ECE 2100 Frequency Response & Bode Plot Examples

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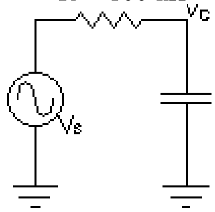
Ex. 1

$R := 100 \cdot k\Omega$



$C := .04 \cdot \mu F$

$$\frac{v_C}{v_S} = \frac{\frac{1}{j \cdot \omega \cdot C}}{\frac{1}{j \cdot \omega \cdot C} + R} = \frac{1}{1 + R \cdot (j \cdot \omega \cdot C)} = H(\omega)$$ = The transfer function

corner frequency is where real = imaginary (in denominator in this case)

$$1 = \omega_c \cdot R \cdot C \qquad \omega_c := \frac{1}{R \cdot C} \qquad \omega_c = 250 \cdot \frac{rad}{sec}$$

So... $H(\omega) := \dfrac{1}{1 + j \cdot \dfrac{\omega}{250 \cdot \frac{rad}{sec}}}$

ω_c is also called a "pole" frequency

The transfer function is said to have one "pole" at ω_c

To make a straight-line approximation of the magnitude of H(ω) we'll approximate |H(ω)| in two regions, one below the corner frequency, and one above the corner frequency. Keep only the real or only the imaginary part of the denominator, depending on which is greater.

below the corner frequency: $\omega < \omega_c \qquad H(\omega) \simeq \frac{1}{1} \qquad |H(\omega)| \simeq 1 \qquad 20 \cdot \log(1) = 0 \cdot dB$

above the corner frequency: $\omega > \omega_c \qquad H(\omega) \simeq \dfrac{1}{j \cdot \dfrac{\omega}{250 \cdot \frac{rad}{sec}}} \qquad |H(\omega)| \simeq \frac{1}{\omega} \cdot \left(250 \cdot \frac{rad}{sec}\right)$ inversely proportional to ω

That's all you need to make the straight-line approximation shown in the plot below

Try some values above the corner frequency:

$$20 \cdot \log\left[\frac{1}{10 \cdot \omega_c} \cdot \left(250 \cdot \frac{rad}{sec}\right)\right] = -20 \cdot dB \qquad 20 \cdot \log\left[\frac{1}{100 \cdot \omega_c} \cdot \left(250 \cdot \frac{rad}{sec}\right)\right] = -40 \cdot dB$$

The slope above the corner frequency is -20 dB per "decade"

Let's find the actual magnitude of H(ω) right at the corner frequency (H(ωc)):

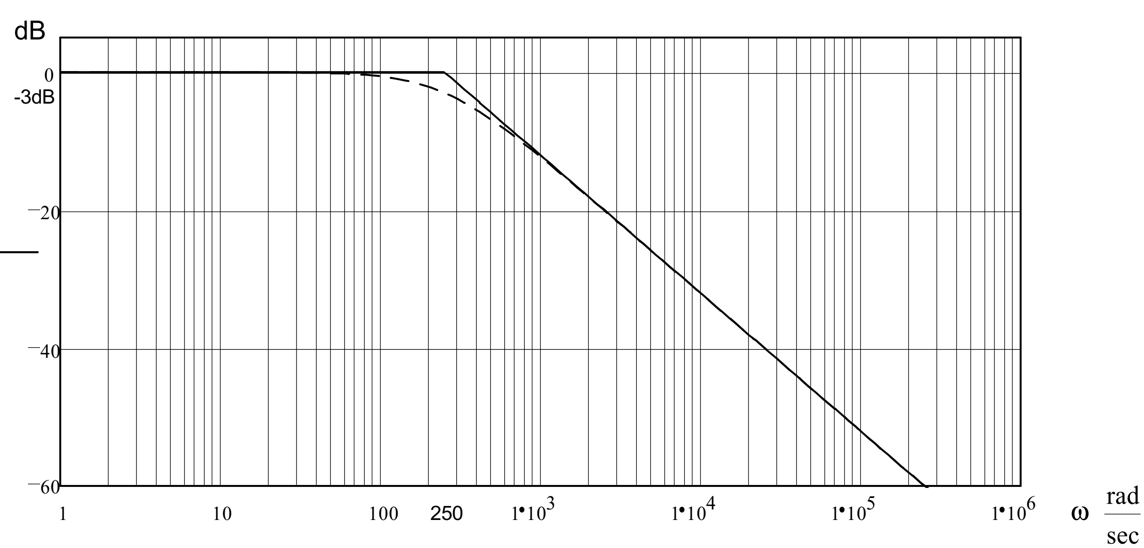
$$\omega = \omega_c \qquad H(\omega) = \frac{1}{1 + j \cdot \dfrac{\omega_c}{250 \cdot \frac{rad}{sec}}} = \frac{1}{1 + j \cdot 1}$$

$$|H(\omega)| = \frac{1}{\sqrt{2}} \qquad 20 \cdot \log\left(\frac{1}{\sqrt{2}}\right) = -3.01 \cdot dB$$

Magnitude plot $|H(\omega)|$

Straight-line approximation ____

Actual _ _ _ _



But the magnitude plot is only half the picture. To get the whole picture, you'll need a phase angle plot as well.

