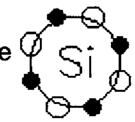
Semiconductor Physics Notes EE 2100 A.Stolp 3/27/01, rev 3/26/02

Silicon atoms & crystals



Silicon atoms each have 4 valence electrons (electrons in their outermost shell). In its crystalline form, each atom covalently bonds with four neighboring atoms to form a tetrahedral crystal. In the pure, "intrinsic" crystal, practically all the electrons are used in bonds and all the spaces are filled, which leaves very few electrons free to move and thus no way to make current flow.

By the effects of heat, light and/or large electric fields, a few electrons do break free of the bonds and become "free" carriers. That is, they're free to move about crystal and "carry" an electrical current. For each free electron created, there will also be a free "hole". A hole is the space left behind by the now free electron and is considered to be a carrier (+) in its own right.

By heat:

Intrinsic free electron concentration (elect/cm³): $n_i := \sqrt{B \cdot T^3 \cdot e^{-\frac{E_g}{k \cdot T}}}$

Bandgap Energy: $E_g := 1.12 \cdot eV$

Electron volt: $eV := 1.60 \cdot 10^{-19} \cdot joule$

Absolute temperature: T = °C + 273

Boltzmann's constant: $k := 8.63 \cdot 10^{-5} \cdot \frac{eV}{K}$

Material-dependant parameter: $B := 5.4 \cdot 10^{31} \cdot \frac{1}{K^3 \cdot cm^6}$ for Silicon

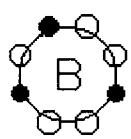
$$n_i^2 = 5.4 \cdot 10^{31} \cdot \frac{1}{K^3 \cdot cm^6} \cdot T^3 \cdot e^{-\frac{1.3 \cdot 10^4 \cdot K}{T}} \qquad \frac{E_g}{k} = 1.298 \cdot 10^4 \cdot K$$

Intrinsic free hole concentration (holes/cm³): $p_i = n_i$ (One for each free electron)

These are the *equilibrium* concentrations: Rate of free carrier creation = Rate of *recombination*.

Doping

p-type

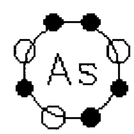


Replace some of the silicon atoms in an intrinsic crystal with trivalent atoms like boron or aluminum. These atoms are called "acceptor" atoms.

Acceptor atom concentration (atoms/cm³): N_A

free hole concentration (holes/cm³): $p \simeq N_A$ Majority carriers

n-type



Replace some of the silicon atoms in an intrinsic crystal with pentavalent atoms like arsenic or phosphorus . These atoms are called "donor" atoms.

Donor atom concentration (atoms/cm³): N_D

free electron concentration (elect/cm³): $n \simeq N_D$

If you make one concentration larger, the other becomes proportionately smaller:

If there are 10 times as many free electrons, then free holes will be filled 10 times faster and exist only 1/10th as long before recombining.

$$p \cdot n = n_i^2$$

If there are 10 times as many holes, then free electrons only last 1/10th as long before recombining.

free electron concentration (elect/cm³): $n_{po} \simeq \frac{n_i^2}{N_A}$ (Minority carrier) in p-type equilibrium

free hole concentration (holes/cm³): $p_{no} \simeq \frac{n_i^2}{N_D}$ (Minority carrier) in n-type equilibrium

Drift Current

Drift current is the common kind of current that you're used to seeing in wires. Under the influence of an electric field, the free negative carriers drift towards the positive charge and the free positive carriers drift towards the negative volta

Drift current density (amp/cm²): $J_{drift} = q \cdot (p \cdot \mu_p + n \cdot \mu_n) \cdot E$

Electric field (V/cm)

