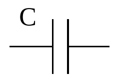


Notes, Second Order Transients

ECE 2210

A.Stolp
4/6/00,
2/25/16

Laplace impedances

Resistor	Capacitor	Inductor
		
$\mathbf{Z_R} = R$	$\mathbf{Z_C} = \dfrac{1}{C\cdot s}$	$\mathbf{Z_L} = L\cdot s$

Transfer function

Use Laplace impedances, manipulate your circuit equation(s) to find a transfer function:

May be $\mathbf{I_X}$ or any desired variable

a_1, b_1, k_1 coefficients may be zero

Rearrange circuit equation to: $\mathbf{H}(s) = \dfrac{\text{output}}{\text{input}} = \dfrac{\mathbf{V_X}(s)}{\mathbf{V_{in}}(s)} = \dfrac{a_1\cdot s^2 + b_1\cdot s + k_1}{s^2 + b\cdot s + k}$ = transfer function

May be $\mathbf{I_{in}}$ or any forcing function

Characteristic equation

$$s^2 + b\cdot s + k = 0$$

To find the poles of the transfer function — characteristic equation

Complete solution

Solutions to the characteristic equation: $s_1 = -\dfrac{b}{2} + \dfrac{\sqrt{b^2 - 4\cdot k}}{2}$ $\quad s_2 = -\dfrac{b}{2} - \dfrac{\sqrt{b^2 - 4\cdot k}}{2}$

Find initial Conditions (v_C and/or i_L)

Find conditions of just before time $t = 0$, v_C(0-) and i_L(0-). These will be the same just after time $t = 0$, v_C(0+) and i_L(0+) and will be your initial conditions.

Use normal circuit analysis to find your desired variable: $v_X(0)$ or $i_X(0)$

Also find: $\dfrac{d}{dt}v_X(0)$ or $\dfrac{d}{dt}i_X(0)$ The trick to finding these is to see that: $\dfrac{d}{dt}v_C(0) = \dfrac{i_C(0)}{C}$ and $\dfrac{d}{dt}i_L(0) = \dfrac{v_L(0)}{L}$

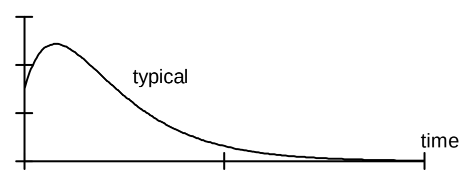
Find final conditions ("steady-state" or "forced" solution)

DC inputs: Inductors are shorts Capacitors are opens Solve by DC analysis $\quad v_X(\infty)$ or $i_X(\infty)$

AC inputs: Solve by AC steady-state analysis using $j\omega$

X(t) may be replaced by $v_X(t)$, $i_X(t)$ or any desired variable in the equations below

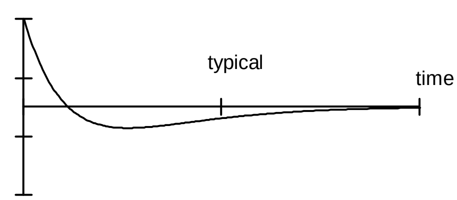
<u>Overdamped</u> $\quad b^2 - 4\cdot k > 0$ $\quad s_1$ and s_2 are real and negative



$$X(t) = X(\infty) + B\cdot e^{s_1\cdot t} + D\cdot e^{s_2\cdot t}$$

$X(0) = X(\infty) + B + D$ $\quad \dfrac{d}{dt}X(0) = B\cdot s_1 + D\cdot s_2$ $\quad$ Solve simultaneously for B and D

<u>Critically damped</u> $\quad b^2 - 4\cdot k = 0$ $\quad s_1 = s_2 = -\dfrac{b}{2} = s$ $\quad s_1$ and s_2 are real, equal and negative



$$X(t) = X(\infty) + B\cdot e^{s\cdot t} + D\cdot t\cdot e^{s\cdot t}$$

$X(0) = X(\infty) + B$

so.. $B = X(0) - X(\infty)$ $\quad \dfrac{d}{dt}X(0) = B\cdot s + D$ $\quad$ so.. $D = \dfrac{d}{dt}X(0) - B\cdot s$

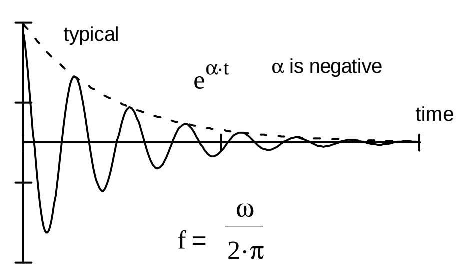
<u>Underdamped</u> $\quad b^2 - 4\cdot k < 0$ $\quad s_1 = \alpha + j\cdot\omega \quad s_2 = \alpha - j\cdot\omega$ $\quad \alpha$ is negative

complex s_1 and s_2

$$X(t) = X(\infty) + e^{\alpha\cdot t}\cdot(B\cdot\cos(\omega\cdot t) + D\cdot\sin(\omega\cdot t))$$

$X(0) = X(\infty) + B$

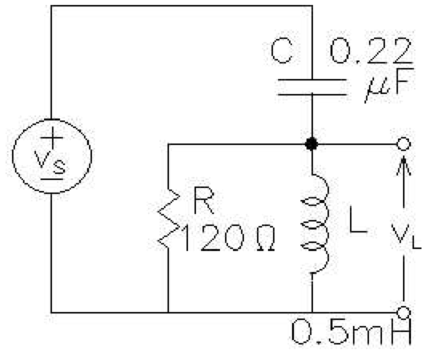
so.. $B = X(0) - X(\infty)$ $\quad \dfrac{d}{dt}X(0) = B\cdot\alpha + D\cdot\omega$ $\quad$ so.. $D = \dfrac{\dfrac{d}{dt}X(0) - B\cdot\alpha}{\omega}$



ECE 2210 Lecture 18 notes Second order Transient examples

A. Stolp
10/30/06
2/19/10

Ex. 1 For the circuit shown:



a) Find the transfer function v_L.

$$\mathbf{V_L(s)} = \frac{\dfrac{1}{\dfrac{1}{L\cdot s}+\dfrac{1}{R}}}{\dfrac{1}{\dfrac{1}{L\cdot s}+\dfrac{1}{R}}+\dfrac{1}{C\cdot s}}\cdot\mathbf{V_S(s)}$$

$$= \frac{1}{1+\dfrac{1}{C\cdot s}\cdot\left(\dfrac{1}{L\cdot s}+\dfrac{1}{R}\right)}\cdot\mathbf{V_S(s)} = \frac{1}{1+\dfrac{1}{C\cdot s}\cdot\dfrac{1}{L\cdot s}+\dfrac{1}{C\cdot s}\cdot\dfrac{1}{R}}\cdot\mathbf{V_S(s)} = \frac{s^2}{s^2+\dfrac{1}{C\cdot R}\cdot s+\dfrac{1}{L\cdot C}}\cdot\mathbf{V_S(s)}$$

$$\mathbf{H(s)} = \frac{\mathbf{V_L(s)}}{\mathbf{V_S(s)}} = \frac{s^2}{s^2+\dfrac{1}{C\cdot R}\cdot s+\dfrac{1}{L\cdot C}}$$

$$= \frac{s^2}{s^2+\dfrac{3.788\cdot 10^4}{sec}\cdot s+\dfrac{9.091\cdot 10^9}{sec^2}}$$

$R := 120\cdot\Omega$ $\quad C := 0.22\cdot\mu F$ $\quad L := 0.5\cdot mH$

$\dfrac{1}{C\cdot R} = 3.788\cdot 10^4 \cdot\dfrac{1}{sec}$ $\qquad \dfrac{1}{L\cdot C} = 9.091\cdot 10^9 \cdot\dfrac{1}{sec^2}$

b) Find the characteristic equation for this circuit.

$$0 = s^2+\frac{1}{C\cdot R}\cdot s+\frac{1}{L\cdot C} = s^2+\frac{3.788\cdot 10^4}{sec}\cdot s+\frac{9.091\cdot 10^9}{sec^2}$$

Just the denominator set to zero. The solutions of the characteristic equation are the "poles" of the transfer function.

c) Find the differential equation for v_L.

Cross-multiply the transfer function

$$s^2\cdot\mathbf{V_S(s)} = \left(s^2+\frac{1}{C\cdot R}\cdot s+\frac{1}{L\cdot C}\right)\cdot\mathbf{V_L(s)}$$

$$s^2\cdot\mathbf{V_S(s)} = s^2\cdot\mathbf{V_L(s)}+\frac{1}{C\cdot R}\cdot s\cdot\mathbf{V_L(s)}+\frac{1}{L\cdot C}\cdot\mathbf{V_L(s)}$$

$$\frac{d^2}{dt^2}v_S(t) = \frac{d^2}{dt^2}v_L(t)+\frac{1}{C\cdot R}\cdot\frac{d}{dt}v_L(t)+\frac{1}{L\cdot C}\cdot v_L(t)$$

$$\frac{d^2}{dt^2}v_S(t) = \frac{d^2}{dt^2}v_L(t)+\frac{3.788\cdot 10^4}{sec}\cdot\frac{d}{dt}v_L(t)+\frac{9.091\cdot 10^9}{sec^2}\cdot v_L(t)$$

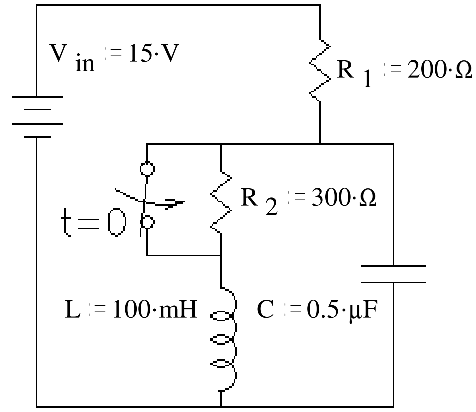
d) What are the solutions to the characteristic equation?

$$s_1 = \frac{-3.788\cdot 10^4}{2}+\frac{1}{2}\cdot\sqrt{\left(3.788\cdot 10^4\right)^2-4\cdot\left(9.091\cdot 10^9\right)} = -1.894\cdot 10^4 + 9.345\cdot 10^4 j$$

$$s_2 = \frac{-3.788\cdot 10^4}{2}-\frac{1}{2}\cdot\sqrt{\left(3.788\cdot 10^4\right)^2-4\cdot\left(9.091\cdot 10^9\right)} = -1.894\cdot 10^4 - 9.345\cdot 10^4 j$$

e) What type of response do you expect from this circuit?

The solutions to the characteristic equation are complex so the response will be **underdamped**.



Ex. 2 Analysis of the circuit shown yields the characteristic equation below. The switch has been in the open position for a long time and is closed (as shown) at time $t = 0$. Find the initial and final conditions and write the full expression for $i_L(t)$, including all the constants that you find.

$$s^2 + \left(\frac{1}{C \cdot R_1}\right) \cdot s + \left(\frac{1}{L \cdot C}\right) = 0$$

$$\left(\frac{1}{C \cdot R_1}\right) = 1 \cdot 10^4 \cdot \frac{1}{sec} \qquad \left(\frac{1}{L \cdot C}\right) = 2 \cdot 10^7 \cdot \frac{1}{sec^2}$$

$$s^2 + 10000 \cdot \frac{1}{sec} \cdot s + 2 \cdot 10^7 \cdot \frac{1}{sec^2} = 0$$

$$s_1 := \left[\frac{-10000}{2} + \frac{1}{2} \cdot \sqrt{(10000)^2 - 4 \cdot \left(2 \cdot 10^7\right)}\right] \cdot sec^{-1} \qquad s_2 := \left[\frac{-10000}{2} - \frac{1}{2} \cdot \sqrt{(10000)^2 - 4 \cdot \left(2 \cdot 10^7\right)}\right] \cdot sec^{-1}$$

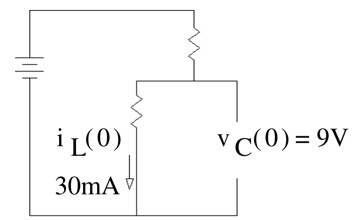
$$s_1 = -2764 \cdot sec^{-1} \qquad s_2 = -7236 \cdot sec^{-1}$$

s_1 and s_2 are both real and distinct, overdamped

Find the initial conditions:

Before the switch closed, the inductor current was: $\dfrac{15 \cdot V}{R_1 + R_2} = 30 \cdot mA \; = \; i_L(0)$

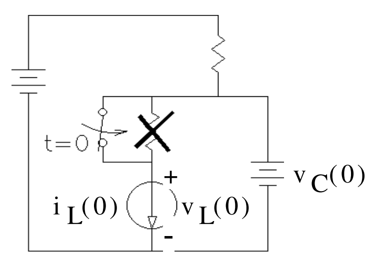
Before the switch closed, the capacitor voltage was: $\dfrac{R_2}{R_1 + R_2} \cdot (15 \cdot V) = 9 \cdot V \; = \; v_C(0)$



When the switch is closed, the inductor is suddenly in parallel with the capacitor, and:

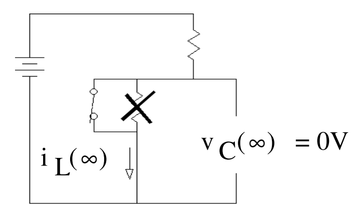
$$v_L(0) = v_C(0)$$

$$\frac{d}{dt} i_L(0) = \frac{1}{L} \cdot v_L(0) = \frac{1}{L} \cdot 9 \cdot V = 90 \cdot \frac{A}{sec}$$



Find the final condition:

$$i_L(\infty) = \frac{15 \cdot V}{R_1} = 75 \cdot mA$$



General solution for the overdamped condition: $i_L(t) = i_L(\infty) + B \cdot e^{s_1 \cdot t} + D \cdot e^{s_2 \cdot t}$

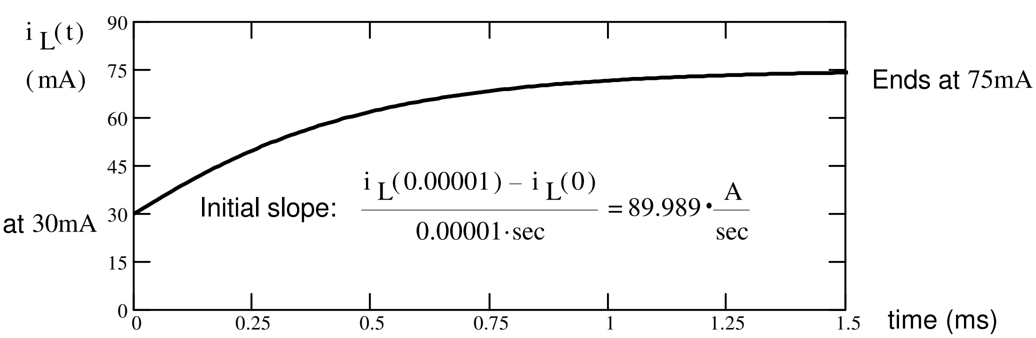
Initial conditions: $i_L(0) = \dfrac{15 \cdot V}{R_1 + R_2} = i_L(\infty) + B + D$, so $B = i_L(0) - i_L(\infty) - D = 30 \cdot mA - 75 \cdot mA - D$

$$= -45 \cdot mA - D$$

$$\frac{d}{dt} i_L(0) = 90 \cdot \frac{A}{sec} = s_1 \cdot B + s_2 \cdot D = s_1 \cdot (-45 \cdot mA - D) + s_2 \cdot D = s_1 \cdot (-45 \cdot mA) - s_1 \cdot D + s_2 \cdot D$$

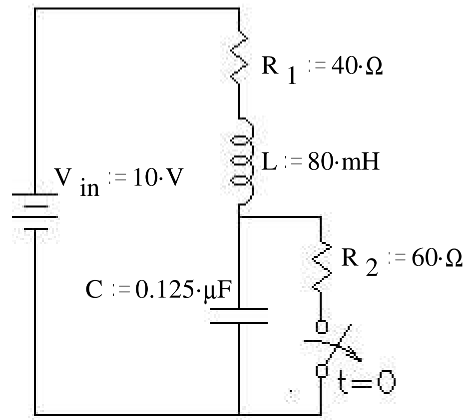
solve for D & B: $D := \dfrac{90 \cdot \frac{A}{sec} - s_1 \cdot (-45 \cdot mA)}{-s_1 + s_2}$ $\qquad D = 7.69 \cdot mA \qquad B := -45 \cdot mA - D \qquad B = -52.7 \cdot mA$

Plug numbers back in: $i_L(t) := 75 \cdot mA - 52.7 \cdot mA \cdot e^{-2764 t} + 7.69 \cdot mA \cdot e^{-7236 t}$



Ex. 3

Analysis of the circuit shown yields the characteristic equation and s values below. The switch has been in the closed position for a long time and is opened (as shown) at time $t = 0$. Find the initial and final conditions and write the full expression for $v_C(t)$, including all the constants.

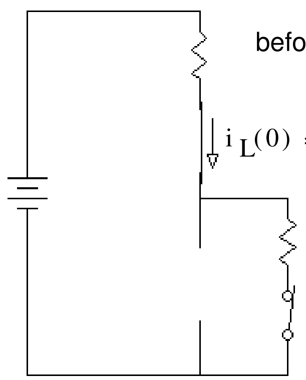


$$0 = s^2 + \frac{R_1}{L}\cdot s + \frac{1}{L\cdot C}$$

$$s_1 := \left(-250 + 10^4\cdot j\right)\cdot\frac{1}{sec} \quad , \qquad s_2 := \left(-250 - 10^4\cdot j\right)\cdot\frac{1}{sec}$$

Solution: $\alpha := -250\cdot\frac{1}{sec}$ $\qquad \omega := 10000\cdot\frac{rad}{sec}$

Initial conditions:

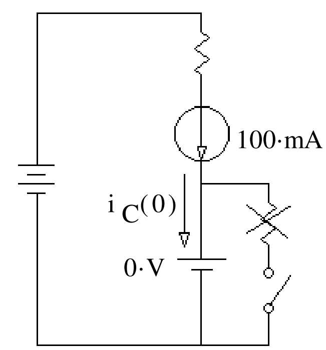


before switch opens

$$i_L(0) = \frac{V_{in}}{R_1 + R_2} = 100\cdot mA$$

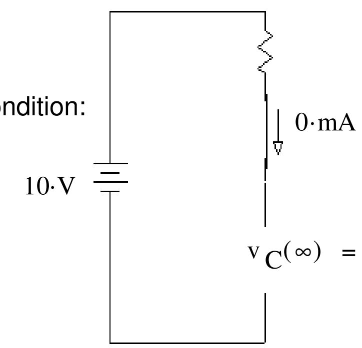
$$v_C(0) = V_{in}\cdot\frac{R_2}{R_1 + R_2} = 6\cdot V$$

just after the switch opens



$$\frac{d}{dt}v_C(0) = \frac{i_C(0)}{C} = \frac{100\cdot mA}{C} = 8\cdot 10^5\cdot\frac{V}{sec}$$

Find final condition:



$$v_C(\infty) = V_{in} = 10\cdot V$$

Find constants: $v_C(0) = v_C(\infty) + B$ $\qquad B = v_C(0) - v_C(\infty)$ $\qquad B := 6\cdot V - 10\cdot V$ $\qquad B = -4\cdot V$

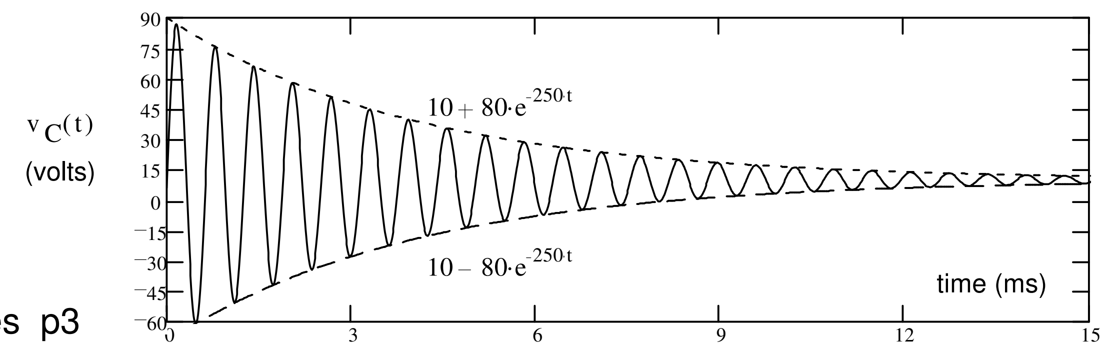
$$\frac{d}{dt}v_C(0) = \alpha\cdot B + D\cdot\omega \qquad D := \frac{8\cdot 10^5\cdot\frac{V}{sec} - \alpha\cdot B}{\omega} \qquad D = 79.9\cdot V$$

Write the full expression for $v_C(t)$, including all the constants that you find.

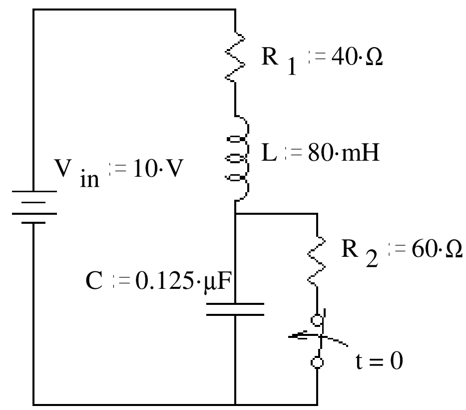
$$v_C(t) = e^{\alpha t}\cdot(B\cdot\cos(\omega\cdot t) + D\cdot\sin(\omega\cdot t)) + v_C(\infty)$$

$$v_C(t) := e^{-250t}\cdot\left(-4\cdot V\cdot\cos\left(10^4\cdot t\right) + 79.9\cdot V\cdot\sin\left(10^4\cdot t\right)\right) + 10\cdot V$$

$$\sqrt{D^2 + B^2} = 80\cdot V$$



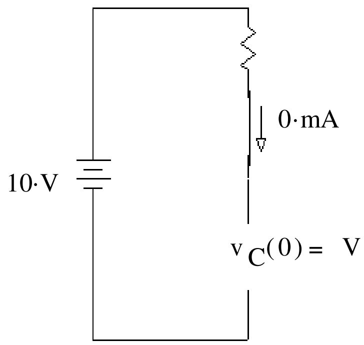
Ex. 4 Ex.3 Backwards, switch closes at $t = 0$



Characteristic eq.: $0 = s^2 + \left(\frac{1}{C \cdot R_2} + \frac{R_1}{L}\right) \cdot s + \left(1 + \frac{R_1}{R_2}\right) \cdot \frac{1}{L \cdot C}$

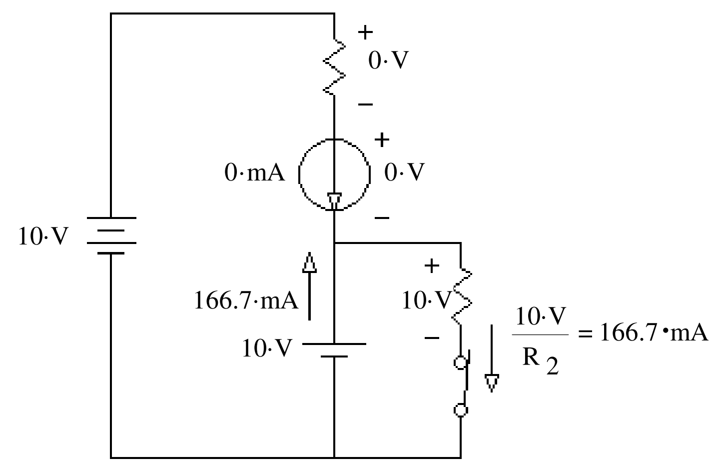
$s_1 := -1.257 \cdot 10^3 \cdot \frac{1}{sec}$ $\qquad s_2 := -1.326 \cdot 10^5 \cdot \frac{1}{sec}$

Initial conditions, same as Ex.3 final:



$v_C(0) = V_{in} = 10 \cdot V$

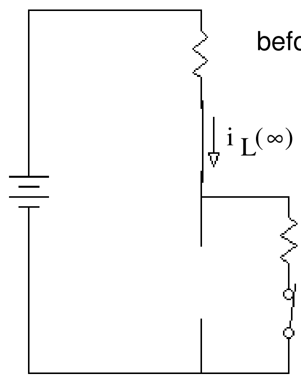
just after the switch opens



$\frac{d}{dt} v_C(0) = \frac{i_C(0)}{C} = \frac{-166.7 \cdot mA}{C} = -1.334 \cdot 10^6 \cdot \frac{V}{sec}$

Find final condition:

before switch opens



$i_L(\infty) = \frac{V_{in}}{R_1 + R_2} = 100 \cdot mA$

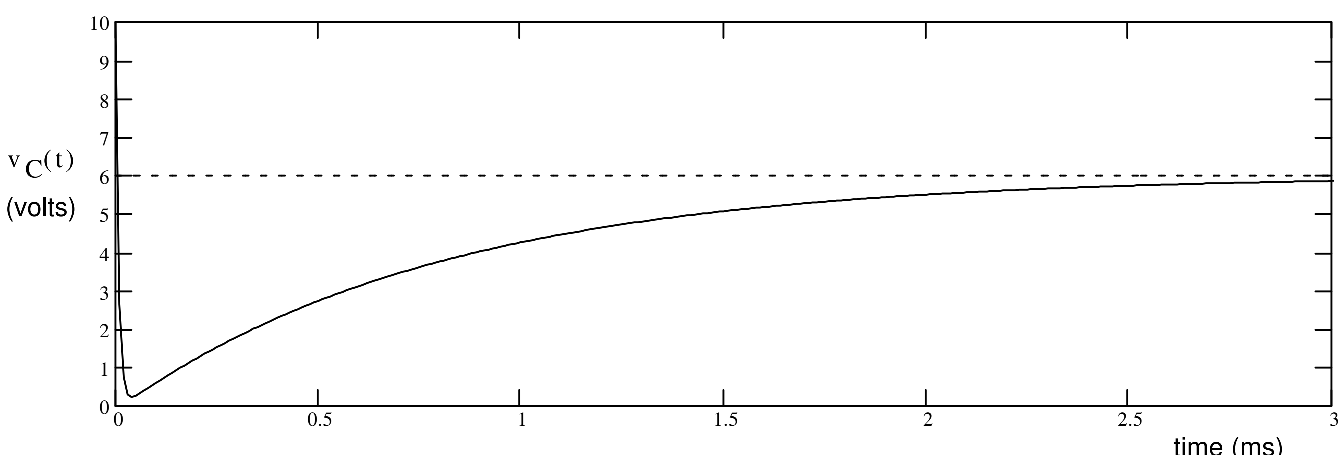
$v_C(\infty) = V_{in} \cdot \frac{R_2}{R_1 + R_2} = 6 \cdot V$

Find constants: $v_C(0) = v_C(\infty) + B + D$, so $B = v_C(0) - v_C(\infty) - D = 10 \cdot V - 6 \cdot V - D = 4 \cdot V - D$

$\frac{d}{dt} v_C(0) = -1.334 \cdot 10^6 \cdot \frac{V}{sec} = s_1 \cdot B + s_2 \cdot D = s_1 \cdot (4 \cdot V - D) + s_2 \cdot D = s_1 \cdot (4 \cdot V) - s_1 \cdot D + s_2 \cdot D$

$D := \frac{-1.334 \cdot 10^6 \cdot \frac{V}{sec} - s_1 \cdot (4 \cdot V)}{-s_1 + s_2}$ $\qquad D = 10.12 \cdot V$ $\qquad B := 4 \cdot V - D$ $\qquad B = -6.12 \cdot V$

$v_C(t) := 6 \cdot V - 6.12 \cdot V \cdot e^{-1257 t} + 10.12 \cdot V \cdot e^{-132600 \cdot t}$



Ex. 5 Analysis of a circuit (not pictured) yields the characteristic equation below.