To make a straight-line approximation of the phase of Hω) we'll approximate $\angle H(\omega)$ in two regions, just like for the magnitude.

below the corner frequency: $\omega < \omega_c$ $\quad H(\omega) \simeq \frac{1}{1}$ $\quad$ angle $\simeq 0$

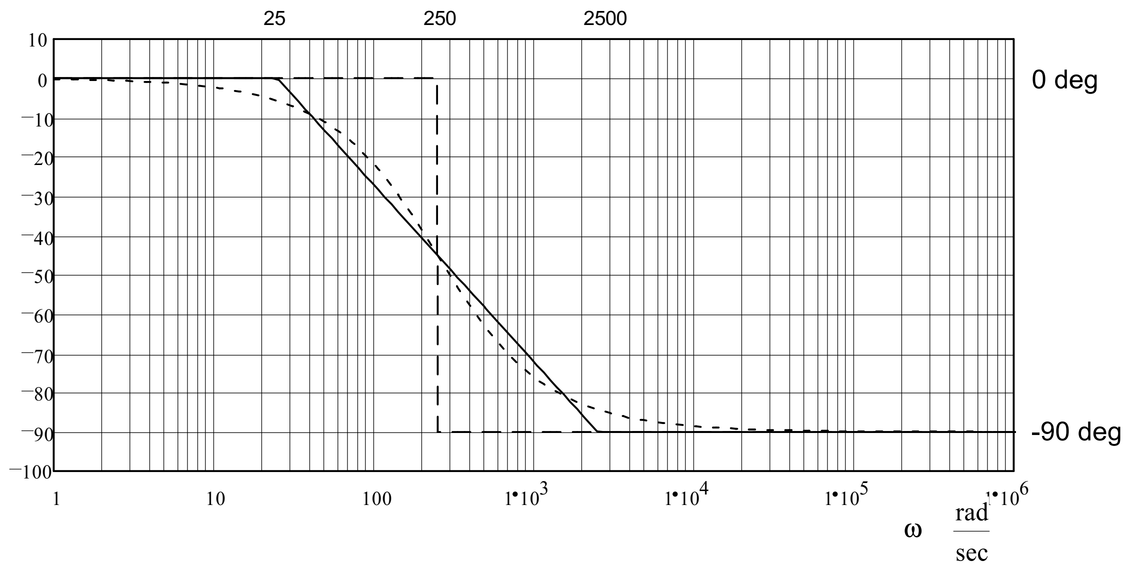
above the corner frequency: $\omega > \omega_c$ $\quad H(\omega) \simeq \dfrac{1}{j \cdot \dfrac{\omega}{250 \cdot \frac{rad}{sec}}}$ $\quad$ angle $\simeq -90 \quad \left(\frac{1}{j}\right)$

However, as a straight-line approximation, this stinks, look at the dashed line below and compare it to the actual phase plot, dotted line. Its much better if you draw a slanted line from one decade below the corner frequency to one decade above.

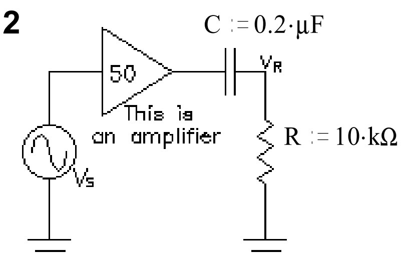
Phase angle plot

Straight-line approximation ____

Actual - - - - - -



Ex. 2



$$\frac{v_C}{v_S} = 50 \cdot \frac{R}{\frac{1}{j \cdot \omega \cdot C} + R} = \frac{50 \cdot (R \cdot (j \cdot \omega \cdot C))}{1 + R \cdot (j \cdot \omega \cdot C)} = H(\omega)$$

Transfer function has one pole at ω_c

corner frequency is where real = imaginary

$$1 = \omega_c \cdot R \cdot C \qquad \omega_c := \frac{1}{R \cdot C} \qquad \omega_c = 500 \cdot \frac{rad}{sec}$$

So... $$H(\omega) := \frac{50 \cdot j \cdot \dfrac{\omega}{500 \cdot \frac{rad}{sec}}}{1 + j \cdot \dfrac{\omega}{500 \cdot \frac{rad}{sec}}} = \frac{50 \cdot j \cdot \omega}{500 \cdot \frac{rad}{sec} + j \cdot \omega}$$

OR: $$H(\omega) := \frac{50 \cdot j \cdot \dfrac{\omega}{\omega_c}}{1 + j \cdot \dfrac{\omega}{\omega_c}}$$

below the corner frequency:

$$\omega < \omega_c \qquad H(\omega) \simeq \frac{50 \cdot j \cdot \dfrac{\omega}{500 \cdot \frac{rad}{sec}}}{1} = \frac{0.1 \cdot \frac{sec}{rad} \cdot j \cdot \omega}{1}$$

$$|H(\omega)| \simeq 0.1 \cdot \frac{sec}{rad} \cdot \omega$$

Proportional to ω. That's all we need to know here. This proportionality to ω will result in a +20 dB per decade slope for all frequencies below the corner frequency

angle $\simeq$ 90 $\quad$ (j) $\quad$ because of the j in the numerator above the corner frequency:

$$\omega > \omega_c \quad H(\omega) \approx \frac{50 \cdot j \cdot \dfrac{\omega}{500 \cdot \frac{rad}{sec}}}{j \cdot \dfrac{\omega}{500 \cdot \frac{rad}{sec}}} \qquad |H(\omega)| \approx 50 \qquad 20 \cdot \log(50) = 33.98 \cdot dB$$