Note: $\frac{cm}{10^4 \cdot \mu m} = 1$

Electron mobility: $\mu_n := 1350 \cdot \frac{cm^2}{V \cdot s}$ typical

Electron charge: $q := 1.60 \cdot 10^{-19} \cdot coul$

Hole mobility: $\mu_p := 480 \cdot \frac{cm^2}{V \cdot s}$ typical

Resistivity (Ωcm): $\rho = \frac{1}{q \cdot (p \cdot \mu_p + n \cdot \mu_n)}$

Current: $I = J \cdot (Cross\_sectional\_area)$

Diffusion Current

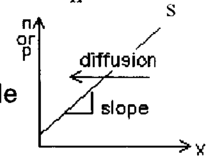
If you add some colored dye to one end of a tub filled with water the color would eventually diffuse to the other side until all the water is equally colored. There will be a net flow of colored water from the side of high concentration to the side of low concentration. The same will happen if you add carriers at one end of a piece of semiconductor, there will be a net flow away from the area of higher concentration. (Diffusion current also happens in regular conductors, but it happens so fast we rarely consider it and just assume the charges equalize instantly.)

Diffusion constant for electrons: $D_n := 34 \cdot \frac{cm^2}{s}$ typical

Diffusion electron current density: $J_{ndiff} = q \cdot D_n \cdot \frac{dn}{dx}$

Electron charge: $q := 1.60 \cdot 10^{-19} \cdot coul$

slope of concentration profile



Diffusion hole current density: $J_{pdiff} = -q \cdot D_p \cdot \frac{dp}{dx}$

Diffusion constant for holes: $D_p := 12 \cdot \frac{cm^2}{s}$ typical

Current: $I = J \cdot (Cross\_sectional\_area)$

Positive slope results in leftward movement of holes and thus a negative current.

Diffusion constant / mobility relationship

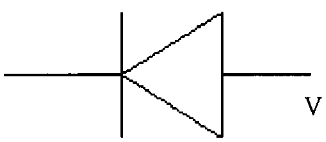
Not surprisingly the diffusion constant of a carrier is related to the mobility of the carrier.

$$\frac{\text{Diffusion constant}}{\text{mobility}} = \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T = \text{Thermal voltage} = \frac{k \cdot T}{q} \simeq 25 \cdot mV$$

(Einstein relationship)

Diodes, no bias or Reverse bias

Note that the diode is pointed leftward, which may seem a little weird right now, but is done for a reason.

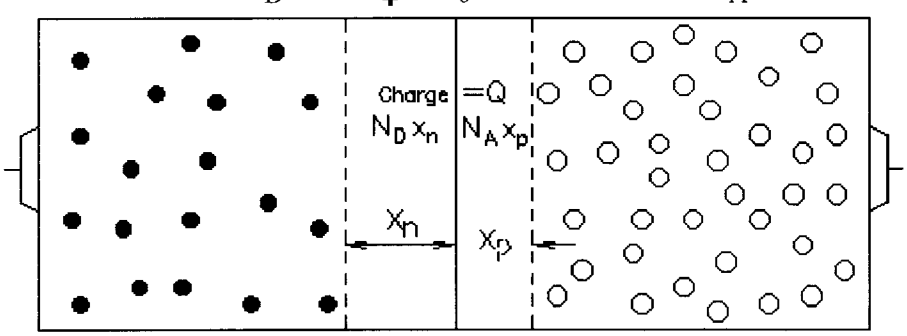


Since the doping drives the creation of the depletion region, The barrier potential and the width of the depletion region depend on the doping.

$$V_0 = V_T \cdot \ln\left(\frac{N_A \cdot N_D}{n_i^2}\right)$$

= Barrier potential (built-in voltage) typical: 0.6V to 0.8V

n-type N_D $+$ V_0 $-$ p-type N_A



dots represent free electrons $\leftarrow W_{dep} \rightarrow$ circles represent free holes

Note: This drawing is NOT to scale. The depletion region is actually very small compared to the size of the diode. Also, there are far more free carriers than shown.

$$\frac{x_n}{x_p} = \frac{N_A}{N_D}$$

$x_n > x_p$ because $N_A > N_D$ in this particular representation

Permittivity of silicon: $\varepsilon_s := 1.035 \cdot 10^{-12} \cdot \frac{F}{cm}$

Width of depletion region: $$W_{dep} = \sqrt{\frac{2 \cdot \varepsilon_s}{q} \cdot \left(\frac{1}{N_A} + \frac{1}{N_D}\right) \cdot (V_0 + V_R)}$$