$0 = s^2 + 400\cdot s + 400000$ $\qquad R := 80\cdot\Omega \qquad L := 20\cdot mH \qquad C := 2\cdot\mu F$

Further analysis yields the followiing initial and final conditions:

$i_L(0) = 120\cdot mA \qquad v_L(0) = -3\cdot V \qquad v_C(0) = 7\cdot V \qquad i_C(0) = -80\cdot mA$

$i_L(\infty) = 800\cdot mA \qquad v_L(\infty) = 0\cdot V \qquad v_C(\infty) = 12\cdot V \qquad i_C(\infty) = 0\cdot mA$

Write the full expression for $i_L(t)$, including all the constants that you find. $\qquad i_L(t) = ?$

Solution:

$$\frac{400}{2} = 200 \qquad \frac{\sqrt{400^2 - 4\cdot 400000}}{2} = 600j$$

$$s_1 := (-200 + 600\cdot j)\cdot\frac{1}{sec} \qquad \text{and} \qquad s_2 := (-200 - 600\cdot j)\cdot\frac{1}{sec}$$

$$\alpha := Re(s_1) \qquad \alpha = -200\cdot sec^{-1} \qquad \omega := Im(s_1) \qquad \omega = 600\cdot sec^{-1}$$

Initial slope: $\quad \frac{d}{dt}i_L(0) = \frac{v_L(0)}{L} = \frac{-3\cdot V}{L} = -150\cdot\frac{A}{sec}$

General solution for the underdamped condition: $i_L(t) = i_L(\infty) + e^{\alpha\cdot t}\cdot(B\cdot\cos(\omega\cdot t) + D\cdot\sin(\omega\cdot t))$

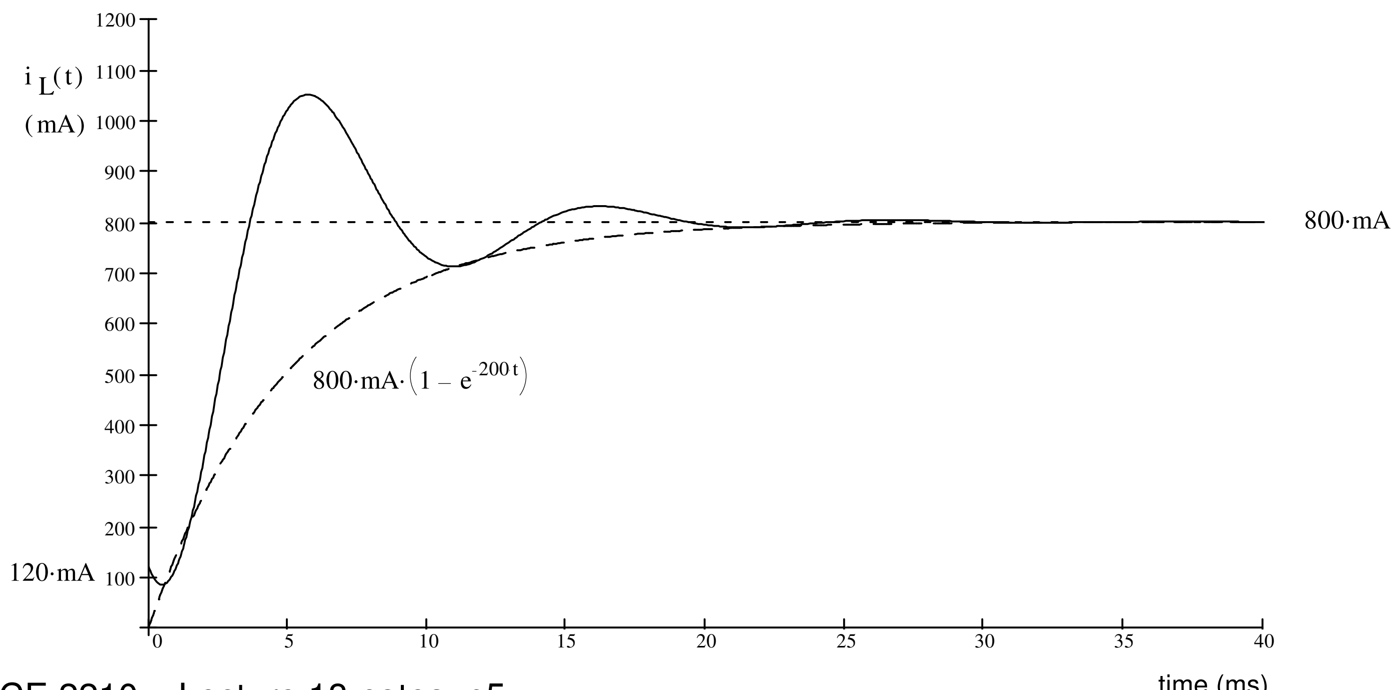
Find constants: $\quad i_L(0) = i_L(\infty) + B \qquad B = i_L(0) - i_L(\infty) \qquad B := 120\cdot mA - 800\cdot mA$

$B = -680\cdot mA$

$$\frac{d}{dt}i_L(0) = \alpha\cdot B + D\cdot\omega \qquad D := \frac{-150\cdot\frac{A}{sec} - \alpha\cdot B}{\omega} \qquad D = -476.667\cdot mA$$

Write the full expression for $i_L(t)$, including all the constants that you find.

$$i_L(t) := 800\cdot mA + e^{-200t}\cdot(-680\cdot mA\cdot\cos(600\cdot t) - 477\cdot mA\cdot\sin(600\cdot t))$$



Ex. 6

Analysis of a circuit (not pictured) yields the characteristic equation below.

$$0 = s^2 + 800\cdot s + 160000 \qquad R := 60\cdot\Omega \quad L := 350\cdot mH \quad C := 20\cdot\mu F \quad V_{in} := 12\cdot V$$

Further analysis yields the following initial and final conditions:

$i_L(0) = 30\cdot mA$ $\quad v_L(0) = -7\cdot V$ $\quad v_C(0) = 5\cdot V$ $\quad i_C(0) = 70\cdot mA$

$i_L(\infty) = 90\cdot mA$ $\quad v_L(\infty) = 0\cdot V$ $\quad v_C(\infty) = 12\cdot V$ $\quad i_C(\infty) = 0\cdot mA$

Write the full expression for $i_L(t)$, including all the constants that you find. $\quad i_L(t) = ?$

Include **units** in your answer

Solution:

$$\frac{-800 + \sqrt{800^2 - 4\cdot 160000}}{2} = -400 \qquad s_1 := -400\cdot\frac{1}{sec} \qquad s_2 := -400\cdot\frac{1}{sec}$$

s_1 and s_2 are the same, **critically damped**

Initial slope: $\quad \frac{d}{dt}i_L(0) = \frac{v_L(0)}{L} = \frac{-7\cdot V}{L} = -20\cdot\frac{A}{sec}$

General solution for the critically damped condition: $\quad i_L(t) = i_L(\infty) + B\cdot e^{s_1\cdot t} + D\cdot t\cdot e^{s_2\cdot t}$

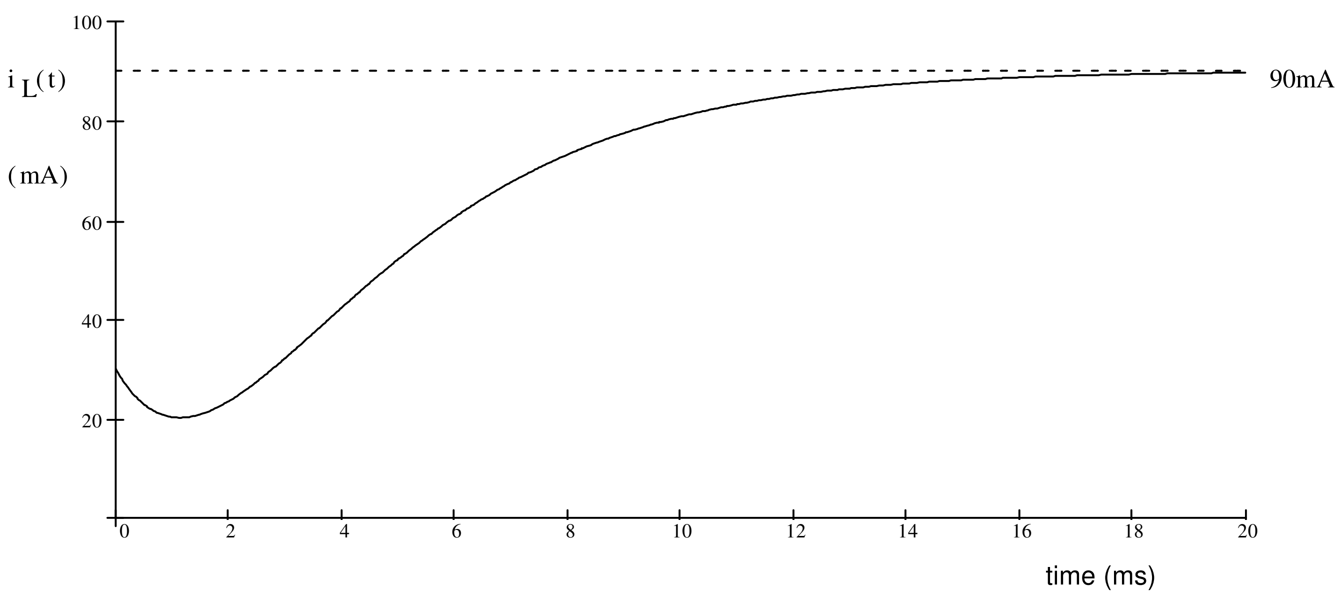
Find constants: $\quad i_L(0) = i_L(\infty) + B \qquad B = i_L(0) - i_L(\infty) \qquad B := 30\cdot mA - 90\cdot mA$

$B = -60\cdot mA$

$\frac{d}{dt}i_L(0) = B\cdot s + D \qquad D := -20\cdot\frac{A}{sec} - B\cdot s_1 \qquad D = -44\cdot\frac{A}{sec}$

Write the full expression for $i_L(t)$, including all the constants that you find.

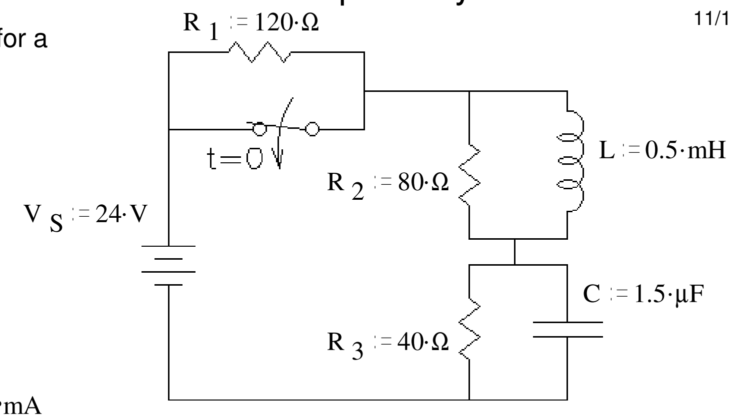
$$i_L(t) := 90\cdot mA - 60\cdot mA\cdot e^{-\frac{400}{sec}\cdot t} - 44\cdot\frac{A}{sec}\cdot t\cdot e^{-\frac{400}{sec}\cdot t}$$



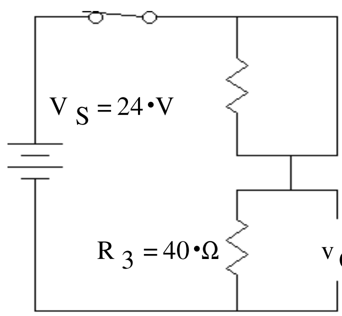
ECE 2210 Lecture 19 notes Second order Transient example & Systems

A. Stolp
11/1/06

Ex 1. The switch at right has been in the open position for a long time and is closed (as shown) at time $t = 0$.



a) What are the final conditions of i_L and the v_C?



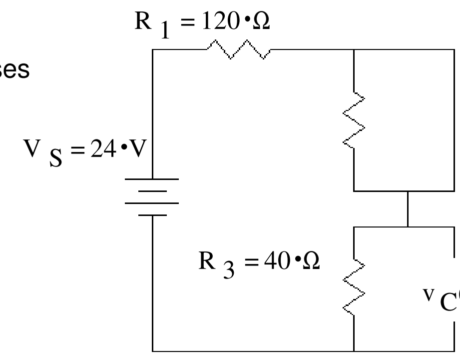
$R_3 := 40\cdot\Omega$

$$i_L(\infty) = \frac{V_S}{R_3} = 600\cdot\text{mA}$$

$$v_C(\infty) = V_S = 24\cdot\text{V}$$

b) Find the initial condition and intial slope of i_L so that you could find all the constants in $i_L(t)$.
Don't find $i_L(t)$ or it's constants, just the initial conditions.

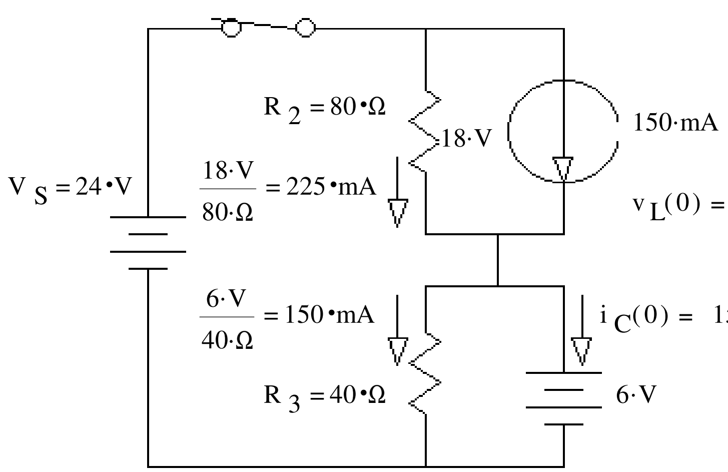
Before the switch closes



$$i_L(0) = \frac{V_S}{R_1 + R_3} = 150\cdot\text{mA}$$

$$v_C(0) = V_S\cdot\frac{R_3}{R_1 + R_3} = 6\cdot\text{V}$$

Just after the switch closes:



$$v_L(0) = 24\cdot\text{V} - 6\cdot\text{V} = 18\cdot\text{V}$$

$$\frac{d}{dt}i_L(0) = \frac{18\cdot\text{V}}{L} = 36000\cdot\frac{\text{A}}{\text{sec}}$$

$$i_C(0) = 150\cdot\text{mA} + \frac{18\cdot\text{V}}{R_2} - \frac{6\cdot\text{V}}{R_3} = 225\cdot\text{mA}$$

c) Find the initial condition and intial slope of v_C so that you could find all the constants in $v_C(t)$.
Don't find $v_C(t)$ or it's constants, just the initial conditions.

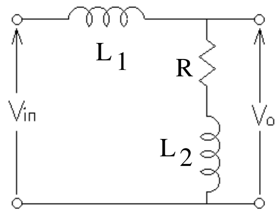
$$v_C(0) = V_S\cdot\frac{R_3}{R_1 + R_3} = 6\cdot\text{V}$$

$$\frac{d}{di}v_C(0) = \frac{225\cdot\text{mA}}{C} = 150000\cdot\frac{\text{V}}{\text{sec}}$$

Systems

Now that we' ve developed the concept of the transfer function, we can now develop system block diagrams using blocks which contain transfer functions.

Consider a circuit:



$$\mathbf{H}(s) = \frac{\mathbf{V}_{\mathbf{o}}(s)}{\mathbf{V}_{\mathbf{in}}(s)} = \frac{R + L_2 \cdot s}{R + L_1 \cdot s + L_2 \cdot s} = \frac{R + L_2 \cdot s}{R + \left(L_1 + L_2\right) \cdot s}$$

$$= \frac{L_2 \cdot s + R}{\left(L_1 + L_2\right) \cdot s + R}$$

This could be represented in as a block operator:

$$\mathbf{V}_{\mathbf{in}}(s) \longrightarrow \boxed{\frac{L_2 \cdot s + R}{\left(L_1 + L_2\right) \cdot s + R}} \longrightarrow \mathbf{V}_{\mathbf{o}}(s) = \mathbf{V}_{\mathbf{in}}(s) \cdot \mathbf{H}(s)$$

Transfer functions can be written for all kinds of devices and systems, not just electric circuits and the input and output do not have to be similar. For instance, the potentiometers used to measure angular position in the lab servo can be represented like this:

$$\boldsymbol{\theta}_{\mathbf{in}}(s) \longrightarrow \boxed{Kp = 0.7 \cdot \frac{V}{rad} = 0.012 \cdot \frac{V}{deg}} \longrightarrow \mathbf{V}_{\mathbf{out}}(s) = K_p \cdot \boldsymbol{\theta}_{\mathbf{in}}(s)$$

In general:

$$\mathbf{H}(s) = \frac{\mathbf{X}_{\mathbf{out}}(s)}{\mathbf{X}_{\mathbf{in}}(s)} \qquad \mathbf{X}_{\mathbf{in}}(s) \longrightarrow \boxed{\mathbf{H}(s)} \longrightarrow \mathbf{X}_{\mathbf{out}}(s) = \mathbf{X}_{\mathbf{in}}(s) \cdot \mathbf{H}(s)$$

$\mathbf{X}_{\mathbf{in}}$ and $\mathbf{X}_{\mathbf{out}}$ could be anything from small electrical signals to powerful mechanical motions or forces.

Two blocks with transfer functions $\mathbf{A}(s)$ and $\mathbf{B}(s)$ in a row would look like this:

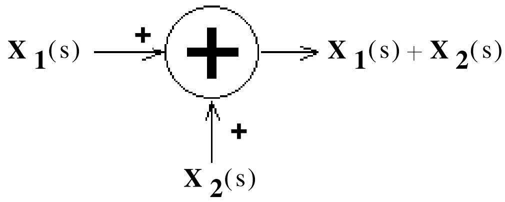
$$\underbrace{\mathbf{X}_{\mathbf{in}}(s) \longrightarrow \boxed{\mathbf{A}(s)} \xrightarrow{\mathbf{X}_{\mathbf{in}}(s) \cdot \mathbf{A}(s)} \boxed{\mathbf{B}(s)}} \longrightarrow \mathbf{X}_{\mathbf{out}}(s) = \mathbf{X}_{\mathbf{in}}(s) \cdot \mathbf{A}(s) \cdot \mathbf{B}(s)$$

The two blocks could be replaced by a single equivalent block:

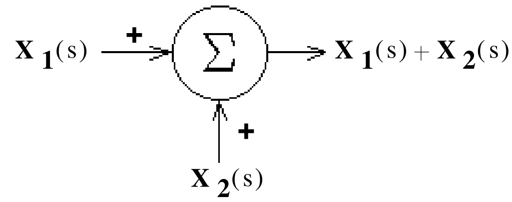
$$\mathbf{X}_{\mathbf{in}}(s) \longrightarrow \boxed{\mathbf{A}(s) \cdot \mathbf{B}(s)} \longrightarrow \mathbf{X}_{\mathbf{out}}(s) = \mathbf{X}_{\mathbf{in}}(s) \cdot \mathbf{A}(s) \cdot \mathbf{B}(s)$$

$$\mathbf{X}_{\mathbf{in}}(s) \cdot \mathbf{A}(s)$$

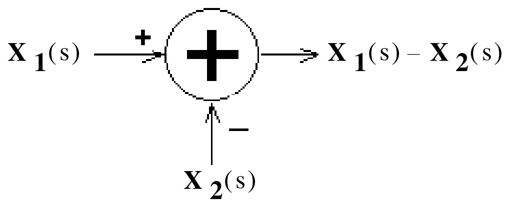
Summer blocks can be used to add signals:



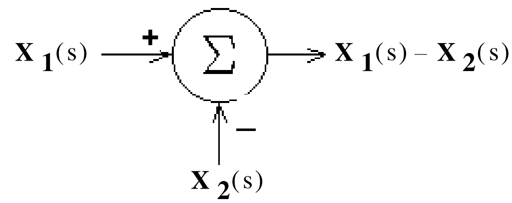
OR



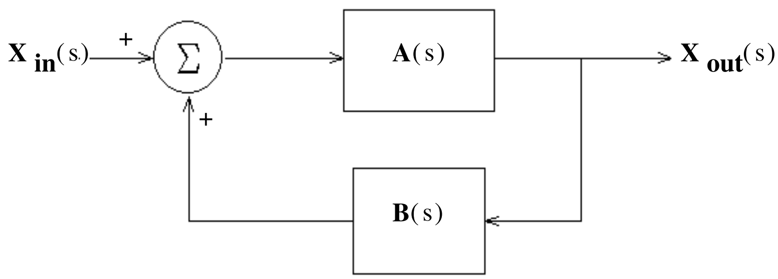
or subtract signals:



OR

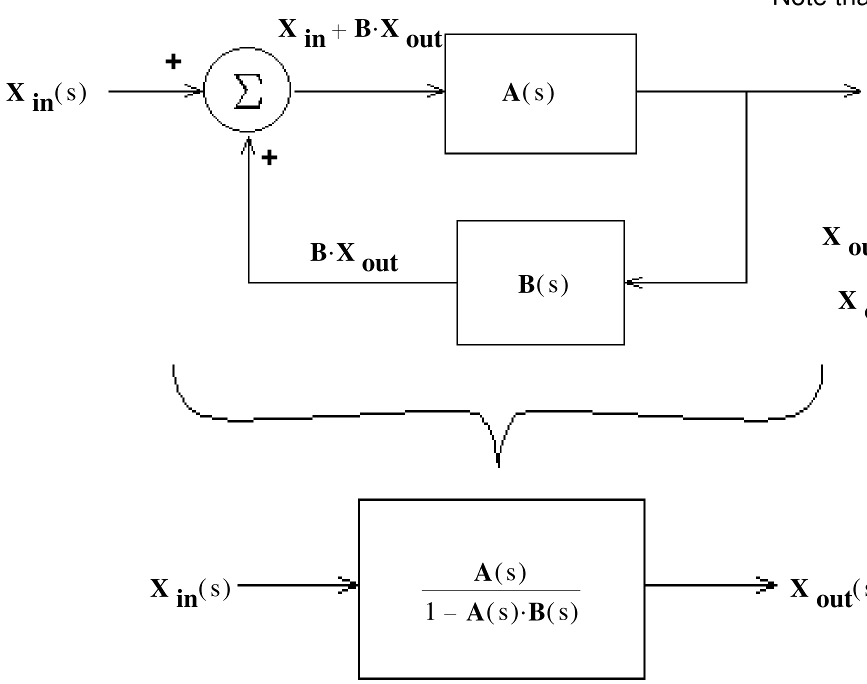


A feedback loop system is particularly interesting and useful:



The entire loop can be replaced by a single equivalent block:

Note that I' ve begun to drop the (s)



$$X_{out}(s) = A \cdot \left(X_{in} + B \cdot X_{out}\right)$$

$$= A \cdot X_{in} + A \cdot B \cdot X_{out}$$

$$X_{out} - A \cdot B \cdot X_{out} = A \cdot X_{in}$$

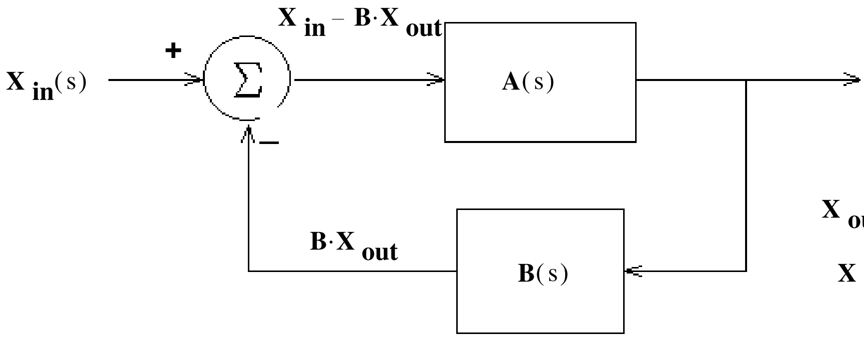
$$X_{out} \cdot (1 - A \cdot B) = A \cdot X_{in}$$

$$\frac{X_{out}}{X_{in}} = \frac{A}{1 - A \cdot B} = H(s)$$

The equivalent transfer function

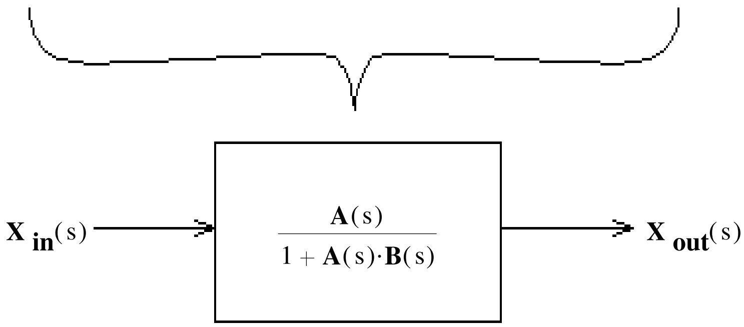
$A(s) \cdot B(s)$ is called the "loop gain" or "open loop gain"

Negative feedback is more common and is used as a control system:



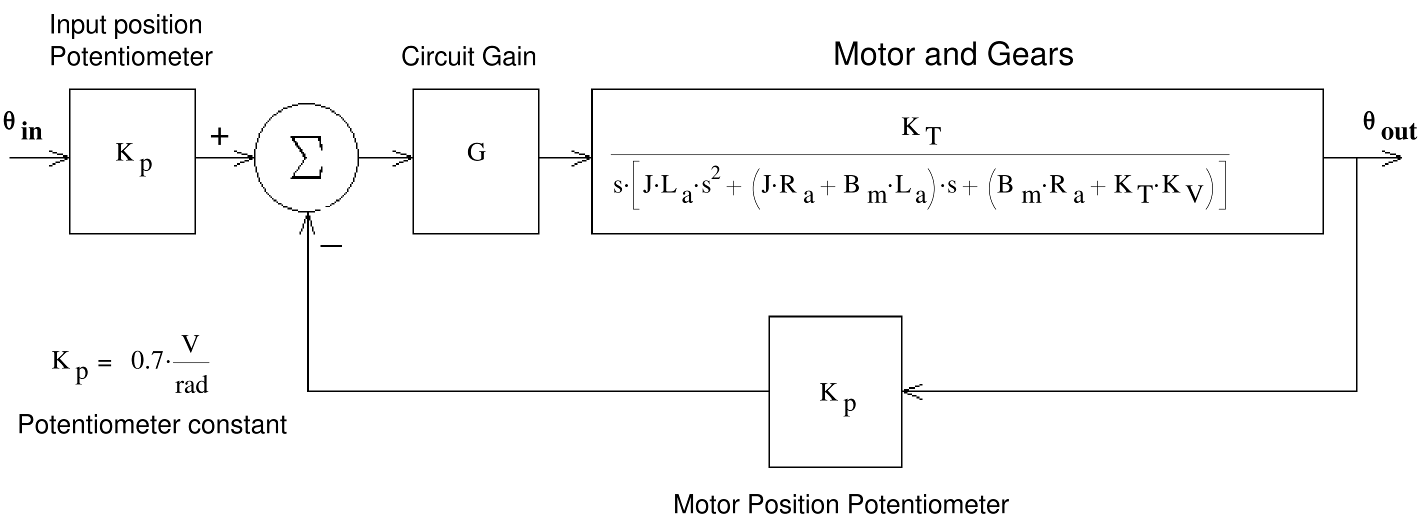
$$X_{out}(s) = A \cdot \left(X_{in} - B \cdot X_{out}\right)$$
$$= A \cdot X_{in} - A \cdot B \cdot X_{out}$$
$$X_{out} + A \cdot B \cdot X_{out} = A \cdot X_{in}$$
$$X_{out} \cdot (1 + A \cdot B) = A \cdot X_{in}$$
$$\frac{X_{out}}{X_{in}} = \frac{A}{1 + A \cdot B} = H(s)$$

The equivalent transfer function



This is called a "closed loop" system, whereas a a system without feedback is called "open loop".
The term "open loop" is often used to describe a system that is out of control.

The servo used in our lab can be represented by:



$K_p = 0.7 \cdot \frac{V}{rad}$

Potentiometer constant

$$H(s) = \frac{\theta_{out}(s)}{\theta_{in}(s)} = \frac{G \cdot K_T \cdot K_p}{s \cdot \left[J \cdot L_a \cdot s^2 + \left(J \cdot R_a + B_m \cdot L_a\right) \cdot s + \left(B_m \cdot R_a + K_T \cdot K_V\right)\right] + K_p \cdot G \cdot K_T}$$

See the appendix to lab 9 for the complete analysis

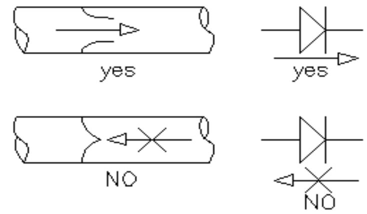
ECE 2210 Lecture 19 notes p4

Diodes Notes

ECE 2210

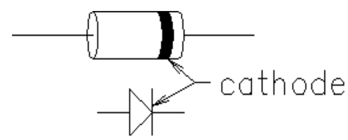
A. Stolp
4/8/03,
2/27/07

Diodes are basically electrical check valves. They allow current to flow freely in one direction, but not the other. Check valves require a small forward pressure to open the valve. Similarly, a diode requires a small forward voltage (bias) to "turn on". This is called the forward voltage drop. There are many different types of diodes, but the two that you are most likely to see are silicon diodes and light-emitting diodes (LEDs). These two have forward voltage drops of about 0.7V and 2V respectively.



Mechanical check valve   Diode

silicon diode   LED



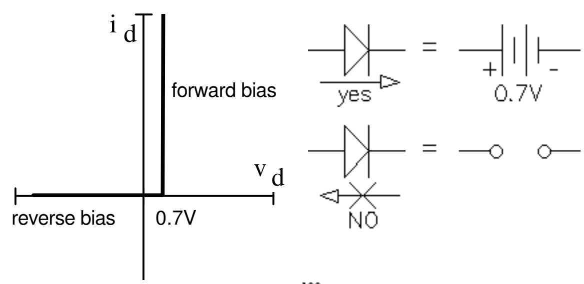
The electrical symbol for a diode looks like an arrow which shows the forward current direction and a small perpendicular line. The two sides of a diode are called the "anode" and the "cathode" (these names come from vacuum tubes). Most small diodes come in cylindrical packages with a band on one end that corresponds to the small perpendicular line, and shows the polarity, see the picture. Normal diodes are rated by the average forward current and the peak reverse voltage that they can handle. Diodes with significant current ratings are known as "rectifier" or "power" diodes. (Rectification is the process of making AC into DC.) Big power diodes come in a variety of packages designed to be attached to heat sinks. Small diodes are known as "signal" diodes because they're designed to handle small signals rather than power.

Diodes are nonlinear parts

So far in this class we've only worked with linear parts. The diode is definitely NOT linear, but it can be modeled as linear in its two regions of operation. If it's forward biased, it can be replaced by battery of 0.7V (2V for LEDs) which opposes the current flow. Otherwise it can be replaced by an open circuit. These are "models" of the actual diode. If you're not sure of the diode's state in a circuit, guess. Then replace it with the appropriate model and analyze the circuit. If you guessed the open, then the voltage across the diode model should come out less than +0.7V (2V for LEDs). If you guessed the battery, then the current through the diode model should come out in the direction of the diode's arrow. If your guess doesn't work out right, then you'll have to try the other option. In a circuit with multiple diodes (say "n" diodes), there will be 2^n possible states, all of which may have to be tried until you find the right one. Try to guess right the first time.

Constant-voltage-drop model

This is the most common diode model and is the only one we'll use in this class. It gives quite accurate results in most cases.



1 Assume the diode is operating in one of the linear regions (make an educated guess).

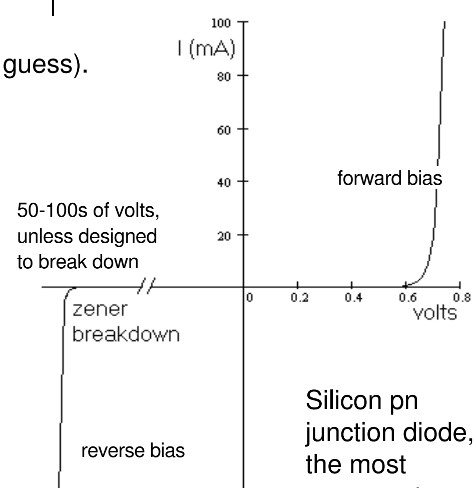
2 Analyze circuit with a linear model od the diode.

3 Check to see if the diode was really in the assumed region.

4 Repeat if necessary.

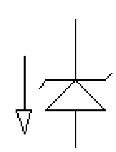
Actual diode curve

The characteristics of real diodes are actually more complicated than the constant-voltage-drop model. The forward voltage drop is not quite constant at any current and the diode "leaks" a little current when the voltage is in the reverse direction. If the reverse voltage is large enough, the diode will "breakdown" and let lots of current flow in the reverse direction. A mechanical check valve will show similar characteristics. Breakdown does not harm the diode as long as it isn't overheated.



Silicon pn junction diode, the most common type.

Zener diodes are special diodes designed to operate in the reverse breakdown region. Since the reverse breakdown voltage across a diode is very constant for a large range of current, it can be used as a voltage reference or regulator. Zener diodes are also used for over-voltage protection. In the forward direction zeners work the same as regular diodes.

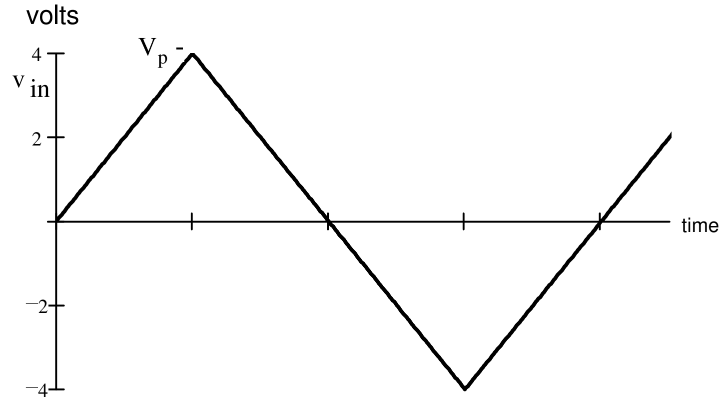


V_Z = zener voltage

I recommend that you try some of the DC analysis in the Diode Circuit Examples handout before you proceed here.

Diodes in AC Circuits

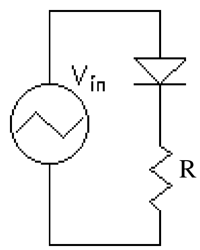
Diodes are often used to manipulate AC waveforms. We'll start with some triangular waveforms to get the general idea.

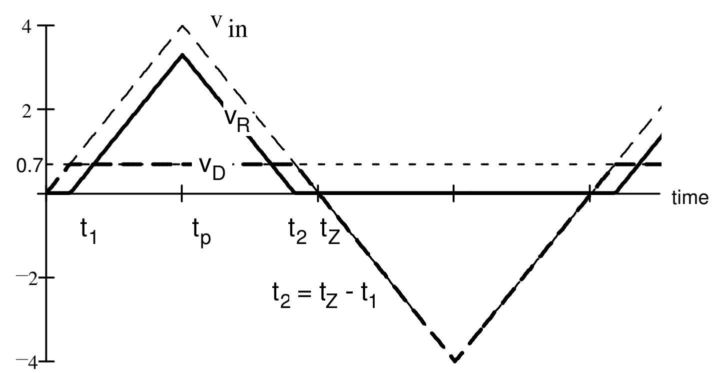


Diode doesn't conduct until v_{in} reaches 0.7V, so 0.7V is a dividing line between the two models of the diode.