The "pass band" angle $\approx$ 0

Actual value at the corner frequency

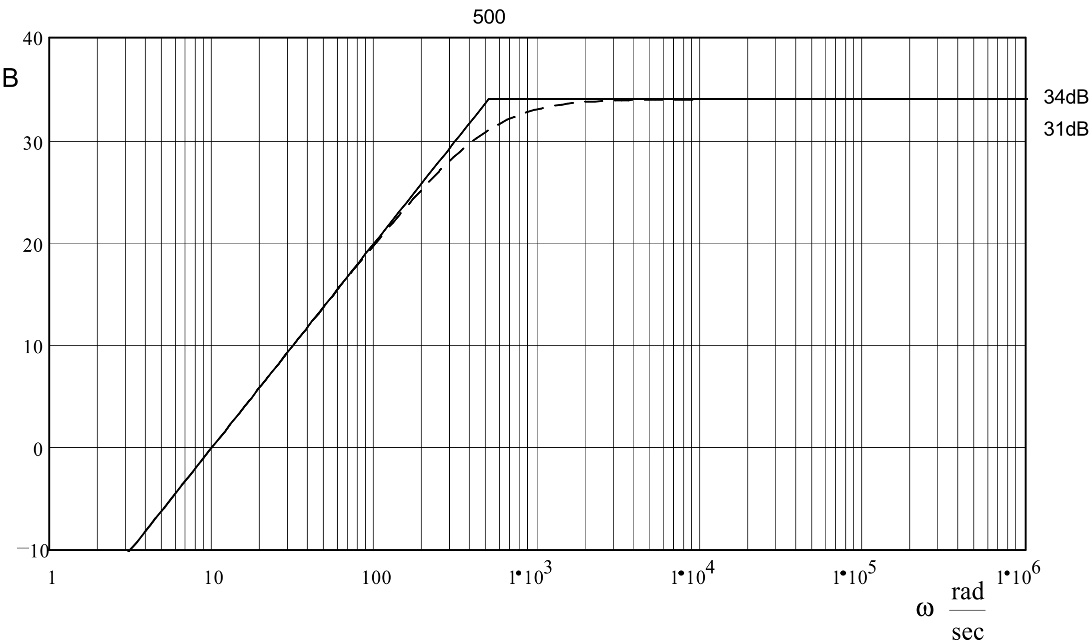
$$\omega = \omega_c \quad H(\omega) = \frac{50 \cdot j \cdot \omega}{500 \cdot \frac{rad}{sec} + j \cdot \omega} = \frac{50 \cdot j}{1 + j \cdot 1} = 25 + 25j \qquad |25 + 25 \cdot j| = 35.355 \qquad 20 \cdot \log(35.355) = 30.97 \cdot dB$$

3 dB lower than the magnitude in the pass band

Magnitude plot $|H(\omega)|$ dB

Straight-line approximation ____

Actual _ _ _ _

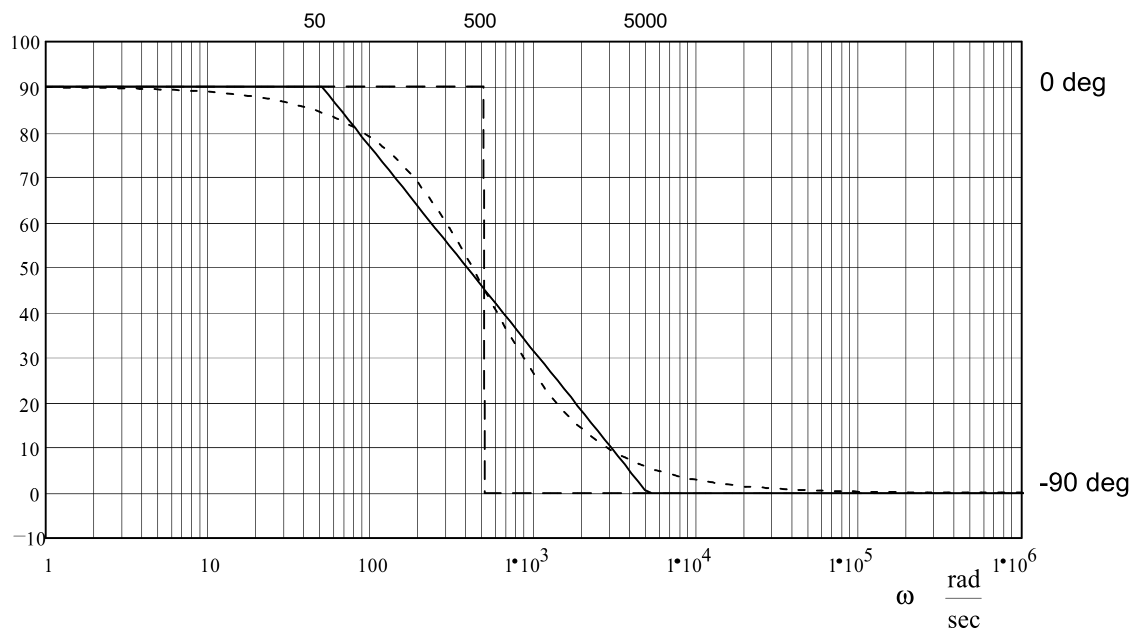


Like before, first use the 90° and 0° angles to get a basic plot, then draw a slanted line from one decade below the corner frequency to one decade above.