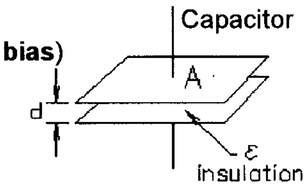
$$\frac{2 \cdot \varepsilon_s}{q} = 1.294 \cdot 10^7 \cdot \frac{1}{V \cdot cm}$$

Reverse bias voltage

$$x_n + x_p = W_{dep} = \sqrt{1.294 \cdot 10^7 \cdot \frac{1}{V \cdot cm} \cdot \left(\frac{1}{N_A} + \frac{1}{N_D}\right) \cdot (V_0 + V_R)}$$

Extent into n-region: $x_n = \frac{N_A}{N_A + N_D} \cdot W_{dep}$ Extent into p-region: $x_p = \frac{N_D}{N_A + N_D} \cdot W_{dep}$

Depletion or Junction Capacitance (due to depletion region, primarily for **reverse bias**)



Junction capacitance
no bias, forward or reverse: $C_{j0} = \dfrac{\varepsilon_s \cdot \text{Area}}{W_{dep}} = A \cdot \sqrt{\dfrac{\varepsilon_s \cdot q}{2 \cdot \left(\dfrac{1}{N_A} + \dfrac{1}{N_D}\right) \cdot V_0}}$

C_{j0} is often given on part data sheets

Barrier potential, see previous page

$C = \varepsilon \cdot \dfrac{A}{d} = \dfrac{Q}{V} = \dfrac{dq}{dv}$

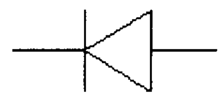
Permittivity of silicon: $\varepsilon_s := 1.035 \cdot 10^{-12} \cdot \dfrac{F}{cm}$

reverse bias: $C_j = A \cdot \sqrt{\dfrac{\varepsilon_s \cdot q}{2 \cdot \left(\dfrac{1}{N_A} + \dfrac{1}{N_D}\right) \cdot (V_0 + V_R)}} = A \cdot \sqrt{\dfrac{8.28 \cdot 10^{-32} \cdot \dfrac{F^2 \cdot V}{cm}}{\left(\dfrac{1}{N_A} + \dfrac{1}{N_D}\right) \cdot (V_0 + V_R)}}$

$C_j = \dfrac{C_{j0}}{\left(1 + \dfrac{V_R}{V_0}\right)^m}$ m: 1/2 or Fudge, 1/3 to 1/2

forward bias: $C_j \simeq 2 \cdot C_{j0}$

Diodes, forward bias

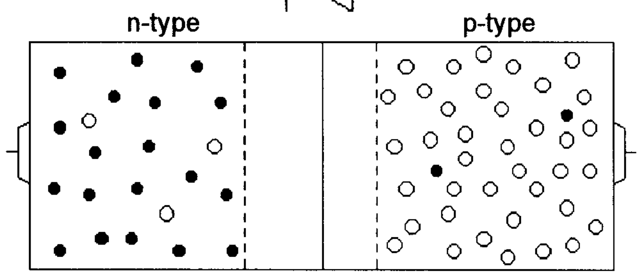


Drawings are NOT to scale, not even close.

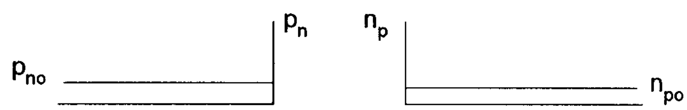
Minority Carriers make the forward bias current

Even in an n-type region, there will be some free holes: $p_{no} \simeq \dfrac{n_i^2}{N_D}$

Likewise, there will be some free electrons in the p-type region: $n_{po} \simeq \dfrac{n_i^2}{N_A}$

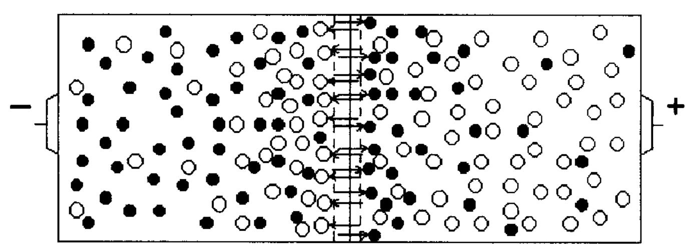


The graph at right shows these minority carrier concentrations as functions of the position in the diode. Since these lines are flat, no diffusion current will flow. The p_{no} line is a little higher than the n_{po} line, which indicates that the n region is more heavily doped.



