

EE2100 Lecture Notes 1/16/02

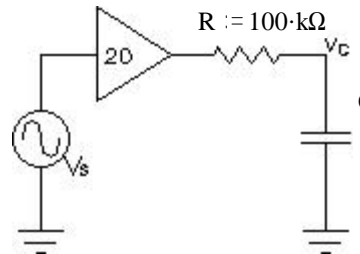
Lab 2 corrections

Two 100Ω & two more 100kΩ resistors not listed on parts list.

Both mic amp and IR receiver should show 100kΩ resistors hooked from noninverting input of opamp to ground.

Bode Plot Examples

1



$$\frac{v_C}{v_S} = 20 \cdot \frac{\frac{1}{j \cdot \omega \cdot C}}{\frac{1}{j \cdot \omega \cdot C} + R} = \frac{20}{1 + R \cdot (j \cdot \omega \cdot C)} = H(\omega)$$

Transfer function has one pole at $j\omega_c$
denominator = 0 at the pole

corner frequency is where real = imaginary (in denominator in this case)

$$1 = \omega_c \cdot R \cdot C$$

$$\omega_c := \frac{1}{R \cdot C}$$

$$\omega_c = 250 \cdot \frac{\text{rad}}{\text{sec}}$$

$$\text{So... } H(\omega) := \frac{20}{1 + j \cdot \frac{\omega}{250 \cdot \frac{\text{rad}}{\text{sec}}}}$$

$$\omega < \omega_c \quad H(\omega) \simeq \frac{20}{1} \quad |H(\omega)| \simeq 20 \quad 20 \cdot \log(20) = 26.021 \cdot \text{dB} \quad \text{angle} \simeq 0$$

$$\omega > \omega_c \quad H(\omega) \simeq \frac{20}{j \cdot \frac{\omega}{250 \cdot \frac{\text{rad}}{\text{sec}}}} \quad |H(\omega)| \simeq \frac{20}{\omega} \cdot \left(250 \cdot \frac{\text{rad}}{\text{sec}} \right) \quad \text{inversely proportional to } \omega \quad \text{angle} \simeq -90 \left(\frac{1}{j} \right)$$

Try some values:

$$20 \cdot \log \left[\frac{20}{\omega_c} \cdot \left(250 \cdot \frac{\text{rad}}{\text{sec}} \right) \right] = 26.02 \cdot \text{dB}$$

$$20 \cdot \log \left[\frac{20}{10 \cdot \omega_c} \cdot \left(250 \cdot \frac{\text{rad}}{\text{sec}} \right) \right] = 6.02 \cdot \text{dB}$$

$$20 \cdot \log \left[\frac{20}{100 \cdot \omega_c} \cdot \left(250 \cdot \frac{\text{rad}}{\text{sec}} \right) \right] = -13.98 \cdot \text{dB}$$