Phase angle plot

Straight-line approximation ____

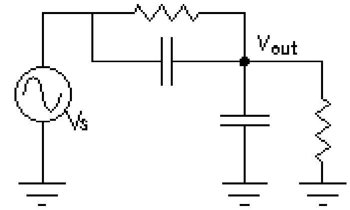
Actual - - - - - -



Ex. 3 The transfer function may already be worked out:

$$H(f) := 10^{-2} \cdot \frac{1 + j \cdot \frac{f}{10 \cdot Hz}}{1 + j \cdot \frac{f}{500 \cdot Hz}}$$

Could come from a circuit like this:



The real and imaginary parts of the numerator are equal at the one corner frequency (called a "zero") $\quad 1 = j \cdot \frac{f_c}{10 \cdot Hz} \qquad f_{c1} := 10 \cdot Hz$

The real and imaginary parts of the denominator are equal at the other corner frequency (pole) $\quad 1 = j \cdot \frac{f_c}{500 \cdot Hz} \qquad f_{c2} := 500 \cdot Hz$

There are now three regions to approximate |H(f)|

Below the first corner frequency: $\quad f < 10 \cdot Hz \qquad |H(f)| \cong \left| 10^{-2} \cdot \frac{1}{1} \right| = 10^{-2}$

$20 \cdot \log\left(10^{-2}\right) = -40 \cdot dB$

angle $\cong$ 0

Between the corner frequencies: $\quad 10 \cdot Hz < f < 500 \cdot Hz \qquad |H(f)| \cong \left| 10^{-2} \cdot \frac{j \cdot \frac{f}{10 \cdot Hz}}{1} \right| = \frac{10^{-3}}{Hz} \cdot f$

proportional to f

angle $\cong$ 90 $\quad (j)$

Above the second corner frequency: $\quad 500 \cdot Hz < f \qquad |H(f)| \cong \left| 10^{-2} \cdot \frac{j \cdot \frac{f}{10 \cdot Hz}}{j \cdot \frac{f}{500 \cdot Hz}} \right| = 0.5$