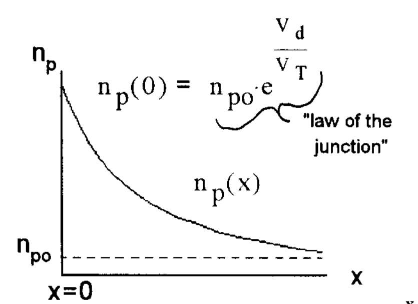
Forward Bias

When the diode is forward biased, holes are "injected" into the n region and electrons are injected into the p region. These carriers are called "excess minority carriers". Excess because there are more than p_{no} and n_{po}. Minority because they're in the wrong region.



There are two minority-carrier currents, the holes in the n region and the electrons in the p region. These two together make up the diode current. There are also majority-carrier currents on each side, but they don't cross the junction and don't limit the diode current, so we don't have to consider those.

Hole concentration graph not shown



The graph at right shows the distribution of minority electrons in the p region. The dotted line is the base level (n_{po}). The exponential curve shows how the number of free electrons tapers off as you move further away from where they're injected. They're being "swallowed" by the holes in this p-type region.

$$n_p(x) = n_{po} + \left(n_p(0) - n_{po}\right) \cdot e^{-\frac{x}{L_n}}$$

Diffusion length

The number of injected electrons is dependent on the voltage across the diode and so is the concentration of free electrons at x=0. That concentration ($n_p(0)$) is shown on this graph and given by a relation known as the "law of the junction".

There are two ways to determine the excess-electron current. One would be to find the total number of excess electrons, divide by their average lifetime, and multiply by q to get the current. The other way would be to find the slope of the $n_p(x)$ curve at x=0 and and use that to find the diffusion current at x=0. Both methods yield the same results, which are shown on the next page.

Hole injection current: $I_p = \mathrm{Area}\cdot q\cdot n_i^2\cdot\dfrac{D_p}{L_p\cdot N_D}\cdot\left(e^{\frac{V_d}{V_T}}-1\right)$ Electron injection current: $I_n = \mathrm{Area}\cdot q\cdot\dfrac{n_i^2}{N_A}\cdot\dfrac{D_n}{L_n}\cdot\left(e^{\frac{V_d}{V_T}}-1\right)$

Usually drop this 1 in forward bias

For a total diode current of:

Diffusion constant for holes in n region $D_p := 12\cdot\dfrac{\mathrm{cm}^2}{\mathrm{s}}$ typical

Diffusion constant for electrons in p region $D_n := 34\cdot\dfrac{\mathrm{cm}^2}{\mathrm{s}}$ typical

Diode Saturation Current
(AKA scale current)

$$I_d = \underbrace{\mathrm{Area}\cdot q\cdot n_i^2\cdot\left(\frac{D_p}{L_p\cdot N_D}+\frac{D_n}{L_n\cdot N_A}\right)}_{I_S}\cdot\left(e^{\frac{V_d}{V_T}}-1\right)$$

