

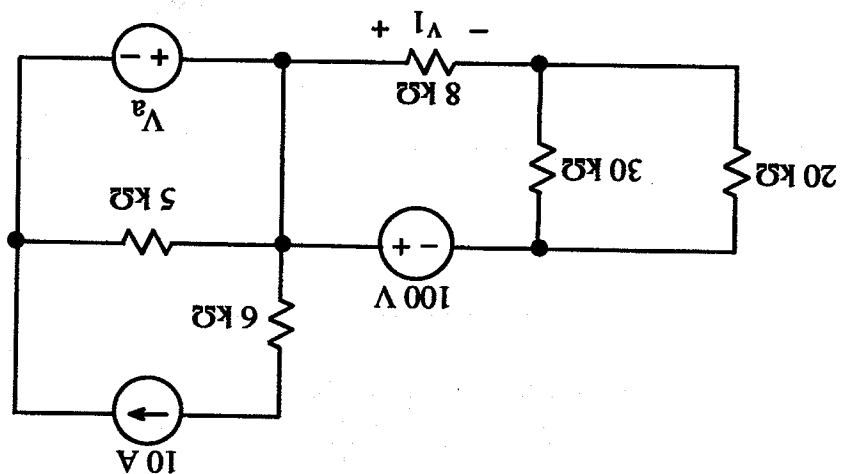
To pass the unit exam, you must be able to do the following (using books and notes):

| CONCEPTUAL TOOLS                                                                                                                                        | Learning Objective                                                                                                                                                                                                                                                               | Reading                |
|---------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|
| <b>CIRCUITS</b><br>BASIC DC CIRCUITS<br>Passive sign convention<br>EXAMPLE 1 (PDF)<br>EXAMPLE 2 (PDF)                                                   | 1.1 Consistently label directions of currents and polarity of potential differences in any given circuit (including independent voltage sources and independent current sources, and dependent sources).                                                                         | Chap 1<br>App A        |
| <b>CIRCUITS</b><br>BASIC DC CIRCUITS<br>Sources: v, i independ, depend<br>EXAMPLE 1 (PDF)                                                               | 1.2 Use the concepts of both independent and dependent voltage sources and current sources in designing and analyzing circuits.                                                                                                                                                  | Chap 2:<br>Sec 2.1     |
| <b>CIRCUITS</b><br>KIRCHHOFF'S LAWS<br>Current sums at nodes (pdf)<br>Voltage loops (pdf)<br>Writing v and i eqns<br>EXAMPLE 1 (PDF)<br>EXAMPLE 2 (PDF) | 1.3 Apply Kirchhoff's voltage and current laws to obtain equations relating voltages and currents in any given circuit.                                                                                                                                                          | Chap 2:<br>Sec 2.2     |
| <b>CIRCUITS</b><br>OHM'S LAW<br>Statement<br>Series resistors<br>Parallel resistors<br>Resistor networks<br>EXAMPLE 1 (PDF)                             | 1.4 Calculate and/or estimate the equivalent resistance (conductance) of a combination of resistances (conductances). Use equivalent resistances (conductances) to reduce circuits.                                                                                              | Chap 2:<br>Sec 2.3-2.4 |
| <b>ALGEBRA</b><br>SIMULTANEOUS EQUATIONS<br>Example (pdf)                                                                                               | 1.5 Solve linear simultaneous equations.                                                                                                                                                                                                                                         | -                      |
| <b>CIRCUITS</b><br>KIRCHHOFF'S LAWS<br>Solving circuits<br>EXAMPLE 1 (PDF)<br>EXAMPLE 2 (PDF)                                                           | 1.6 Apply Kirchhoff's and Ohm's laws to linear resistive circuits, solve for specified voltages and currents, and design circuits to given specifications, making consistency checks when appropriate. Apply Kirchhoff's and Ohm's laws qualitatively as well as quantitatively. | Chap 2:<br>Sec 2.4-2.5 |
| <b>CIRCUITS</b><br>OHM'S LAW<br>Statement<br>Series resistors<br>Parallel resistors<br>Resistor networks<br>EXAMPLE 1 (PDF)                             | 1.7 Calculate and/or estimate the equivalent resistance (conductance) of a combination of resistances (conductances) to reduce circuits.                                                                                                                                         | Chap 3:<br>Sec 3.1-3.2 |

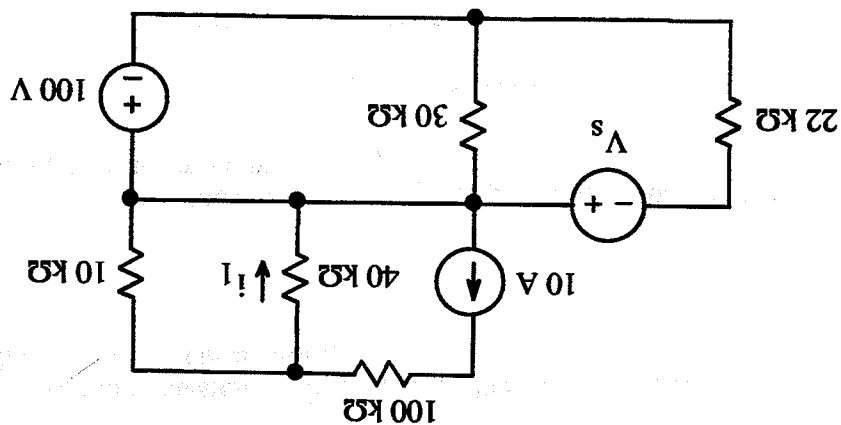
\* The material in this handout is based extensively on concepts developed by C. H. Durney, Professor Emeritus of the University of Utah.

|                                                                                              |                                                                                                                                |                                              |
|----------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------|
| <b>CIRCUITS</b><br><b>VANDIVIDERS</b><br>Voltage divider<br>Current divider<br>Example (pdf) | <b>OP-AMPS</b><br><b>GOLDEN RULES</b><br>$V_+ = V_-$<br>Zero input current<br><b>EXAMPLE 1 (PDF)</b><br><b>EXAMPLE 2 (PDF)</b> | <b>CIRCUITS</b><br><b>CONSISTENCY CHECKS</b> |
| 1.8 Apply voltage divider and current divider relations.<br>Chap 3:<br>Sec 3.3-3.4           | 1.9 Analyze and design elementary op-amp circuits.<br>Chap 5                                                                   | 1.10 Make consistency checks.                |
|                                                                                              |                                                                                                                                |                                              |

1. a. (5 points)  
Calculate  $v_1$ .

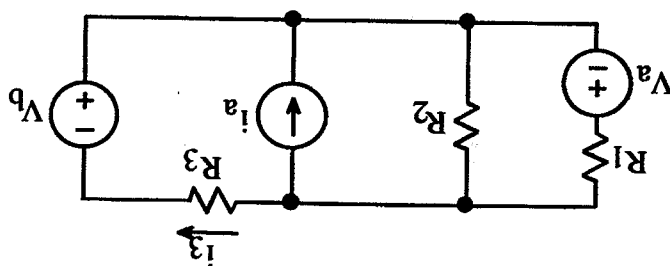


- b. (5 points)  
Calculate  $i_1$ .