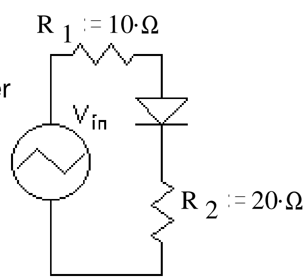
$$\text{slope} = \frac{0.7\cdot V}{t_1} = \frac{V_p}{t_p}$$

$$t_1 = \frac{0.7\cdot V}{V_p}\cdot t_p$$

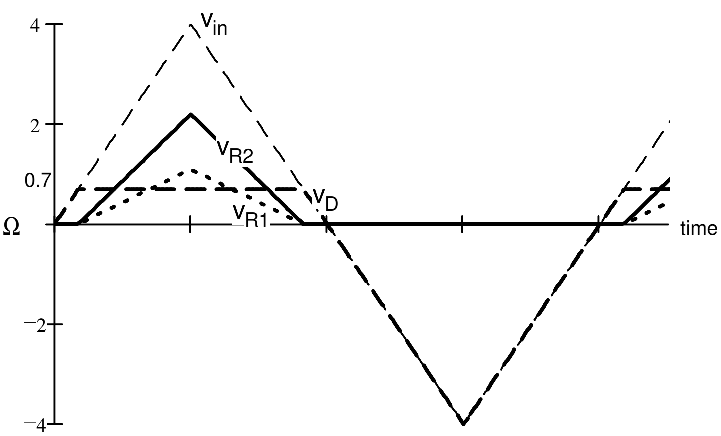




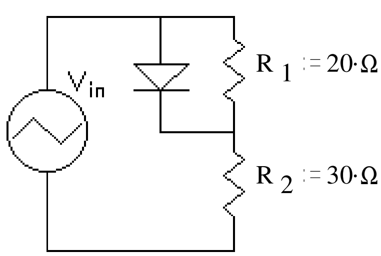
When the diode conducts, you're left with a voltage divider



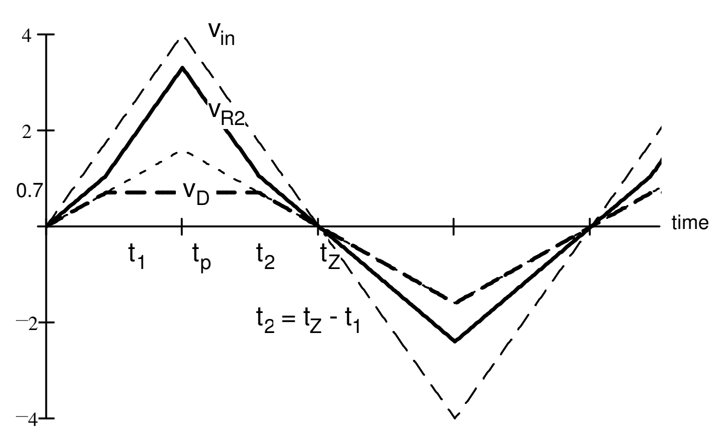
$$V_{R2peak} = \left(V_p - 0.7\cdot V\right)\cdot\frac{R_2}{R_1 + R_2}$$



Sometimes it's helpful to figure out what the voltage across the diode would be if it never conducted (light dotted line).



$$t_1 = \frac{0.7\cdot V}{V_p\cdot\frac{R_1}{R_1 + R_2}}\cdot t_p$$

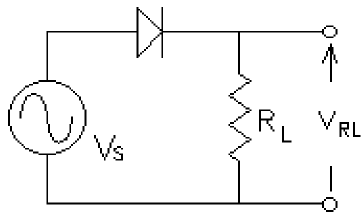


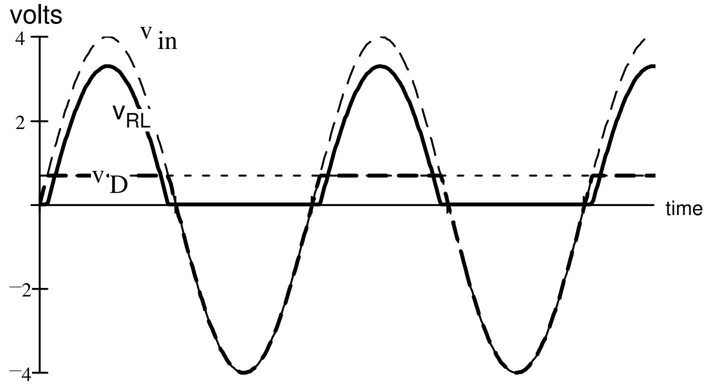
Rectifier Circuits & Power Supplies

Half-wave rectification

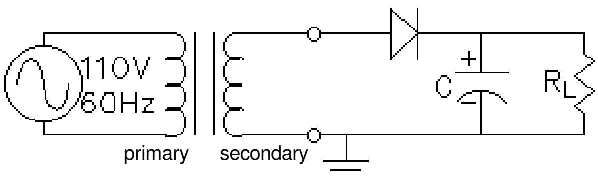
What if the input is a sine wave?

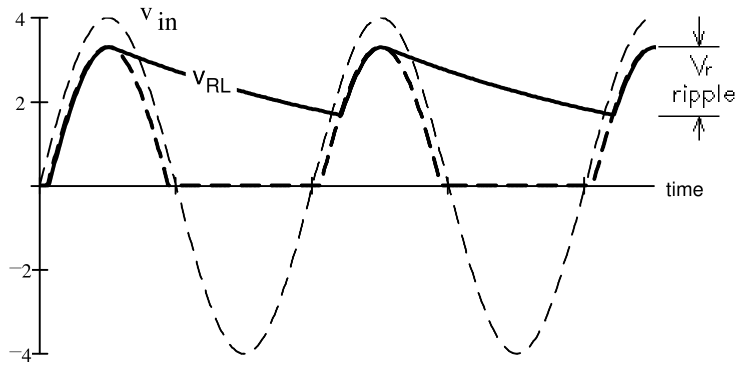
V_{RL} is now DC, although a bit bumpy. Some things are better if they' re bumpy, but not roads and not DC voltages.





Rectification is the process of making DC from AC. Usually the AC is derived from the AC wall outlet (often through a transformer) and the DC is needed for electronic circuitry modeled by R_L here.

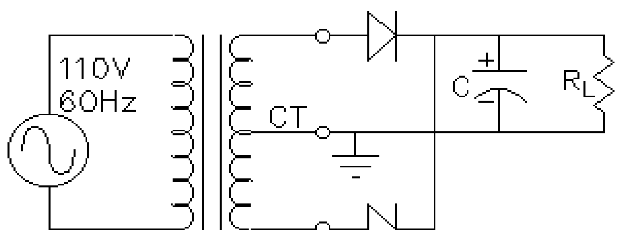


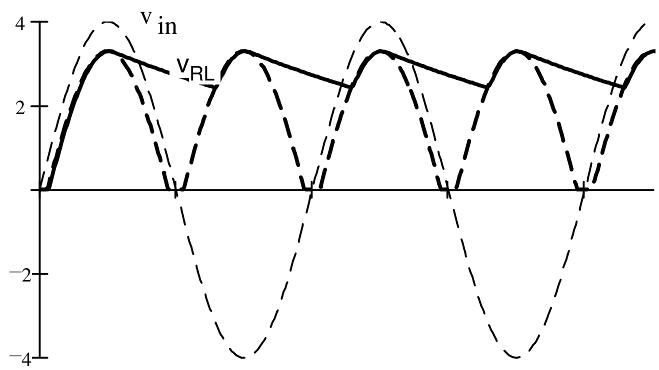


A "filter" capacitor (usually a big electrolytic) helps smooth out the bumps, although it sure looks like we could a bit bigger one here. The remaining bumpiness is called "ripple", V_r is peak-to-peak ripple

Full-wave rectification

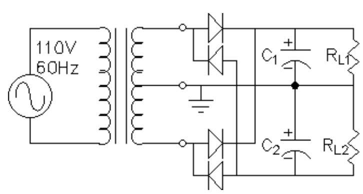
The "center tap" in the secondary of this transformer makes it easy to get full-wave rectification.



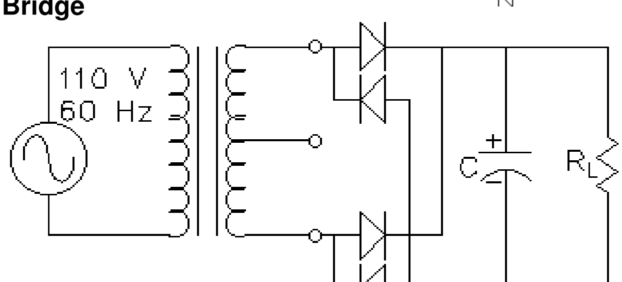


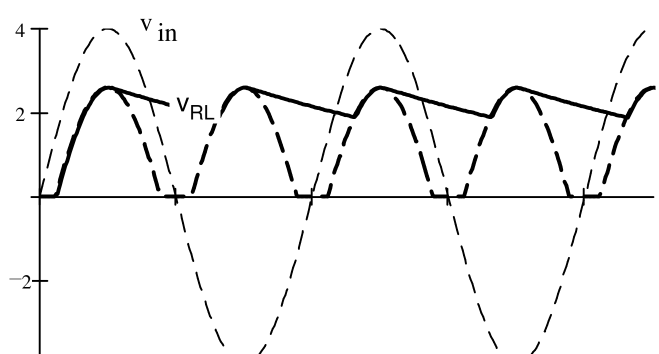
ripple is less

The center-tap transformer is also good for making $\pm$ supplies



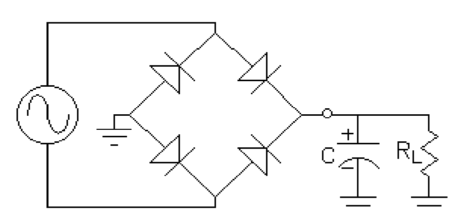
Bridge





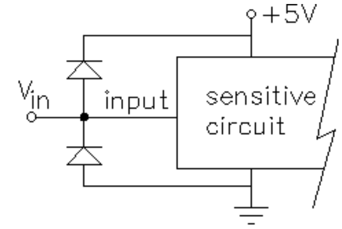
A "bridge" circuit or "bridge rectifier" can give you full-wave rectification without a center-tap transformer, but now you loose another "diode drop"

Bridge rectifiers are often drawn like this:



Other Useful Diode Circuits

Simple limiter circuits can be made with diodes.
A common input protection to protect circuit from excessive input voltages such as static electricity.



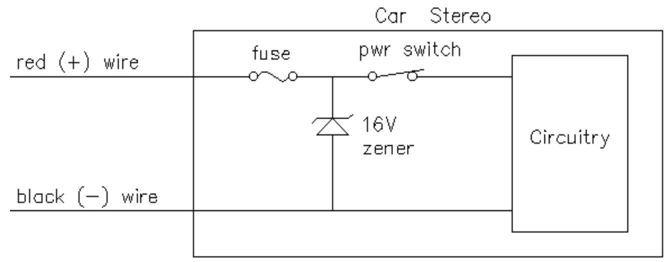
The input to the box marked "sensitive circuit" can' t get higher than the positive supply + 0.7V or lower than the negative supply - 0.7V.

Put a fuse in the V_{in} line and the diodes can make it blow, providing what' s known as "crowbar" protection.

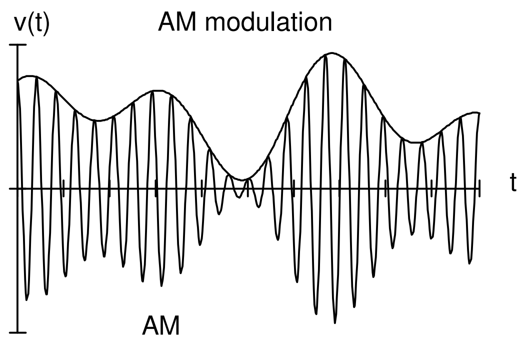
Another example of crowbar protection:

If the input voltage goes above 16 V. the fuse will blow, protecting the circuitry.

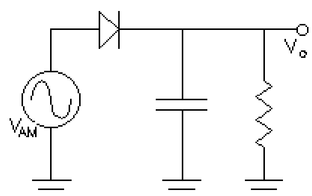
Or, If the input voltage is hooked up backwards the fuse will blow, protecting the circuitry.



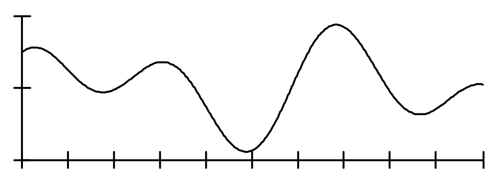
AM detector



A simple rectifier circuit



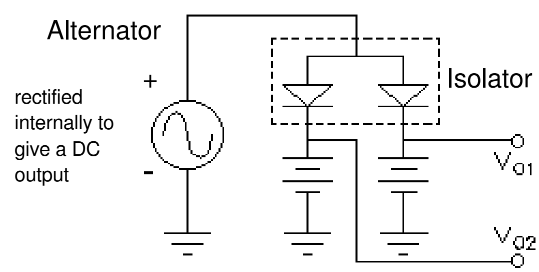
Returns the modulation signal



And a coupling capacitor can remove the DC

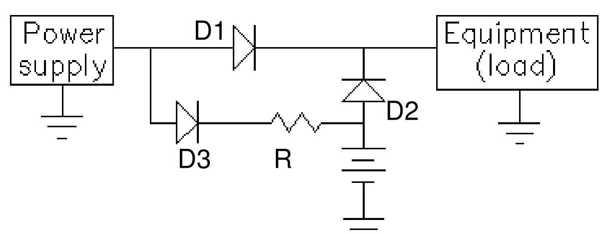
Battery Isolator

Like you might find in an RV. One alternator is used to charge two batteries. When the alternator is **not** charging, the batteries, the circuits they are hooked to should be isolated from one another. If not, then one battery might discharge through the second, especially if second is bad. Also, you wouldn' t want the accessories in the RV to drain the starting battery, or your uncle George from South Dakota might never leave your driveway.



Battery Backup Power

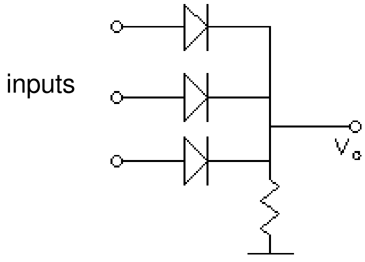
Normally the power supply powers the load through D1. However, if it fails, the load will remain powered by the battery through D2. Finally, D3 and R may be added to keep the battery charged when the power supply is working. These sorts of circuits are popular in hospitals.



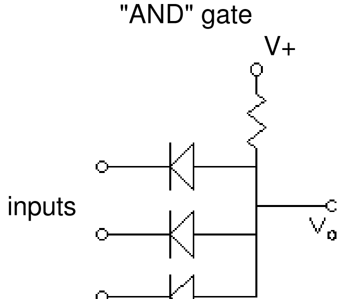
Diode Logic Circuits

Actually, both of the previous circuits are logic circuits as well.

"OR" gate

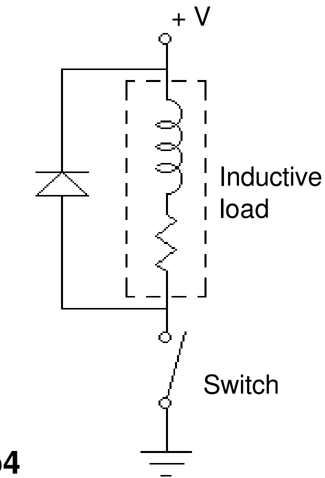


"AND" gate



"Flyback" Diode

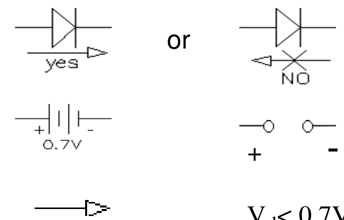
Every time the switch opens the inductor current continues to flow through the diode for a moment. If the diode weren' t there, then the current would arc across the switch.



ECE 2210 Diode Circuit Examples

Basic diode circuit analysis

	conducting		not conducting
1 Make an educated guess about each diode' s state.	yes	or	NO
2 Replace each diode with the appropriate model:	+ 0.7V -		+ -
3 Redraw and analyze circuit.			
4 Make sure that each diode is actually in the state you assumed:	current		$V_d < 0.7V$ Check

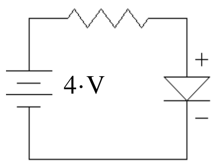


Note: 0.7V is for silicon junction diodes & will be different for other types. (2V for LED)

If any of your guesses don' t work out right, then you' ll have to start over with new guesses. In a circuit with n diodes there will be 2^n possible states, all of which may have to be tried until you find the right one. Try to guess right the first time.

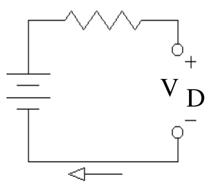
Ex1

$R := 1\cdot k\Omega$



I_D

Try reverse-biased, non-conducting model



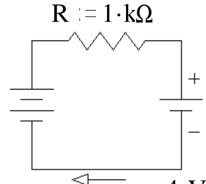
$V_D := 4\cdot V \quad > 0.7V$

$I_D := 0$

Doesn' t work, diode must be forward biased.

Try forward-biased, conducting model

$R := 1\cdot k\Omega$



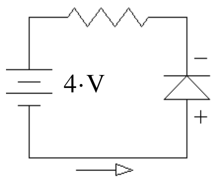
$V_D := 0.7\cdot V$

$$I_D = \frac{4\cdot V - 0.7\cdot V}{1\cdot k\Omega} = 3.3\cdot mA \quad > 0$$

The current is in the forward direction, confirming the assumption.

Ex2

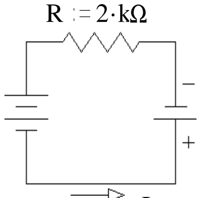
$R := 2\cdot k\Omega$



I_D

Try forward-biased, conducting model

$R := 2\cdot k\Omega$

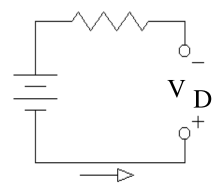


$V_D := 0.7\cdot V$

$$I_D = \frac{-4\cdot V - 0.7\cdot V}{1\cdot k\Omega} = -4.7\cdot mA < 0$$

Doesn' t work, diode must be reverse biased.

Try reverse-biased, non-conducting model



Check the diode voltage

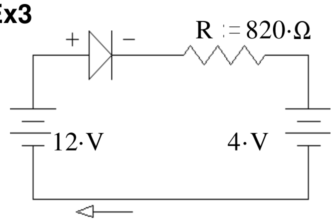
$V_D := -4\cdot V \quad < 0.7V$

Confirms diode is reverse biased

$I_D := 0\cdot mA$

Ex3

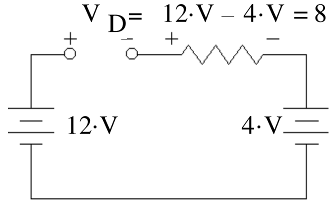
$R := 820\cdot\Omega$



Try reverse-biased, non-conducting model

Doesn' t work, diode must be forward biased.

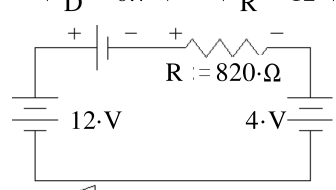
$V_D = 12\cdot V - 4\cdot V = 8\cdot V \quad > 0.7V$



Try forward-biased, conducting model

$V_D := 0.7\cdot V \qquad V_R := 12\cdot V - 4\cdot V - V_D$

$V_R = 7.3\cdot V$



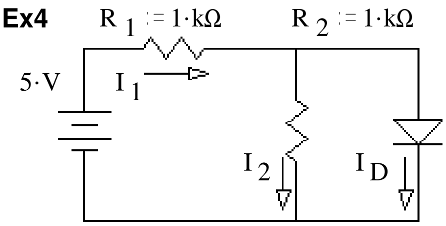
Check the diode current

$$I = \frac{V_R}{R} = 8.9\cdot mA \quad > 0$$

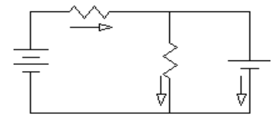
Confirms diode is forward biased

In each of these examples, my first guess was pretty stupid. I did that intentionally to show the process. I expect that you can make better guess and thus save yourself some work.

Ex4



Assume diode conducts:
Analyze



$V_D := 0.7 \cdot V \; = \; V_{R2}$

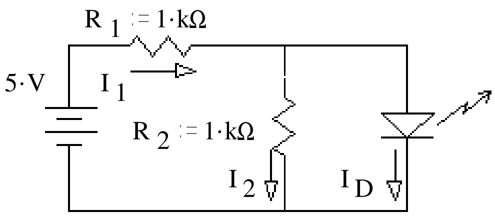
$V_{R2} := V_D \qquad I_2 := \dfrac{V_{R2}}{R_2} \qquad I_2 = 0.7 \cdot mA$

$V_{R1} := 5 \cdot V - V_D \qquad V_{R1} = 4.3 \cdot V \qquad I_1 := \dfrac{V_{R1}}{R_1} \qquad I_1 = 4.3 \cdot mA$

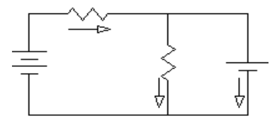
We assumed conducting (assuming a voltage), so check the current.

$I_D := I_1 - I_2 \qquad I_D = 3.6 \cdot mA \quad > 0$, so assumption was correct

Ex5 Now with an LED



Assume diode conducts



$V_D := 2 \cdot V \; = \; V_{R2}$

Analyze

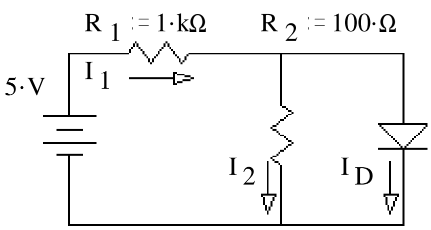
$V_{R2} := V_D \qquad I_2 := \dfrac{V_{R2}}{R_2} \qquad I_2 = 2 \cdot mA$

$V_{R1} := 5 \cdot V - V_D \qquad V_{R1} = 3 \cdot V \qquad I_1 := \dfrac{V_{R1}}{R_1} \qquad I_1 = 3 \cdot mA$

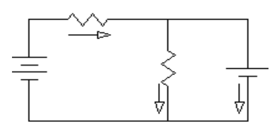
We assumed conducting (assuming a voltage), so check the current.

$I_D := I_1 - I_2 \qquad I_D = 1 \cdot mA \quad > 0$, so assumption was correct, but the current is probably too small to create noticeable light

Ex6 Regular diode, but smaller R_1



Assume diode conducts



$V_D := 0.7 \cdot V = \; V_{R2}$

Analyze

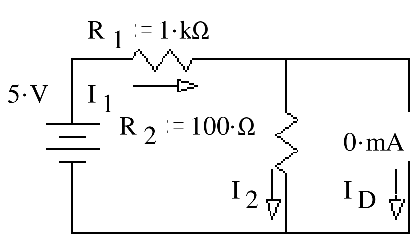
$V_{R2} := V_D \qquad I_2 := \dfrac{V_{R2}}{R_2} \qquad I_2 = 7 \cdot mA$

$V_{R1} := 5 \cdot V - V_D \qquad V_{R1} = 4.3 \cdot V \qquad I_1 := \dfrac{V_{R1}}{R_1} \qquad I_1 = 4.3 \cdot mA$

We assumed conducting (assuming a voltage), so check the current.

$I_D := I_1 - I_2 \qquad I_D = -2.7 \cdot mA \quad < 0$, so assumption was **WRONG !**

Assume diode does not conduct $\quad I_D := 0 \cdot mA$

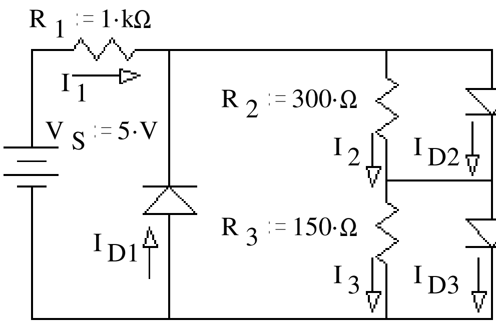


Analyze $\qquad I_1 := \dfrac{5 \cdot V}{R_1 + R_2} \qquad I_2 := I_1$

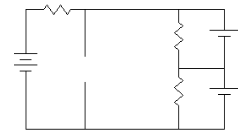
We assumed not conducting (assuming a current), so check the voltage.

$V_{R2} := I_2 \cdot R_2 \qquad V_{R2} = 0.455 \cdot V \quad < 0.7V$, so assumption was correct

Actually, this final check isn' t necessary, since first assumption didn' t work, so this one had to.



You can safety say that diode D_1 doesn' t conduct without rechecking later because no supply is even trying to make current flow through that diode the right way.



$V_{D2} := 0.7 \cdot V$

$V_{D3} := 0.7 \cdot V$

Assume both D_2 and D_3 conduct.

Analyze $V_{R1} := V_S - V_{D2} - V_{D2}$

$V_{R1} = 3.6 \cdot V$

$I_1 := \dfrac{V_{R1}}{R_1}$ $\quad I_1 = 3.6 \cdot mA$

$I_2 := \dfrac{V_{D2}}{R_2}$ $\quad I_2 = 2.333 \cdot mA$

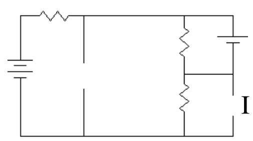
$I_3 := \dfrac{V_{D3}}{R_3}$ $\quad I_3 = 4.667 \cdot mA$

We assumed D_1 & D_2 conduct (assumed a voltage), so check currents.

$I_{D2} := I_1 - I_2$ $\quad I_{D2} = 1.267 \cdot mA \quad > 0$, so assumption OK

$I_{D3} := I_1 - I_3$ $\quad I_{D3} = -1.067 \cdot mA \quad < 0$, so assumption **wrong**

Assume D_2 conducts and D_3 doesn' t.



$V_{D2} := 0.7 \cdot V$

$I_{D3} := 0 \cdot mA$

Analyze $I_2 := \dfrac{V_{D2}}{R_2}$ $\quad I_2 = 2.333 \cdot mA$

$I_1 := \dfrac{V_S - V_{D2}}{R_1 + R_3}$ $\quad I_1 = 3.739 \cdot mA$

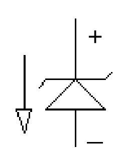
Assumed D_2 conducts, so check D_2 current. $\quad I_{D2} := I_1 - I_2$ $\quad I_{D2} = 1.406 \cdot mA \quad > 0$, so assumption OK

Assumed D_3 doesn' t conduct, so check D_3 voltage. $\quad V_{R3} := I_1 \cdot R_3$ $\quad V_{R3} = 0.561 \cdot V \quad < 0.7V$, so OK

Once you find one case that works, you don' t have to try any others.

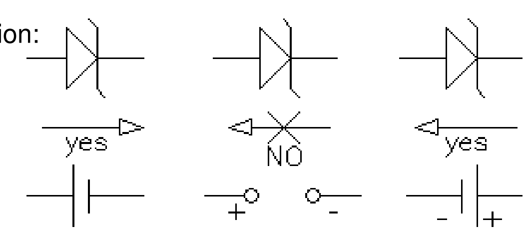
Zener Diodes

Zener diodes are special diodes designed to operate in the reverse breakdown region. Since the reverse breakdown voltage across the diode is very constant for a large range of current, it can be used as a voltage reference or regulator. Diodes are not harmed by operating in this region as long as their power rating isn' t exceeded. In the forward direction zeners work the same as regular diodes.



V_Z = zener voltage

Now there are three possible regions of operation:



$V_D = 0.7V$ $\quad 0.7V > V_D > V_Z$ $\quad V_Z$

Same basic diode circuit analysis

1 Make an educated guess about each diode' s state.

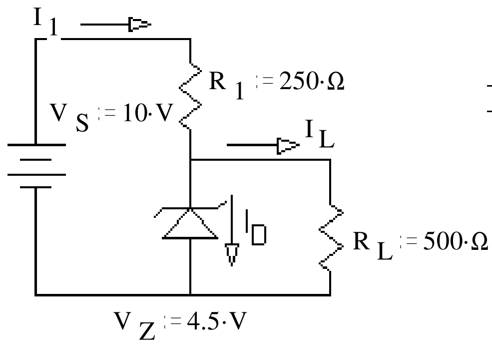
2 Replace each diode with the appropriate model:

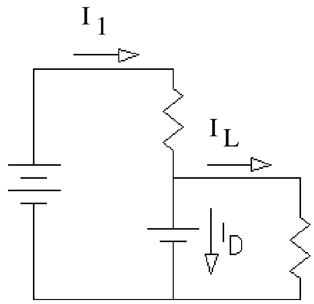
3 Redraw and analyze circuit.

4 Make sure that each diode is actually in the state you assumed:

Zener Diode Circuit Examples

Ex1 Typical shunt regulator circuit:





$V_Z := 4.5\cdot V$ $\qquad$ $R_L := 500\cdot\Omega$

Assume conducting in breakdown region

$V_D := V_Z$

$I_L := \dfrac{V_Z}{R_L}$ $\qquad$ $I_L = 9\cdot mA$

$I_1 := \dfrac{V_S - V_Z}{R_1}$ $\qquad$ $I_1 = 22\cdot mA$

Assumed a conducting region, so check the current to see if the current flows in the direction shown.

$I_D := I_1 - I_L$ $\qquad$ $I_D = 13\cdot mA$ > 0, so assumption OK

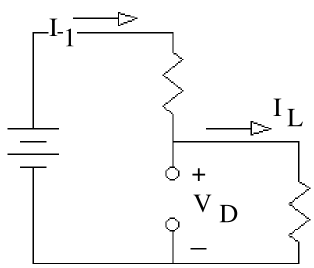
Ex2 What if R_L is smaller? $\qquad$ $R_L := 150\cdot\Omega$

Assume conducting in breakdown region $\qquad$ $V_D := V_Z$ $\qquad$ $I_L := \dfrac{V_Z}{R_L}$ $\qquad$ $I_L = 30\cdot mA$

$I_1 := \dfrac{V_S - V_Z}{R_1}$ $\qquad$ $I_1 = 22\cdot mA$ $\qquad$ $I_D := I_1 - I_L$ $\qquad$ $I_D = -8\cdot mA$ < 0, so assumption is **WRONG !**

Circuit "falls out of regulation"

Assume not conducting



$I_L \;=\; I_1 := \dfrac{V_S}{R_1 + R_L}$ $\qquad$ $I_1 = 25\cdot mA$

Assumed a non-conducting region, so check the voltage to see if it' s in the right range.

$V_D := \dfrac{R_L}{R_1 + R_L}\cdot V_S$ $\qquad$ $V_D = 3.75\cdot V$ $<$ $V_Z = 4.5\cdot V$

so this assumption is OK

Ex3 What if V_S is smaller instead of R_L? $\qquad$ $V_S := 6\cdot V$ $\qquad$ $R_L := 500\cdot\Omega$

Assume conducting in breakdown region $\qquad$ $V_D := V_Z$ $\qquad$ $I_L := \dfrac{V_Z}{R_L}$ $\qquad$ $I_L = 9\cdot mA$

$I_1 := \dfrac{V_S - V_Z}{R_1}$ $\qquad$ $I_1 = 6\cdot mA$ $\qquad$ $I_D := I_1 - I_L$ $\qquad$ $I_D = -3\cdot mA$ < 0, so assumption is **WRONG !**

Circuit "falls out of regulation"

Assume not conducting $\qquad$ $I_L \;=\; I_1 := \dfrac{V_S}{R_1 + R_L}$ $\qquad$ $I_1 = 8\cdot mA$

Assumed a non-conducting region, so check the voltage to see if it' s in the right range. $\qquad$ $V_D := \dfrac{R_L}{R_1 + R_L}\cdot V_S$ $\qquad$ $V_D = 4\cdot V$ $<$ $V_Z = 4.5\cdot V$

so this assumption is OK

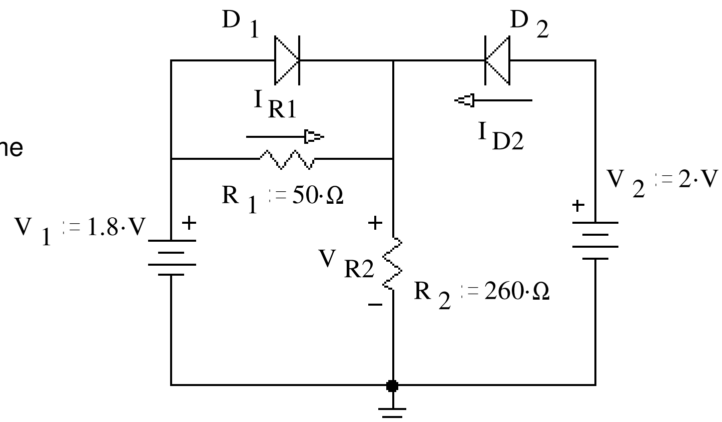
Exam-type Diode Circuit Examples

On an exam, I usually tell you what assumptions to make about the diodes, then you can show that you know how to analyze the circuit and test those assumptions. Since everyone starts with the same assumptions, everyone should do the same work.

In the circuit shown, use the constant-voltage-drop model for the silicon diode.

a) Assume that diode D_1 does NOT conduct.
Assume that diode D_2 does conduct.

Find V_{R2}, V_{R1}, I_{R1}, & I_{D2}, based on these assumptions. Stick with these assumptions even if your answers come out absurd. Hint: think in nodal voltages.