diode voltage

Thermal voltage $= \dfrac{k\cdot T}{q} \simeq 25\cdot\mathrm{mV}$

Usually drop this 1 in forward bias

Junction area

p doping

n doping

Diffusion length of holes in n region

Diffusion length of electrons in p region

Diode Equation

diode voltage

Diode current: $I_d = I_s\cdot\left(e^{\frac{V_d}{n\cdot V_T}}-1\right)$

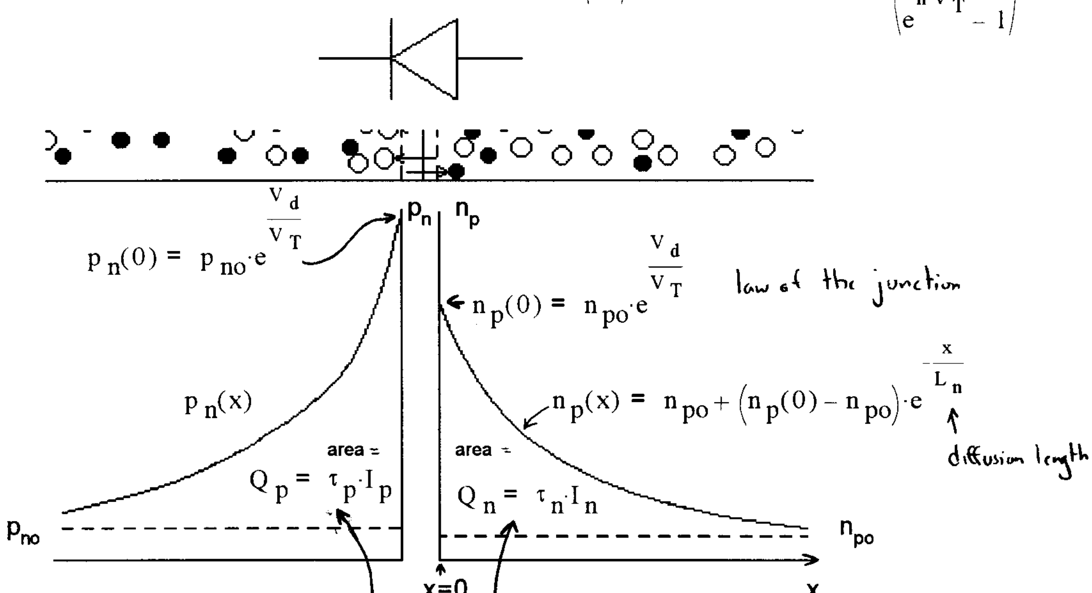
Usually drop this 1 in forward bias

Saturation current
(AKA scale current)

Thermal voltage $= \dfrac{k\cdot T}{q} \simeq 25\cdot\mathrm{mV}$

Fudge factor, assume n = 1 in ICs and n = 2 for discrete parts

Other permutations of the diode equation: $V_d = n\cdot V_T\cdot\ln\left(\dfrac{I_d}{I_s}\right)$ $\qquad I_s = \dfrac{I_d}{\left(e^{\frac{V_d}{n\cdot V_T}}-1\right)}$



Minority carrier charge storage

(excess minority charges waiting to be diffused and absorbed

Excess hole charge in the n region

Excess electron charge in the p region

The areas under the curves

Total excess minority charge: $Q = Q_n + Q_p$

Mean transit time: $\tau_T = \dfrac{Q}{I_d}$

Minority Carrier Lifetime (Average lifetime of an excess minority carrier)

Excess hole lifetime in n region: $\tau_p = \dfrac{L_p^2}{D_p} = \dfrac{\text{(Diffusion length of holes in n region)}^2}{\text{Diffusion constant for holes in n region}: D_p := 12 \cdot \frac{cm^2}{s} \text{ typical}}$