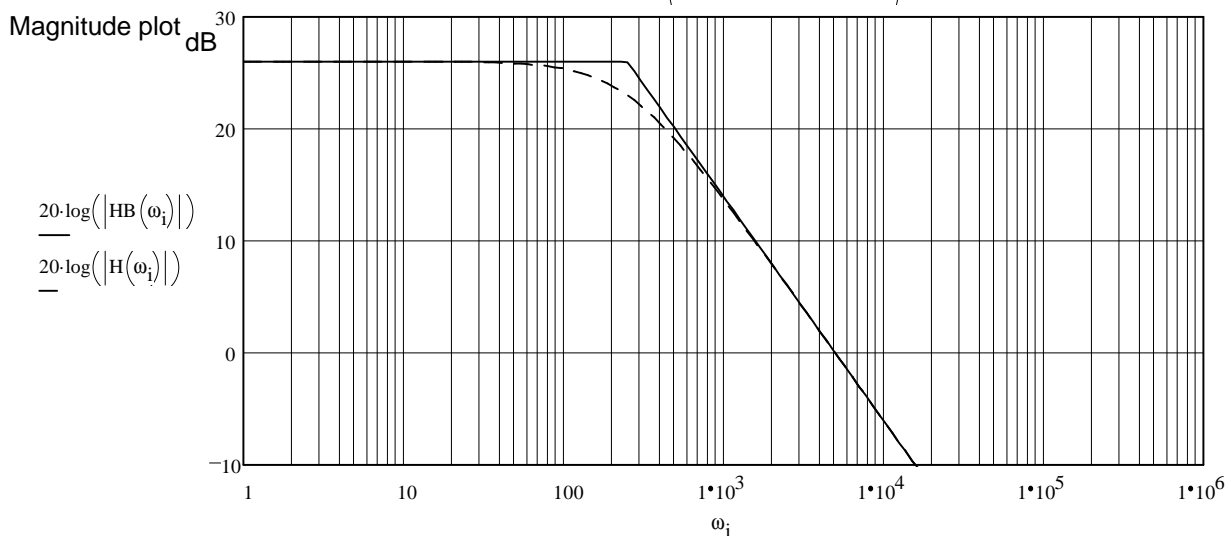
$$20 \cdot \log \left[\frac{20}{200 \cdot \omega_c} \cdot \left(250 \cdot \frac{\text{rad}}{\text{sec}} \right) \right] = -20 \cdot \text{dB}$$

$$\omega = \omega_c \quad H(\omega) = \frac{20}{1 + j \cdot \frac{\omega_c}{250 \cdot \frac{\text{rad}}{\text{sec}}}} = \frac{20}{1 + j \cdot 1} = 10 - 10j \quad |10 - 10j| = 14.142 \quad 20 \cdot \log(14.142) = 23.01 \cdot \text{dB}$$

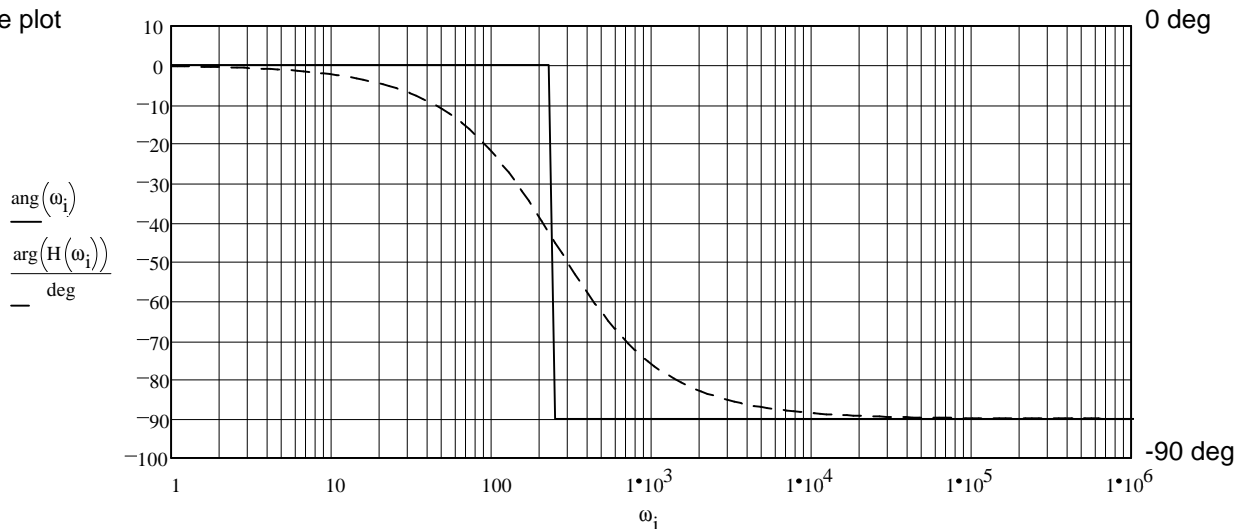
angle: $\arg(10 - 10j) = -45 \cdot \text{deg}$