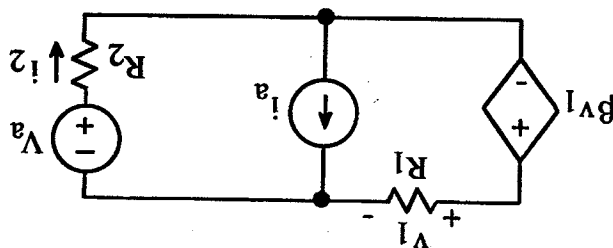
2. (30 points)

Derive an expression for  $i_3$ . The expression must not contain more than the circuit parameters  $V_a$ ,  $V_b$ ,  $i_a$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .



3. (30 points)

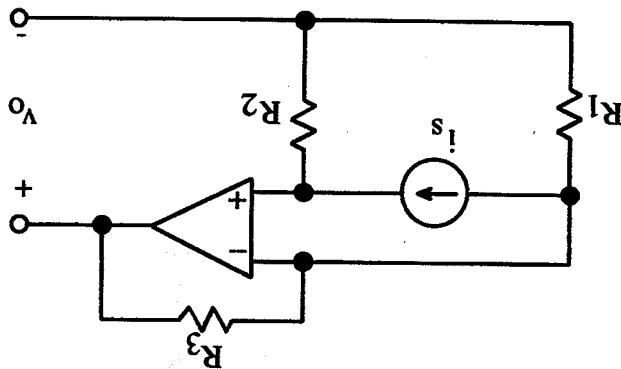
a. Derive an expression for  $i_2$ . The expression must not contain more than the circuit parameters  $B$ ,  $V_a$ ,  $i_a$ ,  $R_1$ , and  $R_2$ .



b. Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

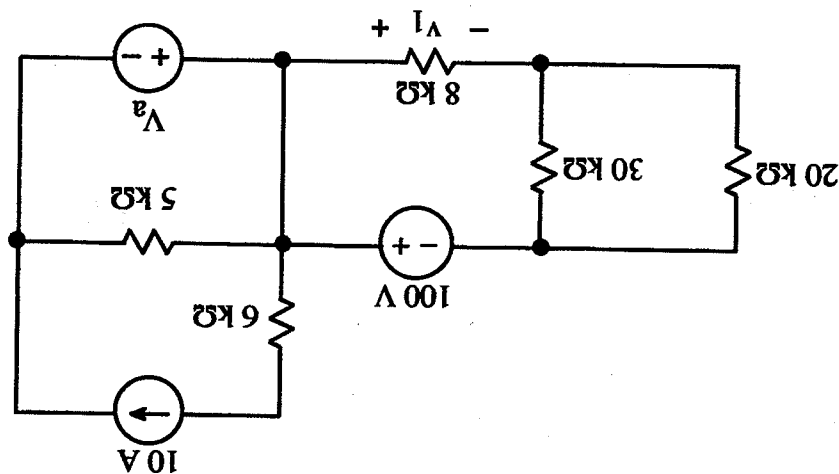
4. (30 points)

The op amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for  $V_o$  in terms of  $i_s$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .



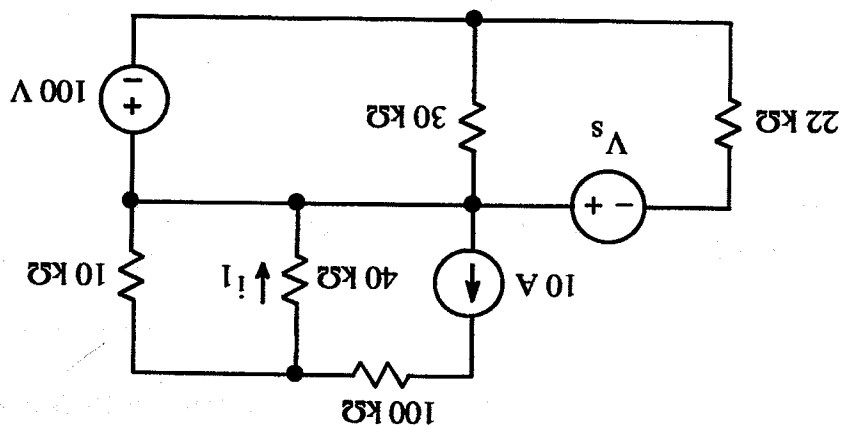
1. a. (5 points)

Calculate  $V_1$ .



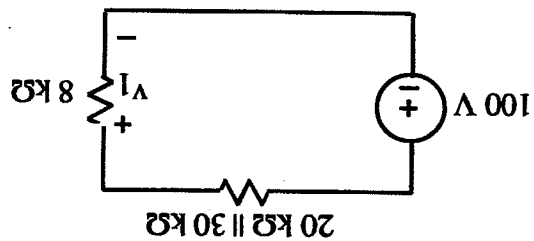
b. (5 points)

Calculate  $i_1$ .



ans: a) 40 V  
b) 2 A

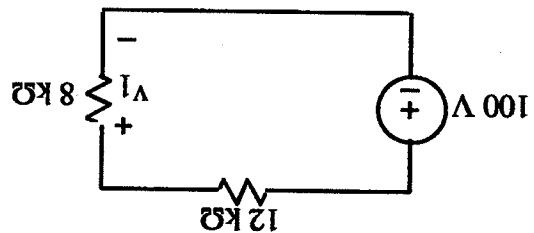
sol'n: a) The 100 V source is directly across  $20\text{ k}\Omega \parallel 30\text{ k}\Omega$  in series with  $8\text{ k}\Omega$ . Thus, the rest of the circuit is irrelevant in the calculation of  $V_1$ .



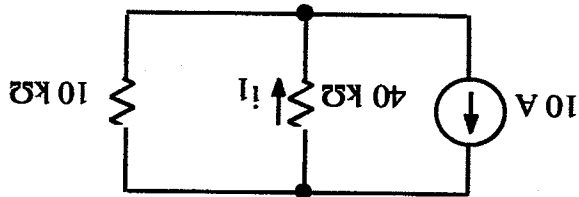
$$20\text{ k}\Omega \parallel 30\text{ k}\Omega = 10\text{ k}\Omega \cdot 2 \parallel 3 = 10\text{ k}\Omega \cdot \frac{2 \cdot 3}{2+3} = 10\text{ k}\Omega \cdot \frac{6}{5} = 12\text{ k}\Omega$$

Now we have a voltage divider.

$$V_1 = 100\text{ V} \cdot \frac{8\text{ k}\Omega}{8\text{ k}\Omega + 12\text{ k}\Omega} = 40\text{ V}$$



(b) The 10 A source current is in series with  $40\text{ k}\Omega \parallel 10\text{ k}\Omega$ . Thus, all of the 10 A must flow through the  $40\text{ k}\Omega \parallel 10\text{ k}\Omega$ , and the rest of the circuit is irrelevant in the calculation of  $I_1$ . We use the current divider formula, and we may ignore the  $100\text{ k}\Omega$  resistor.