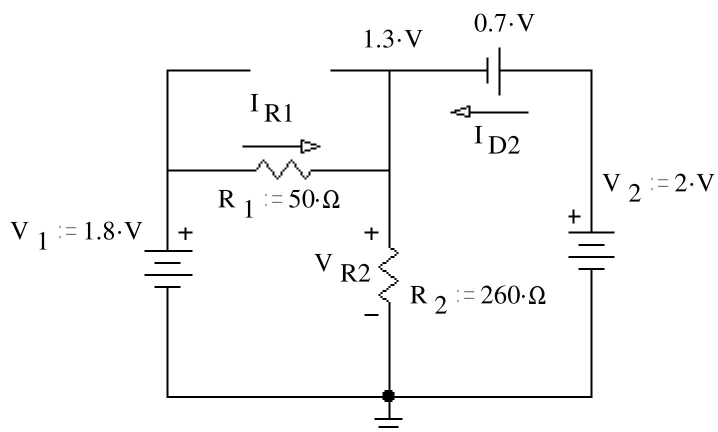
$V_{R2} =$ ________

$V_{R1} =$ ________

$I_{R1} =$ ________

$I_{D2} =$ ________

Solution to a)



$V_{R2} := V_2 - 0.7 \cdot V$ $\qquad V_{R2} = 1.3 \cdot V$

$V_{R1} := V_1 - V_{R2}$ $\qquad V_{R1} = 0.5 \cdot V$

$I_{R1} := \dfrac{V_{R1}}{R_1}$ $\qquad I_{R1} = 10 \cdot mA$

$I_{R2} := \dfrac{V_{R2}}{R_2}$ $\qquad I_{R2} = 5 \cdot mA$

$I_{D2} := I_{R2} - I_{R1}$ $\qquad I_{D2} = -5 \cdot mA$

b) Based on your numbers above, does it look like the assumption about D_1 was correct? yes no (circle one)
How do you know? (Specifically show a value which is or is not within a correct range.)

yes $\quad V_{D1} = V_{R1} = 0.5 \cdot V < 0.7V$

c) Based on your numbers above, does it look like the assumption about D_2 was correct? yes no (circle one)
How do you know?

no $\quad I_{D2} = -5 \cdot mA < 0$

d) Based on your answers to b) and c), which (if any) of the following was not correctly calculated in part a.

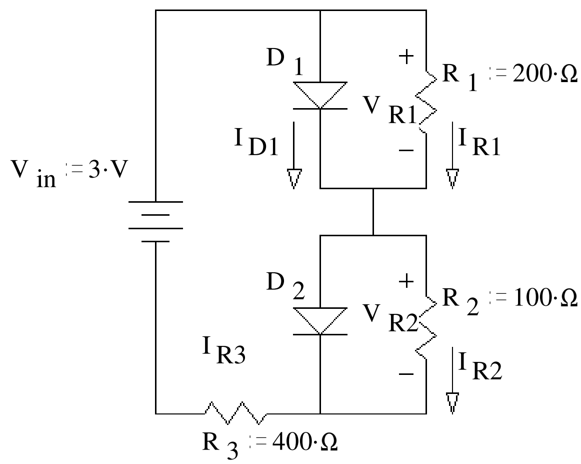
V_{R2} $\qquad V_{R1}$ $\qquad I_{R1}$ $\qquad I_{D2}$

(circle any number of answers)

Circle all in this case

Assume that diode D_1 is conducting and that diode D_2 is not conducting.

a) Find V_{R1}, I_{R1}, I_{R3}, I_{D1}, V_{R2} based on these assumptions.
Do not recalculate if you find the assumptions are wrong.



$V_{R1} =$ ____________

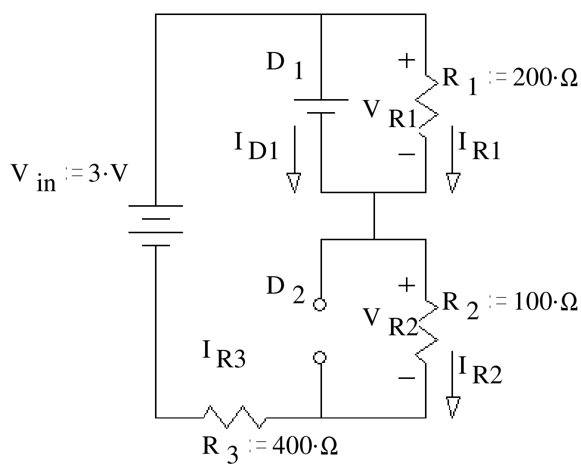
$I_{R1} =$ ____________

$I_{R3} =$ ____________

$I_{D1} =$ ____________

$V_{R2} =$ ____________

Solution:



$V_{R1} := 0.7\cdot V$

$I_{R1} := \dfrac{V_{R1}}{R_1}$ $\qquad I_{R1} = 3.5\cdot mA$

$I_{R3} := \dfrac{V_{in} - 0.7\cdot V}{R_2 + R_3}$ $\qquad I_{R3} = 4.6\cdot mA$

$I_{D1} := I_{R3} - I_{R1}$ $\qquad I_{D1} = 1.1\cdot mA$

$I_{R2} := I_{R3}$

$V_{R2} := I_{R2}\cdot R_2$ $\qquad V_{R2} = 0.46\cdot V$

(circle one)

b) Was the assumption about D_1 correct? yes no

How do you know? (Specifically show a value which is or is not within a correct range.)

yes $I_{R2} = 4.6\cdot mA > 0$

c) Was the assumption about D_2 correct? yes no

(circle one)

How do you know?

yes $V_{D2} = V_{R2} = 0.46\cdot V < 0.7V$

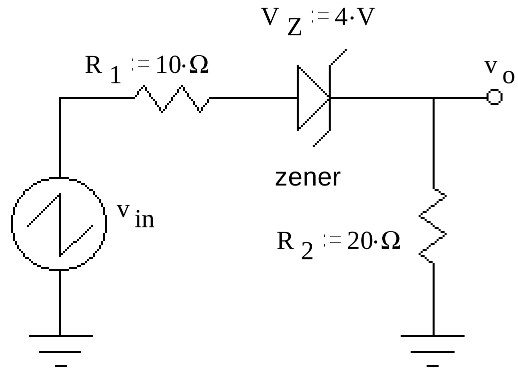
d) Based on your answers to b) and c), which (if any) of the following was not correctly calculated in part a.

V_{R1} $\quad I_{R1}$ $\quad I_{R3}$ $\quad I_{D2}$ $\quad V_{R2}$

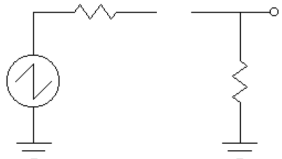
(circle any number of answers)

Circle none in this case

A voltage waveform (dotted line) is applied to the circuit shown.
Accurately draw the output waveform (v_o) you expect to see.
Label important times **and** voltage levels.



If diode doesn't conduct:

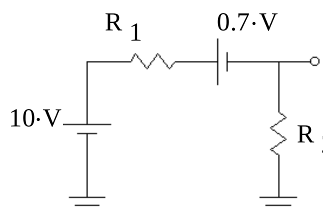


$v_o := 0$

Positive half

Diode conducts at: $0.7 \cdot V$ input at time: $\frac{0.7 \cdot V}{10 \cdot V} \cdot 10 \cdot ms = 0.7 \cdot ms$

Maximum:

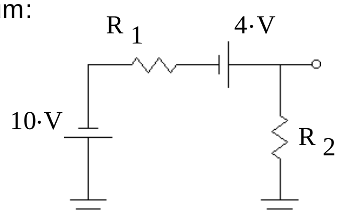


$v_o := (10 \cdot V - 0.7 \cdot V) \cdot \frac{R_2}{R_1 + R_2}$ $v_o = 6.2 \cdot V$

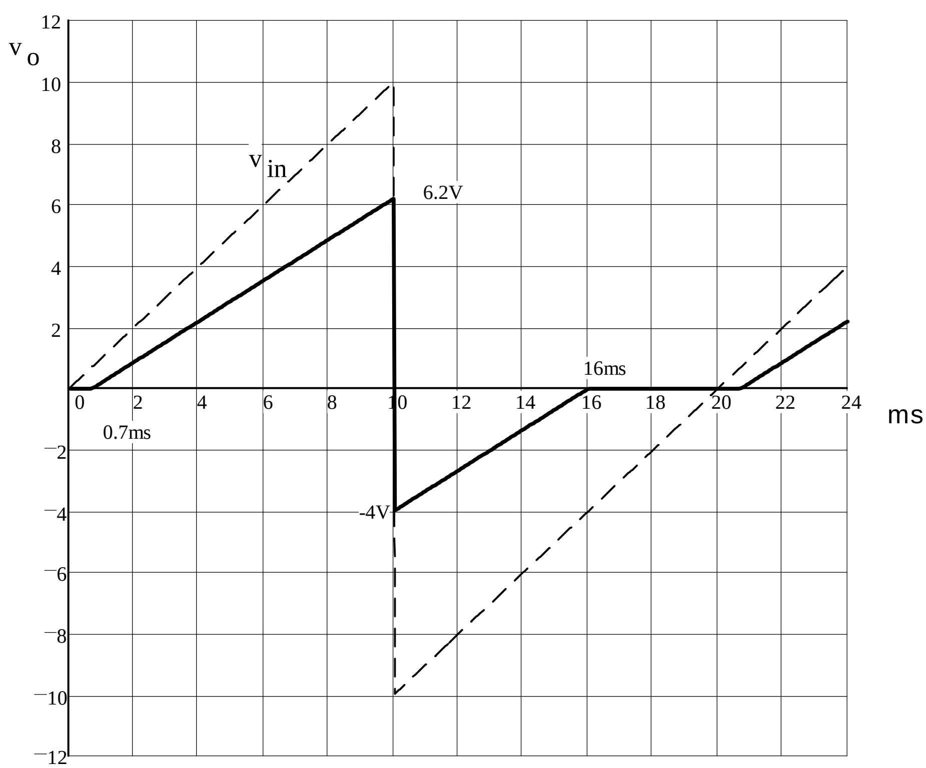
Negative half

Diode conducts at: $-4 \cdot V$ input at time: $20 \cdot ms - \frac{4 \cdot V}{10 \cdot V} \cdot 10 \cdot ms = 16 \cdot ms$

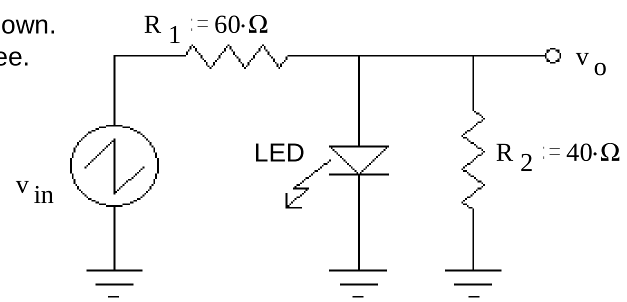
Maximum:



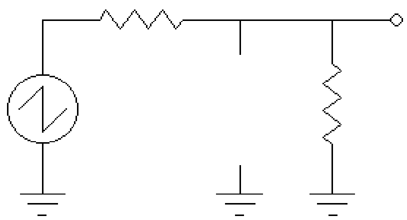
$v_o := -(10 \cdot V - 4 \cdot V) \cdot \frac{R_2}{R_1 + R_2}$ $v_o = -4 \cdot V$



A voltage waveform (dotted line) is applied to the circuit shown. Accurately draw the output waveform (v_o) you expect to see. Label important times **and** voltage levels.



If diode doesn't conduct:

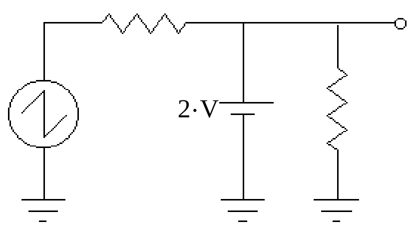


$$v_o = \frac{R_2}{R_1 + R_2}\cdot v_{in}$$

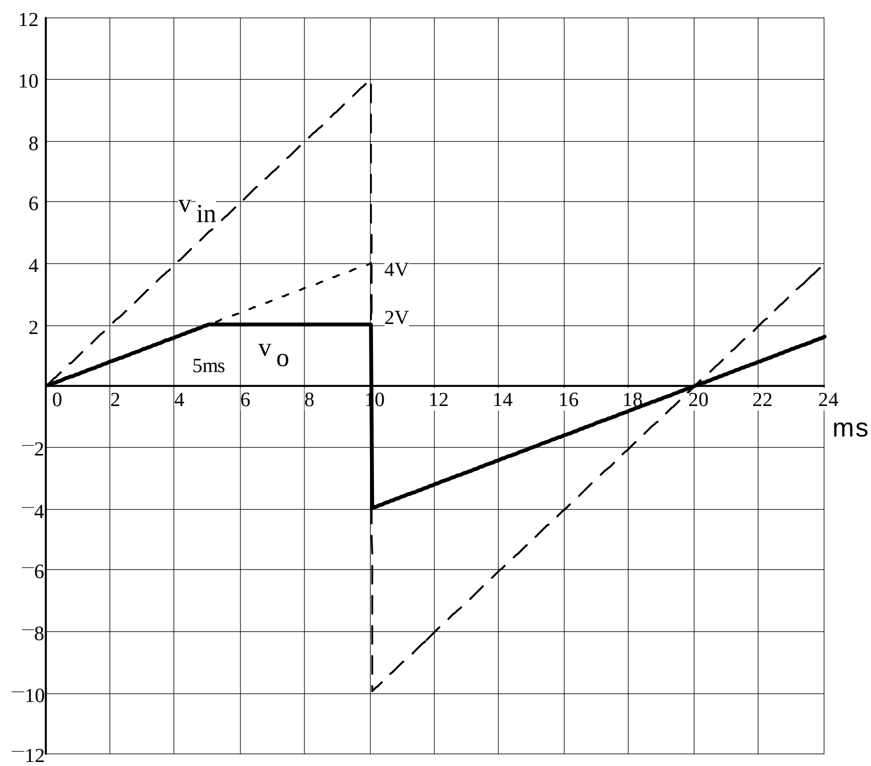
$$\frac{R_2}{R_1 + R_2}\cdot 10\cdot V = 4 \cdot V$$

When: $v_{in} := \frac{R_1 + R_2}{R_2}\cdot 2\cdot V$ $\quad v_{in} = 5 \cdot V$ at: $5\cdot ms$ Diode begins to conduct

When diode conducts:



$$v_o = 2\cdot V$$

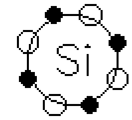


Diode Physics (The simple version) EE 2210 A.Stolp 3/22/01, rev, 2/25/16

FYI Only, You don't need to know this

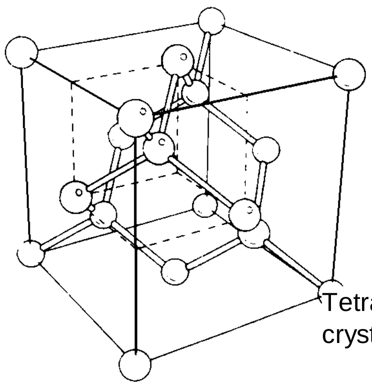
Silicon atoms

Silicon atoms each have 4 valence electrons (electrons in their outermost shell). That leaves 4 spaces in the outer shell of 8. This makes silicon a very reactive chemical, like carbon, which has the same valence configuration.

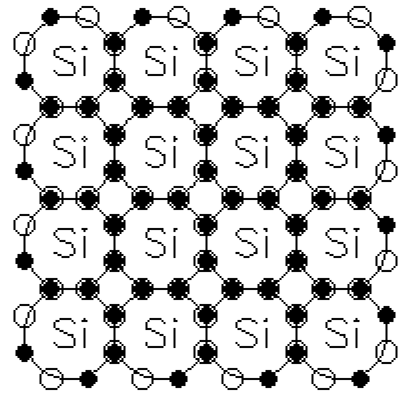


Silicon crystals

Each atom covalently bonds with four neighboring atoms to form a tetrahedral crystal, which we'll represent in 2D.



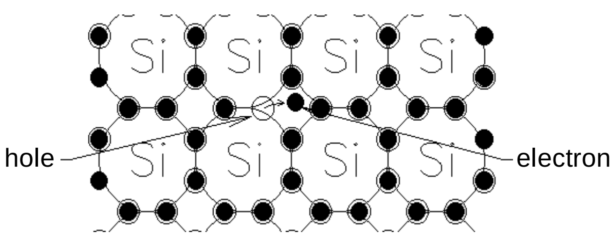
Tetrahedral crystal

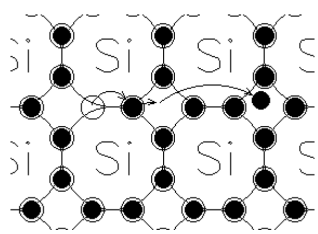


2-dimensional representation

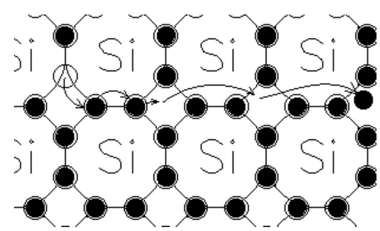
In the pure, "intrinsic" crystal, practically all the electrons are used in bonds and all the spaces are filled, which leaves almost no electrons free to move and thus no way to make current flow.

By the effects of heat, light and/or large electric fields, a few electrons do break free of the bonds and become "free" carriers. That is, they're free to move about crystal and "carry" an electrical current.





carrier movement



more carrier movement

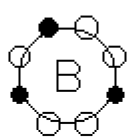
Interestingly, the space that was vacated by the electron also acts like a carrier. This pseudo-carrier is called a "hole" and it acts like a positively charged carrier.

Unless there's a lot of heat or light, the intrinsic silicon is still a very bad conductor.
Silicon is considered a semiconductor.

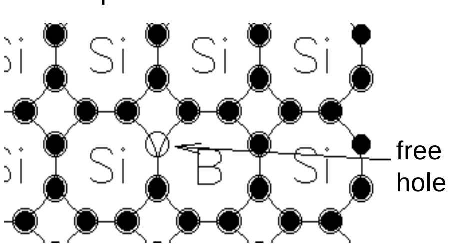
Doping

p-type

Some atoms, like boron and aluminum naturally have 3 valence electrons in their outer shells.

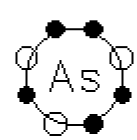


If you replace some of the silicon atoms in a crystal with boron there won't be quite enough electrons to fill the crystalline bond structure and unfilled spaces will act just like free holes. This "doped" silicon crystal is now called an p-type semiconductor. The p refers to the "extra" "positive" carriers.

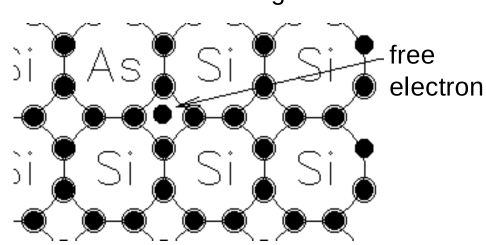


n-type

Some atoms, like arsenic and phosphorus naturally have 5 valence electrons in their outer shells.

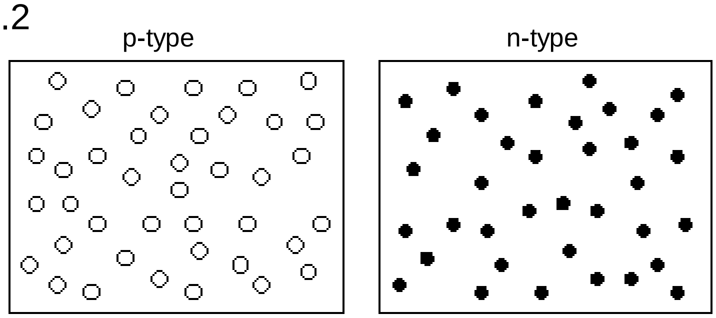


If you replace some of the silicon atoms in a crystal with arsenic the 5th electron doesn't fit into the crystalline bond structure and is therefore free to roam about and be a carrier. This "doped" silicon crystal is now called an n-type semiconductor. The n refers to the "extra" negative carriers.



Diode Physics (The simple version) p.2

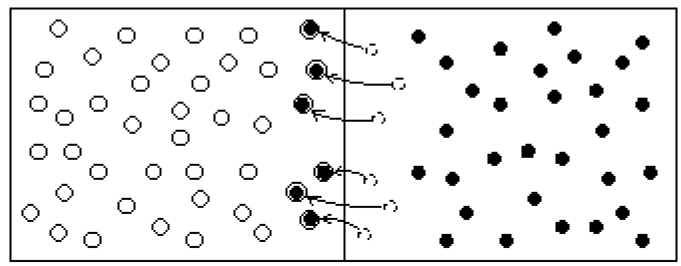
It turns out that the free carriers are the most important things in the semiconductor crystals, so we can simplify the drawings to show only these free carriers.



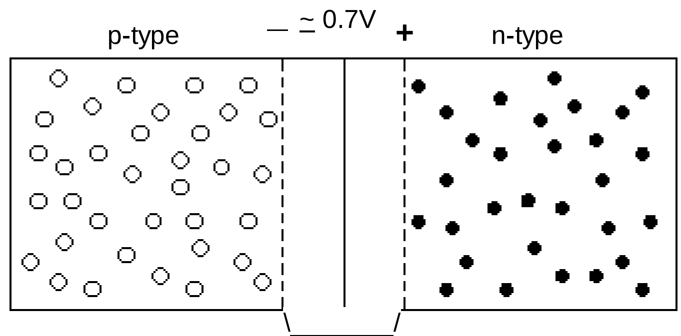
circles represent free holes   dots represent free electrons

PN Junction

When a p-type semiconductor is created next to an n-type, some of the free electrons from the n side will cross over and fill some of the free holes on the p side. This makes the p side negatively charged and leaves the n side positively charged. When the voltage across the junction reaches about 0.7 V the electrons find it too difficult to move against the charge and the process stops.



A region near the junction is now depleted of carriers and (surprise) is called the depletion region.



Reverse bias

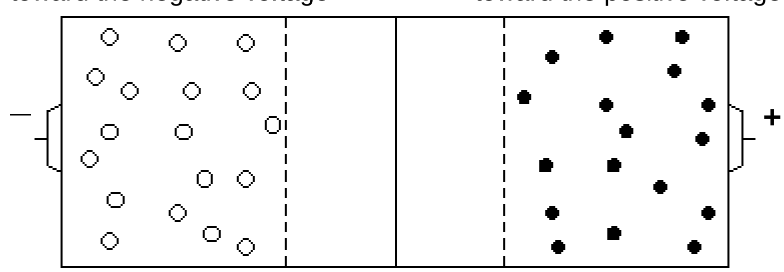
This pn junction is now a diode. If you place an external voltage across the diode in the reverse bias direction, the depletion region gets bigger and no current flows.

This reverse bias region can be used as a heat or light sensor since the only current flow should be due to a few carriers produced by these effects.

The reverse biased diode can also be used as a voltage variable capacitor since it is essentially an insulator (the depletion region) sandwiched between two conducting regions.

"positive" holes move toward the negative voltage

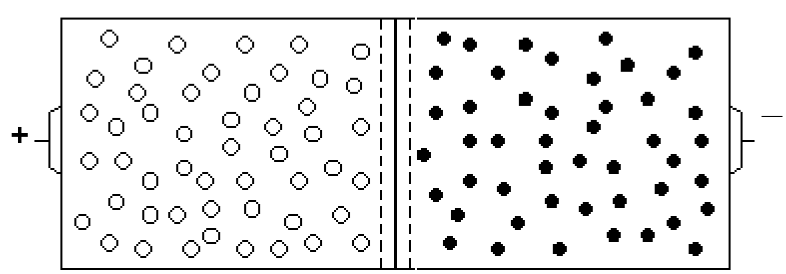
negative electrons move toward the positive voltage



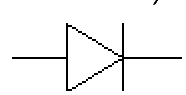
With reverse bias the depletion region gets bigger

Forward bias

If you place an external voltage across the diode in the forward bias direction, the depletion region shrinks until your external voltage reaches about 0.7V. After that the diode conducts freely..



With forward bias the depletion region gets smaller and eventually (at about 0.7V) conducts freely.

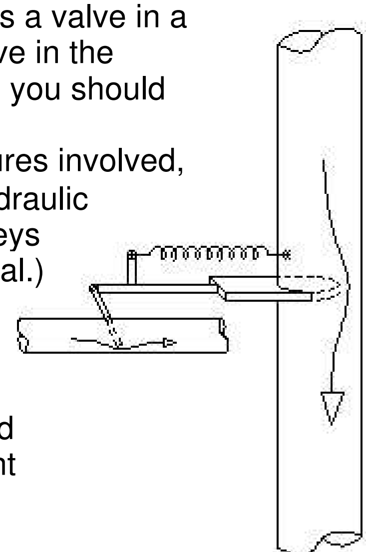


Diode

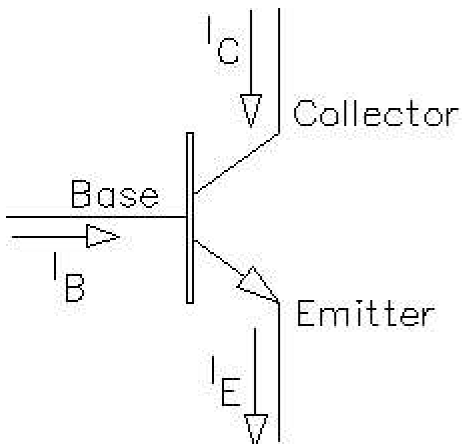
Transistor Notes (BJT) ECE 2210

A.Stolp
3/25/00 rev, 11/21/06

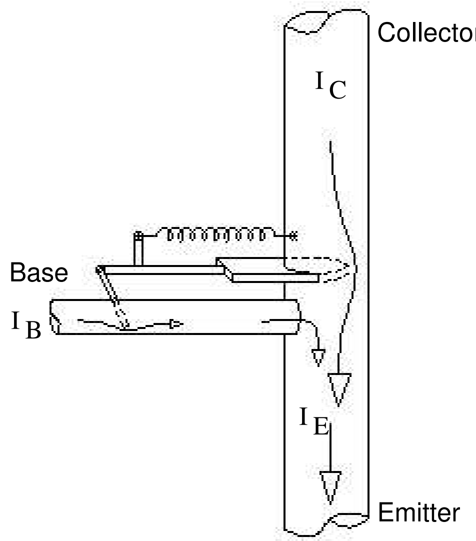
Imagine, if you will, a hydraulic device where the flow in a small pipe controls a valve in a larger pipe. The greater the flow in the small pipe the more it opens the valve in the large pipe. Take a look at the figure to the right. As an engineering student you should immediately see that this could be a useful device. One use might be as a flow-controlled on/off valve (switch). Or, depending on the flows and pressures involved, it could be used as an *amplifier*. That is, it could be used to make some hydraulic signal larger and more powerful. (A signal is a flow or pressure which conveys information and an amplifier is a device which increases the power of a signal.)



The electrical equivalent of this flow-controlled valve is a transistor. Specifically the NPN bipolar junction transistor (BJT). (There are other types.) The symbol for a transistor is shown below. Notice that it's a three-terminal device. That's because the control current (into the base) and the controlled current (into the collector) join together to form a single current out of the bottom (the emitter current). The valve drawn below is a more accurate analogy for the electrical transistor.



NPN transistor

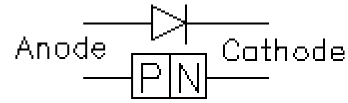


A transistor has three terminals-- the base, the collector, and the emitter. The current flow from the collector to the emitter (through the transistor) is controlled by the current flow from the base to the emitter. A small base current can control a much larger collector current. Often they are related by a simple factor, called beta (β). For a given base current, the transistor will allow β times as much collector current. The key word here is *allow*. The transistor doesn't make the current flow-- some outside power source does that. It simply regulates the current like the valve above. Big power transistors usually have a βs between 20 and 100. For little signal transistors, β is usually between 100 and 400. Darlington transistors (really two transistors in one package) can have βs in the 1000s.

A transistor can be used as a current controlled switch. When there's no base current, it's off, like an open switch. When there is a base current, it's on. If something outside of the transistor is limiting the collector current to less than β times the base current then the transistor will turn on as much as it can, like a closed switch. A transistor that is off is operating in its "cutoff" region. A transistor that is fully on is operating in its "saturation" region. A transistor that is partially on is in active control of its collector current (β times the base current) and is operating in its "active" region. (Note the valve analogy has a problem with the "open" and "closed" terms.)

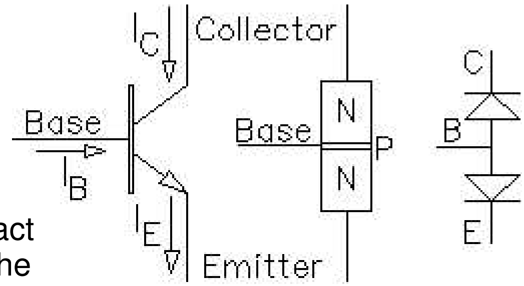
There are many types of transistors. PNP transistors work like the NPN transistors, except that all the currents and voltages are backwards. Field-effect transistors (FETs) are are controlled by voltage instead of current and come in many varieties. In this class we'll only work with NPN transistors.

Silicon diodes are made of two layers of doped silicon, a P layer is the anode and an N layer is the cathode. A P-N junction is a diode.



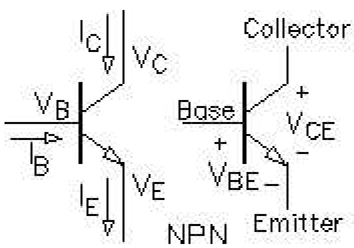
Bipolar junction transistors (BJTs) consist of three layers of doped silicon. The NPN transistor has a thin layer of P-doped silicon sandwiched between two layers of N-doped silicon. Each P-N junction can act like a diode. In fact, this is a fairly good way to check a transistor with an ohmmeter (set to the diode setting).

The base-emitter junction always acts like a diode, but because the base is very thin, it makes the other junction act like a controlled valve (you probably don' t want to know the details, so call it magic).

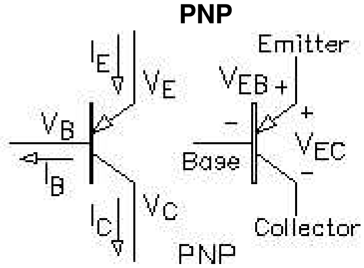


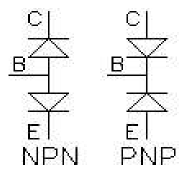
A bipolar junction transistor contains two diode junctions

Transistor Symbols



PNP





Cutoff	Active	Saturation
reverse	reverse	forward
reverse	forward	forward

Junction bias

Notice the subscripts

$v_{BE} = v_B - v_E$

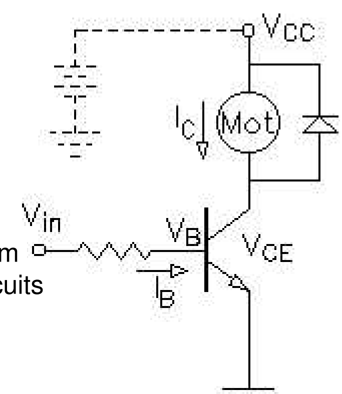
$v_{CE} = v_C - v_E$

Replace v_{BE} with v_{EB} and v_{CE} with v_{EC} in equations below

Modes or regions of operation (v_{BE} and v_{CE} are approximate)

Cutoff (off)	Active (partially on)	Saturation (fully on)
$v_{BE} < 0.7V$	$v_{BE} \simeq 0.7V$	$v_{BE} \simeq 0.7V$
$i_B = 0$	$i_B > 0$	$i_B > 0$
	$v_{CE} \geq 0.2V$	$v_{CE} \simeq 0.2V$
$i_C = 0$	$i_C = \beta i_B = \alpha i_E$ $\alpha \simeq 1$ controlled by the transistor	$i_C < \beta i_B$ limited by something outside of the transistor

input from logic circuits



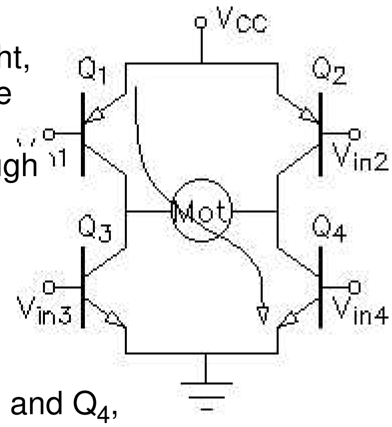
Motor Driver

The Transistor as a switch

One of the most common uses of a transistor is as a current-controlled switch. Transistor switches are the basis for all digital circuits, but that' s probably not where you' ll use the transistor. More likely, you' ll want to control a high-current device, like a motor, with arintegrated-circuit output from a computer or logic circuit . The small integrated circuit won' t be able to supply enough current to run the motor, so you' ll use a transistor to switch the larger current that flows through the motor. The input is hooked to the base of the transistor. (Often through a current limiting resistor, since V_B will only be 0.7V when the transistor is on.) A small I_B can switch on the much larger I_C and V_{CE} can be as low as 0.2V.