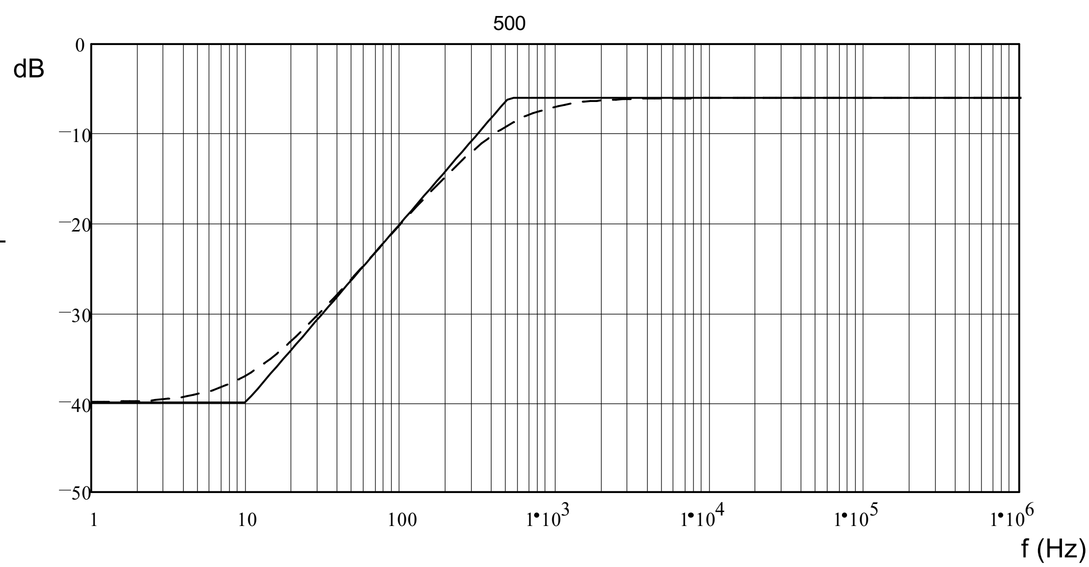
$20 \cdot \log(0.5) = -6.02 \cdot dB$

angle $\cong$ 0

Magnitude plot $|H(f)|$

Straight-line approximation ____

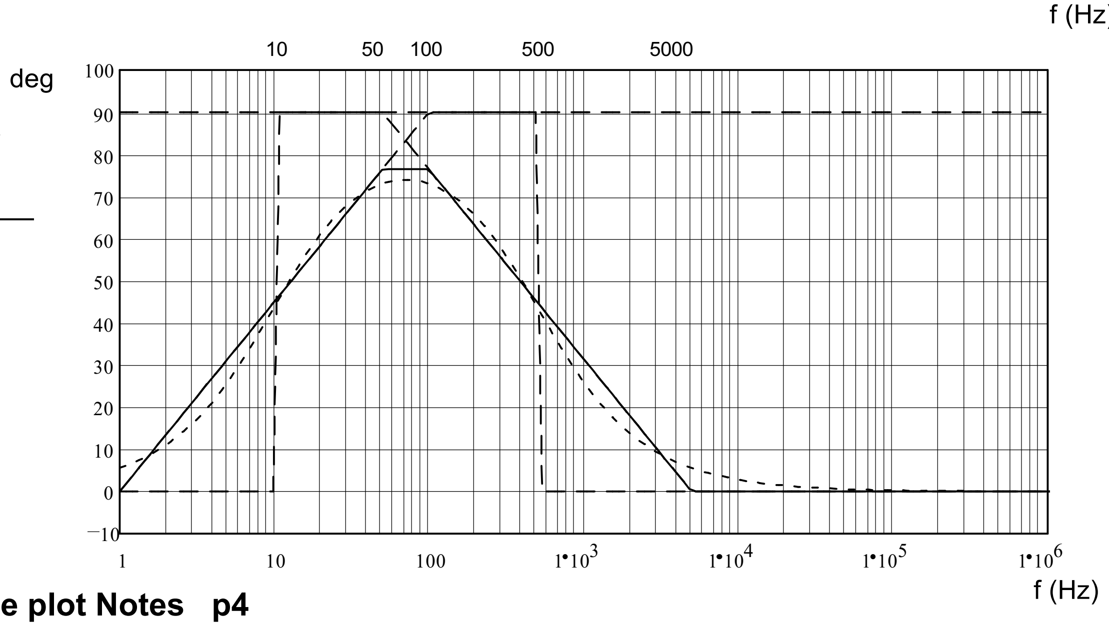
Actual _ _ _ _



Phase angle plot

Straight-line approximation ____

Actual - - - - - -



ECE 2100 Frequency Response & Bode Plot Examples, part 2

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Ex. 4 A Transfer function of a typical amplifier:

$$H(\omega) := \frac{j\cdot\omega\cdot 0.182\cdot\text{sec}}{\left(1+\frac{j\cdot\omega}{416.67\cdot\frac{\text{rad}}{\text{sec}}}\right)\cdot\left(1+\frac{j\cdot\omega}{6.875\cdot 10^4\cdot\frac{\text{rad}}{\text{sec}}}\right)}$$

$$\omega_{C1} := 416.67\cdot\frac{\text{rad}}{\text{sec}} \qquad \omega_{C2} := 6.875\cdot 10^4\cdot\frac{\text{rad}}{\text{sec}}$$

Below ω_{C1} $\quad H(\omega) \;\underset{\sim}{-}\; \frac{j\cdot\omega\cdot 0.182}{(1)\cdot(1)}$ $\quad$ proportional to ω
angle $\underset{\sim}{-}$ 90

Between the two poles (passband): $\quad H(\omega) \;\underset{\sim}{-}\; \frac{j\cdot\omega_i\cdot 0.182}{(1)\cdot\left(\frac{j\cdot\omega_i}{416.67}\right)} = 75.834$ $\quad 20\cdot\log(75.834) = 37.6$
angle $\underset{\sim}{-}$ 0

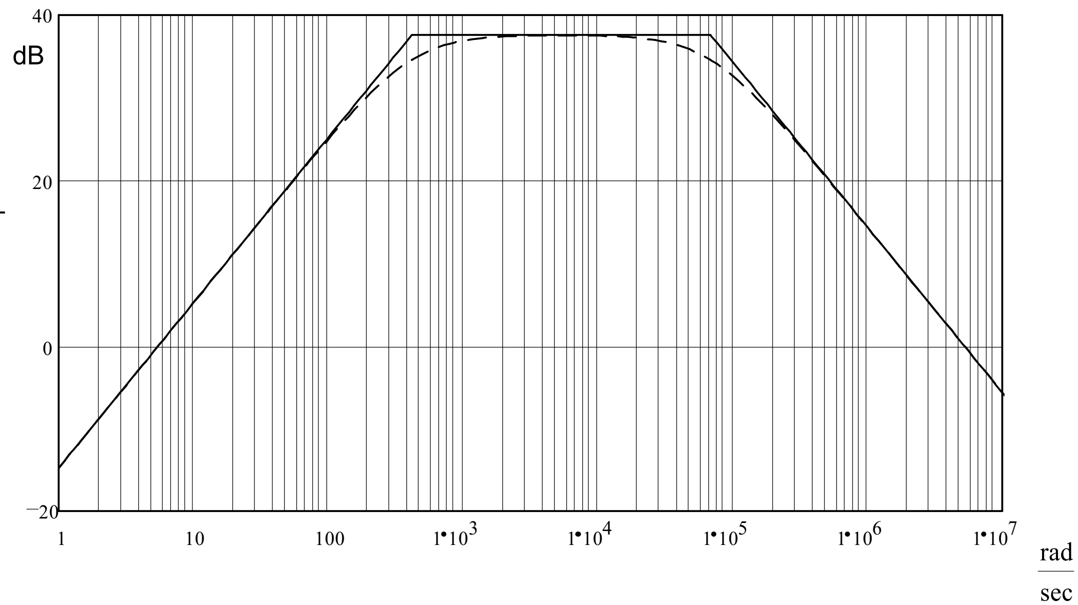
Above ω_{C2} $\quad H(\omega) \;\underset{\sim}{-}\; \frac{j\cdot\omega\cdot 0.182}{\left(\frac{j\cdot\omega}{6.875\cdot 10^4}\right)\cdot\left(\frac{j\cdot\omega}{416.67}\right)}$ $\quad$ inversely proportional to ω
angle $\underset{\sim}{-}$ -90