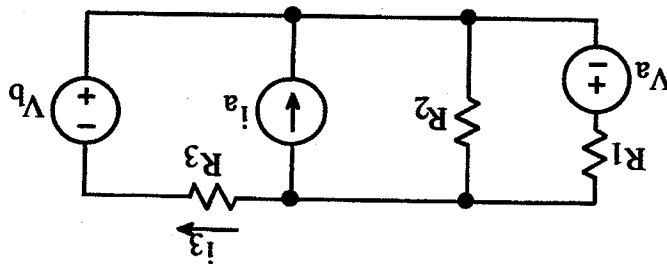


$$I_1 = 10\text{ A} \cdot \frac{10\text{ k}\Omega}{10\text{ k}\Omega + 40\text{ k}\Omega} = 2\text{ A}$$

UNIT 1  
PRACTICE EXAM SOLUTION Prob 2

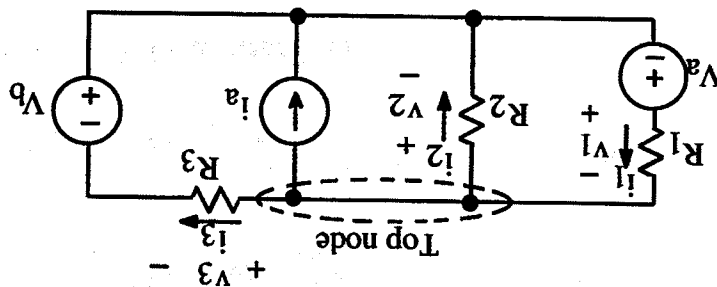
2. (30 points)

Derive an expression for  $i_3$ . The expression must not contain more than the circuit parameters  $V_a$ ,  $V_b$ ,  $i_a$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .



ans: 
$$i_3 = \frac{V_a R_2 + V_b (R_1 + R_2) - i_a R_1 R_2}{R_1 R_2 + R_3 (R_1 + R_2)}$$

sol'n: Using passive sign convention, label voltage drop and current measurement polarities.



Use Kirchhoff's laws:

sum  $v$  drops around loop = 0  
sum  $i$  out of node = 0

$v$  drops for loop on left, using Ohm's law for  $v_1$  and  $v_2$ :

$$V_a - i_1 R_1 - i_2 R_2 = 0 \text{ V}$$

Middle loop would include current source, so use slightly larger loop with  $R_2$  on left and  $V_b$  on right:

$$i_2 R_2 - i_3 R_3 + V_b = 0 \text{ V}$$

Now sum currents out of top node (that consists of the two top nodes connected by a wire). Note: We are always allowed to combine nodes connected by wires.

$$-i_1 + i_2 + i_3 = 0 \text{ A}$$

We now have three equations in three unknowns. We solve for  $i_3$ . Use the second equation to eliminate  $i_2$ :

$$i_2 = \frac{i_3 R_3 - V_b}{R_2}$$

Use the first equation to eliminate  $i_1$ :

$$i_1 = \frac{V_a - i_2 R_2}{R_1} = \frac{R_1}{1} \left[ V_a - \left( \frac{i_3 R_3 - V_b}{R_2} \right) R_2 \right] = \frac{R_1}{1} (V_a + V_b - i_3 R_3)$$

Substitute for  $i_1$  and  $i_2$  in the third equation:

$$-\frac{R_1}{1} (V_a + V_b - i_3 R_3) + \frac{i_3 R_3 - V_b}{R_2} + i_3 + i_3 = 0 \text{ A}$$

Solve for  $i_3$ :

$$-\frac{R_1}{1} (V_a + V_b) - \frac{R_2}{1} + i_3 R_3 + \frac{R_1}{1} i_3 R_3 + \frac{i_3 R_3}{R_2} + i_3 = 0 \text{ A}$$

$$i_3 \left( \frac{R_1}{R_3} + \frac{R_2}{R_3} + 1 \right) = \frac{R_1}{1} (V_a + V_b) + \frac{R_2}{V_b} - i_a$$

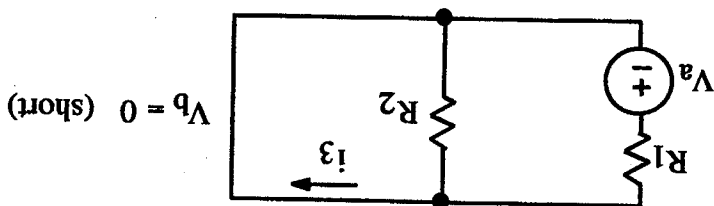
Multiply both sides by  $R_1 R_2$  to clear fractions:

$$i_3 (R_3 R_2 + R_3 R_1 + R_1 R_2) = R_2 (V_a + V_b) + R_1 V_b - i_a R_1 R_2$$

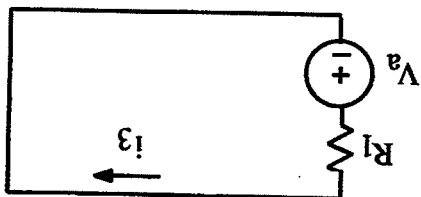
or

$$i_3 = \frac{V_a R_2 + V_b (R_1 + R_2) - i_a R_1 R_2}{R_1 R_2 + R_3 (R_1 + R_2)}$$

1) Now for consistency checks to verify our answer. (Optional)  
Consider  $i_a = 0$ ,  $V_b = 0$ , and  $R_3 = 0$ :



Since  $R_2$  is bypassed by a short, no current flows in  $R_2$ . Therefore, we can remove  $R_2$  without changing  $i_3$ :



$i_3 = \frac{V_a}{R_1}$  by Ohm's law

Our formula gives  $i_3 = \frac{V_a}{R_1}$ .

2) Consider  $i_a = 0$  (open circuit) and  $R_2 = \infty$  (open circuit):

Removing  $R_2$  and  $i_a$  leaves total voltage  $V_a + V_b$  across  $R_1 + R_3$  in outside loop.

Therefore, we have

$$i_3 = \frac{(V_a + V_b)}{(R_1 + R_3)}$$

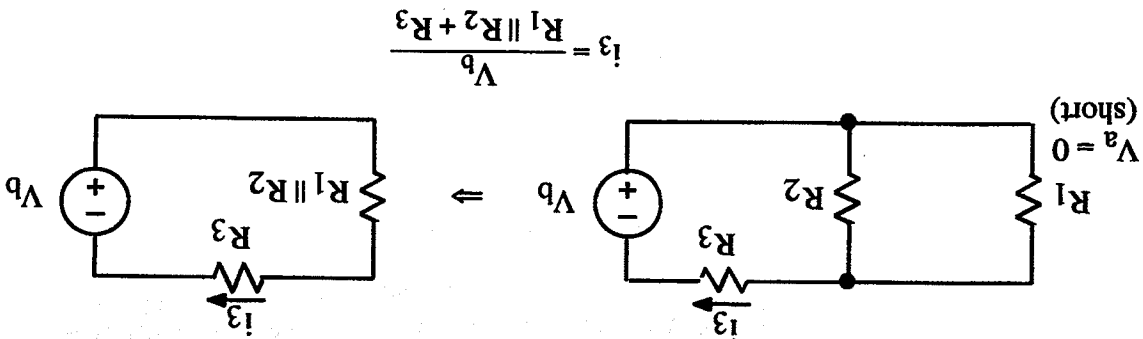
For our formula, we use the following identities:

$$\lim_{R_2 \rightarrow \infty} \frac{R_2}{R_1 R_2 + R_3 (R_1 + R_2)} = \frac{R_1 + R_3}{1} \quad \lim_{R_2 \rightarrow \infty} \frac{R_1 R_2 + R_3 (R_1 + R_2)}{R_1 + R_2} = \frac{R_1 + R_3}{1}$$