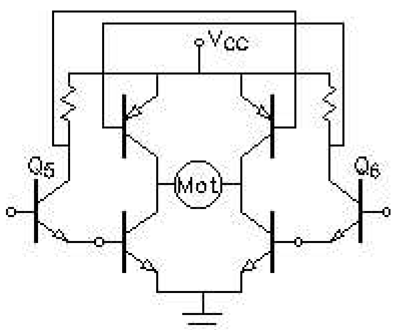
V_{CC}: The terminal marked V_{CC} above is just a circuit terminal hooked to a power supply, drawn in dotted lines here, but usually not shown at all. Power supply wires, like ground wires are often not shown explicitly on schematics. It makes the schematics a little less cluttered and easier to read.

Diode: If you' re switching an inductive load, like a motor, you should add a diode so that you' re not trying to switch off the motor current instantly. The diode (called a *flyback* diode when used like this) provides a path for the current still flowing through the motor when the transistor is switched off.



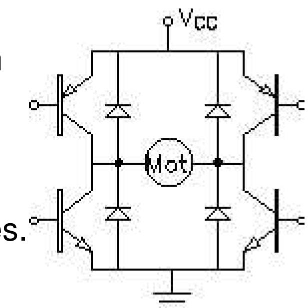
H-bridge: Of course, if you want to make the motor turn in both directions you' ll need a more complex circuit. Look at the circuit at right, it' s has the shape of an H, hence the name. If transistors Q_1 and Q_4 are on, then the current flows as shown, left-to-right through the motor. If transistors Q_2 and Q_3 are on, then the current flows the other way through the motor and the motor will turn in the opposite direction. (The motor here is a permanent-magnet DC motor.) In my circuit, the top two transistors are PNPs, which makes the circuit more efficient. The H-bridge could also be made with all NPNs or with power MOSFET transistors.

An H-bridge requires four inputs, all operated in concert. To turn on Q_1 and Q_4, as shown, V_{in1} would have to be low and V_{in4} would have to be high. At the same time, the other two transistors would have to be off, so V_{in2} would have to be high and V_{in3} would have to be low.



If the control circuit makes a mistake and turns on Q_1 and Q_3 (or Q_2 and Q_4) at the same time you' ll have a toaster instead of a motor driver, at least for a short while.

The circuit at left requires only two inputs. Transistors Q_5 and Q_6 work as *inverters*, when their inputs are high, their outputs are low and vice-versa. The resistors are known as *pull-up* resistors.



The H-bridge should also include flyback diodes.

Linear Amplifiers

The objective of a linear amplifier is to output a faithful reproduction of an input signal, only bigger. A voltage amplifier makes the signal voltage bigger. A current amplifier makes the signal current bigger. Many amplifiers do both. All amplifiers should make the signal power bigger (depends somewhat on the load). Of course that means that they need a source of power, generally DC power from a battery or power supply. The signals are usually AC.

Unlike transistor switches, which operate in cutoff and saturation, linear amplifiers must operate in the active region.

Important relations: (active region)

$$v_{BE} = v_B - v_E = 0.7 \cdot V \qquad v_{CE} = v_C - v_E > 0.7 \cdot V \text{ (} \simeq 0.2\text{V if saturated)}$$

$$i_C = \beta \cdot i_B \qquad i_C = \alpha \cdot i_E \simeq i_E$$

Bias:

Outside of the active region the input (base current) doesn' t linearly control the output (collector current). To work as an linear amplifier, a transistor must operate in the active region. That means that the transistor must be turned on part way even when there' s no signal at al Look back at the valve analogy, if small fluctuations in the horizontal pipe flow (i_B) should produce larger but similar fluctuations in the vertical pipe flow (i_C), then there must always be *some* flow. If either flow ever stops, the horizontal pipe flow (i_B) is no longer in control.

To work in the active region i_B and i_C must be positive for all values of the AC signals. i_B and i_C must be *biased* to some positive DC value. We use capital letters (I_B and I_C) for these DC bias values and lower case letters (i_b and i_c) for the AC signals that will appear as fluctuations of these DC values

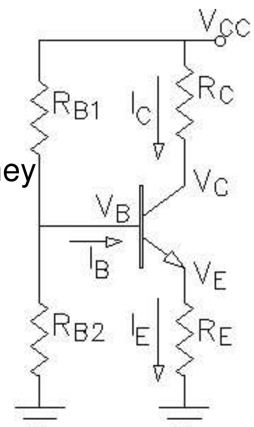
All voltages and currents can be shown in three different ways

	examples		meaning
CAP_{CAP}	V_B	I_C	DC, Bias
sm_{sm}	v_b	i_c	AC, signal
sm_{CAP}	v_B	i_C	DC and AC Together

The objective of bias then, is to partially turn on the transistor, to turn it, sort-of, half-way on. Now if I twiddle i_B, i_C will show a similar, but bigger, twiddle-- that' s the whole idea. The transistor should never go into cutoff for any expected input signal, otherwise you' ll ge*clipping* at the output. Clipping is a form of distortion, where the output no longer looks like the input.

Furthermore, the transistor must not saturate. That will also cause clipping at the output.

Because β can vary widely from transistor to transistor of the same part number and V_{BE} changes with temperature, achieving a stable bias can be a bit of a problem. Usually an emitter resistor (R_E) is needed to stabilize the bias.



DC Analysis in the active region

DC analysis applies to both switching and bias, although the circuits we' ll look at here will include an R_E and we' ll be working in the active region, meaning they are bias circuits. The key to DC analysis with an R_E is usually finding V_B.

The circuit at right shows a typical bias arrangement. The equations below are for that circuit, adapt them as necessary to fit your actual circuit.

If you can neglect I_B:

Often in quick-and-dirty analysis you can neglect the base current, I_B. In that case:

$$V_B = V_{CC} \cdot \frac{R_{B2}}{R_{B1} + R_{B2}} \qquad V_E = V_B - 0.7 \cdot V \qquad I_E = \frac{V_E}{R_E} \simeq I_C \qquad V_C = V_{CC} - I_C \cdot R_C$$

This assumption is OK if: $R_{B1} \| R_{B2} << \beta \cdot R_E$

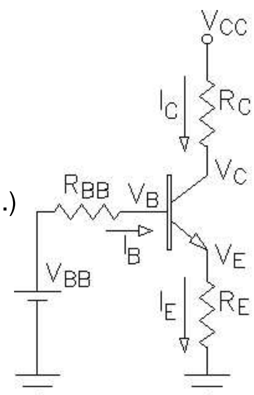
Quick check: $R_{B1} < 10 \cdot R_E$ and/or $R_{B2} < 10 \cdot R_E$ Should result in <10% error if $\beta = 100$

If you can't neglect I_B:

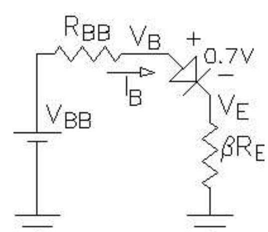
Then you need to make a Thevenin equivalent of the base bias resistors.

$$V_{BB} = V_{CC} \cdot \frac{R_{B2}}{R_{B1} + R_{B2}} \qquad R_{BB} = \frac{1}{\dfrac{1}{R_{B1}} + \dfrac{1}{R_{B2}}}$$

(Thevenin Eq.)



From the base' s point-of-view, the emitter resistor will look β + 1) times bigger than it really is. This is because (β + 1) times as much current flows through R_E than into the base. We can ignore the fact that the current is bigger if we pretend that the resistor is bigger. That leads to the simplified circuit. (Usually we use β as the factor rather than (β + 1), after all β just isn' t that well known anyway.)



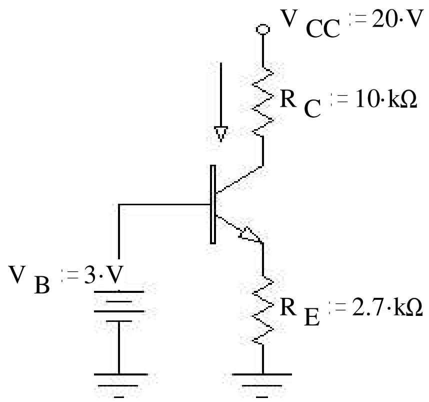
$$I_B = \frac{V_{BB} - 0.7 \cdot V}{R_{BB} + \beta \cdot R_E} \qquad I_C = \beta \cdot I_B \simeq I_E \qquad V_E = I_E \cdot R_E \simeq I_C \cdot R_E \qquad V_B = V_E + 0.7 \cdot V$$

$$V_C = V_{CC} - I_C \cdot R_C$$

OR: $V_B = I_B \cdot \beta \cdot R_E + 0.7 \cdot V$ $\quad V_E = V_B - 0.7 \cdot V$ $\quad I_E = \dfrac{V_E}{R_E} \simeq I_C$ $\quad V_C = V_{CC} - I_C \cdot R_C$

Examples, DC (Bias) Analysis

1)



Given:

$V_B := 3 \cdot V$, regardless of current into base

$V_{CC} := 20 \cdot V \quad R_C := 10 \cdot k\Omega \quad R_E := 2.7 \cdot k\Omega$

Find I_C, V_C, V_{CE}, and P_Q:

Solution:

$V_E := V_B - 0.7 \cdot V \qquad V_E = 2.3 \cdot V$

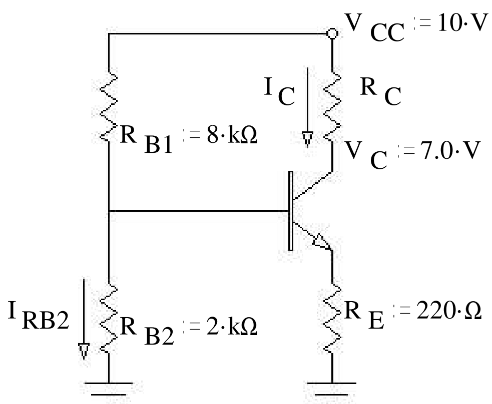
$I_E := \dfrac{V_E}{R_E} \qquad I_E = 0.852 \cdot mA \quad \simeq \quad I_C := I_E$

$V_C := V_{CC} - I_C \cdot R_C \qquad V_C = 11.48 \cdot V$

$V_{CE} := V_C - V_E \qquad V_{CE} = 9.18 \cdot V$ > 0.2V, OK, is in active region

$P_Q := V_{CE} \cdot I_C \qquad P_Q = 7.82 \cdot mW$

2)



Given: may neglect I_B

$V_{CC} := 10 \cdot V \quad V_C := 7.0 \cdot V \quad R_{B1} := 8 \cdot k\Omega \quad R_{B2} := 2 \cdot k\Omega \quad R_E := 220 \cdot \Omega$

Find V_B, V_E, I_C, R_C, V_{CE}, I_{RB2}, and P_Q:

Solution:

$V_B := V_{CC} \cdot \dfrac{R_{B2}}{R_{B1} + R_{B2}} \qquad V_B = 2 \cdot V$

$V_E := V_B - 0.7 \cdot V \qquad V_E = 1.3 \cdot V$

$I_E := \dfrac{V_E}{R_E} \qquad I_E = 5.91 \cdot mA \quad \simeq \quad I_C := I_E$

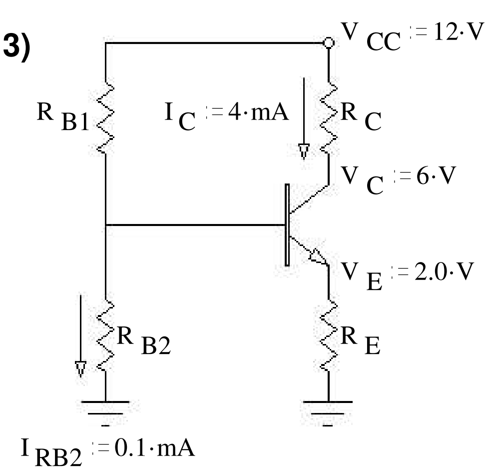
$R_C := \dfrac{V_{CC} - V_C}{I_C} \qquad R_C = 508 \cdot \Omega$

$V_{CE} := V_C - V_E \qquad V_{CE} = 5.7 \cdot V$ > 0.2V, OK, is in active region

$I_{RB2} := \dfrac{V_B}{R_{B2}} \qquad I_{RB2} = 1 \cdot mA$

$P_Q := V_{CE} \cdot I_C \qquad P_Q = 33.68 \cdot mW$

3)



Given: may NOT neglect I_B $\quad \beta := 150$

$V_{CC} := 12 \cdot V \quad V_E := 2.0 \cdot V \quad V_C := 6 \cdot V \quad I_{RB2} := 0.1 \cdot mA \quad I_C := 4 \cdot mA$

Find R_E, R_C, V_B, I_B, R_{B2}, and R_{B1}:

Solution:

$V_{CE} := V_C - V_E \qquad V_{CE} = 4 \cdot V$ > 0.2V, is in active region

$I_E \simeq I_C \quad I_E := I_C \qquad R_E := \dfrac{V_E}{I_E} \qquad R_E = 500 \cdot \Omega$

$R_C := \dfrac{V_{CC} - V_C}{I_C} \qquad R_C = 1.5 \cdot k\Omega$

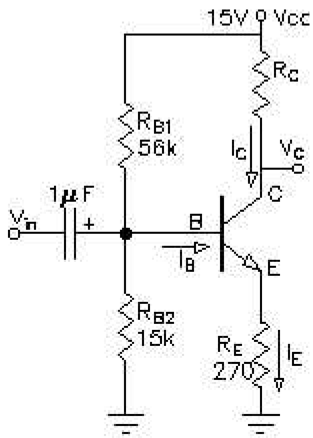
$V_B := V_E + 0.7 \cdot V \qquad V_B = 2.7 \cdot V \qquad I_B := \dfrac{I_C}{\beta} \qquad I_B = 0.027 \cdot mA$

$R_{B2} := \dfrac{V_B}{I_{RB2}} \qquad R_{B2} = 27 \cdot k\Omega \qquad R_{B1} := \dfrac{V_{CC} - V_B}{I_{RB2} + I_B} \qquad R_{B1} = 73.4 \cdot k\Omega$

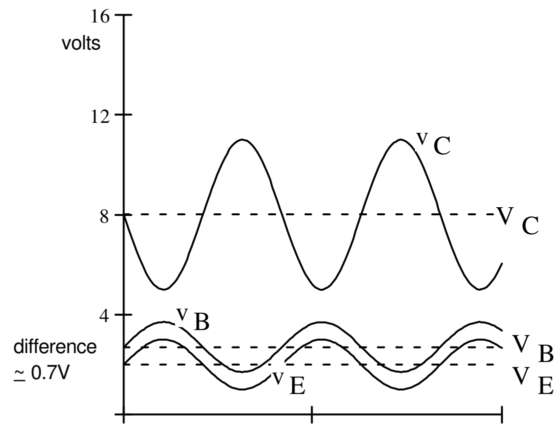
AC Analysis of Common emitter (CE) amplifier

With an R_E, any AC signal applied to the base will then also appear just as big at the emitter (just lower by 0.7V DC). The AC signal current through R_E, will be about the same as through R_C, so the AC signal voltage across R_C will be bigger than that across R_E by the ratio of R_C/R_E. Recalling that the signal at the emitter is about the same as the signal at the base...

base to collector AC gain $= \dfrac{v_c}{v_b} = \dfrac{R_C}{R_E}$



Signals



v_E follows v_B, just 0.7V DC lower

If a capacitor is placed in parallel with R_E then the effective AC resistance in the emitter goes way down and the gain goes way up. In that case we need a way to estimate the AC resistance within the base-emitter junction itself.

This is called the small-signal emitter resistance: $r_e = \dfrac{25 \cdot mV}{I_C}$

To find the gains when the input has a source resistance and the output is connected to a load resistor, the calculations become a little more complex. YOU DON' T NEED TO KNOW THE FOLLOWING MATERIAL.

R_E is the DC resistance from emitter to ground

R_e is the AC signal resistance from emitter to ground, may be zero

Input impedance: $R_i = R_{B1} \| R_{B2} \| \beta \cdot (r_e + R_e)$

Output impedance: $R_o = R_C \| r_o$ <-- r_o Often neglected

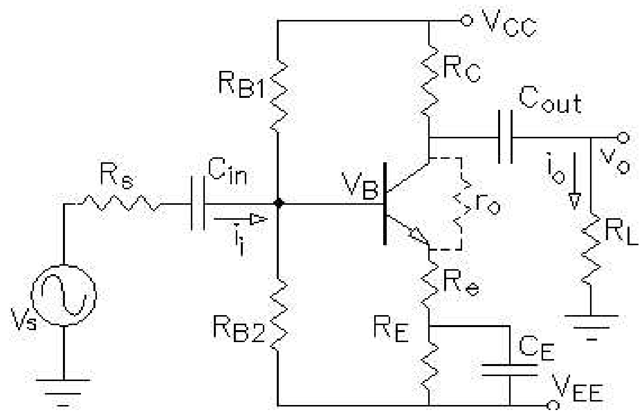
AC collector resistance: $r_c = R_C \| R_L \| r_o$

r_o is a characteristic of the transistor, and is often neglected

Voltage gain: $A_v = \dfrac{v_o}{v_b} = \dfrac{r_c}{r_e + R_e}$

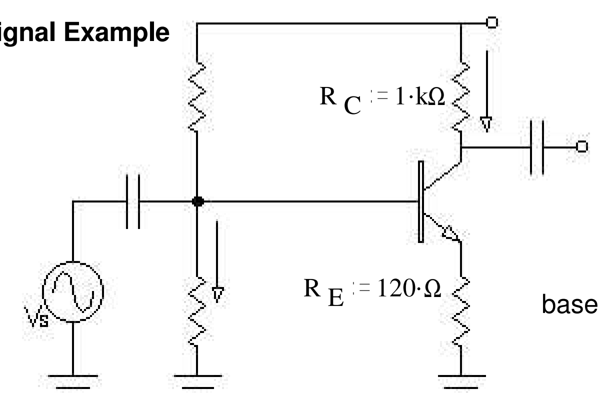
OR: $\dfrac{v_o}{v_s} = \dfrac{R_i}{R_S + R_i} \cdot \dfrac{r_c}{r_e + R_e}$

Current gain: $A_i = \dfrac{i_o}{i_i} = \dfrac{r_c}{r_e + R_e} \cdot \dfrac{R_i}{R_L} = A_v \cdot \dfrac{R_i}{R_L}$



There are several other types of transistor amplifiers, but we won' t look at them here.

AC Signal Example



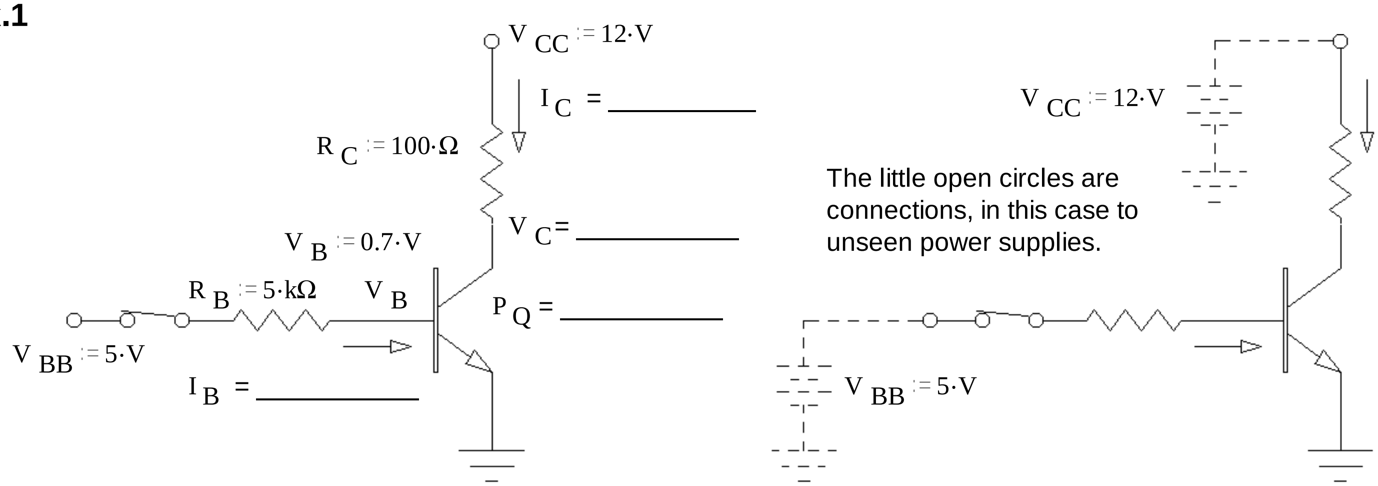
If the v_s signal were applied at the base, an AC signal would also appear at the collector. How much larger would it be? (Voltage gain).

base to collector AC gain $= \dfrac{v_c}{v_b} = \dfrac{R_C}{R_E} = 8.33$ times bigger

ECE 2210 Transistor Switching Circuit Examples

A.Stolp 11/21/06
rev 12/3/14

Ex.1



If: $\beta := 100$ $\quad I_B := \dfrac{V_{BB} - 0.7 \cdot V}{R_B}$ $\quad I_B = 0.86 \cdot mA$

$I_C := \beta \cdot I_B$ $\quad I_C = 86 \cdot mA$

$V_C := V_{CC} - R_C \cdot I_C$ $\quad V_C = 3.4 \cdot V$

$P_Q := V_C \cdot I_C$ $\quad P_Q = 292.4 \cdot mW$

If: $\beta := 200$ $\quad I_B := \dfrac{V_{BB} - 0.7 \cdot V}{R_B}$ $\quad I_B = 0.86 \cdot mA$

$I_C := \beta \cdot I_B$ $\quad I_C = 172 \cdot mA$

$V_C := V_{CC} - R_C \cdot I_C$ $\quad V_C = -5.2 \cdot V$

must be in saturation: $\quad V_C := 0.2 \cdot V$

$I_C := \dfrac{V_{CC} - V_C}{R_C}$ $\quad I_C = 118 \cdot mA$

$P_Q := V_C \cdot I_C$ $\quad P_Q = 23.6 \cdot mW$

Since saturation can depend on β, You usually assume a small β when designing a circuit that should saturate (a switching circuit).

If: $R_C := 50 \cdot \Omega$ $\quad I_C := \beta \cdot I_B$ $\quad I_C = 172 \cdot mA$ $\quad V_C := V_{CC} - R_C \cdot I_C$ $\quad V_C = 3.4 \cdot V$

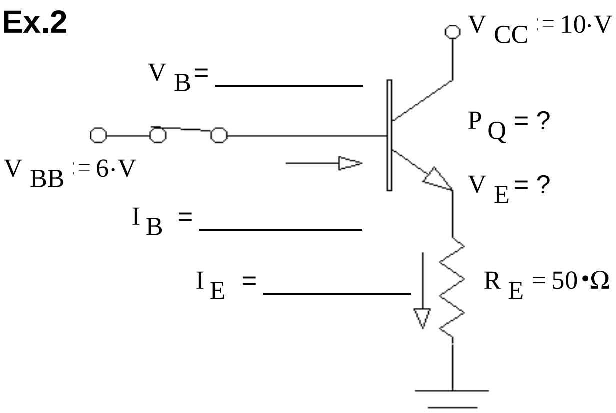
same $\beta := 200$ $\quad P_Q := V_C \cdot I_C$ $\quad P_Q = 584.8 \cdot mW$

Saturation also depends on R_C and V_{CC}.

What is the largest value that R_B could be and still keep the transistor in saturation?

$I_{Csat} := \dfrac{V_{CC} - 0.2 \cdot V}{R_C}$ $\quad I_{Csat} = 236 \cdot mA$ $\quad I_B := \dfrac{I_{Csat}}{\beta}$ $\quad I_B = 1.18 \cdot mA$ $\quad R_{Bmax} = \dfrac{5 \cdot V - 0.7 \cdot V}{I_B} = 3.644 \cdot k\Omega$

Ex.2



$V_B := 6 \cdot V$ $\quad V_E := 5.3 \cdot V$ $\quad R_E := 50 \cdot \Omega$

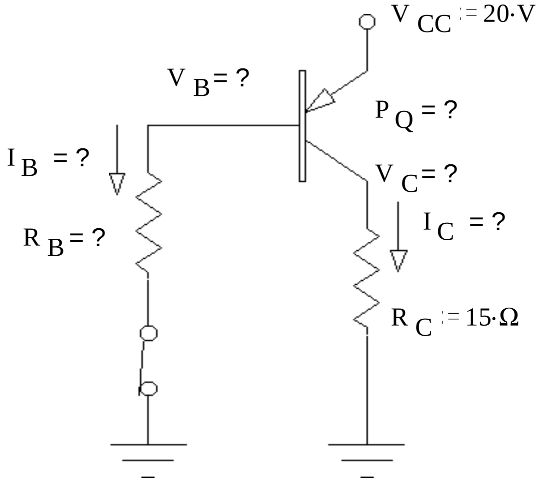
$I_E := \dfrac{V_E}{R_E}$ $\quad I_E = 106 \cdot mA$ Doesn't depend on β

$I_B := \dfrac{I_E}{\beta + 1}$ $\quad I_B = 0.5 \cdot mA$ $\quad \simeq \dfrac{I_E}{\beta} = 0.53 \cdot mA$

If β is big enough.

$P_Q := (V_{CC} - V_E) \cdot I_E$ $\quad P_Q = 0.498 \cdot W$

Ex.3 If the load must be connected to ground, a PNP transistor is often a better choice.



Let's assume a a small β and saturation and find the R_B necessary.

a small β: $\beta := 20$

$V_C := V_{CC} - 0.2 \cdot V$ $\qquad V_C = 19.8 \cdot V$

$R_C := 15 \cdot \Omega$

$I_{Csat} := \dfrac{V_C}{R_C}$ $\qquad I_{Csat} = 1.32 \cdot A$

$I_B := \dfrac{I_{Csat}}{\beta}$ $\qquad I_B = 66 \cdot mA$

$V_B := V_{CC} - 0.7 \cdot V$ $\qquad V_B = 19.3 \cdot V$

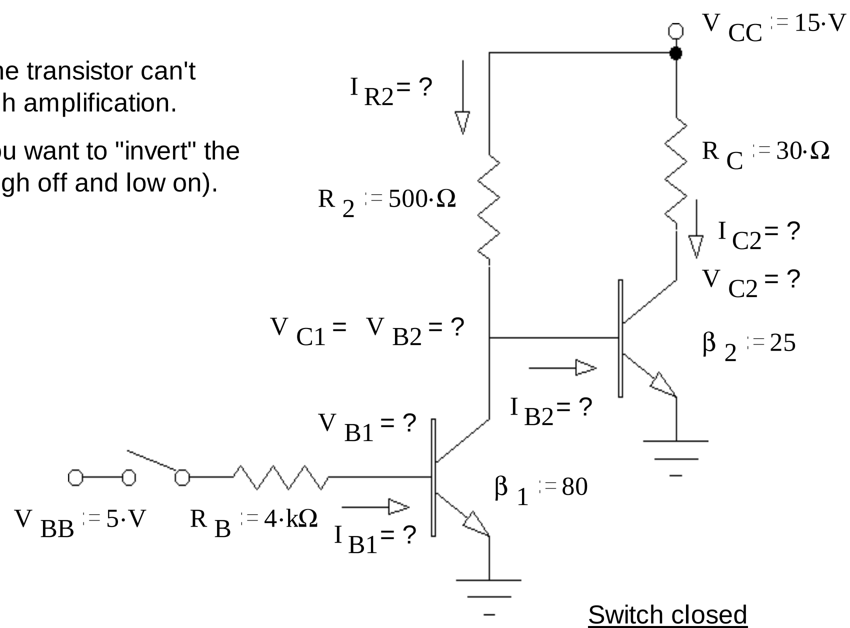
$R_B := \dfrac{V_B}{I_B}$ $\qquad R_B = 292 \cdot \Omega$

$P_Q := 0.2 \cdot V \cdot I_C$ $\qquad P_Q = 34 \cdot mW$

Ex.4

Sometimes one transistor can't provide enough amplification.

Sometimes you want to "invert" the input (make high off and low on).



<u>Switch open</u>

$I_{B1o} := 0$ $\qquad V_{B1o} := 0 \cdot V$

$V_{B2o} := 0.7 \cdot V$

$I_{B2o} := \dfrac{V_{CC} - 0.7 \cdot V}{R_2}$ $\qquad I_{B2o} = 28.6 \cdot mA$

$I_{R2o} := I_{B2o}$ $\qquad I_{R2o} = 28.6 \cdot mA$

$I_{C2o} := \beta_2 \cdot I_{B2o}$ $\qquad I_{C2o} = 715 \cdot mA$

$V_{C2o} := V_{CC} - R_C \cdot I_{C2o}$ $\qquad V_{C2o} = -6.45 \cdot V$

Q_2 must be in saturation: $\qquad V_{C2o} := 0.2 \cdot V$

$I_{C2o} := \dfrac{V_{CC} - V_{C2o}}{R_C}$ $\qquad I_{C2o} = 493.3 \cdot mA$

<u>Switch closed</u>

$I_{B1c} := \dfrac{5 \cdot V - 0.7 \cdot V}{R_B}$ $\qquad I_{B1c} = 1.08 \cdot mA$

assume Q_1 is in saturation $\qquad V_{C1c} := 0.2 \cdot V$

$I_{C1c} := \dfrac{V_{CC} - V_{C1c}}{R_2}$ $\qquad I_{C1c} = 29.6 \cdot mA$

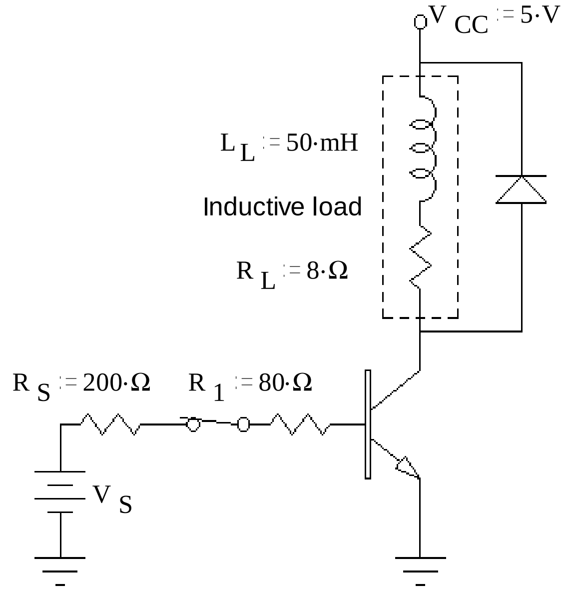
$\beta_1 \cdot I_{B1c} = 86 \cdot mA$ $\qquad I_{C1}$ is controlled by R_2

$V_{B2c} := V_{C1c}$ $\qquad V_{B2c} = 0.2 \cdot V$

$I_{B2c} := 0$ $\qquad I_{C2c} := 0 \quad = \quad I_{RCc}$

When the switch is open, current flows in through the load resistor, R_C, When it is closed, no current flows though the load. This is an example of logical "inversion".

Ex.5 Modified from F07 Final



A transistor is used to control the current flow through an inductive load (in the dotted box, it could be a relay coil or a DC motor).

a) Assume the transistor is in saturation (fully on) and that switch has been closed for a long time. What is the load current?

$I_C = ?$

$I_{Csat} := \dfrac{V_{CC} - 0.2\cdot V}{R_L}$ $\quad I_{Csat} = 600\cdot mA$

b) $\beta := 80$ find the minimum value of V_S, so that the transistor will be in saturation.

$I_{Bmin} := \dfrac{I_{Csat}}{\beta}$ $\quad I_{Bmin} = 7.5\cdot mA$

$V_{Smin} := I_{Bmin}\cdot(R_S + R_1) + 0.7\cdot V$ $\quad V_{Smin} = 2.8\cdot V$

Use this V_S for the rest of the problem.

c) Does the diode in this circuit ever conduct a significant current? If yes, when and how much?

When the switch opens. $\quad I_{Dmax} = I_{Csat} = 600\cdot mA$ $\quad$ from part a)

d) You got a bad transistor. $\beta := 60$ Find the new I_C, and V_{CE} and P_Q.

$I_C = ?$ $\quad I_C := \beta\cdot I_{Bmin}$ $\quad I_C = 450\cdot mA$ $\quad$ Now operating in active region

$V_{CE} = ?$ $\quad V_{CE} := V_{CC} - R_L\cdot I_C$ $\quad V_{CE} = 1.4\cdot V$

$P_Q = ?$ $\quad P_Q := V_{CE}\cdot I_C$ $\quad P_Q = 0.63\cdot W$

$\beta = 60$ Use this for the rest of the problem.

c) Find the minimum value of R_L so that the transistor will be in saturation.