Expressed in terms that Mathcad can understand, for plotting below:

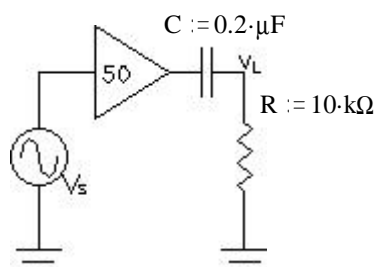
$$HB(\omega) := \text{if} \left(\omega \leq \omega_c, 20, \frac{5000 \cdot \frac{\text{rad}}{\text{sec}}}{\omega} \right) \quad \text{ang}(\omega) := \text{if}(\omega < \omega_c, 0, -90)$$



Phase angle plot



2



$$\frac{v_C}{v_S} = 50 \cdot \frac{R}{\frac{1}{j \cdot \omega \cdot C} + R} = \frac{50 \cdot (R \cdot (j \cdot \omega \cdot C))}{1 + R \cdot (j \cdot \omega \cdot C)} = H(\omega)$$

Transfer function has one pole at $j\omega_c$
denominator = 0 at the pole

corner frequency is where real = imaginary
 $1 = \omega_c \cdot R \cdot C$

$$\omega_c := \frac{1}{R \cdot C} \quad \omega_c = 500 \cdot \frac{\text{rad}}{\text{sec}}$$

$$\text{So... } H(\omega) := \frac{50 \cdot j \cdot \frac{\omega}{500 \cdot \frac{\text{rad}}{\text{sec}}}}{1 + j \cdot \frac{\omega}{500 \cdot \frac{\text{rad}}{\text{sec}}}} = \frac{50 \cdot j \cdot \omega}{500 \cdot \frac{\text{rad}}{\text{sec}} + j \cdot \omega}$$

$$\omega < \omega_c \quad H(\omega) \simeq \frac{0.1 \cdot \frac{\text{sec}}{\text{rad}} \cdot (j \cdot \omega)}{1} \quad |H(\omega)| \simeq 0.1 \cdot \frac{\text{sec}}{\text{rad}} \cdot \omega \quad \text{proportional to } \omega \quad \text{angle} \simeq 90$$

$$\text{Try some values: } 20 \cdot \log\left(0.1 \cdot \frac{\text{sec}}{\text{rad}} \cdot \omega_c\right) = 33.98 \cdot \text{dB} \quad 20 \cdot \log\left(0.1 \cdot \frac{\text{sec}}{\text{rad}} \cdot \frac{\omega_c}{10}\right) = 13.98 \cdot \text{dB}$$

$$20 \cdot \log\left(0.1 \cdot \frac{\text{sec}}{\text{rad}} \cdot \frac{\omega_c}{100}\right) = -6.02 \cdot \text{dB} \quad 20 \cdot \log\left(0.1 \cdot \frac{\text{sec}}{\text{rad}} \cdot \frac{\omega_c}{200}\right) = -12.041 \cdot \text{dB}$$

$$\omega > \omega_c \quad H(\omega) \simeq \frac{50 \cdot j \cdot \frac{\omega}{500 \cdot \frac{\text{rad}}{\text{sec}}}}{j \cdot \frac{\omega}{500 \cdot \frac{\text{rad}}{\text{sec}}}} \quad |H(\omega)| \simeq 50 \quad 20 \cdot \log(50) = 33.98 \cdot \text{dB} \quad \text{angle} \simeq 0$$