Making these substitutions in our formula gives

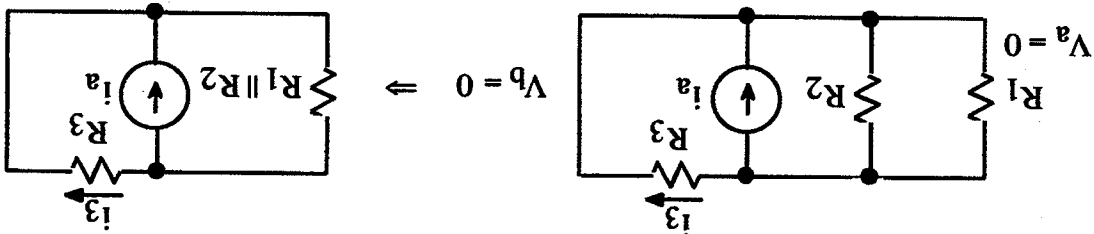
$$i_3 = \frac{(V_a + V_b)}{(R_1 + R_3)}$$

(3) Consider  $V_a = 0$ ,  $i_a = 0$ :



Our formula gives  $i_3 = \frac{V_b(R_1 + R_2)}{R_1 R_2 + R_3(R_1 + R_2)}$  or  $i_3 = \frac{R_1 \parallel R_2 + R_3}{V_b}$

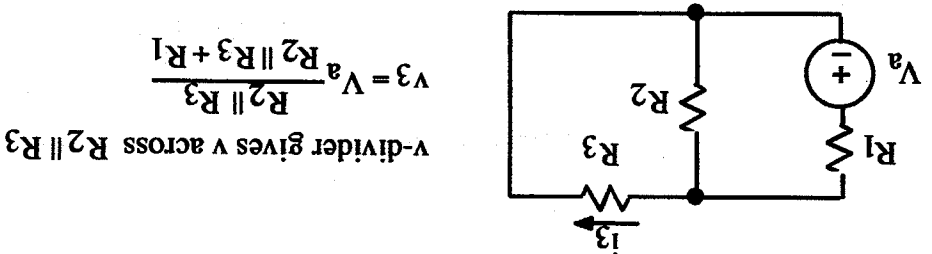
(4) Consider  $V_a = 0, V_b = 0$ :



By current divider formula, we have  $i_3 = -\frac{i_a R_1 \parallel R_2}{R_1 \parallel R_2 + R_3}$

Our formula gives  $i_3 = -\frac{i_a R_1 R_2}{R_1 R_2 + R_3(R_1 + R_2)}$  or  $i_3 = -\frac{i_a R_1 \parallel R_2}{R_1 \parallel R_2 + R_3}$

(5) Consider  $i_a = 0, V_b = 0$ .

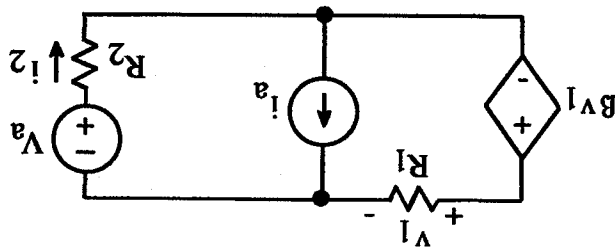


Now 
$$i_3 = \frac{V_3}{R_3} = \frac{V_a R_2 \parallel R_3}{R_3(R_1 + R_2 \parallel R_3)} = \frac{V_a \frac{R_2 R_3}{R_2 + R_3}}{R_3 \left( R_1 \frac{R_2 + R_3}{R_2 + R_3} + \frac{R_2 R_3}{R_2 + R_3} \right)} = \frac{V_a R_2}{R_1(R_2 + R_3) + R_2 R_3}$$

Our formula gives  $i_3 = \frac{V_a R_2}{R_1 R_2 + R_3(R_1 + R_2)}$

3. (30 points)

- a. Derive an expression for  $i_2$ . The expression must not contain more than the circuit parameters  $\beta$ ,  $V_a$ ,  $i_a$ ,  $R_1$ , and  $R_2$ .

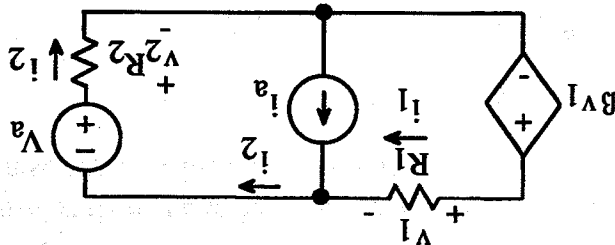


- b. Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

ans: a)  $i_3 = \frac{V_a + V_b(R_1 + R_2) + i_a R_1 R_2}{R_1 R_2 + R_3(R_1 + R_2)}$

- b) Many possible answers. See solution below.

sol'n: (a) Use Kirchhoff's laws to write several equations. Then eliminate unwanted variables.



We sum currents out of top-center node:

$$-i_1 - i_a + i_2 = 0$$

Note that summing currents out of bottom-center node does not give us anything new. By Ohm's law, we also have

$$i_1 = \frac{V_1}{R_1}$$

Now, we sum voltages around a loop. We choose the outer loop because the inner loops have a current source with unknown voltage drop.

$$\beta v_1 - v_1 + V_a - v_2 = 0 \quad \text{or} \quad (\beta - 1)v_1 + V_a - v_2 = 0$$

By Ohm's law, we also have

$$v_2 = i_2 R_2$$

After the Ohm's law substitutions, we have two equations, and we may eliminate  $v_1$ .

Use the simpler equation first:

$$\frac{v_1}{R_1} + i_a - i_2 = 0 \quad \text{or} \quad v_1 = (i_2 - i_a) R_1$$

Substitute for  $v_1$  in the second equation:

$$(\beta - 1)R_1(i_2 - i_a) + V_a - i_2 R_2 = 0$$

After some algebra, we get

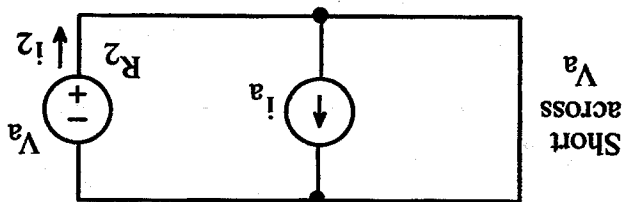
$$i_2 = \frac{(1 - \beta)i_a R_1 + V_a}{(1 - \beta)R_1 + R_2} \quad \text{units consistent}$$

(b)

There are many possible consistency checks.