$I_B := \dfrac{V_{Smin} - 0.7\cdot V}{R_S + R_1}$ $\quad I_B = 7.5\cdot mA$

$I_{Cmax} := \beta\cdot I_B$ $\quad I_{Cmax} = 450\cdot mA$

$R_{Lmin} := \dfrac{V_{CC} - 0.2\cdot V}{I_{Cmax}}$ $\quad R_{Lmin} = 10.7\cdot\Omega$

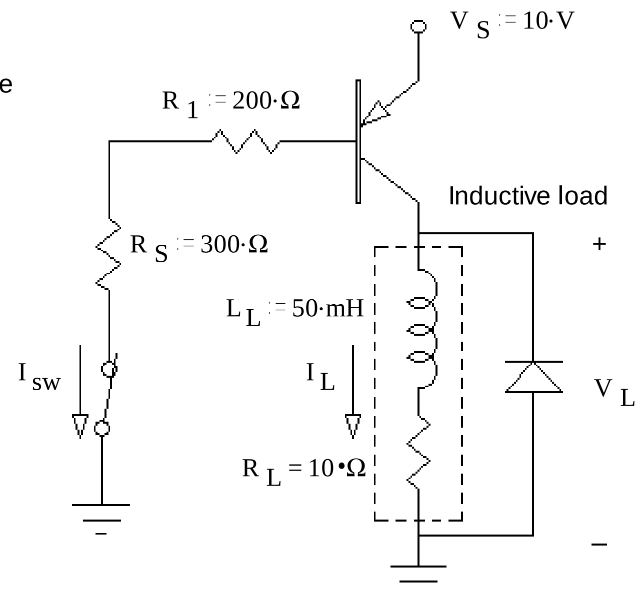
d) R_L, can't be changed, so find the maximum value of R_1 so that the transistor will be in saturation.

$I_{Csat} = 600\cdot mA$ $\quad$ from part a)

$I_{Bmin} := \dfrac{I_{Csat}}{\beta}$ $\quad I_{Bmin} = 10\cdot mA$

$R_{1max} = \dfrac{V_{Smin} - 0.7\cdot V}{I_{Bmin}} - R_S = 10\cdot\Omega$

Ex.6 From F05 Final with modifications from F06 Final



A transistor is used to control the current flow through an inductive load (in the dotted box, it could be a relay coil or a DC motor).

a) $\beta := 25$ Assume the transistor is in the active region, find I_{sw}, I_L, V_L, V_{EC} and P_Q.

$$I_B := \frac{V_S - 0.7\cdot V}{R_S + R_1} \qquad I_B = 18.6\cdot mA = I_{sw}$$

$$I_L := \beta \cdot I_B \qquad I_L = 465\cdot mA$$

$$R_L := 10\cdot\Omega$$

$$V_L := I_L \cdot R_L \qquad V_L = 4.65\cdot V$$

$$V_{EC} := V_S - V_L \qquad V_{EC} = 5.35\cdot V$$

$$P_Q := V_{EC}\cdot I_L \qquad P_Q = 2.488\cdot W$$

b) Was the transistor actually operating in the active region? yes no (circle one) yes

How do you know? (Specifically show a value which is or is not within a correct range.)

$$V_{EC} = 5.35\cdot V \;>\; 0.2\cdot V$$

c) Find the maximum value of R_1, so that the transistor will be in saturation.

If saturated: $V_{EC} := 0.2\cdot V$

$$I_{Csat} := \frac{V_S - 0.2\cdot V}{R_L} \qquad I_{Csat} = 0.98\cdot A$$

$$I_{Bmin} := \frac{I_{Csat}}{\beta} \qquad I_{Bmin} = 39.2\cdot mA$$

$$R_{1max} = \frac{V_S - 0.7\cdot V}{I_{Bmin}} - R_S = -63\cdot\Omega$$ **NOT POSSIBLE**

d) $R_1 = 200\cdot\Omega$ and can't be changed, find the minimum value of β so that the transistor will be in saturation.

$$I_{Csat} = 0.98\cdot A \qquad \beta_{min} := \frac{I_{Csat}}{I_B} \qquad \beta_{min} = 52.7$$

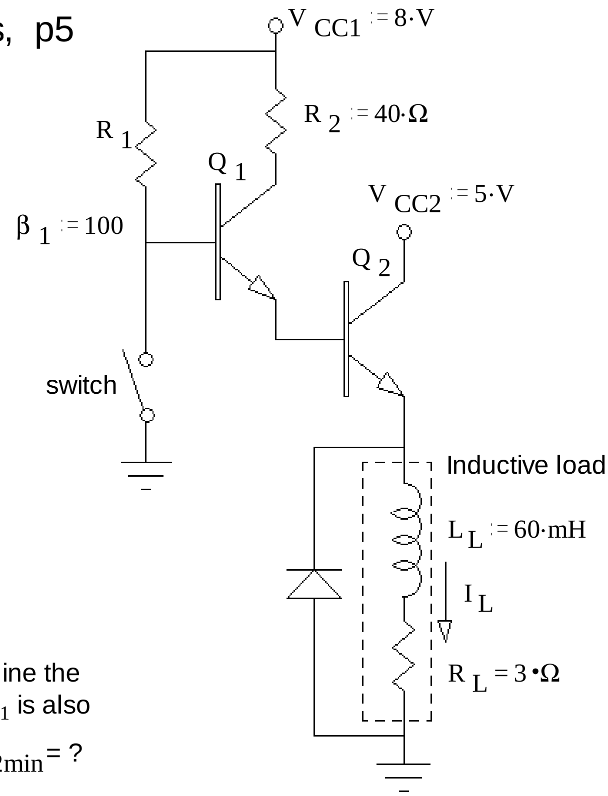
e) How much power is dissipated by the transistor if it has the β you found in part d)

$$P_Q := 0.2\cdot V\cdot I_{Csat} \qquad P_Q = 0.196\cdot W$$

f) Does the diode in this circuit ever conduct a significant current? If yes, when and how much?

When the switch opens. $I_{Dmax} = I_{Csat} = 0.98\cdot A$ from part a)

g) The switch is open for a while. What is the load current (I_L) now? 0



Ex.7 From F13 Final

A transistor is used to control the current flow through an inductive load (in the dotted box, it could be a relay coil or a DC motor).

a) In order for current to flow in through the load, the switch should be: i) closed or ii) open (Circle one)

b) Assume the switch has been in the position you circled above for a long time. I_L is 1.3A. Find the power dissipated by transistor Q_2 (neglect base current and V_{BE}).

$I_L := 1.3 \cdot A$ $\quad P_{Q2} = ?$ $\quad R_L := 3 \cdot \Omega$

$V_{CE2} := V_{CC2} - I_L \cdot R_L$ $\quad V_{CE2} = 1.1 \cdot V$

$P_{Q2} := V_{CE2} \cdot I_L$ $\quad P_{Q2} = 1.43 \cdot W$

c) This is an unacceptable power loss, so you would like to determine the minimum β_2 needed so that Q_2 will be in saturation. Assume Q_1 is also in saturation. You may assume $I_E = I_C$ for both traistors. $\quad \beta_{2min} = ?$

$$I_L := \frac{V_{CC2} - 0.2 \cdot V}{R_L} \qquad I_L = 1.6 \cdot A = I_{C2}$$

$V_{E2} := V_{CC2} - 0.2 \cdot V$ $\quad V_{E2} = 4.8 \cdot V$

$V_{B2} := V_{E2} + 0.7 \cdot V$ $\quad V_{B2} = 5.5 \cdot V$

$V_{C1} := V_{B2} + 0.2 \cdot V$ $\quad V_{C1} = 5.7 \cdot V$

$$I_{C1} := \frac{V_{CC1} - V_{C1}}{R_2} \qquad I_{B2} := I_{C1} \qquad I_{B2} = 57.5 \cdot mA \qquad \beta_{2min} = \frac{I_L}{I_{B2}} = 27.826$$

Better answer $\quad I_{B2} := I_{C1} \cdot \left(\frac{\beta_1 + 1}{\beta_1}\right)$ $\quad I_{B2} = 58.075 \cdot mA$ $\quad \beta_{2min} = \frac{I_L}{I_{B2}} - 1 = 26.551$

You replace Q_2 with a new transistor that has a β greater than what you just calculated.

d) How much power is dissipated by the new transistor Q_2 (neglect base current and V_{BE})? $\quad P_{Q2} = ?$

$P_{Q2} := 0.2 \cdot V \cdot I_L$ $\quad P_{Q2} = 320 \cdot mW$

e) What is the maximum value of R_1 needed to saturate Q_1? $\quad \beta_1 = 100$

$$I_{B1min} := \frac{I_{C1}}{\beta_1} \qquad I_{B1min} = 0.575 \cdot mA \qquad V_{B1} := V_{B2} + 0.7 \cdot V \qquad V_{B1} = 6.2 \cdot V$$

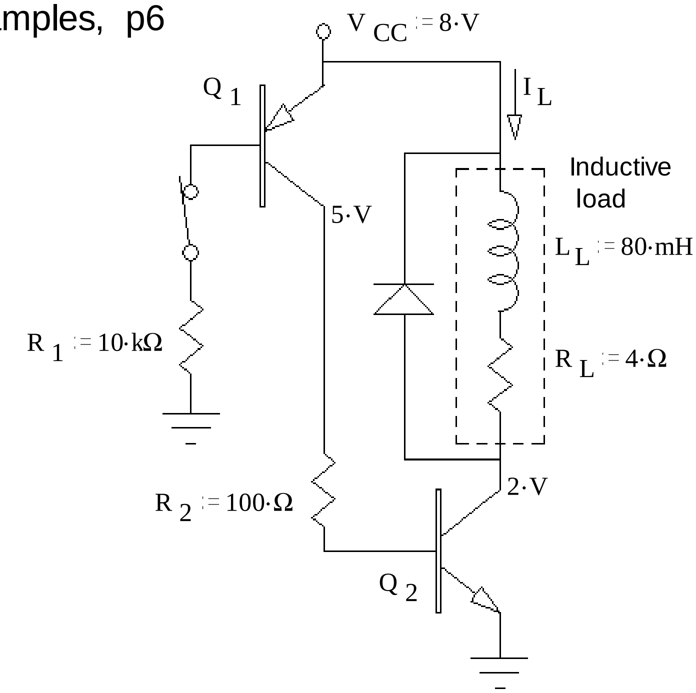
$$R_{1max} := \frac{V_{CC1} - V_{B1}}{I_{B1min}} \qquad R_{1max} = 3.13 \cdot k\Omega$$

f) Does the diode in this circuit ever conduct a significant current? If yes, when and how much?

When the switch closes. $\quad I_{Dmax} = I_L = 1.6 \cdot A$ $\quad$ from part c)

Ex.8 From F12 Final

A couple of transistors are used to control the current flow through an inductive load. The switch has been closed, as shown, for a long time.



a) You measure the voltage at each collector (referenced to ground) as shown on the drawing. Find the power dissipated by transistor Q_2 .

$V_{C1} := 5 \cdot V$ $\qquad V_{C2} := 2 \cdot V$

$I_L := \dfrac{V_{CC} - 2 \cdot V}{R_L}$ $\qquad I_L = 1.5 \cdot A$

$P_{Q2} := V_{C2} \cdot I_L$ $\qquad P_{Q2} = 3 \cdot W$

b) Find the β of transistor Q_2 .

$V_{R2} := 5 \cdot V - 0.7 \cdot V$ $\qquad V_{R2} = 4.3 \cdot V$

$I_{R2} := \dfrac{V_{R2}}{R_2}$ $\qquad I_{R2} = 43 \cdot mA$

$\beta_2 := \dfrac{I_L}{I_{R2}}$ $\qquad \beta_2 = 34.884$

c) Find the β of transistor Q_1 .

$I_{R1} := \dfrac{V_{CC} - 0.7 \cdot V}{R_1}$ $\qquad \beta_1 := \dfrac{I_{R2}}{I_{R1}}$ $\qquad \beta_1 = 58.9$

d) Find the minimum β for transistor Q_1 to be in saturation. $\beta_{1min} = ?$

If Q_1 is saturated: $V_{R2} := V_{CC} - 0.2 \cdot V - 0.7 \cdot V$ $\qquad V_{R2} = 7.1 \cdot V$

If Q_1 is saturated: $I_{R2} := \dfrac{V_{R2}}{R_2}$ $\qquad I_{R2} = 71 \cdot mA$ $\qquad \beta_{1min} := \dfrac{I_{R2}}{I_{R1}}$ $\qquad \beta_{1min} = 97.3$

You replace Q_1 with a different transistor so that now: $\beta_1 := 200$ Use this from now on.

e) Find the new load current (I_L) assuming transistor Q_2 is in the active region.

Q_1 is saturated: $I_{R2} = 71 \cdot mA$ $\qquad I_L := I_{R2} \cdot \beta_2$ $\qquad I_L = 2.477 \cdot A$

f) Check the assumption that Q_2 is in the active region and recaculate I_L if necessary.

$I_{R2} \cdot \beta_2 \cdot R_L = 9.907 \cdot V$ $\qquad V_{CE2} := V_{CC} - I_{R2} \cdot \beta_2 \cdot R_L$ $\qquad V_{CE2} = -1.907 \cdot V$ **Not possible**

Q_2 is saturated: $I_L := \dfrac{V_{CC} - 0.2 \cdot V}{R_L}$ $\qquad I_L = 1.95 \cdot A$

g) Does the diode in this circuit ever conduct a significant current? If yes, when and how much?

When the switch opens. $I_{Dmax} = 1.95 \cdot A$ from part f)

Operational Amplifiers

A. Stolp, 4/22/01
rev, 12/5/05

An *operational amplifier* is basically a complete high-gain voltage amplifier in a small package. Op-amps were originally developed to perform mathematical operations in analog computers, hence the odd name. They are now made using integrated circuit technology, so they come in the typical multi-pin IC packages. With the proper external components, the operational amplifier can perform a wide variety of “operations” on the input voltage. It can multiply the input voltage by nearly any constant factor, positive or negative, it can add the input voltage to other input voltages, and it can integrate or differentiate the input voltage. The respective circuits are called *amplifiers*, *summers*, *integrators*, and *differentiators*. Op-amps are also used to make active frequency filters, current-to-voltage converters, voltage-to-current converters, current amplifiers, voltage comparators, etc. etc.. These little parts are so versatile, useful, handy, and cheap that they’re kind of like electronic Lego blocks — although somewhat drably colored.

Op-amp characteristics

Operational amplifiers have several very important characteristics that make them so useful:

1. An op-amp has *two* inputs and it amplifies the *voltage difference* between those two inputs. These two inputs are known as the noninverting input, labeled (+), and the inverting input, labeled (-), as shown in Fig. 1. The output voltage is a function of the noninverting input voltage minus the inverting input voltage.

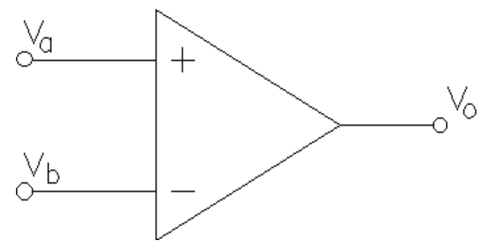


Figure 1 Op-amp symbol

That is: $v_o = G(v_a - v_b)$ Where G = voltage gain of the op-amp.

2. The op-amp must be connected to external sources of power (not shown on the drawing above). The output voltage (v_O) cannot be more positive than the positive power source or more negative than the negative power source. The gain (G) is very high, typically more than 100,000. Together that means that if the output (v_o) is in the *active* range (somewhere between its physical limits, often called “rails”), then $v_a - v_b \approx 0$, and $v_a \approx v_b$. This is a very important point. If you don’t see this, look back at the equation above, v_O is limited, G is very big, so $(v_a - v_b)$ must be very small.

<u>If the output is</u>:	**<u>The inputs must be</u>:**
In active range - rail < v_O < +rail	$v_a \approx v_b$

<u>If the inputs are</u>:	**<u>The output must be</u>:**
$v_a > v_b$	+ rail
$v_a < v_b$	- rail

3. In fact, $v_a - v_b$ must be so small that it’s very difficult to make v_a & v_b close enough

without using some *negative feedback*. Negative feedback makes the op-amp maintain $v_a \approx v_b$ for itself. With the proper negative feedback the op-amp keeps $v_a \approx v_b$ so close that you can assume that $v_a = v_b$. Without this negative feedback the op-amp output will almost certainly be at one of its limits, either high or low, i.e. NOT in its active, or *linear*, range. Incidentally, circuits without negative feedback are also useful, but then the output is either high or low (digital) and not linearly related to the input. These types of circuits are called *nonlinear* circuits.

4. Op-amps amplify DC as well as AC.

5. The input currents are almost zero. In more technical terms, the op-amp has very high input impedance. As long as you use reasonable resistor values in your circuits (say ≤ 1 MΩ), you can neglect the input currents.

Simple, isn' t it? OK, so it doesn't sound so simple yet, but the application of these characteristics really isn't hard. Let's look at some circuits.

Linear Circuits

Linear circuits employ negative feedback to keep $v_a \approx v_b$. If a circuit has a connection from the output to the inverting (-) input, then it has negative feedback.

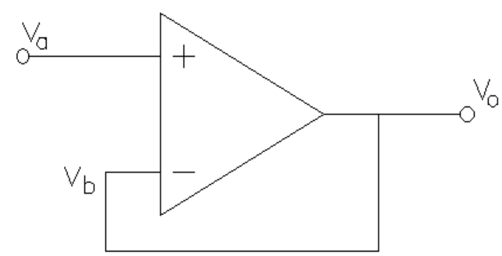


Figure 2 Voltage follower

Voltage follower

The voltage follower shown in Fig. 2 is probably the simplest linear op-amp circuit. Notice the feedback from the output to the inverting (-) input. If we were to hook the circuit input (v_a) up to some voltage source, say 2 volts DC, what would happen? If the output was lower than 2 V, then the input voltage difference (v_a - v_b) would be positive and the huge gain of the op-amp would drive the output higher. If the output was higher than 2 V, then v_a - v_b would be negative the output would go down. Very quickly the output voltage v_o would change until v_a - v_b becomes very small. Or basically, until v_o = 2 V. This is the concept of negative feedback! A fraction (in this case all) of the output voltage is "fed back" to the input in order to control the gain of the op-amp. The op-amp works very hard to maintain a very small difference between the voltages on its inputs. This circuit is known as a voltage follower because the output "follows" the input.

Negative feedback is an important concept. It is used in almost all systems, including all natural systems. A very simple example is the heating system in your house. If the air temperature is too low the thermostat detects a difference between its setting and the air temperature and turns on the heater. When the air temperature reaches the set temperature the thermostat turns off the heater—negative feedback. The servo system that you've seen in lab is another example of negative feedback. When the motor position sensor senses a different position than the input position sensor the circuit makes the motor turn in such a way that the difference is minimized and the positions line up.

Noninverting amplifier

Now suppose we feed back only a fraction of the output voltage rather than all of it. The method used for this is shown in Fig. 3. R_1 and R_f constitute a voltage divider. Remember, the current flowing into an op-amp input is virtually nil, so we can neglect its effect on the voltage divider. This is one of the very nice features of an op-amp. In this circuit, as in the voltage follower, the op-amp works very hard to keep $v_a - v_b$ very small. Only now v_b is a fraction of v_o and the op-amp has to make v_o that much larger.

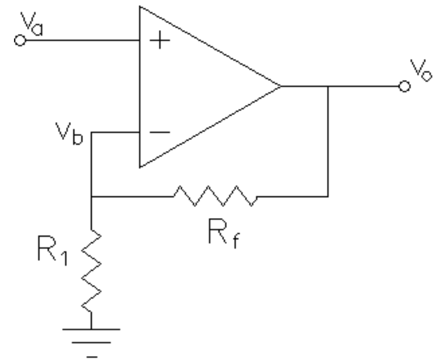


Figure 3 Noninverting amplifier

$$v_{in} = v_a \approx v_b = \frac{R_1}{R_1+R_f} v_o$$

For all practical purposes:

$$v_o = \frac{R_1+R_f}{R_1} v_{in} = (1+\frac{R_f}{R_1}) v_{in}$$

Notice that by adjusting the ratio of R_f and R_1, we can make the gain of the op-amp circuit almost anything we want. Isn' t that neat? The circuit in Fig. 3 is called a noninverting amplifier because the output voltage is in phase with the input voltage; that is, it is *not* inverted. When the input voltage increases, the output voltage will also increase and vice-versa. Yes, noninverting is a double negative and kind-of a dumb name.

Inverting amplifier

Before going on, observe that I've swapped the positions of the two inputs(- & +) on my op-amp symbol. Either way of drawing the op-amp is OK, whatever makes the whole schematic look better. The noninverting input is on the bottom in this case because it's hooked to ground. Draw the op-amp so that the surrounding circuitry is clear.

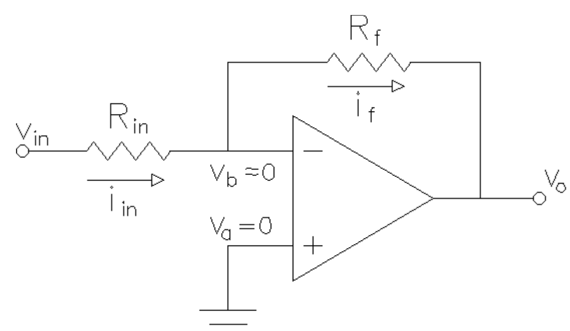


Figure 4 Inverting amplifier

The op-amp output is still connected to the inverting (-) input, so again we have negative feedback. If $v_b > v_a$ then the output will go down, taking v_b with it, until $v_b \approx v_a$. If $v_b < v_a$ then the output will go up until $v_b \approx v_a$. Negative feedback makes the op-amp do its best to equalize its inputs. In this circuit $v_a = 0$, which means that the op-amp will try to keep $v_b \approx 0$ as well. The current into the op-amp is zero, so i_{in} and i_f must be the same ($i_f = i_{in}$). Using these two ideas together:

$$i_{in} = \frac{v_{in} - 0}{R_{in}} = i_f = \frac{0 - v_o}{R_f} \qquad v_o = -\frac{R_f}{R_{in}} v_{in}$$

The minus sign means that v_o will be inverted with respect to v_{in}, hence the name of this amplifier. When v_{in} is positive, v_o is negative, and when v_{in}. is negative, v_o is positive. The gain of the inverting amplifier, like that of the noninverting amplifier, is completely dependent on our choices of R_f and R_{in}.

Summer

The inverting amplifier can also be used as a summing amplifier; that is, it can be made to add the effects of several input voltages together. Look at the circuit in Fig. 5.

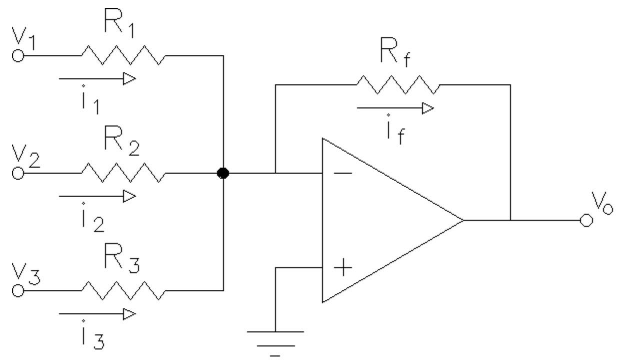


Figure 5 Summer

$$i_f = i_1 + i_2 + i_3$$

$$i_f = -\frac{v_o}{R_f} = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}$$

$$v_o = -\frac{R_f}{R_1}v_1 - \frac{R_f}{R_2}v_2 - \frac{R_f}{R_3}v_3$$

The summer can be expanded to any number of inputs. See? This is getting easier.

Differentiator

The differentiator looks an awful lot like the inverting amplifier, and is analyzed in a very similar way.

$$i_C = C\frac{dv_{in}}{dt} = i_f = -\frac{v_o}{R_f}$$

$$v_o = -CR_f\frac{dv_{in}}{dt}$$

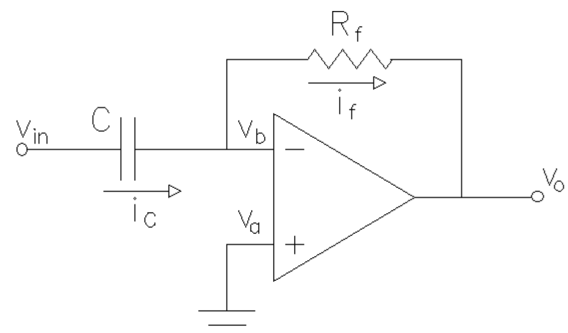


Figure 6 Differentiator

Integrator

Another useful op-amp circuit is the integrator, shown in Fig. 7. For this circuit:

$$i_{in} = \frac{v_{in}}{R_{in}} = i_C = -C\frac{dv_o}{dt}$$

$$v_o = -\frac{1}{CR_{in}}\int v_{in}dt$$

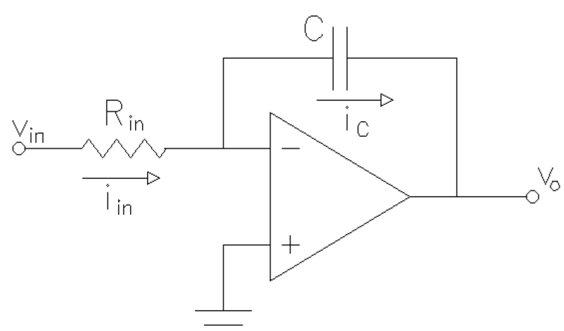


Figure 7 Integrator

Unfortunately, The simple integrator does have one little practical problem. Notice that if the input voltage has any dc component, the output voltage will soon try to run off to infinity. (Actually it will stop when the op-amp reaches one of its output limits, either negative or positive.) A resistor is usually placed in parallel with the capacitor to eliminate this rather annoying effect. The circuit in Fig. 8 has such a resistor. This is a *running-average* or *Miller* integrator.

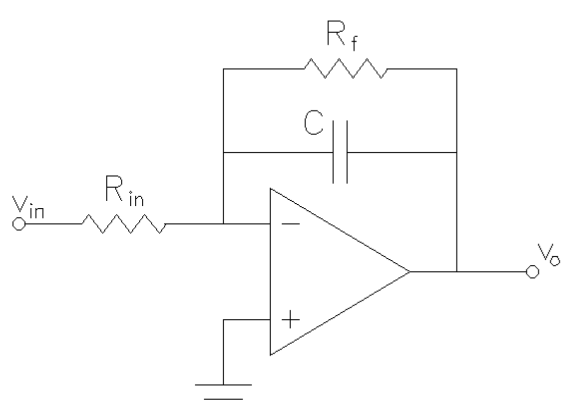


Figure 8 Practical integrator

Active Filters

If you replace the resistors in the inverting and noninverting amplifiers with frequency dependant impedances (capacitors and/or inductors), you can make all sorts of frequency dependant circuits, including filters. In fact, the differentiator and integrator circuits can be thought of as filters.

One of the main advantages of active filters is that you don't need to use inductors. Real inductors are far from ideal, as you've no doubt observed in lab. Real capacitors are much closer to ideal capacitors and they're cheaper than inductors. Entire books are devoted to these *active* filters and we won't cover them any further here.

Differential amplifier

This circuit amplifies only the difference between the two inputs. In this circuit there are two resistors labeled R_{in}, which means that their values are equal. Same goes for the two R_f's.

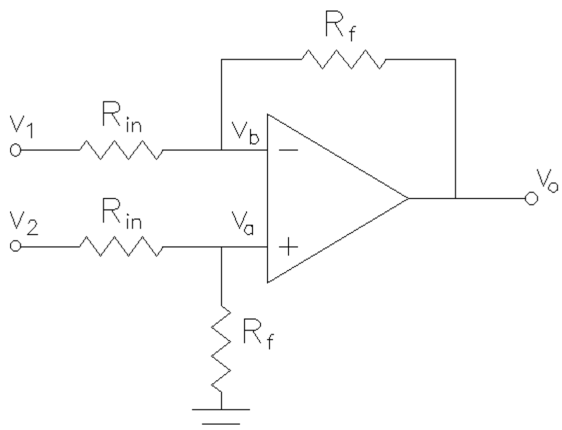


Figure 10 Differential amplifier

$$v_a = \frac{R_f}{R_{in}+R_f}v_2 = v_b = \frac{R_f}{R_{in}+R_f}(v_1-v_o) + v_o$$

$$\frac{R_f}{R_{in}+R_f}v_2 = \frac{R_f}{R_{in}+R_f}(v_1-v_o) + \frac{R_{in}+R_f}{R_{in}+R_f}v_o$$

$$R_f v_2 = R_f v_1 - R_f v_o + R_{in} v_o + R_f v_o$$

$$R_f v_2 = R_f v_1 + R_{in} v_o$$

$$v_o = \frac{R_f}{R_{in}}(v_2 - v_1)$$

Don't confuse the differential amplifier with the differentiator. The differential amplifier amplifies the <u>difference</u> of two inputs while the differentiator amplifies the <u>slope</u> of an input.

Instrumentation Amplifier

The differential amplifier isn't really very practical. The current that flows into the top input depends on the voltage applied to the bottom input. This may not seem that bad, but it is. It means that the input characteristics of this circuit are not constant. One way to get around this would be to place a voltage follower on each input, as shown here.

$$v_o = \frac{R_f}{R_{in}}(v_2 - v_1)$$

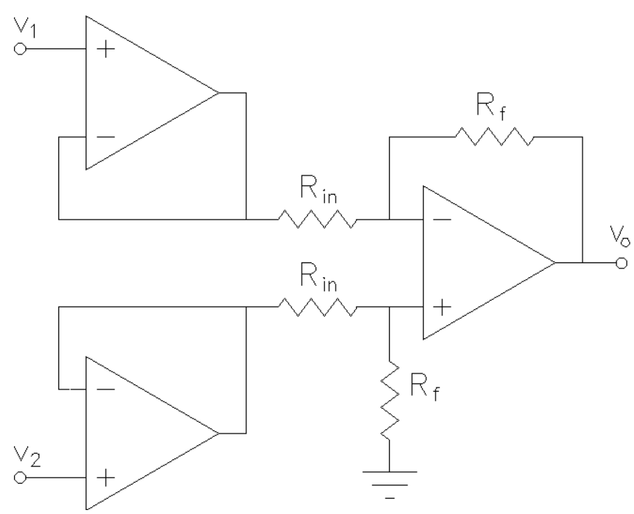


Figure 9 Buffered differential amplifier

Now this is a perfectly good circuit. If the two R_fs are closely matched and the two R_{in}s are also closely matched, then this circuit will amplify differential voltages very well and reject *commo*n voltages (a voltage that is common to both inputs should subtract out of the equation. In EE terms, it has a good Common-Mode-Rejection-Ration (CMRR).

But what if you want to change the gain? You'd have to change two resistors at the same time. By adding two more matched resistors and variable resistor we'll get the instrumentation amplifier shown at right. The equation for this circuit is:

$$v_o = (1 + \frac{2R_2}{R_1})\frac{R_4}{R_3}(v_2 - v_1)$$

This is an important circuit and you will probably see it again many times. For instance, if you had to amplify the output of a wheatstone bridge of stain gages, this would be the amp for the job.

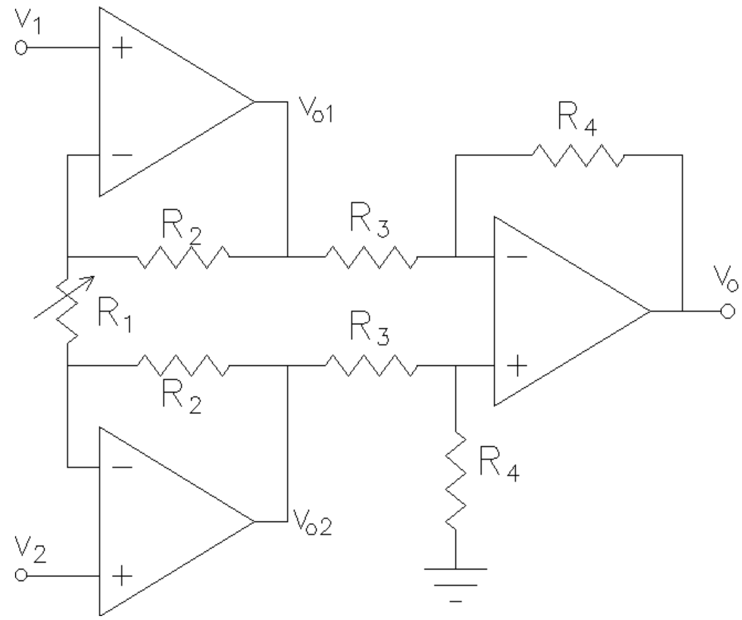


Figure 11 Instrumentation amplifier

Op-amp with extra current amplification
Most op-amps cannot supply much current to the load. They are often limited to 10 or 20 mA, about enough to light an LED, but not much more. That can br very limiting. The circuit at right shows a quick and dirty way to use two transistors to greatly increase the load current (at a small cost in output voltage swing). Notice that the feedback is taken from the output of the transistors, so they sort-of become part of the op-amp and the op-amp will do a pretty good job of eliminating the "crossover" dead-zone that occurs as one transistor turns off and the other turn on.