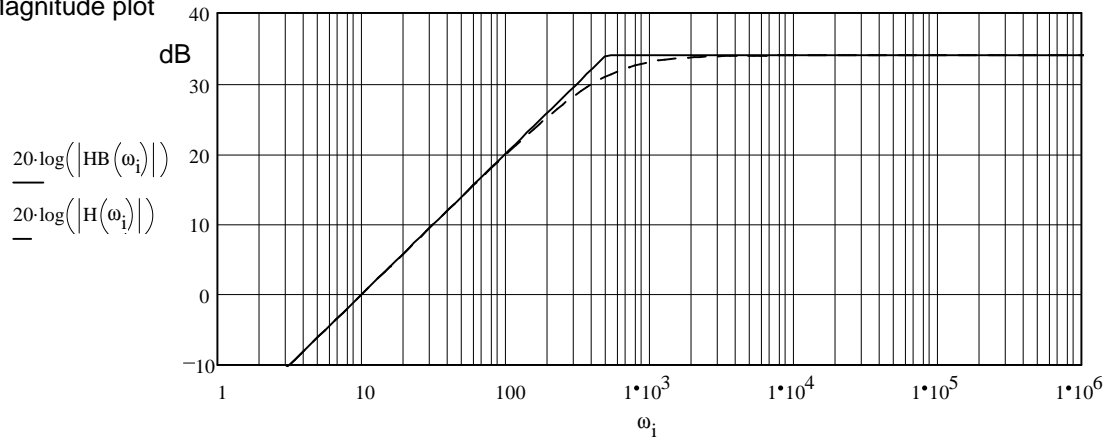
$$\omega = \omega_c \quad H(\omega) = \frac{50 \cdot j \cdot \omega}{500 \cdot \frac{\text{rad}}{\text{sec}} + j \cdot \omega} = \frac{50 \cdot j}{1 + j \cdot 1} = 25 + 25j \quad |25 + 25j| = 35.355 \quad 20 \cdot \log(35.355) = 30.97 \cdot \text{dB}$$

angle: $\arg(25 + 25j) = 45 \cdot \text{deg}$

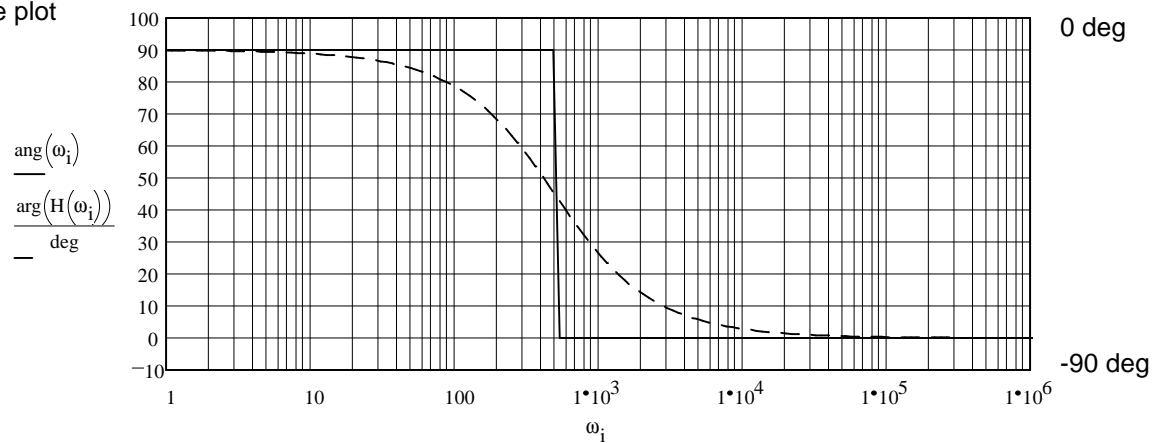
Expressed in terms that Mathcad can understand, for plotting:

$$HB(\omega) := \text{if}\left(\omega \leq \omega_c, 0.1 \cdot \frac{\text{sec}}{\text{rad}} \cdot \omega, 50\right) \quad \text{ang}(\omega) := \text{if}\left(\omega < \omega_c, 90, 0\right)$$