- 1)  $i_a = 0$  and  $R_1 = 0$ . Then  $v_1 = 0$ ,  $\beta v_1 = 0$ , and sum  $v$ 's around outer loop gives  $i_2 = V_a / R_2$ . Our formula also gives  $V_a / R_2$ .

- 2) Consider  $R_1 = 0$  and  $R_2 = 0$ . As in (1),  $v_1 = 0$  and  $\beta v_1 = 0$ . Since  $R_2 = 0$  we also end up with a short across  $V_a$ :

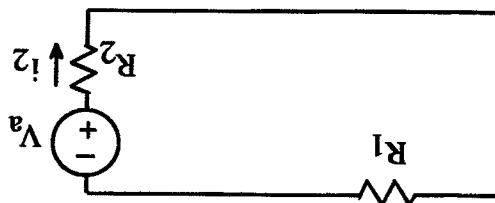


We expect  $i_2 \rightarrow \infty$  for short across  $V_a$

Our formula gives

$$\lim_{R_2 \rightarrow 0} \frac{V_a}{R_2} = \infty \quad \text{from (1) for } i_2$$

3) Consider  $i_a = 0, \beta = 0$ :

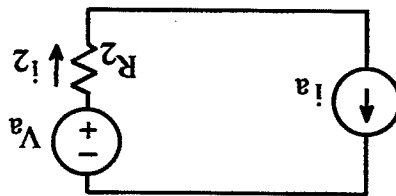


$$i_2 = \frac{V_a}{R_1 + R_2} \text{ by Ohm's law}$$

Our formula gives

$$i_2 = \frac{V_a}{R_1 + R_2}$$

4) Consider  $R_1 \rightarrow \infty$  (open circuit)



Clearly  $i_a = i_2$  since the same current flows through elements in series.

Our formula gives:

$$i_2 = \lim_{R_1 \rightarrow \infty} \frac{(1-\beta)i_a R_1 + V_a}{(1-\beta)i_a R_1} = \lim_{R_1 \rightarrow \infty} \frac{(1-\beta)R_1 + R_2}{(1-\beta)R_1}$$

$$= i_a$$

5) Consider  $R_2 \rightarrow \infty$  (open circuit): We have  $i_2 = 0$  since no current flows through the open circuit. Our formula gives:

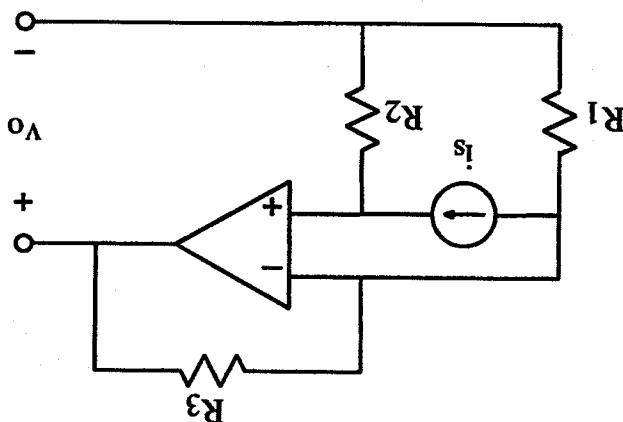
$$i_2 = \lim_{R_2 \rightarrow \infty} \frac{(1-\beta)i_a R_1 + V_a}{(1-\beta)R_1 + R_2} = \frac{\text{const}}{\infty} = 0$$

Many more consistency checks are possible.



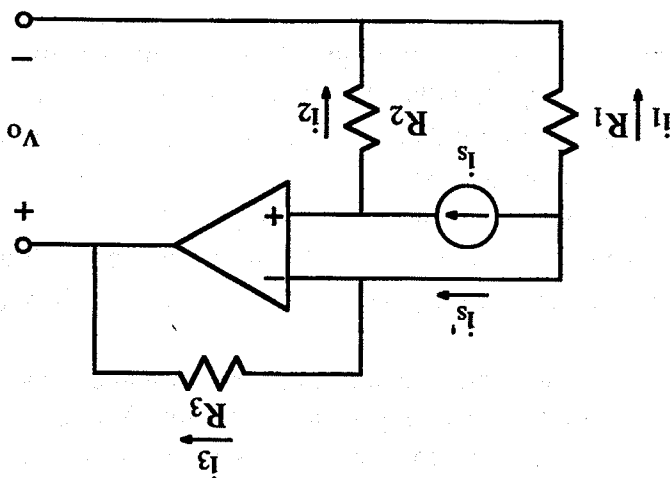
4. (30 points)

The op amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for  $v_o$  in terms of  $i_s$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .



ans:  $v_o = i_s(R_2 + R_3 + \frac{R_2 R_3}{R_1})$

sol'n: Using passive sign convention, label voltage drop and current measurement polarities.



Assume an ideal op-amp since we are in linear mode and we are solving for DC conditions.

Use the standard procedure to find  $v_o$ :

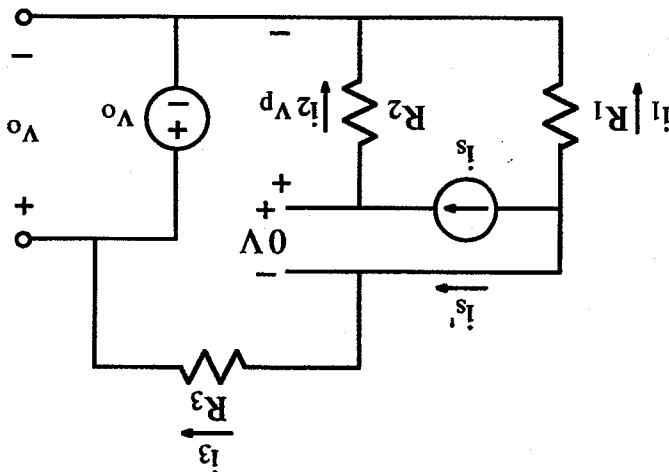
1) Calculate the voltage drop,  $v_p$ , from the - input of the op amp to the bottom wire, (i.e. across  $R_2$ ). We use Kirchhoff's law for summation of currents out of the node next to the + terminal of the op-amp. Since  $i_p = 0$ , (i.e., no current flows into the + terminal), we conclude that

$$i_2 = i_s.$$

By Ohm's law, it follows that

$$v_p = i_s R_2$$

2) Remove the op amp, put a source called  $v_o$  across the terminals where we measure the output, and assume a 0V drop from + input to - input of op amp. (Because of negative feedback, the op-amp output voltage,  $v_o$ , reaches an equilibrium level that results in a nearly 0V drop from + input to - input.) Now we have the following model circuit:



Note that we model the - terminal of the op-amp as a voltage source with zero current flowing into it. We add this constraint on the current with the understanding that  $v_o$  must have a value that results in  $i_n = 0$ .

3) Calculate  $i_s$ , the current flowing toward the - terminal from the left. Note that current  $i_s'$  has a name similar to the current source only by coincidence.) Now we observe that the voltage drop from the - terminal of the op amp to the bottom wire is  $v_p - 0V = v_p$ . It is as though we have a voltage source of voltage  $v_p$  from the - terminal of the op amp to the bottom wire. This voltage source separates the two sides of the circuit. What happens on the right side of the circuit will not affect the value of  $i_s'$ . Thus, we utilize the left side of the circuit to calculate  $i_s$ .