$L_p = \sqrt{D_p \cdot \tau_p}$

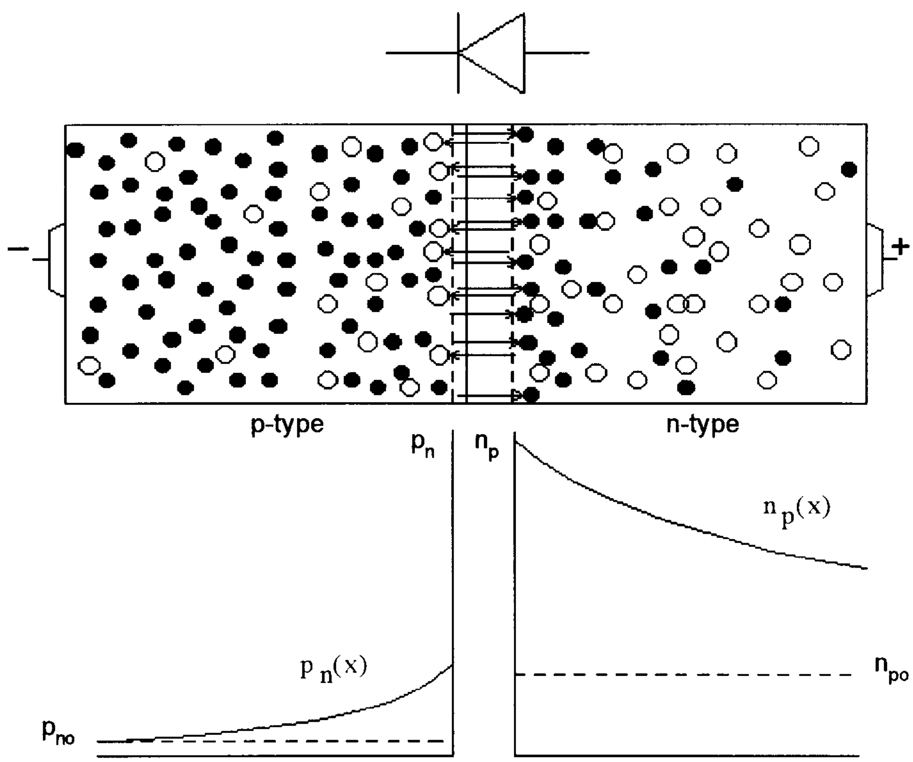
Excess electron lifetime in p region: $\tau_n = \dfrac{L_n^2}{D_n} = \dfrac{\text{(Diffusion length of electrons in p region)}^2}{\text{Diffusion constant for electrons in p region}: D_n := 34 \cdot \frac{cm^2}{s} \text{ typical}}$

$L_n = \sqrt{D_n \cdot \tau_n}$

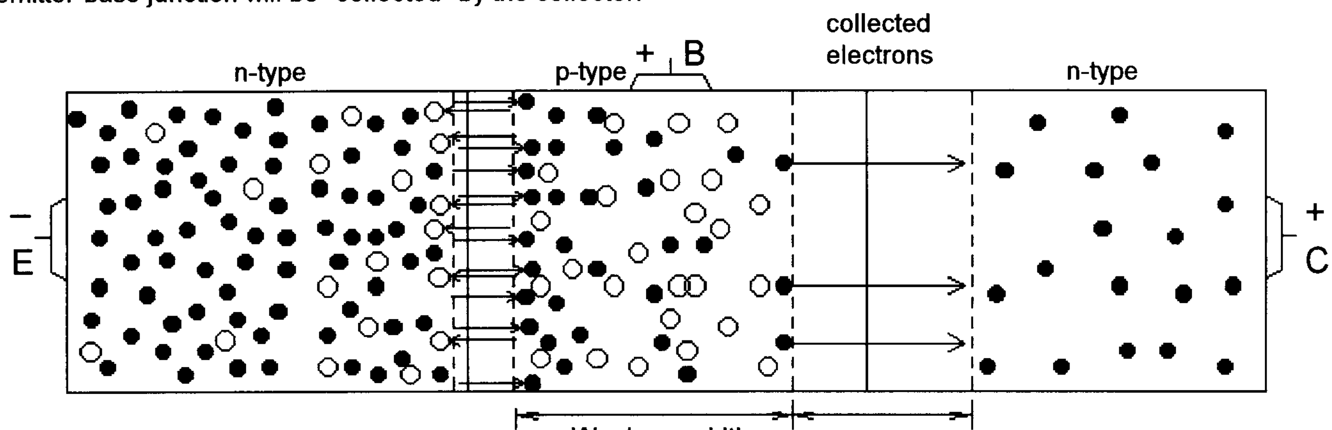
Diffusion Capacitance (due to Minority carrier charge storage) $C_d = \left(\dfrac{\tau_T}{V_T}\right) \cdot I_d$ diode current