Magnitude plot



Phase angle plot



3 Or, the transfer function may already be worked out: $H(\omega) := \frac{50 \cdot \text{sec} \cdot j \cdot \omega}{100 + j \cdot 4 \cdot \text{sec} \cdot \omega}$

The real and imaginary parts of the denominator are equal at the corner frequency

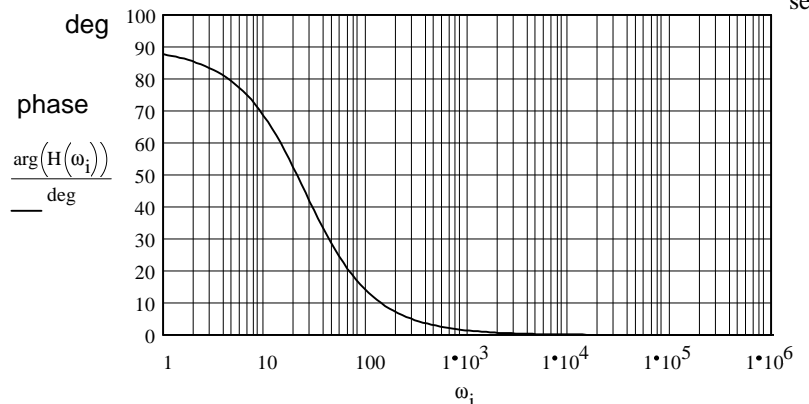
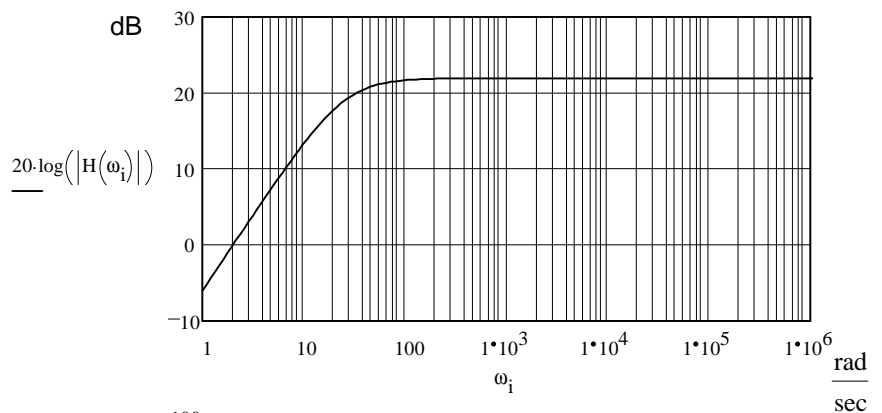
$$100 = j \cdot 4 \cdot \omega_c \quad \omega_c := 25 \cdot \frac{\text{rad}}{\text{sec}}$$

$$\omega < \omega_c \quad |H(\omega)| \simeq \left| \frac{50 \cdot j \cdot \omega}{100} \right| = 0.5 \cdot \omega$$

Proportional to frequency.

$$\omega > \omega_c \quad |H(\omega)| \simeq \left| \frac{50 \cdot j \cdot \omega}{100 + j \cdot 4 \cdot \omega} \right| = 12.5$$

$$20 \cdot \log(12.5) = 21.94 \cdot \text{dB}$$



4

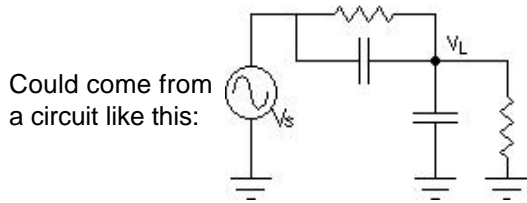
$$H(f) := 10^{-2} \cdot \frac{1 + j \cdot \frac{f}{10}}{1 + j \cdot \frac{f}{500}}$$

The real and imaginary parts of the numerator are equal at the one corner frequency (zero)