We use a summation of node currents at the node above  $R_1$  to calculate the value of  $i_s$ :

$$i_s' + i_1 + i_s = 0 \quad \text{or} \quad i_s' = -(i_1 + i_s)$$

But we can also calculate  $i_1$  because we have  $v_n$  across  $R_1$ :

$$i_1 = v_p / R_1 = i_s R_2 / R_1$$

We can now solve for  $i_s'$ :

$$i_s' = i_s (R_2 / R_1 + 1) = i_s (1 + R_2 / R_1)$$

4) Calculate  $i_3$ , the current flowing in the negative feedback path on the right side of the circuit. Again, it is as though we had a voltage source of voltage  $v_p$  from the - terminal of the op amp to the bottom wire. We utilize this voltage source and a voltage loop around the right side of the circuit to calculate  $v_o$ :

$$v_p - i_3 R_3 - v_o = 0 \quad \text{or} \quad i_3 = (v_o - v_p) / R_3 = (v_o - i_s R_2) / R_3$$

5) Since  $i_n = 0$ , set  $i_s' = i_3$ .

$$i_s' = i_s (1 + R_2 / R_1) = i_3 = (v_o - i_s R_2) / R_3$$

Equate the 2nd and 4th expressions and solve for  $v_o$ .

$$i_s (1 + R_2 / R_1) = (v_o - i_s R_2) / R_3 \quad \text{or} \quad v_o = i_s R_3 (1 + R_2 / R_1) + i_s R_2 \quad \text{or}$$

$$v_o = i_s (R_2 + R_3 + R_2 R_3 / R_1)$$

Consistency checks:

- 1) By linearity and fact that we have one source,  $i_s$ , we expect  $v_o$  to be directly proportional to  $i_s$ . Agrees with our formula for  $v_o$ . ✓
- 2) Consider  $R_2 = 0$ : Then  $v_p = 0$ . Therefore,  $i_1 = 0$  and  $i_3 = -i_s$ . Then,  $v_o = -i_3 R_3 = i_s R_3$ . Agrees with our formula for  $v_o$  when we plug in  $R_2 = 0$ . ✓
- 3) Consider  $R_1 = \infty$  (open circuit):  $v_p = i_s R_2$ ,  $i_3 = -i_s$ , and  $v_o = v_n - i_3 R_3 = i_s R_2 + i_s R_3$ . Agrees with our formula for  $v_o$  when we plug in  $R_1 = \infty$ . ✓
- 4) Consider  $R_3 = 0$ : Then  $v_o = v_n = v_p = i_s R_2$ . Agrees with our formula for  $v_o$  when we plug in  $R_3 = 0$ . ✓

To pass the unit exam, you must be able to do the following (using books and notes):

| CONCEPTUAL TOOLS |                                                                                                                         | Learning Objective |                                                                                                                                               | Reading                  |
|------------------|-------------------------------------------------------------------------------------------------------------------------|--------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|--------------------------|
| CIRCUITS         | BASIC DC QUANTITIES<br>Power                                                                                            | 2.1                | Calculate power dissipated by circuit elements.                                                                                               | Chap 1:<br>Sec 1.6       |
|                  | CIRCUITS<br>NODE-VOLTAGE METHOD<br>Example 1 (pdf)<br>Example 2 (pdf)<br>Example 3 (pdf) Matlab (x1)<br>Example 4 (pdf) | 2.2                | Use the node-voltage method in analyzing and designing circuits.                                                                              | Chap 4:<br>Sec 4.1-4.4   |
|                  | CIRCUITS<br>MESH-CURRENT METHOD<br>Example 1 (pdf)<br>Example 2 (pdf)<br>Example 3 (pdf)                                | 2.3                | Use the method of mesh currents in analyzing and designing circuits.                                                                          | Chap 4:<br>Sec 4.5-4.8   |
|                  | CIRCUITS<br>THEVENIN EQUIVALENT<br>Thevenin->Norton xform<br>EXAMPLE (pdf)                                              | 2.4                | Transform current sources to voltage sources and vice versa and take advantage of these transformations in analyzing and designing circuits.  | Chap 4:<br>Sec 4.9       |
|                  | CIRCUITS<br>THEVENIN EQUIVALENT<br>Example (pdf)                                                                        | 2.5                | Apply Thevenin's theorem and construct a Thevenin's model for a given circuit. Use Thevenin's models to find specified voltages and currents. | Chap 4:<br>Sec 4.10-4.11 |

\* The material in this handout is based extensively on concepts developed by C. H. Durney, Professor Emeritus of the University of Utah.

1. The first part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom.

2. The second part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom.

3. The third part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom.

4. The fourth part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom.

5. The fifth part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom.

6. The sixth part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom.

7. The seventh part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom.

8. The eighth part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom.

9. The ninth part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom.

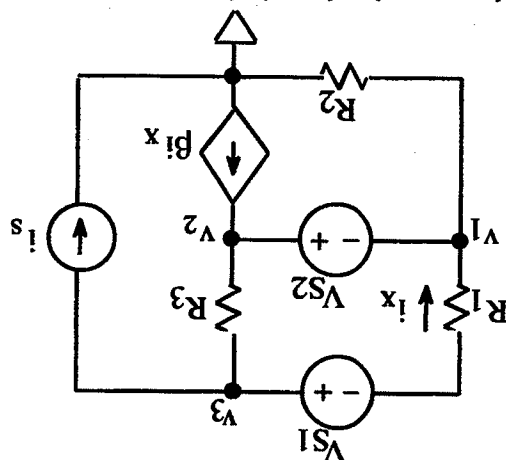
10. The tenth part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom.

11. The eleventh part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom.

UNIT 2  
PRACTICE EXAM



1. (a) (10 points)

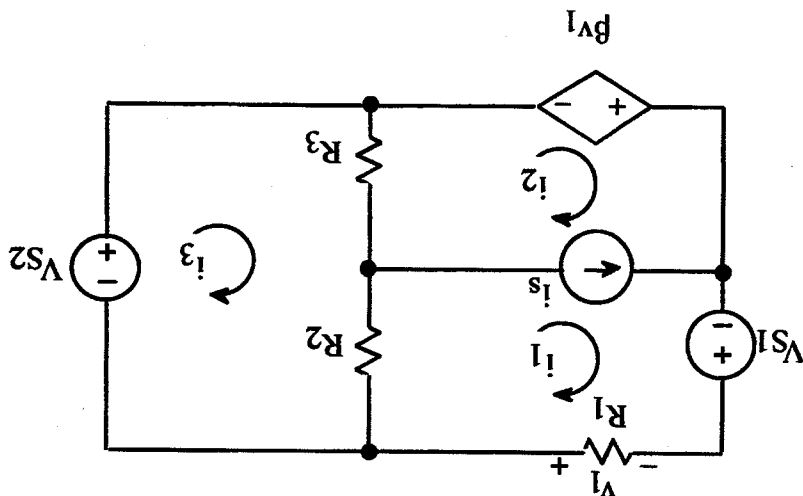


For the circuit shown, write three independent equations for the node voltages  $v_1$ ,  $v_2$ , and  $v_3$ . The quantity  $i_x$  must not appear in the equations.