Magnitude plot

$|H(\omega)|$

Straight-line approximation ____

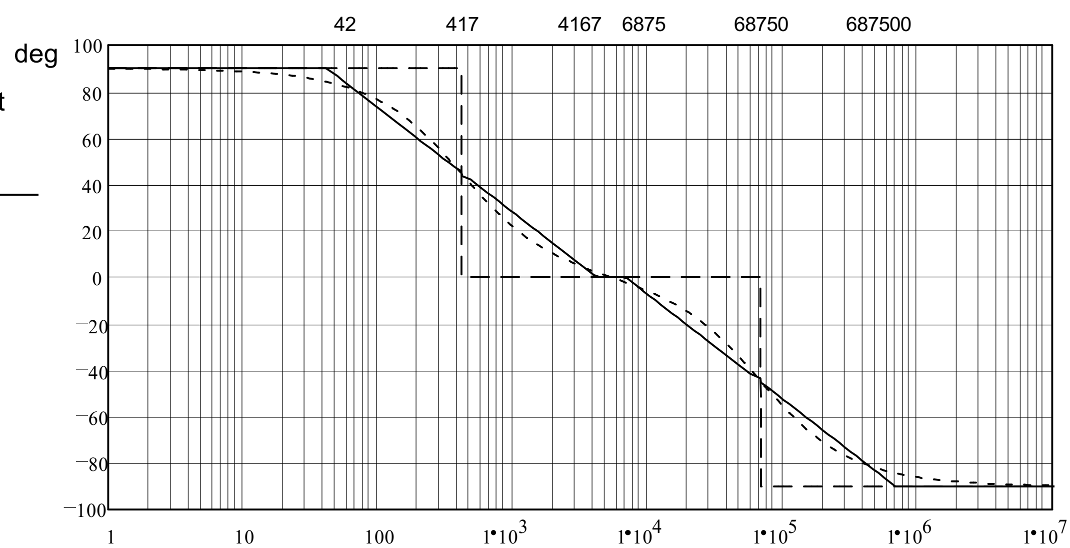
Actual _ _ _ _



Phase angle plot

Straight-line approximation ____

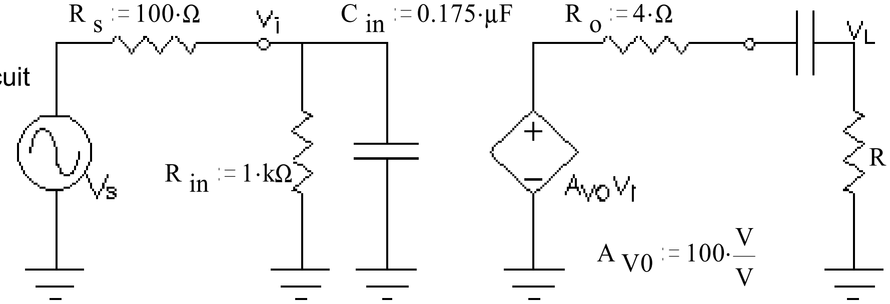
Actual - - - - - -



Ex. 5

How about an amplifier circuit

$R_s := 100\cdot\Omega$ $C_{in} := 0.175\cdot\mu F$ $R_o := 4\cdot\Omega$ $C_o := 22\cdot\mu F$



$R_{in} := 1\cdot k\Omega$ $R_L := 20\cdot\Omega$ $A_{V0} := 100\cdot\frac{V}{V}$

$$Z_{in} = \frac{1}{\frac{1}{R_{in}} + j\cdot\omega\cdot C_{in}} = \frac{R_{in}}{1 + j\cdot\omega\cdot C_{in}\cdot R_{in}} \qquad Z_o = R_o + \frac{1}{j\cdot\omega\cdot C_o}$$

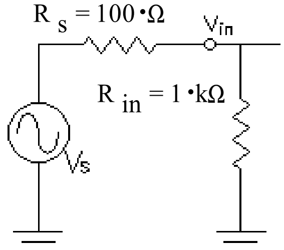
$$\frac{v_o}{v_s} = \frac{Z_{in}}{R_s + Z_{in}}\cdot A_{Vo}\cdot\frac{R_L}{Z_o + R_L} = \frac{\frac{R_{in}}{1 + j\cdot\omega\cdot C_{in}\cdot R_{in}}}{R_s + \frac{R_{in}}{1 + j\cdot\omega\cdot C_{in}\cdot R_{in}}}\cdot A_{V0}\cdot\frac{R_L}{R_o + \frac{1}{j\cdot\omega\cdot C_o} + R_L}$$

$$\frac{v_o}{v_s} = \frac{R_{in}}{R_s\cdot(1 + j\cdot\omega\cdot C_{in}\cdot R_{in}) + R_{in}}\cdot A_{V0}\cdot\frac{R_L\cdot(j\cdot\omega\cdot C_o)}{(R_o + R_L)\cdot(j\cdot\omega\cdot C_o) + 1} = \frac{R_{in}}{(R_s + R_{in}) + j\cdot\omega\cdot C_{in}\cdot R_{in}\cdot R_s}\cdot A_{V0}\cdot\frac{R_L\cdot(j\cdot\omega\cdot C_o)}{1 + j\cdot\omega\cdot C_o\cdot(R_o + R_L)}$$