The real and imaginary parts of the denominator are equal at the other corner frequency (pole)

$$1 = j \cdot \frac{f_c}{10} \quad f_{c1} := 10 \cdot \text{Hz}$$

$$1 = j \cdot \frac{f_c}{500} \quad f_{c2} := 500 \cdot \text{Hz}$$



$$f < 10 \quad |H(f)| \simeq \left| 10^{-2} \cdot \frac{1}{1} \right| = 10^{-2}$$

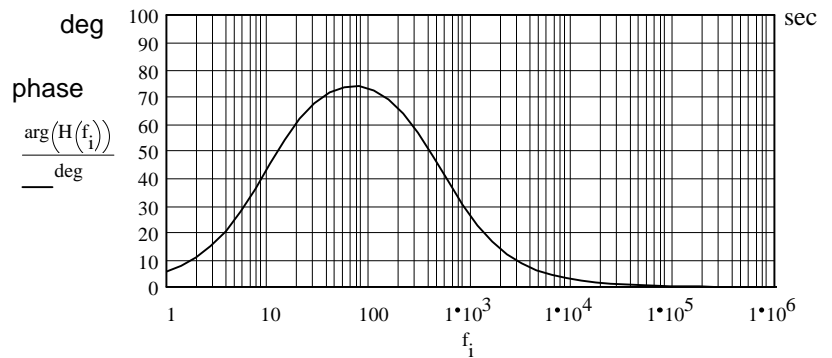
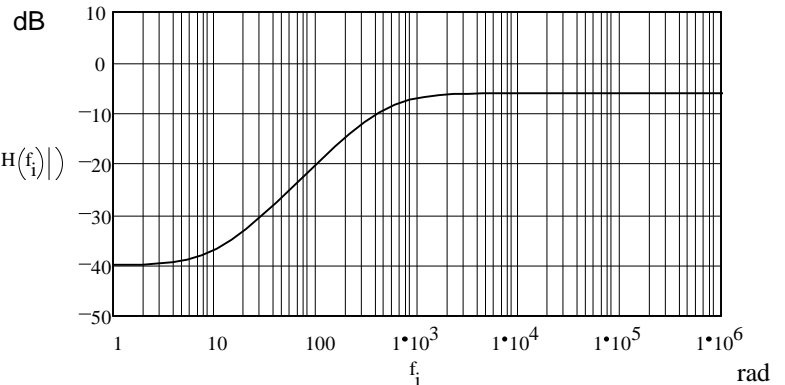
$$20 \cdot \log(10^{-2}) = -40 \cdot \text{dB}$$

$$10 < f < 500 \quad |H(f)| \simeq \left| 10^{-2} \cdot \frac{j \cdot \frac{f}{10}}{1} \right| = 10^{-3} \cdot f$$

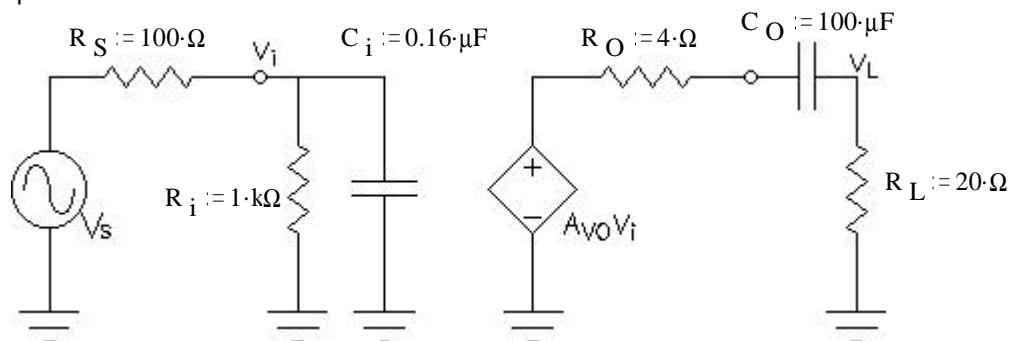
proportional to f

$$1000 < f \quad |H(f)| \simeq \left| 10^{-2} \cdot \frac{j \cdot \frac{f}{10}}{j \cdot \frac{f}{500}} \right| = 1$$