(b) (10 points)

Make a consistency check on your equations by setting one or more resistor values to 0 or  $\infty$  and setting other sources and resistor to values for which  $v_1$ ,  $v_2$ , and  $v_3$  are obvious.

(c) (10 points)

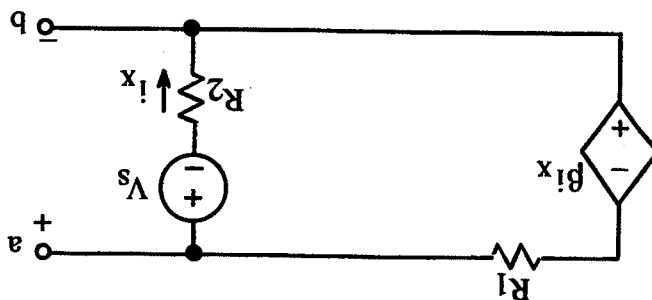


For the circuit shown, write three independent equations for the three mesh currents  $i_1$ ,  $i_2$ , and  $i_3$ . The quantity  $v_1$  must not appear in the equations.

(d) (10 points)

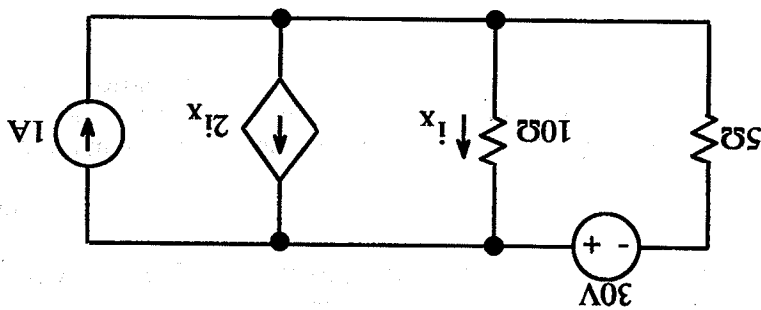
Make a consistency check on your equations by setting one or more sources to zero and using convenient resistor and source values.

2. (30 points)



Find the Thevenin's equivalent circuit at terminals a-b. Hint: Use node voltage method to find  $v_{th}$ .

3. (30 points)

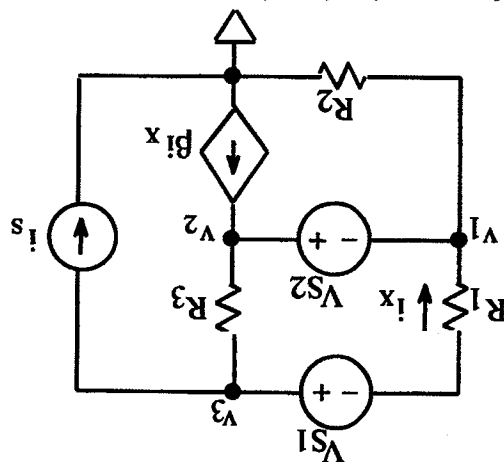


Calculate the power furnished or absorbed by the 30V voltage source, and state whether it is furnished or absorbed.

UNIT 2  
PRACTICE EXAM SOLUTION Prob 1



1. (a) (10 points)

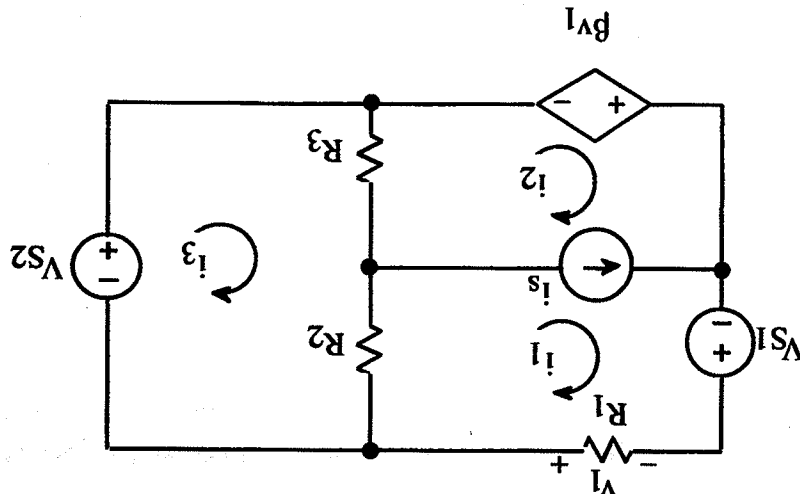


For the circuit shown, write three independent equations for the node voltages  $v_1$ ,  $v_2$ , and  $v_3$ . The quantity  $i_x$  must not appear in the equations.

(b) (10 points)

Make a consistency check on your equations by setting one or more resistor values to 0 or  $\infty$  and setting other sources and resistor to values for which  $v_1$ ,  $v_2$ , and  $v_3$  are obvious.

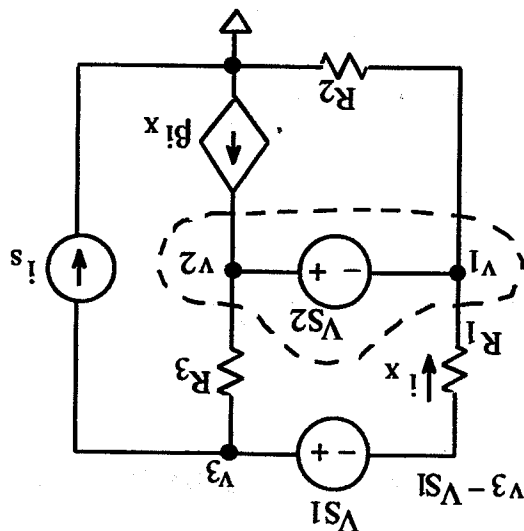
(c) (10 points)



For the circuit shown, write three independent equations for the three mesh currents  $i_1$ ,  $i_2$ , and  $i_3$ . The quantity  $v_1$  must not appear in the equations.

(d) Make a consistency check on your equations by setting one or more sources to zero and using convenient resistor and source values.