$$H(\omega) = \frac{j\cdot\omega\cdot C_o\cdot A_{V0}\cdot R_{in}\cdot R_L}{\left[(R_s + R_{in}) + j\cdot\omega\cdot C_{in}\cdot R_{in}\cdot R_s\right]\cdot\left[1 + j\cdot\omega\cdot C_o\cdot(R_o + R_L)\right]} = \frac{\frac{j\cdot\omega\cdot C_o\cdot A_{V0}\cdot R_{in}\cdot R_L}{(R_s + R_{in})}}{\left[1 + \frac{j\cdot\omega\cdot C_{in}\cdot R_{in}\cdot R_s}{(R_s + R_{in})}\right]\cdot\left[1 + j\cdot\omega\cdot C_o\cdot(R_o + R_L)\right]} \;....$$

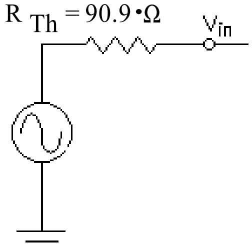
OR, the easy way, first make a Thevenin equivalent circuit:

$R_s = 100\cdot\Omega$ $R_{in} = 1\cdot k\Omega$

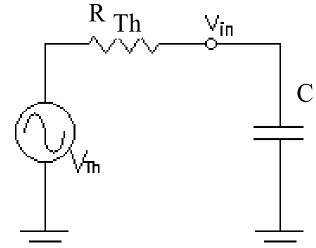


$$R_{Th} := \frac{1}{\left(\frac{1}{R_s} + \frac{1}{R_{in}}\right)} \qquad \frac{v_{Th}}{v_s} = \frac{R_{in}}{R_{in} + R_s}$$

$R_{Th} = 90.9\cdot\Omega$



R_{Th} $C_{in} = 0.175\cdot\mu F$

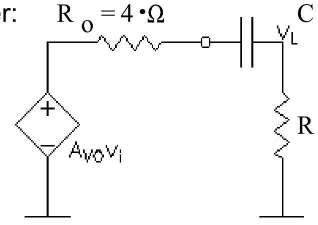


By inspection this is low-pass filter and the corner frequency is where: $\frac{1}{j\cdot\omega_{C1}\cdot C_{in}} = R_{Th}$ $\omega_{C1} := \frac{1}{C_{in}\cdot R_{Th}}$

For left side of amplifier: $H_L(\omega) = \frac{R_{in}}{R_s + R_{in}}\cdot\frac{1}{1 + j\cdot\left(\frac{\omega}{\omega_{C1}}\right)}$ $\omega_{C1} = 62857\cdot\frac{rad}{sec}$

$f_{C1} := \frac{\omega_{C1}}{2\cdot\pi}$ $f_{C1} = 10.004\cdot kHz$

For right side of amplifier: $R_o = 4\cdot\Omega$ $C_o = 22\cdot\mu F$ $R_L = 20\cdot\Omega$



$$\frac{v_o}{v_{in}} = A_{V0}\cdot\frac{R_L}{R_o + R_L + \frac{1}{j\cdot\omega\cdot C_o}}\cdot\frac{(j\cdot\omega\cdot C_o)}{(j\cdot\omega\cdot C_o)}$$

$$H_R(\omega) = A_{V0}\cdot\frac{j\cdot\omega\cdot C_o\cdot R_L}{j\cdot\omega\cdot C_2\cdot C_o\cdot(R_o + R_L) + 1}$$

$\omega_{C2} := \frac{1}{C_o\cdot(R_o + R_L)}$ $\omega_{C2} = 1894\cdot\frac{rad}{sec}$ $f_{C2} := \frac{\omega_{C2}}{2\cdot\pi}$ $f_{C2} = 301.43\cdot Hz$

Complete transfer function

$$H(\omega) = \frac{v_o}{v_s} = H_L(\omega)\cdot H_R(\omega)$$

Fit into a nice clean form:

$$= \frac{j\cdot\omega\cdot K}{\left(1+\frac{j\cdot\omega}{\omega_{C1}}\right)\cdot\left(1+\frac{j\cdot\omega}{\omega_{C2}}\right)} \qquad K = \frac{R_{in}}{R_s+R_{in}}\cdot A_{V0}\cdot C_o\cdot R_L = 0.04\cdot\text{sec}$$

OR, in terms of Hz:

$$H(f) := \frac{j\cdot 2\cdot\pi\cdot f\cdot 0.04\cdot\text{sec}}{\left(1+\frac{j\cdot f}{10000\cdot\text{Hz}}\right)\cdot\left(1+\frac{j\cdot f}{301\cdot\text{Hz}}\right)}$$

Below f_{C2} $\quad H(f) \cong \frac{j\cdot 2\cdot\pi\cdot f\cdot 0.04\cdot\text{sec}}{(1)\cdot(1)}$

proportional to f
angle ≅ 90

Between the two poles (midband): $\quad H(\omega) \cong \frac{j\cdot 2\cdot\pi\cdot f\cdot 0.04\cdot\text{sec}}{(1)\cdot\left(\frac{j\cdot f}{301\cdot\text{Hz}}\right)} = 2\cdot\pi\cdot 301\cdot 0.04 = 75.65$