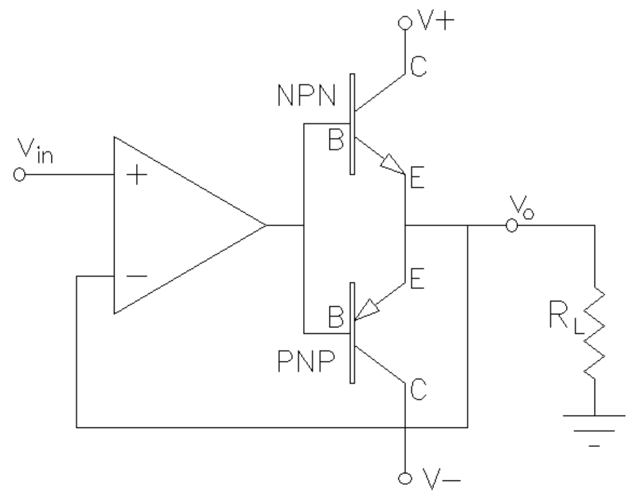


Figure 12 How to get more current

This particular circuit is a simple voltage follower. You can adapt this same current amplification to most of the other op-amp circuits that we have discussed. A few words of warning, however. The extra delay in the feedback can result in instabilities. Try it with the parts you intend to use before you depend on this design. Also, if you use a low quality op-amp (with a slow slew rate) you can get significant crossover distortion.

Nonlinear Circuits

In all cases so far, the feedback signal (voltage) has been applied to the inverting (-) input of the op-amp. This means that the feedback is negative. Negative feedback tends to reduce the difference between the v_a and v_b voltages and make linear circuits. Without negative feedback the op-amp cannot minimize the difference between v_a and v_b and the very high sensitivity of the op-amp results in *switching*, or *nonlinear* circuits.

Comparator

Now look at Fig. 14. This circuit will **not** work as a linear circuit. If $v_a > 0$ the output will be as high as the op-amp can make it, usually a volt or two below the positive power supply. If $v_a < 0$ the output will be as low as the op-amp can make it, usually a volt or two above the negative power supply. The output is no longer linearly related to the input– it's more like a digital signal, high or low depending on how v_{in} compares to ground (0 V). The comparator is a *nonlinear* circuit.

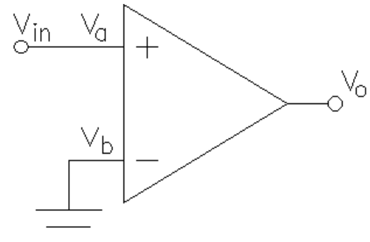


Figure 13 Comparator

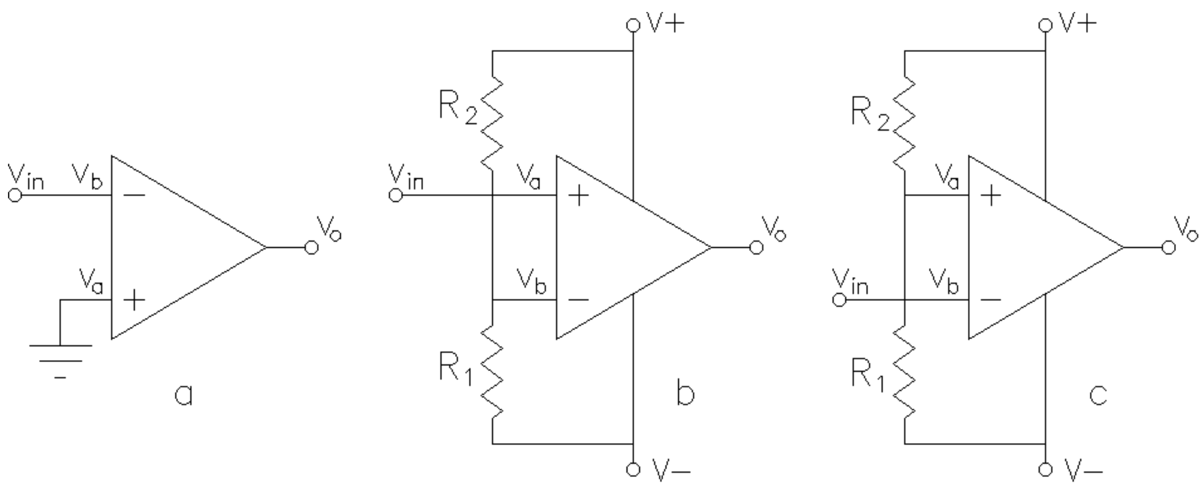


Figure 14 Other comparators

All the circuits above are also comparators. In the first circuit, the input is again compared to ground, but this time the output goes low when the input goes high and vise-versa. In the remaining circuits the input is compared not to 0 V, but to some voltage set by the voltage divider of R_1 and R_2.

Schmitt trigger

The Schmitt trigger is a variation of the simple comparator which has hysteresis, that is, it has a toggle action. When the output is high, positive feedback makes the switching level higher than it is when the output is low. A little positive feedback makes a comparator with better noise immunity. Increase the positive feedback and the Schmitt trigger can be used in other switching applications.

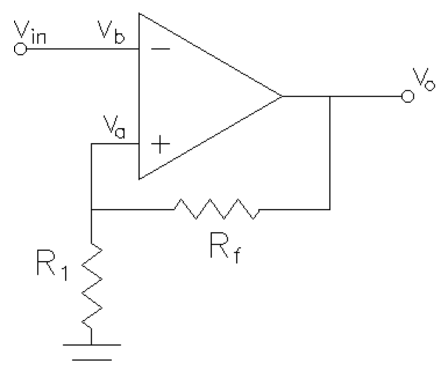


Figure 15 Schmitt trigger

Look at the Schmitt ttrigger circuit shown at right. Notice that $v_a = [R_1/(R_1 + R_f)]v_o$, it depends on the output. Lets say the output is low and the input is decreasing. When $v_{in} < v_a$ the output goes high and suddenly v_a goes a little bit higher with it. That makes the difference between v_b and v_a even bigger. To make the circuit switch again v_{in} has to go back up beyond the original switching level. It has to reach the new v_a before the output will switch low. In this circuit the two switching levels are above and below ground by the same amount (unless you have nonsymmetric power supplies).

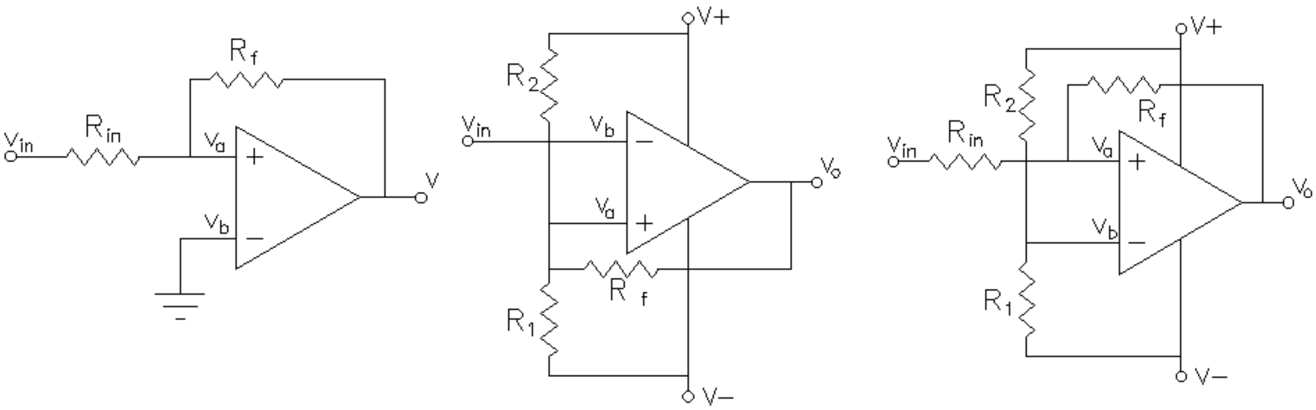


Figure 16 Other Schmitt triggers

The circuits above are variations of the Schmitt trigger. In the first circuit, the input is again compared to levels above and below ground, but this time the output goes high when the input goes high and vise-versa. In the remaining circuits the switching levels are not symmetric about 0 V, but about some voltage set by the voltage divider of R_1 and R_2.

Multivibrator (square wave generator)
The heart of the multivibrator is a Schmitt trigger with lots of positive feedback. Usually $R_2 = R_3$, which set the switching levels at about ½ V+ and ½ V-. When the output is high the capacitor charges through R_1 until it reaches the ½ V+ switching level, the output switches low and the capacitor discharges to zero and then charges up (down) until it reaches the ½ V- switching level. That makes the output switch high and the process repeats.

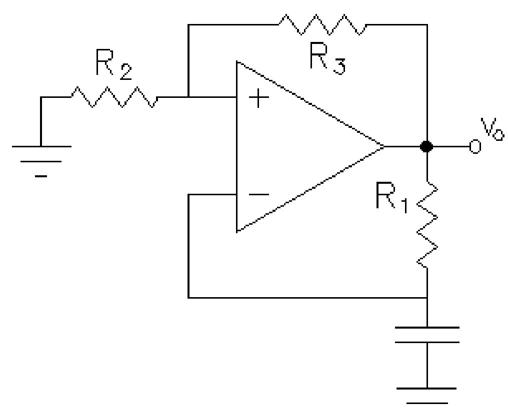


Figure 17 Multivibrator

Conclusion

In all of these circuits, with either negative or positive feedback, the output voltage v_o cannot increase without bounds. It is bounded in the positive direction by V+, the op-amp positive power supply voltage, and is bounded in the negative direction by V-, the op-amp negative supply voltage. If the output voltage is within these bounds, $v_a - v_b$ must be very small. If $v_a - v_b$ were not very small, v_o would soon be forced to one of its limits. Linear circuits use negative feedback to keep this difference small. Without negative feedback you can reasonably assume that the circuit is some kind of switching circuit and that the output is always at one or the other of its limits.

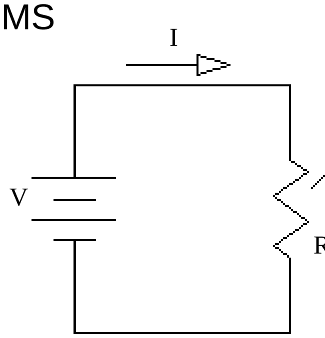
This only scratches the surface of what you can do with op-amps. Get a copy of *The Op-amp Cookbook* for lots more ideas presented in a no-nonsense way.

ECE 2210
RMS

Introduction to AC Power, RMS

A. Stolp
3/31/09,
2/20/10
11/23/13
corrected
p4 '14

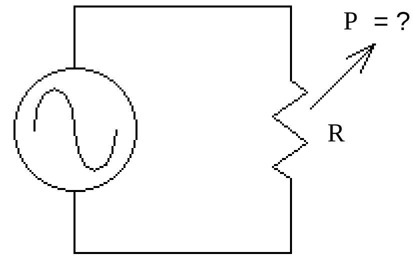
DC Power

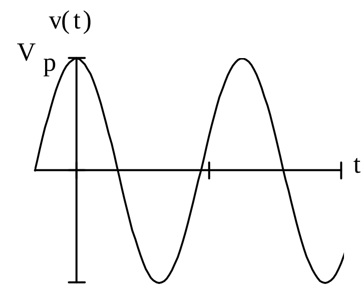


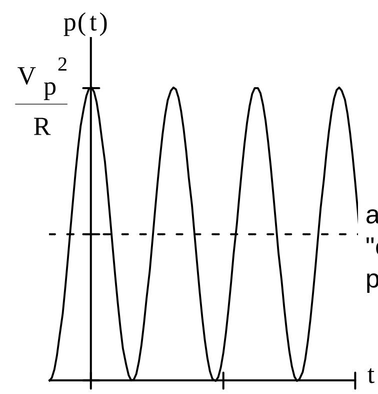
$$P = V \cdot I = \frac{V^2}{R} = I^2 \cdot R$$

AC Power

$$v(t) = V_p \cdot \cos(\omega \cdot t)$$







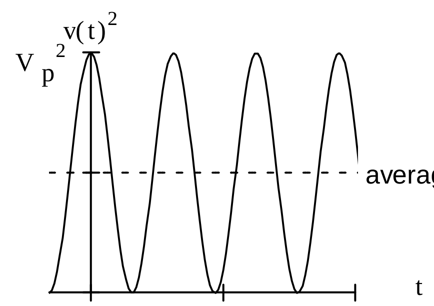
average or "effective" power = $\dfrac{\left(\dfrac{V_p^{\,2}}{R}\right)}{2}$

Couldn't we define an "effective" voltage that would allow us to use the same relationships for AC power as used for DC power?

$$P_{ave} = \frac{\left(\frac{V_p^{\,2}}{R}\right)}{2} = \frac{\left(\frac{V_p^{\,2}}{2}\right)}{R} = \frac{\left(\frac{V_p}{\sqrt{2}}\right)^2}{R}$$

$$V_{eff} = \sqrt{\left(\frac{V_p}{\sqrt{2}}\right)^2} = \frac{V_p}{\sqrt{2}} = V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T (v(t))^2 \, dt}$$

Square
Root
Mean (average)



average = $\dfrac{V_p^{\,2}}{2}$

RMS **R**oot of the **M**ean of the **S**quare

Use RMS in power calculations

Sinusoids

$$V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T (v(t))^2 \, dt} = \sqrt{\frac{1}{T} \cdot \int_0^T \left(V_p \cdot \cos(\omega \cdot t)\right)^2 dt} = \sqrt{\frac{1}{T} \cdot \int_0^T V_p^{\,2} \cdot \left(\frac{1}{2} + \frac{1}{2} \cdot \cos(2 \cdot \omega \cdot t)\right) dt}$$

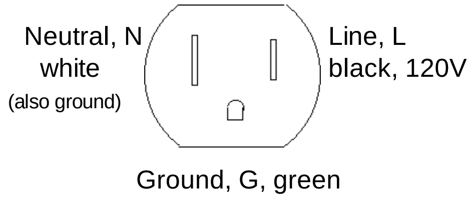
$$= \frac{V_p}{\sqrt{2}} \cdot \sqrt{\frac{1}{T} \cdot \int_0^T (1)\, dt + \frac{1}{T} \cdot \int_0^T \cos(2 \cdot \omega \cdot t)\, dt} = \frac{V_p}{\sqrt{2}} \cdot \sqrt{1 + 0}$$

Common household power

$f = 60 \cdot Hz$

$\omega = 377 \cdot \frac{rad}{sec}$

$T = 16.67 \cdot ms$

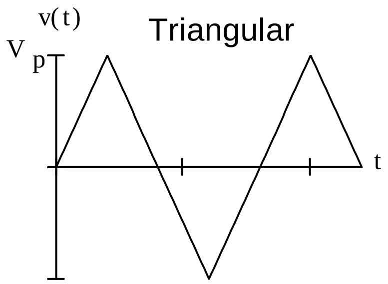


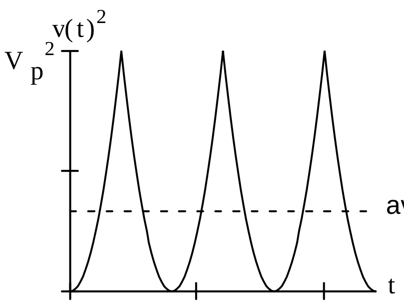
$V_{rms} := 120 \cdot V$

$V_p = V_{rms} \cdot \sqrt{2} = 170 \cdot V$

What about other wave shapes??

Triangular

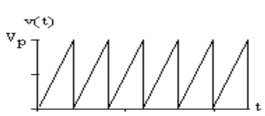


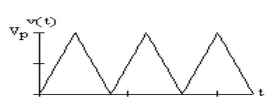


average = $\frac{V_p^2}{3}$

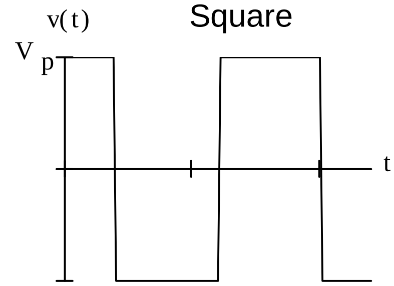
$V_{rms} = \frac{V_p}{\sqrt{3}}$

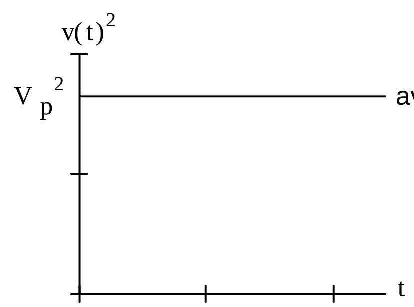
Works for all types of triangular and sawtooth waveforms





Square



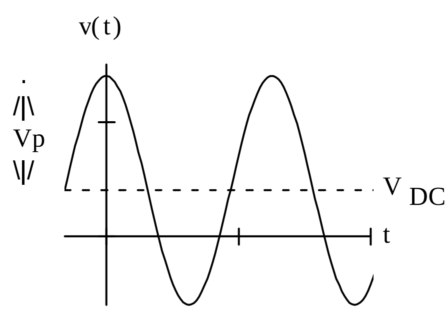


average = V_p^2

$V_{rms} = V_p$

Same for DC

How about AC + DC ?



$$V_{rms} = \sqrt{\frac{1}{T} \cdot \int_0^T (v(t))^2 \, dt}$$

$$= \sqrt{\frac{1}{T} \cdot \int_0^T \left(V_p \cdot \cos(\omega \cdot t) + V_{DC}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T} \cdot \int_0^T \left[\left(V_p \cdot \cos(\omega \cdot t)\right)^2 + 2 \cdot \left(V_p \cdot \cos(\omega \cdot t)\right) \cdot V_{DC} + V_{DC}^2\right] dt}$$

$$= \sqrt{\frac{1}{T} \cdot \int_0^T \left(V_p \cdot \cos(\omega \cdot t)\right)^2 dt + \frac{1}{T} \cdot \int_0^T 2 \cdot \left(V_p \cdot \cos(\omega \cdot t)\right) \cdot V_{DC} \, dt + \frac{1}{T} \cdot \int_0^T V_{DC}^2 \, dt}$$

- - - zero over one period - - -

$$= \sqrt{V_{rmsAC}^2 + 0 + V_{DC}^2} \qquad = \sqrt{V_{rmsAC}^2 + V_{DC}^2}$$

sinusoid: $V_{rms} = \frac{V_p}{\sqrt{2}}$ $\quad I_{rms} = \frac{I_p}{\sqrt{2}}$

triangular: $V_{rms} = \frac{V_p}{\sqrt{3}}$ $\quad I_{rms} = \frac{I_p}{\sqrt{3}}$

square: $V_{rms} = V_p$ $\quad I_{rms} = I_p$

waveform + DC $\quad V_{rms} = \sqrt{V_{rmsAC}^2 + V_{DC}^2}$

rectified average $\quad V_{ra} = \frac{1}{T}\cdot\int_0^T |v(t)|\,dt$

$V_{ra} = \frac{2}{\pi}\cdot V_p$ $\quad I_{ra} = \frac{2}{\pi}\cdot I_p$

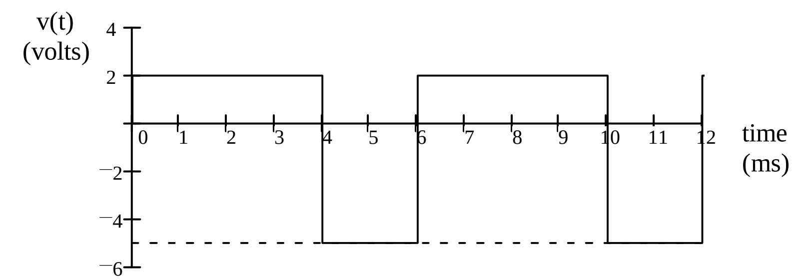
$V_{ra} = \frac{1}{2}\cdot V_p$ $\quad I_{ra} = \frac{1}{2}\cdot I_p$

$V_{ra} = V_{rms} = V_p$ $\quad I_{ra} = I_{rms} = I_p$

Most AC meters don't measure true RMS. Instead, they measure V_{ra} , display $1.11V_{ra}$, and call it RMS. That works for sine waves but not for any other waveform.

Some waveforms don't fall into these forms, then you have to perform the math from scratch

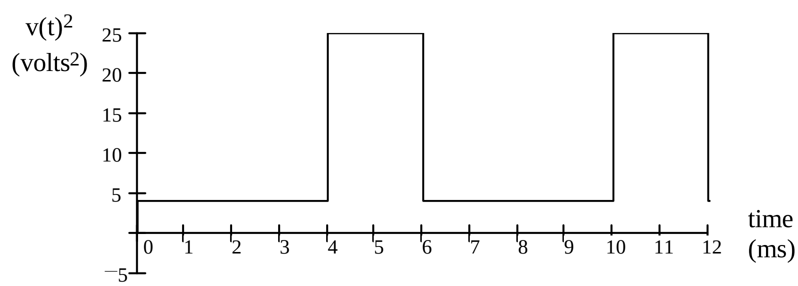
For waveform shown



The average DC (V_{DC}) value

$$\frac{2\cdot V\cdot(4\cdot ms) + (-5\cdot V)\cdot(2\cdot ms)}{6\cdot ms} = -0.333\cdot V$$

The RMS (effective) value



Graphical way

$$\frac{4\cdot V^2\cdot(4\cdot ms) + 25\cdot V^2\cdot(2\cdot ms)}{6\cdot ms} = 11\cdot V^2$$

$V_{RMS} := \sqrt{11\cdot V^2}$ $\qquad V_{RMS} = 3.32\cdot V$

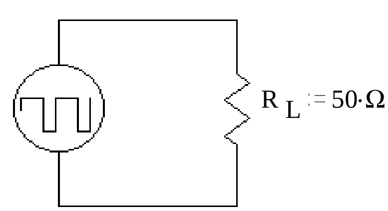
OR...

$$V_{RMS} = \sqrt{\frac{1}{T}\cdot\int_0^T (v(t))^2\,dt}$$

$$= \sqrt{\frac{1}{6\cdot ms}\cdot\left[\int_{0\cdot ms}^{4\cdot ms} (2\cdot V)^2\,dt + \int_{2\cdot ms}^{6\cdot ms} (-5\cdot V)^2\,dt\right]} \quad = \sqrt{\frac{1}{6\cdot ms}\cdot\left[4\cdot ms\cdot(2\cdot V)^2 + 2\cdot ms\cdot(-5\cdot V)^2\right]} = 3.32\cdot V$$

The voltage is hooked to a resistor, as shown, for 6 seconds.

The energy is transfered to the resistor during that 6 seconds:



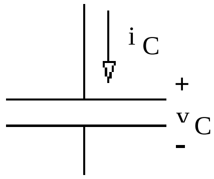
$P_L := \frac{V_{RMS}^2}{R_L}$ $\qquad P_L = 0.22\cdot W$

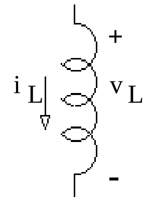
$W_L := P_L\cdot 6\cdot sec$ $\qquad W_L = 1.32\cdot joule$ All converted to heat

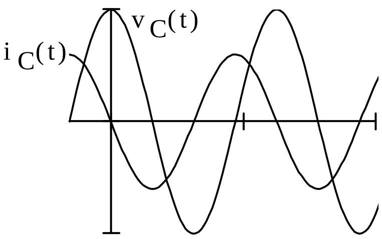
Use RMS in power calculations

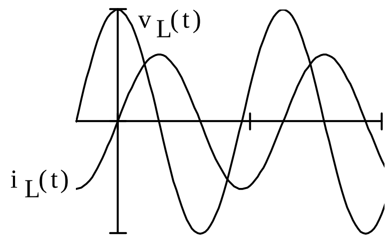
$$P = I_{Rrms}^{2} \cdot R = \frac{V_{Rrms}^{2}}{R} \qquad \text{for Resistors ONLY ! !}$$

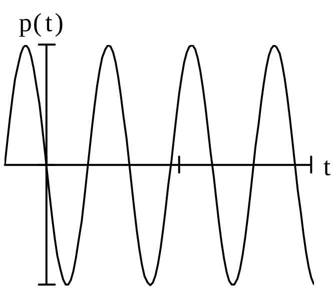
Capacitors and Inductors

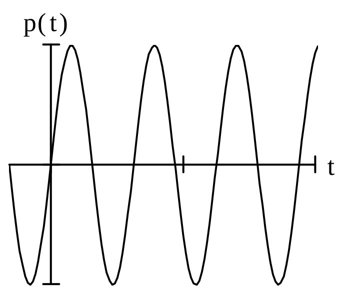












Average power is ZERO $\quad P = 0$

Average power is ZERO $\quad P = 0$

Capacitors and Inductors DO NOT dissipate (real) average power.

Reactive power is negative

$$Q_C = -I_{Crms} \cdot V_{Crms}$$

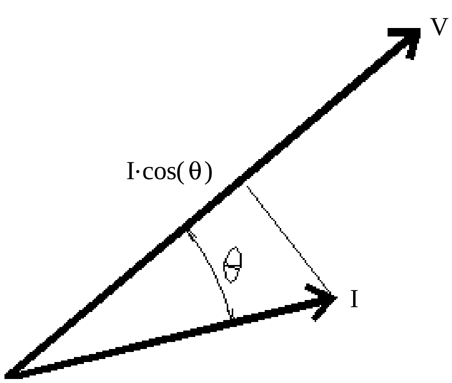
$$= -I_{Crms}^{2} \cdot \frac{1}{\omega \cdot C} = -V_{Crms}^{2} \cdot \omega \cdot C$$

Reactive power is positive

$$Q_L = I_{Lrms} \cdot V_{Lrms}$$

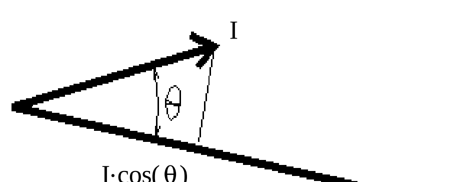
$$= I_{Lrms}^{2} \cdot \omega \cdot L = \frac{V_{Lrms}^{2}}{\omega \cdot L}$$

If current and voltage are not in phase, only the in-phase part of the current matters for the power-- DOT PRODUCT



"Lagging" power

Inductor dominates



"Leading" Power

Capacitor dominates

Real Power

$$P = I_{Rrms}^{2} \cdot R = \frac{V_{Rrms}^{2}}{R} \quad \text{for resistors}$$

other wise....

$$P = V_{rms} \cdot I_{rms} \cdot \cos(\theta) = I_{rms}^{2} \cdot |\mathbf{Z}| \cdot \cos(\theta) = \frac{V_{rms}^{2}}{|\mathbf{Z}|} \cdot \cos(\theta)$$ units: watts, kW, MW, etc.

$$P = \text{"Real" Power (average)} = V_{rms} \cdot I_{rms} \cdot pf = I_{rms}^{2} \cdot |\mathbf{Z}| \cdot pf = \frac{V_{rms}^{2}}{|\mathbf{Z}|} \cdot pf$$

Reactive Power

capacitors -> - Q $\quad Q_C = I_{Crms}^{2} \cdot X_C = \dfrac{V_{Crms}^{2}}{X_C} \qquad X_C = -\dfrac{1}{\omega \cdot C}$ and is a negative number

inductors -> + Q $\quad Q_L = I_{Lrms}^{2} \cdot X_L = \dfrac{V_{Lrms}^{2}}{X_L} \qquad X_L = \omega \cdot L$ and is a positive number

other wise....

$$Q = \text{Reactive "power"} = V_{rms} \cdot I_{rms} \cdot \sin(\theta)$$ units: VAR, kVAR, etc. "volt-amp-reactive"

Complex and Apparent Power

$$\mathbf{S} = \text{Complex "power"} = \mathbf{V}_{\mathbf{rms}} \cdot \overline{\mathbf{I}_{\mathbf{rms}}} = P + jQ = V_{rms} I_{rms} \angle\theta$$ (complex congugate) units: VA, kVA, etc. "volt-amp"

NOT $I_{rms}^{2} \cdot \mathbf{Z}$ **NOR** $\dfrac{V_{rms}^{2}}{\mathbf{Z}}$

$$S = \text{Apparent "power"} = |\mathbf{S}| = V_{rms} \cdot I_{rms} = \sqrt{P^2 + Q^2}$$ units: VA, kVA, etc. "volt-amp"

Power factor

$pf = \cos(\theta)$ = power factor (sometimes expressed in %) $\quad 0 \le pf \le 1$

θ is the **phase angle** between the voltage and the current or the phase angle of the impedance. $\theta = \theta_Z$

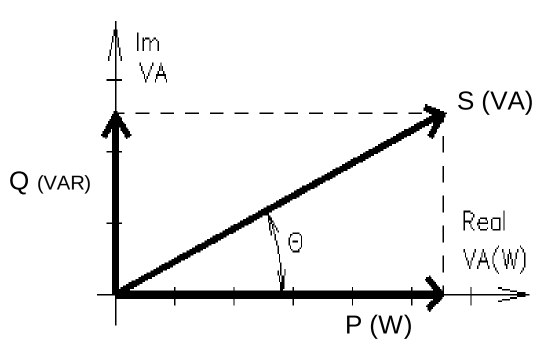
$\theta < 0$ Load is "Capacitive", power factor is "leading". This condition is very rare

$\theta > 0$ Load is "Inductive", power factor is "lagging". This condition is so common you can assume any power factor given is lagging unless specified otherwise. Transformers and motors make most loads inductive.

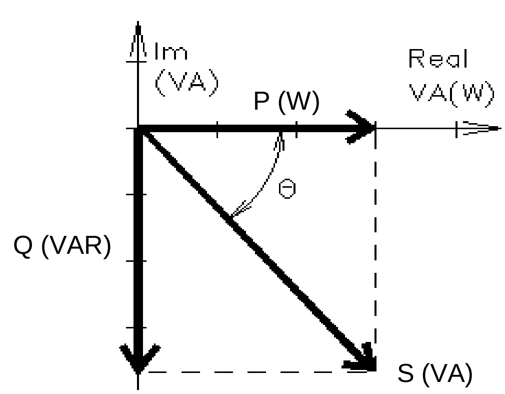
Industrial users are charged for the reactive power that they use, so power factor < 1 is a bad thing.

Power factor < 1 is also bad for the power company. To deliver the same power to the load, they have more line current (and thus more line losses).

Power factors are "corrected" by adding capacitors (or capacitve loads) in parallel with the inductive loads which cause the problems. (In the rare case that the load is capacitive, the pf would be corrected by an inductor.)



"Lagging" power



"Leading" Power

Transformer basics and ratings

A Transformer is two coils of wire that are magnetically coupled.

Transformers are only useful for AC, which is one of the big reasons electrical power is generated and distributed as AC.

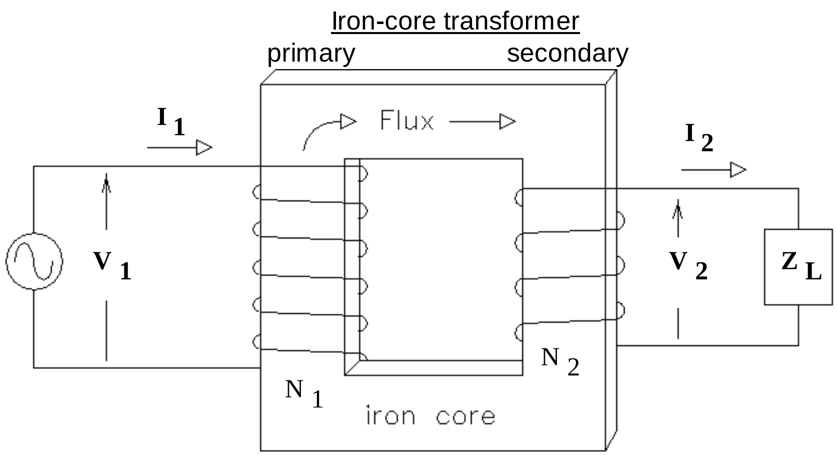
Transformer turns and turns ratios are rarely given, V_p/V_s is much more common where V_p/V_s is the rated primary over rated secondary voltages. You may take this to be the same as N_1/N_2 although in reality N_2 is usually a little bit bigger to make up for losses. Also common: $V_p : V_s$.