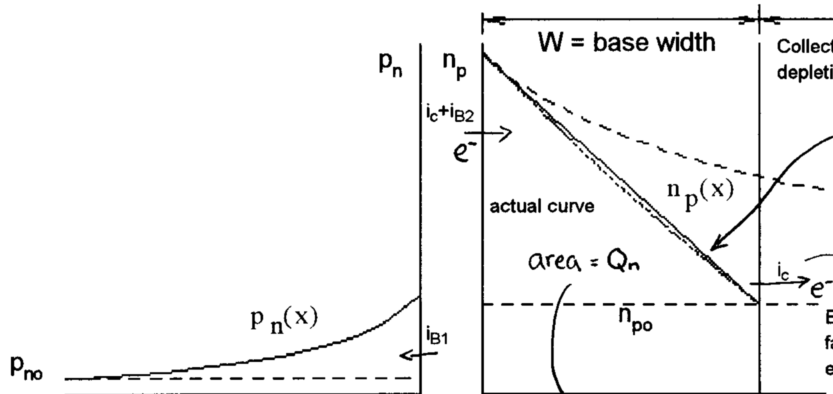
Transistors

Imagine a diode with a heavily doped p region and a lightly doped n region, like this:



Now imagine placing another diode junction very close to the first, most of the electrons that come across the emitter-base junction will be "collected" by the collector.





This slope leads to a diffusion current, nearly all of which flows to the collector.

$$i_C \simeq \frac{A_E \cdot q \cdot D_n}{W} \cdot \frac{n_i^2}{N_A} \cdot e^{\frac{v_{BE}}{V_T}}$$

Because all of the electrons that diffuse this far are swept into the collector, the free electron curve is pulled down to this line.

Hole-injection current is part of the base current

$$i_{B1} = \left(\frac{A_E \cdot q \cdot D_p}{L_p} \cdot \frac{n_i^2}{N_D} \right) \cdot e^{\frac{v_{BE}}{V_T}}$$

emitter hole diffusion constant

emitter doping

emitter hole diffusion length

Excess-minority charge stored in the base: $Q_n = \frac{1}{2} \cdot \left(A_E \cdot q \cdot W \cdot \frac{n_i^2}{N_A} \right) \cdot e^{\frac{v_{BE}}{V_T}}$

Divide Q_n by the average lifetime of the electrons in the base to get the rate that electrons are absorbed in the base, leading to the second part of the base current.

$$i_{B2} = \frac{Q_n}{\tau_b} = \frac{1}{2} \cdot \left(\frac{A_E \cdot q \cdot W}{\tau_b} \cdot \frac{n_i^2}{N_A} \right) \cdot e^{\frac{v_{BE}}{V_T}}$$

base width

base minority carrier lifetime

base-emitter junction effective area

base doping

Collector Current

Ebers-Moll equation: $i_C = I_S \cdot e^{\frac{v_{BE}}{V_T}}$ Saturation current (npn) $I_S = \frac{A_E \cdot q \cdot D_n \cdot n_i^2}{N_A \cdot W}$

$1.6\text{x}10^{-19}$ coul

base electron diffusion constant — $D_n := 34 \cdot \frac{cm^2}{s}$ typ

intrinsic carrier density

base width

base doping

Beta

$$\beta = \frac{i_c}{i_{B1} + i_{B2}} = \frac{1}{\frac{D_p}{D_n} \cdot \frac{N_A}{N_D} \cdot \frac{W}{L_p} + \frac{1}{2} \cdot \frac{W^2}{D_n \cdot \tau_b}}$$

emitter hole diffusion constant

base doping

base width

base minority carrier lifetime $= \frac{L_n^2}{D_n} = \frac{\text{(Diffusion length of electrons in p region)}^2}{\text{Diffusion constant for electrons in p region:}}$

$D_n := 34 \cdot \frac{cm^2}{s}$ typ

emitter hole diffusion length

emitter doping

base electron diffusion constant

β depends on the effective base width, W which depends on V_{CB}. This leads to the Early effect, which is expressed as an output resistance.

PNP Swap p and n in subscripts Swap N_A and N_D $\tau_b = \frac{L_p^2}{D_p} = \frac{\text{(Diffusion length of holes in n region)}^2}{\text{Diffusion constant for holes in n region}}$

$D_p := 12 \cdot \frac{cm^2}{s}$ typical