$20\cdot\log(75.65) = 37.6\cdot\text{dB}$
angle ≅ 0

Above f_{C1} $\quad H(\omega) \cong \frac{j\cdot 2\cdot\pi\cdot f\cdot 0.04\cdot\text{sec}}{\left(\frac{j\cdot f}{10000\cdot\text{Hz}}\right)\cdot\left(\frac{j\cdot f}{301}\right)}$

inversely proportional to f
angle ≅ -90

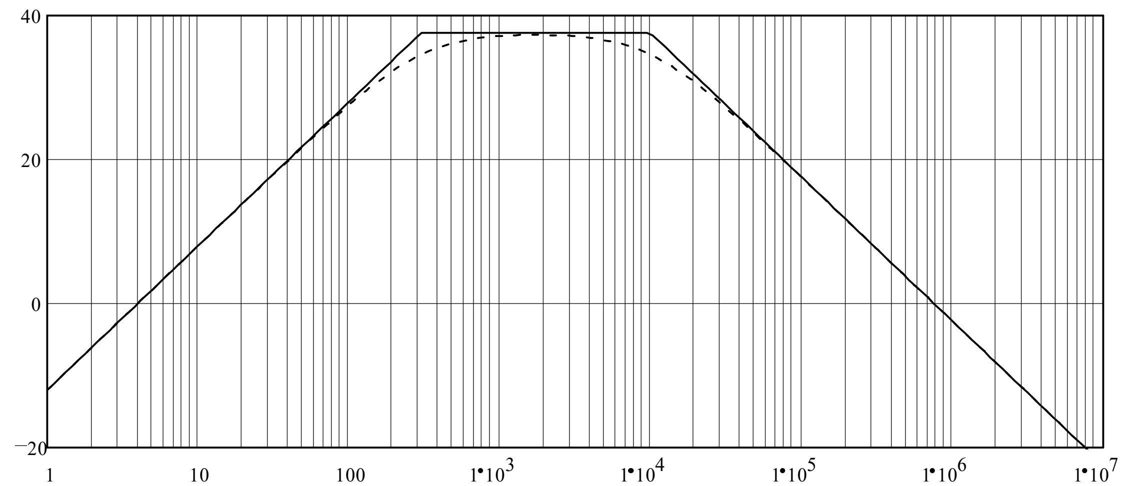
Bandwidth = $f_{C1} - f_{C2} = 9.703\cdot\text{kHz}$

Magnitude plot

$|H(\omega)|$ dB

Straight-line approximation ____

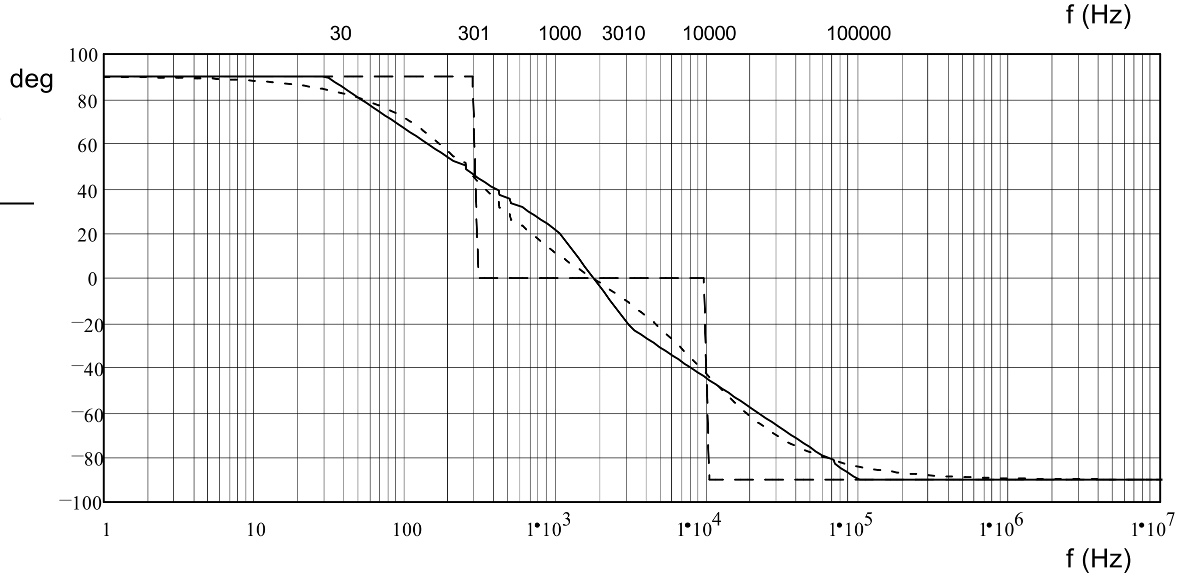
Actual _ _ _ _



Phase angle plot

Straight-line approximation ____

Actual - - - - - -



Warning

The Bode plots that we've covered here are the simplest types and will do for our purposes, where we are primarily interested in the design of amplifiers and the analysis the low and high corner frequencies. When poles and zeroes are too close to each other they can interact or even result in complex poles. If asked in a future classes if you have "covered" Bode plots, do not make the mistake of saying "yes".