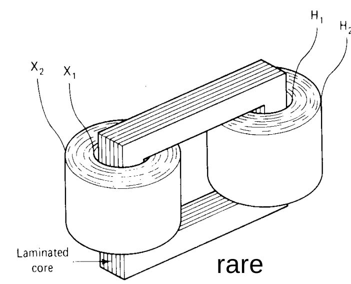
Both RMS

Transformers are rated in VA   Transformer Rating (VA) = (rated V) x (rated I) , on either side.

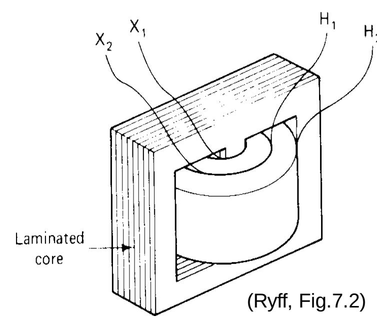
Don't allow voltages over the rated V , regardless of the actual current.
Don't allow currents over the rated I , regardless of the actual voltage.

Ideal Transformers





rare



(Ryff, Fig.7.2)

common

Ideal: $P_1 = P_2$

power in = power out

Transformation of voltage and current

$$\frac{N_1}{N_2} = \frac{\mathbf{V_1}}{\mathbf{V_2}} = \frac{\mathbf{I_2}}{\mathbf{I_1}}$$

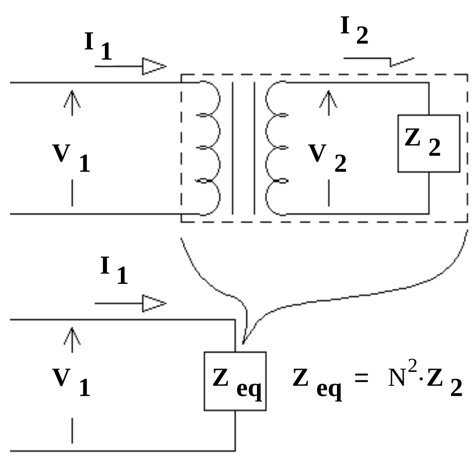
Turns ratio

Turns ratio as defined in Chapman text: $a = \frac{N_1}{N_2}$, same as $N = \frac{N_1}{N_2}$

Note: some other texts define the turns ratio as: $\frac{N_2}{N_1}$

Be careful how you and others use this term

Transformation of impedance



$$\mathbf{Z_{eq}} = N^2 \cdot \mathbf{Z_2} = \left(\frac{N_1}{N_2}\right)^2 \cdot \mathbf{Z_2}$$

You can replace the entire transformer and load with ($\mathbf{Z_{eq}}$). This "impedance transformation" can be very handy.

Transformers can be used for "impedance matching"

This also works the opposite way, to move an impedance from the primary to the secondary, multiply by: $\left(\frac{N_2}{N_1}\right)^2$

Other Transformers

Multi-tap transformers: Many transformers have more than two connections to primary and/or the secondary. The extra connections are called "taps" and may allow you to select from several different voltages or get more than one voltage at the same time.

Isolation Transformers: Allmost all transformers isolate the primary from the secondary. An Isolation transformer has a 1:1 turns ratio and is just for isolation.

Auto Transformers: Auto transformers have only one winding with taps for various voltages. The primary and secondary are simply parts of the same winding. These parts may overlap. Any regular transformer can be wired as an auto transformer. Auto transformers DO NOT provide isolation.

Vari-AC: A special form of auto transformer with an adjustable tap for an adjustable output voltage.

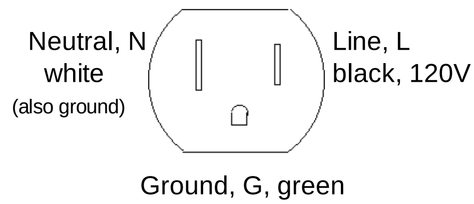
LVDT: A Linear-Variable-Differential-Transformers has moveable core which couples the primary winding to the secondary winding(s) in such a way the the secondary voltage is proportional to the position of the core. LVDTs are used as position sensors.

Home power

Standard 120 V outlet connections are shown at right.

The 3 lines coming into your house are **NOT** 3-phase.
They are +120 V, Gnd, -120 V

(The two 120s are 180^o out-of-phase, allowing for 240 V connections)

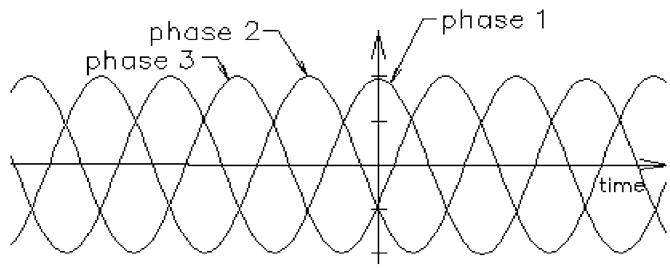


3-Phase Power (FYI ONLY)

Single phase power pulses at 120 Hz. This is not good for motors or generators over 5 hp.

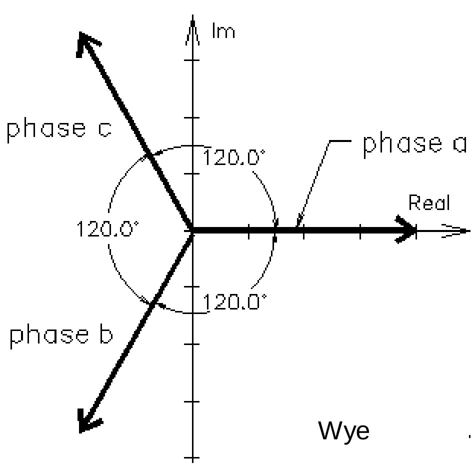
Three phase power is constant as long as the three loads are balanced.

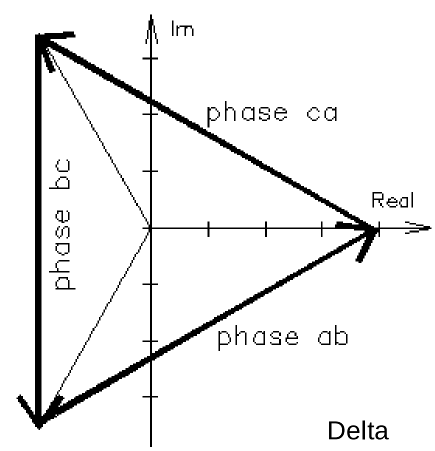
Three lines are needed to transmit 3-phase power.
If loads are balanced, ground return current will be zero.



<u>Wye connection</u>:
Connect each load or generator phase between a line and ground.

<u>Delta connection</u>:
Connect each load or generator phase between two lines.



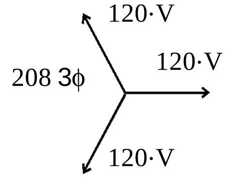


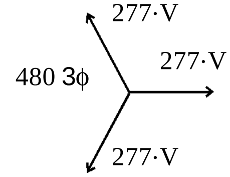
$$V_{LN} = \frac{V_{LL}}{\sqrt{3}} \qquad I_L = \sqrt{3} \cdot I_{LL}$$

$$V_{LL} = \sqrt{3} \cdot V_{LN} \qquad I_{LL} = \frac{I_L}{\sqrt{3}}$$

3-Phase Power (FYI ONLY)

Common 3-phase voltages:

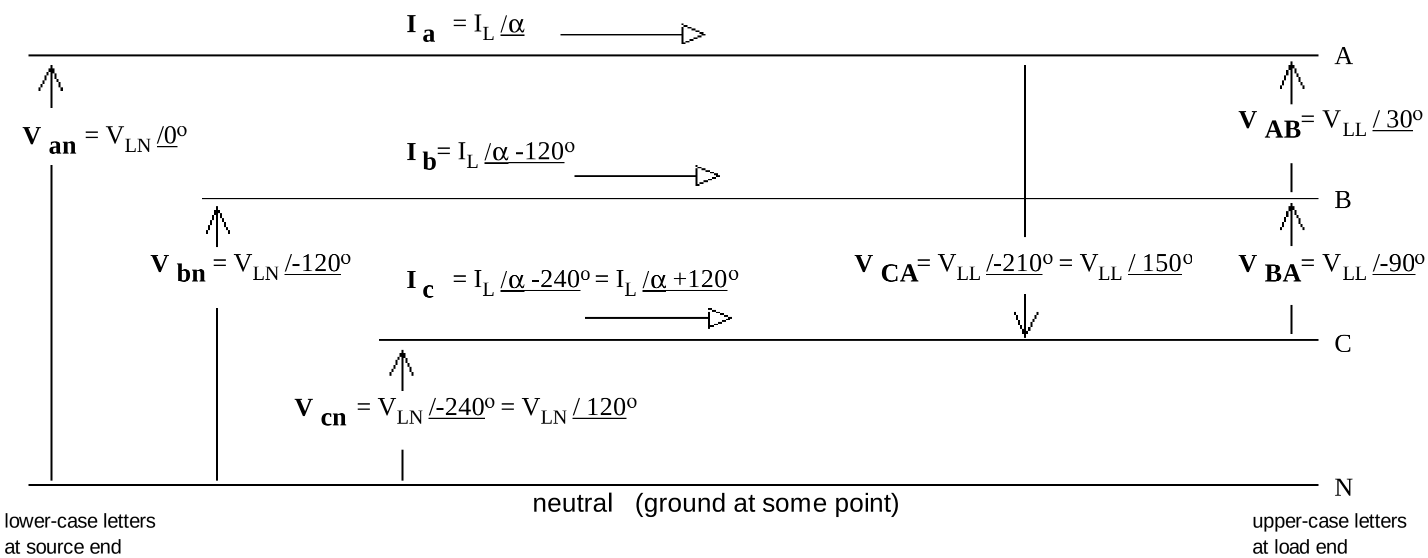


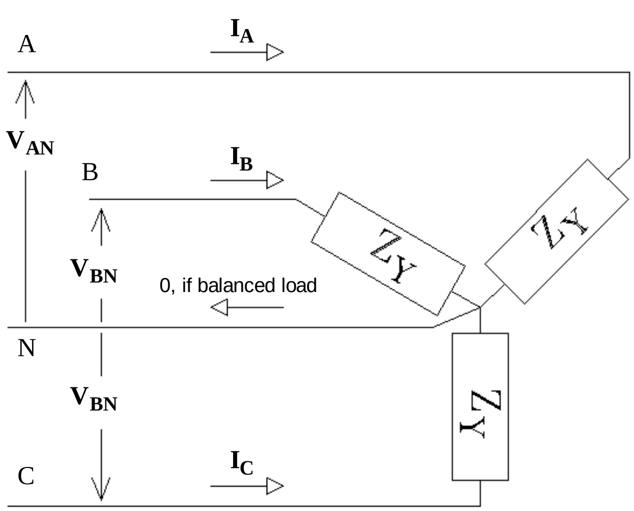


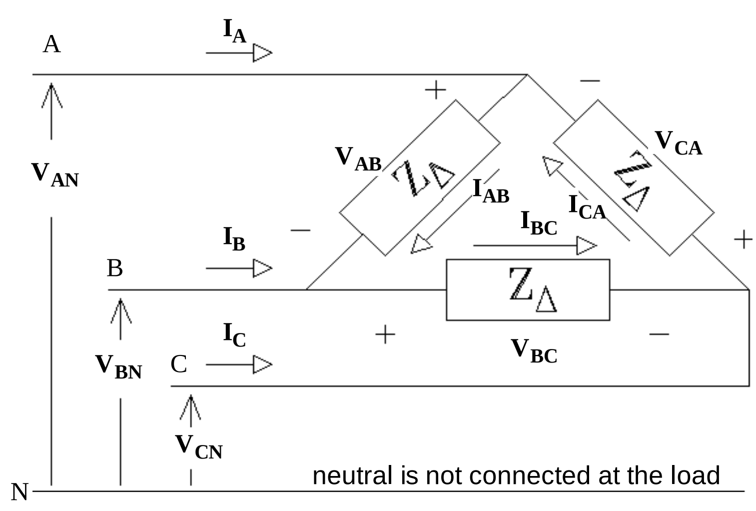
Apparent Power: $|S_{3\phi}| = 3\cdot V_{LN}\cdot I_L = 3\cdot V_{LL}\cdot I_{LL} = \sqrt{3}\cdot V_{LL}\cdot I_L$

Power: $P_{3\phi} = 3\cdot V_{LN}\cdot I_L\cdot pf = 3\cdot V_{LL}\cdot I_{LL}\cdot pf = \sqrt{3}\cdot V_{LL}\cdot I_L\cdot pf = S_{3\phi}\cdot pf$ $\qquad pf = \cos(\theta)$

Reactive power: $Q_{3\phi} = 3\cdot V_{LN}\cdot I_L\cdot \sin(\theta)$ etc... $= \sqrt{\left(|S_{3\phi}|\right)^2 - P_{3\phi}^{\,2}}$







$$|V_{AN}| = |V_{BN}| = |V_{CN}| = V_{LN} = \frac{V_{LL}}{\sqrt{3}} \qquad |V_{AB}| = |V_{BC}| = |V_{CA}| = V_{LL} = \sqrt{3}\cdot V_{LN}$$

$$|I_A| = |I_B| = |I_C| = I_L = \sqrt{3}\cdot I_{LL} \qquad |I_{AB}| = |I_{BC}| = |I_{CA}| = I_{LL} = \frac{I_L}{\sqrt{3}}$$

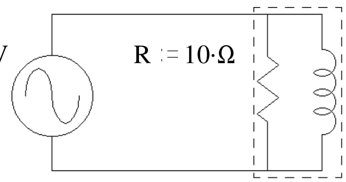
To get equivalent line currents with equivalent voltages

$$Z_Y = \frac{Z_\Delta}{3} \qquad Z_\Delta = 3\cdot Z_y$$

A.Stolp
11/06/02
rev 2/27/07

Ex. 1 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

$\mathbf{V}_{in} := 110 \cdot V$
$\omega := 377 \cdot \frac{rad}{sec}$



$R := 10 \cdot \Omega$ $\quad L := 25 \cdot mH$

$$\mathbf{Z} := \frac{1}{\left(\frac{1}{R} + \frac{1}{j \cdot \omega \cdot L}\right)} = \frac{1}{0.1458 \cdot \frac{1}{\Omega} \cdot e^{-j \cdot 46.7 \cdot deg}}$$

$\mathbf{Z} = 4.704 + 4.991j \ \cdot \Omega$ $\quad |\mathbf{Z}| = 6.859 \cdot \Omega$ $\quad \theta := \arg(\mathbf{Z})$ $\quad \theta = 46.7 \cdot deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.686$

$\mathbf{I} := \frac{\mathbf{V}_{in}}{\mathbf{Z}}$ $\quad \mathbf{I} = 11 - 11.671j \ \cdot A$ $\quad |\mathbf{I}| = 16.038 \cdot A$ $\quad \arg(\mathbf{I}) = -46.7 \cdot deg$

$P := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot pf$ $\quad P = 1.21 \cdot kW$

$Q := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot \sin(\theta)$ $\quad Q = 1.284 \cdot kVAR$ $\quad$ OR... $\quad Q := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot \sqrt{1^2 - pf^2}$ $\quad Q = 1.284 \cdot kVAR$

$\mathbf{S} := \mathbf{V}_{in} \cdot \overline{\mathbf{I}}$ OR.. $\mathbf{S} := P + j \cdot Q$ $\quad \mathbf{S} = 1.21 + 1.284j \ \cdot kVA$ $\quad S := \sqrt{\mathrm{Re}(\mathbf{S})^2 + \mathrm{Im}(\mathbf{S})^2} = |\mathbf{S}| = 1.764 \cdot kVA$

$\mathrm{atan}\left(\frac{\mathrm{Im}(\mathbf{S})}{\mathrm{Re}(\mathbf{S})}\right) = 46.696 \cdot deg$

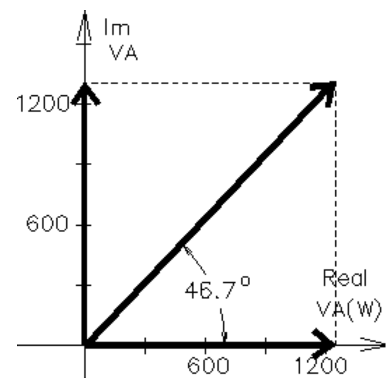
$\mathbf{S} = 1.764kVA \ \underline{/46.7^o}$

OR, since we know that the voltage across each element of the load is V_{in} ...
Real power is dissipated only by resistors

$P := \frac{(|\mathbf{V}_{in}|)^2}{R}$ $\quad P = 1.21 \cdot kW$ $\quad Q := \frac{(|\mathbf{V}_{in}|)^2}{\omega \cdot L}$ $\quad Q = 1.284 \cdot kVAR$

$\mathbf{S} := P + j \cdot Q$

$S = |\mathbf{S}| = \sqrt{P^2 + Q^2} = 1.764 \cdot kVA$ $\quad pf = \frac{P}{|\mathbf{S}|} = 0.686$



What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

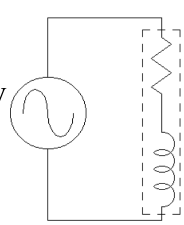
$\mathrm{Im}(\mathbf{I}) = -11.671 \cdot A$ $\quad X_C := \frac{\mathbf{V}_{in}}{\mathrm{Im}(\mathbf{I})}$ $\quad X_C = -9.425 \cdot \Omega = \frac{-1}{\omega \cdot C}$

$\frac{1}{|X_C| \cdot \omega} = 281 \cdot \mu F$ $\quad$ OR.. $\quad \omega = \frac{1}{\sqrt{L \cdot C}}$ $\quad C := \frac{1}{L \cdot \omega^2}$ $\quad C = 281 \cdot \mu F$

Ex. 2 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

Series R & L

$\mathbf{V}_{in} := 110 \cdot V$
$\omega := 377 \cdot \frac{rad}{sec}$



$R := 10 \cdot \Omega$ $\quad \mathbf{Z} := R + j \cdot \omega \cdot L$

$\mathbf{Z} = 10 + 9.425j \ \cdot \Omega$ $\quad |\mathbf{Z}| = 13.742 \cdot \Omega$

$L := 25 \cdot mH$ $\quad \theta := \arg(\mathbf{Z})$ $\quad \theta = 43.304 \cdot deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.728$

$\mathbf{I} := \frac{\mathbf{V}_{in}}{\mathbf{Z}}$ $\quad \mathbf{I} = 5.825 - 5.49j \ \cdot A$

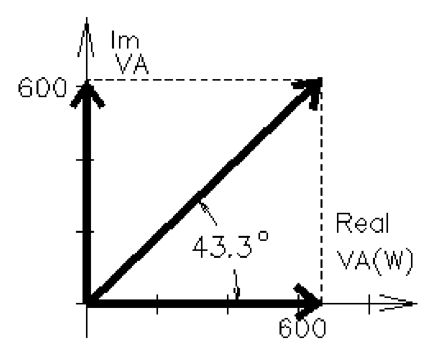
$|\mathbf{I}| = 8.005 \cdot A$ $\quad \arg(\mathbf{I}) = -43.304 \cdot deg$

$P := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot pf$ $\quad P = 0.641 \cdot kW$

$Q := |\mathbf{V}_{in}| \cdot |\mathbf{I}| \cdot \sin(\theta)$ $\quad Q = 0.604 \cdot kVAR$

$\mathbf{S} := \mathbf{V}_{in} \cdot \overline{\mathbf{I}}$ $\quad \mathbf{S} = 0.641 + 0.604j \ \cdot kVA$

$|\mathbf{S}| = 0.881 \cdot kVA$ $\quad \arg(\mathbf{S}) = 43.304 \cdot deg$ $\quad \mathbf{S} = 881VA\underline{/43.3^o}$



OR, if we first find the magnitude of the current which flows through each element of the load...

$$|\mathbf{I}| = \frac{\mathbf{V_{in}}}{\sqrt{R^2 + (\omega \cdot L)^2}} = 8.005 \cdot A$$

$P := (|\mathbf{I}|)^2 \cdot R$ $\quad P = 0.641 \cdot kW$ $\quad Q := (|\mathbf{I}|)^2 \cdot (\omega \cdot L)$ $\quad Q = 0.604 \cdot kVAR$

$\mathbf{S} := P + j \cdot Q$ $\quad |\mathbf{S}| = \sqrt{P^2 + Q^2} = 0.881 \cdot kVA$ $\quad pf = \frac{P}{|\mathbf{S}|} = 0.728$

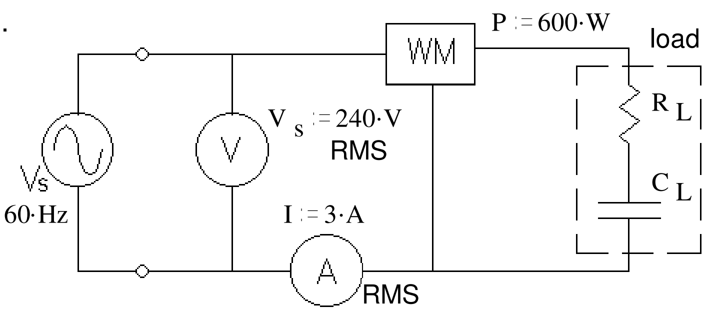
What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

$Q = 603.9 \cdot VAR$ so we need: $Q_C := -Q$ $\quad Q_C = -603.9 \cdot VAR = \frac{V_{in}^2}{X_C}$

$X_C := \frac{\mathbf{V_{in}}^2}{Q_C}$ $\quad X_C = -20.035 \cdot \Omega = \frac{-1}{\omega \cdot C}$ $\quad C := \frac{1}{|X_C| \cdot \omega}$ $\quad C = 132 \cdot \mu F$

Check: $\frac{1}{\frac{1}{R + j \cdot \omega \cdot L} + j \cdot \omega \cdot C} = 18.883 \cdot \Omega$ No j term, so $\theta = 0^o$

Ex. 3 R, & C together are the load in the circuit shown. The RMS voltmeter measures 240 V, the RMS ammeter measures 3 A, and the wattmeter measures 600 W. Find the following: Be sure to show the correct units for each value.



a) The value of the load resistor. R_L = ?

$P = I^2 \cdot R_L$

$R_L := \frac{P}{I^2}$ $\quad R_L = 66.7 \cdot \Omega$

b) The apparent power. |**S**| = ? $\quad S := V_s \cdot I$ $\quad S = 720 \cdot VA$

c) The reactive power. Q = ? $\quad Q := -\sqrt{S^2 - P^2}$ $\quad Q = -398 \cdot VAR$

d) The complex power. **S** = ? $\quad \mathbf{S} := P + j \cdot Q$ $\quad \mathbf{S} = 600 - 398i \cdot VA$

e) The power factor. pf = ? $\quad pf := \frac{P}{V_s \cdot I}$ $\quad pf = 0.833$

f) The power factor is leading or lagging? leading (load is capacitive, Q is negative)

g) The two components of the load are in a box which cannot be opened. Add (draw it) another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and find its value. This component should not affect the real power consumption of the load.

$f = 60 \cdot Hz$ $\quad \omega := 377 \cdot \frac{rad}{sec}$

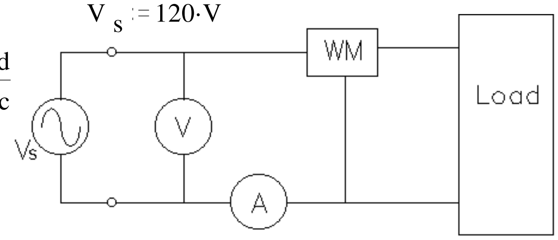
Add an inductor in parallel with load

$Q = -398 \cdot VAR$ so we need: $Q_L := -Q$ $\quad Q_L = 398 \cdot VAR = \frac{V_s^2}{X_L}$

$X_L := \frac{V_s^2}{Q_L}$ $\quad X_L = 144.725 \cdot \Omega = \omega \cdot L$ $\quad L := \frac{|X_L|}{\omega}$ $\quad L = 384 \cdot mH$

Ex. 4 For the 60 Hz load shown in the figure, the RMS voltmeter measures 120 V. The phasor diagram for the power is also shown. Find the following:

$V_s := 120\cdot V$



$\omega := 377\cdot\frac{rad}{sec}$

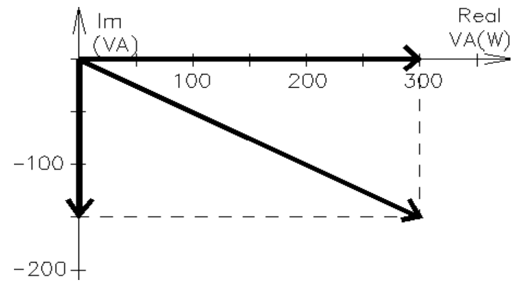
a) The complex power. $\mathbf{S}$ = ?

$P := 300\cdot W$ $\quad$ $Q := -150\cdot VA$

$\mathbf{S} := P + j\cdot Q$ $\quad$ $\mathbf{S} = 300 - 150j \;\cdot VA$

b) The apparent power. $|\mathbf{S}|$ = ? $\quad |\mathbf{S}| = \sqrt{P^2 + Q^2} = 335.4\cdot VA$

c) The power factor. pf = ? $\quad pf := \frac{P}{|\mathbf{S}|}$ $\quad pf = 0.894$



d) The item marked "WM" in the figure is a wattmeter, what does it read? (give a number) $\quad P = 300\cdot W$

e) The item marked "A" in the figure is an RMS ammeter, what does it read? (give a number)

$I := \frac{|\mathbf{S}|}{V_s}$ $\quad I = 2.795\cdot A$ $\quad I = 2.8\cdot A$

f) The power factor is leading or lagging? leading (Q is negative)

g) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and <u>find its value</u>. This component should not affect the real power consumption of the load.

Add an inductor in parallel with load

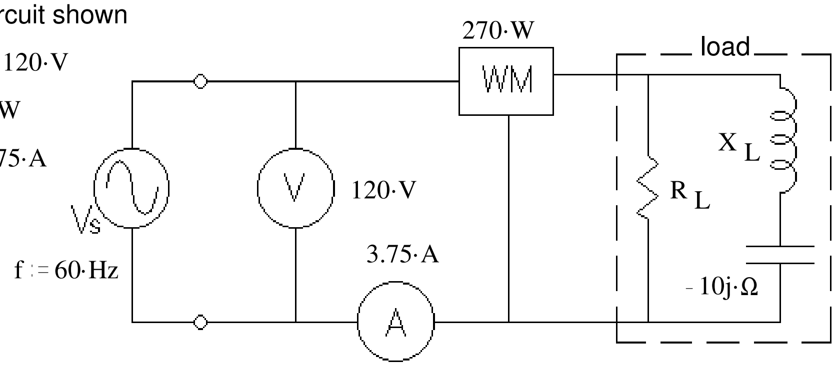
$Q = -150\cdot VAR$ $\quad$ need: $Q_L := -Q$ $\quad Q_L = 150\cdot VAR = \frac{V_s^2}{\omega\cdot L}$ $\quad L := \frac{V_s^2}{\omega\cdot Q_L}$ $\quad L = 255\cdot mH$

Ex. 5 R, L, & C together are the load in the circuit shown

The RMS voltmeter measures 120 V. $V_s := 120\cdot V$

The wattmeter measures 270 W. $P := 270\cdot W$

The RMS ammeter measures 3.75 A. $I := 3.75\cdot A$



Find the following: Be sure to show the correct units for each value.

a) The value of the load resistor. R_L = ?

$P = \frac{V_s^2}{R_L}$ $\quad R_L := \frac{V_s^2}{P}$ $\quad R_L = 53.3\cdot\Omega$

b) The magnitude of the impedance of the load inductor (reactance) . $|\mathbf{Z}_L| = X_L$ = ?

$I_R := \frac{V_s}{R_L}$ $\quad I_R = 2.25\cdot A$ $\quad I_L := \sqrt{I^2 - I_R^2}$ $\quad I_L = 3\cdot A$ $\quad X := \frac{V_s}{I_L}$ $\quad X = 40\cdot\Omega$

$X_C := -10\cdot\Omega$ $\quad X_L := X - X_C$ $\quad X_L = 50\cdot\Omega$

c) The reactive power. Q = ? $\quad Q := \sqrt{(V_s\cdot I)^2 - P^2}$ $\quad Q = 360\cdot VAR$ $\quad$ positive, because the load is primarily inductive

d) The power factor is leading or lagging? lagging (load is inductive, Q is positive)

e) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and <u>find its value</u>. This component should not affect the real power consumption of the load.

Add a capacitor in parallel with load $\qquad f = 60 \cdot \text{Hz} \qquad \omega := 377 \cdot \frac{\text{rad}}{\text{sec}}$

$Q = 360 \cdot \text{VAR}$ so we need: $Q_C := -Q \qquad Q_C = -360 \cdot \text{VAR} = -\frac{V_s^2}{\frac{1}{\omega \cdot C}} = -\omega \cdot C \cdot V_s^2$

$$C := \frac{Q_C}{-\omega \cdot V_s^2} \qquad C = 66.3 \cdot \mu\text{F}$$

Ex. 6 A step-down transformer has an output voltage of 220 V (rms) when the primary is connected across a 560 V (rms) source.

a) If there are 280 turns on the primary winding, how many turns are required on the secondary? $\qquad 280 \cdot \frac{220 \cdot \text{volt}}{560 \cdot \text{volt}} = 110$ turns

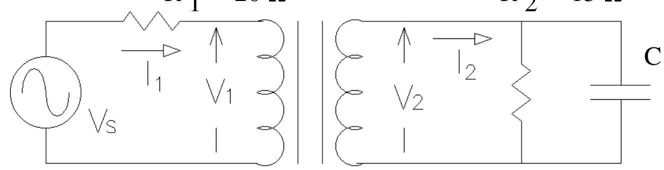
b) If the current in the primary is 2.4 A, what current flows in the load connected to the secondary? $\qquad 2.4 \cdot \text{amp} \cdot \frac{280}{110} = 6.11 \cdot \text{A}$

c) If the transformer is rated at 700/275 V, 2.1 kVA, what are the rated primary and secondary currents? pri: $\frac{2.1 \cdot \text{kVA}}{700 \cdot \text{V}} = 3 \cdot \text{A}$ sec: $\frac{2.1 \cdot \text{kVA}}{275 \cdot \text{V}} = 7.636 \cdot \text{A}$

Ex. 7 The transformer shown in the circuit below is ideal. Find the following:

a) $|\mathbf{I_1}|$ = ?

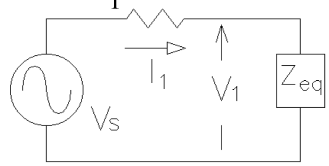
$V_s := 120 \cdot \text{V} \qquad \omega := 377 \cdot \frac{\text{rad}}{\text{sec}} \qquad R_1 := 20 \cdot \Omega \qquad R_2 := 15 \cdot \Omega \qquad C := 40 \cdot \mu\text{F}$



$N_1 := 150 \cdot \text{turns} \qquad N_2 := 50 \cdot \text{turns}$

$$\mathbf{Z_L} := \frac{1}{\frac{1}{R_2} + j \cdot \omega \cdot C}$$

Make an equivalent circuit: $\quad R_1 := 20 \cdot \Omega$



$$\mathbf{Z_L} = 14.27 - 3.228j \ \cdot \Omega$$

$$\mathbf{Z_{eq}} := \left(\frac{N_1}{N_2}\right)^2 \cdot \mathbf{Z_L} \qquad \mathbf{Z_{eq}} = 128.429 - 29.051j \ \cdot \Omega$$

$$R_1 + \mathbf{Z_{eq}} = 148.429 - 29.051j \ \cdot \Omega \qquad \sqrt{148.429^2 + 29.051^2} = 151.245$$

$$\left|\mathbf{I_1}\right| = \frac{V_s}{\left|R_1 + \mathbf{Z_{eq}}\right|} = \frac{V_s}{151.245 \cdot \Omega} = 0.793 \cdot \text{A}$$

b) $|\mathbf{I_2}|$ = ? $= \left(\frac{N_1}{N_2}\right) \cdot \left|\mathbf{I_1}\right| = \frac{150}{50} \cdot .793 \cdot \text{A} = 2.379 \cdot \text{A}$

c) $|\mathbf{V_1}|$ = ? $= V_s \cdot \left|\frac{\mathbf{Z_{eq}}}{R_1 + \mathbf{Z_{eq}}}\right|$ OR.. $\left|\mathbf{V_1}\right| = \left|\mathbf{I_1}\right| \cdot \left|\mathbf{Z_{eq}}\right| = .793 \cdot \text{A} \cdot \sqrt{128.429^2 + 29.051^2} \cdot \Omega = 104.417 \cdot \text{V}$