$$20 \cdot \log(1) = 0 \cdot \text{dB}$$



5 How about an amplifier circuit



$$Z_i = \frac{1}{\frac{1}{R_i} + j \cdot \omega \cdot C_i} = \frac{R_i}{1 + j \cdot \omega \cdot C_i \cdot R_i}$$

$$Z_O = R_O + \frac{1}{j \cdot \omega \cdot C_O}$$

$$\frac{v_o}{v_s} = \frac{Z_i}{R_S + Z_i} \cdot A_{Vo} \cdot \frac{R_L}{Z_O + R_L} = \frac{\frac{R_i}{1 + j \cdot \omega \cdot C_i \cdot R_i}}{R_S + \frac{R_i}{1 + j \cdot \omega \cdot C_i \cdot R_i}} \cdot A_{Vo} \cdot \frac{R_L}{R_O + \frac{1}{j \cdot \omega \cdot C_O} + R_L}$$

$$\frac{v_o}{v_s} = \frac{R_i}{R_s \cdot (1 + j\omega C_i R_i) + R_i} \cdot A_{Vo} \cdot \frac{R_L \cdot (j\omega C_O)}{(R_O + R_L) \cdot (j\omega C_O) + 1}$$

$$\frac{v_o}{v_s} = \frac{R_i}{(R_s + R_i) + j\omega C_i R_i R_s} \cdot A_{Vo} \cdot \frac{R_L \cdot (j\omega C_O)}{1 + j\omega C_O (R_O + R_L)}$$

$$\frac{v_o}{v_s} = \frac{j\omega C_O A_{Vo} R_i R_L}{[(R_s + R_i) + j\omega C_i R_i R_s] [1 + j\omega C_O (R_O + R_L)]} = \frac{\frac{j\omega C_O A_{Vo} R_i R_L}{(R_s + R_i)}}{\left[1 + \frac{j\omega C_i R_i R_s}{(R_s + R_i)}\right] [1 + j\omega C_O (R_O + R_L)]}$$

$$K = \frac{C_O A_{Vo} R_i R_L}{R_s + R_i} = 0.182 \cdot \text{sec} \quad \omega_{C1} = \frac{R_s + R_i}{C_i R_i R_s} = 6.875 \cdot 10^4 \cdot \frac{\text{rad}}{\text{sec}} \quad \omega_{C2} = \frac{1}{C_O (R_O + R_L)} = 416.7 \cdot \frac{\text{rad}}{\text{sec}}$$

$$A(\omega) := \frac{j\omega \cdot 0.182 \cdot \text{sec}}{\left(1 + \frac{j\omega}{6.875 \cdot 10^4 \cdot \frac{\text{rad}}{\text{sec}}}\right) \cdot \left(1 + \frac{j\omega}{416.7 \cdot \frac{\text{rad}}{\text{sec}}}\right)}$$

Between the two poles (midband): $A(\omega) \simeq \frac{j\omega_i \cdot 0.182}{(1) \cdot \left(\frac{j\omega_i}{416.67}\right)} = 75.834 \quad 20 \cdot \log(75.834) = 37.6$
 $\text{angle} \simeq 0$

Below ω_{C1} $A(\omega) \simeq \frac{j\omega \cdot 0.182}{(1) \cdot (1)}$
 proportional to ω
 $\text{angle} \simeq 90$

Above ω_{C2} $A(\omega) \simeq \frac{j\omega \cdot 0.182}{\left(\frac{j\omega}{6.875 \cdot 10^4}\right) \cdot \left(\frac{j\omega}{416.67}\right)}$
 inversely proportional to ω
 $\text{angle} \simeq -90$