**Solution:**



We have a supernode, (dashed box on diagram, above), since  $v_1$  and  $v_2$  are connected by a  $V$  source. Sum of currents out of both nodes = 0 A:

$$\frac{v_1}{R_1} + \frac{v_1 - (v_3 - V_{S1})}{R_2} + \frac{R_1}{v_2 - v_3} - \beta i_x = 0 \text{ A}$$

$v_1$  and  $v_2$  are related by  $V$  drop between them:

$$v_2 - v_1 = V_{S2}$$

Now we need the node- $V$  equation for  $v_3$ :

$$\frac{(v_3 - V_{S1}) - v_1}{R_1} + \frac{R_1}{v_3 - v_2} + i_s = 0 \text{ A}$$

Now we eliminate  $i_x$  by expressing it in terms of node voltages:

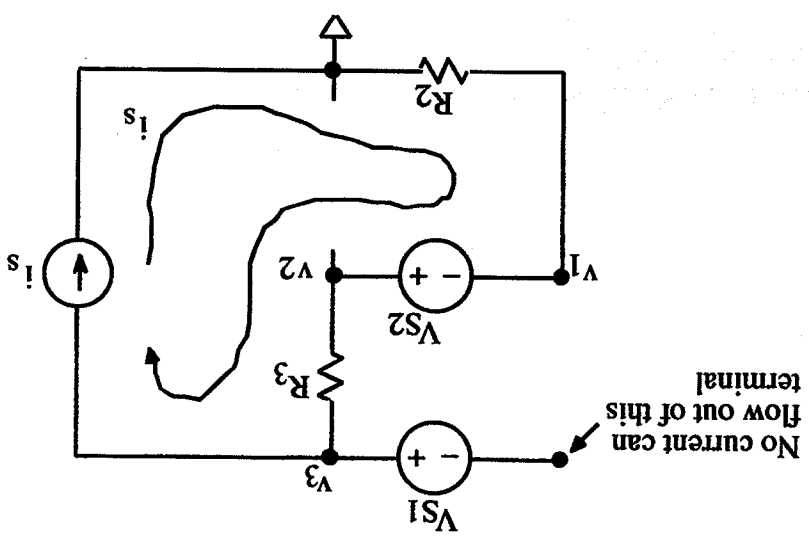
$$i_x = \frac{R_1}{(v_3 - V_{S1}) - v_1}$$

substitute into the first equation to get:

$$\frac{v_1}{R_1} + \frac{R_2}{v_1 - (v_3 - V_{S1})} + \frac{R_1}{v_2 - v_3} - \frac{R_1}{\beta[(v_3 - V_{S1}) - v_1]} = 0 \text{ A}$$

(b) Consistency checks. Choose convenient source and R values, and verify that the equations are satisfied.

Check:  $R_1 = \infty \Omega \Rightarrow i_x = 0, \beta i_x = 0, V_{S1} \text{ disconnected}$



We observe that  $i_s$  flows around the entire loop. Therefore,

$$V_1 = -i_s R_2$$

$$V_2 = V_1 + V_{S2} = -i_s R_2 + V_{S2}$$

$$V_3 = V_2 - i_s R_3 = -i_s (R_2 + R_3) + V_{S2}$$

Plug  $V$ 's into our answer to (a):

$$1) \quad V_2 - V_1 = -i_s R_2 + V_{S2} - (-i_s R_2) = V_{S2} \quad \checkmark$$

2)

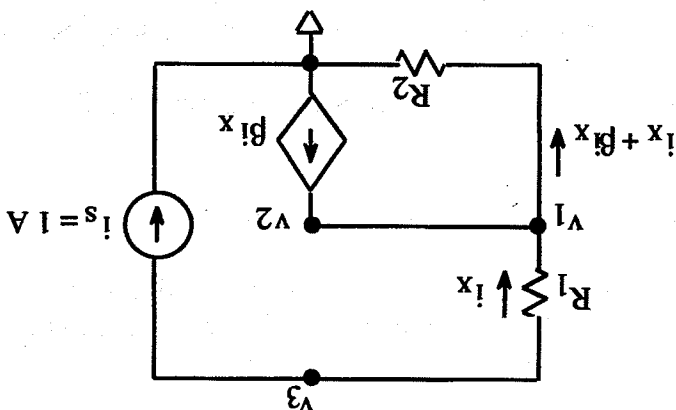
$$\begin{aligned} & \frac{(V_3 - V_{S1}) - V_1}{R_1} + \frac{V_3 - V_2}{R_3} + i_s = \frac{(-i_s (R_2 + R_3) + V_{S2} - V_{S1}) - (-i_s R_2)}{\infty} + i_s \\ & \quad + \frac{-i_s (R_2 + R_3) + V_{S2} - (-i_s R_2 + V_{S2})}{R_3} \\ & \quad = \frac{-i_s R_3}{-i_s R_3} + i_s = 0 \text{ A} \quad \checkmark \end{aligned}$$

3)

$$\frac{V_1}{R_1} + \frac{V_2 - V_3}{R_2} + \frac{V_3 - V_{S1}}{R_3} - \beta[(V_3 - V_{S1}) - V_1] = \frac{-i_s R_2}{-i_s R_2 - (-i_s(R_2 + R_3) + V_{S2} - V_{S1})} + \frac{R_3}{-i_s R_2 + V_{S2} - (-i_s(R_2 + R_3) + V_{S2})} - \frac{\beta[(-i_s(R_2 + R_3) + V_{S2} - V_{S1}) - (-i_s R_2)]}{-i_s + i_s = 0 \text{ A}} \checkmark$$

or

Check:  $R_3 = \infty \Omega$  and  $V_{S1} = 0 \text{ V}$  and  $V_{S2} = 0 \text{ V}$  and  $i_s = 1 \text{ A}$



$$i_x = -i_s = -1 \text{ A}$$

$$V_1 = i_x(1 + \beta)R_2 = (-1 \text{ A})(1 + \beta)R_2$$

$$V_2 = V_1$$

$$V_3 = V_1 + i_x R_1 = (-1 \text{ A})(1 + \beta)R_2 + R_1$$

Plug  $V$ 's into our equations from (a)

$$\begin{aligned} 1) \quad V_2 - V_1 &= V_1 - V_1 = 0 = V_{S2} \\ 2) \quad V_3 - V_1 &= V_1 - V_1 = 0 = V_{S1} \end{aligned} \checkmark$$

$$\frac{R_1}{(V_3 - V_{S1}) - V_1} + \frac{R_3}{V_3 - V_2} + i_s = \frac{\beta[(1 + \beta)R_2 + R_1](-1 \text{ A}) - (-1 \text{ A})}{\frac{R_1}{0} + \frac{R_3}{\infty} + i_s} = \frac{R_1}{0} + \frac{R_3}{\infty} + i_s = -1 \text{ A} + 1 \text{ A} = 0 \text{ A} \checkmark$$

3)

$$\begin{aligned} & \frac{V_1}{R_2} + \frac{V_1 - (V_3 - V_{S1})}{R_1} + \frac{V_2 - V_3}{R_1} - \beta[(V_3 - V_{S1}) - V_1] \\ &= \frac{R_2}{(1 + \beta)R_2(-1A)} + \frac{R_1}{(1 + \beta)R_2(-1A) - [(1 + \beta)R_2 + R_1](-1A)} + \frac{R_1}{(1 + \beta)R_2(-1A) - [(1 + \beta)R_2 + R_1](-1A)} \\ & \quad - \frac{R_1}{(1 + \beta)R_2(-1A) - [(1 + \beta)R_2 + R_1](-1A)} + \frac{R_1}{(1 + \beta)R_2(-1A) - [(1 + \beta)R_2 + R_1](-1A)} \\ &= \frac{R_1}{(1 + \beta)R_2(-1A) - [(1 + \beta)R_2 + R_1](-1A)} - \frac{R_1}{(1 + \beta)R_2(-1A) - [(1 + \beta)R_2 + R_1](-1A)} \\ &= 0A \quad \checkmark \end{aligned}$$

**Note:** It is probably easier to choose actual numerical values for all the components. Where possible, you may wish to use values that are easy to work with and can be uniquely associated with a component.

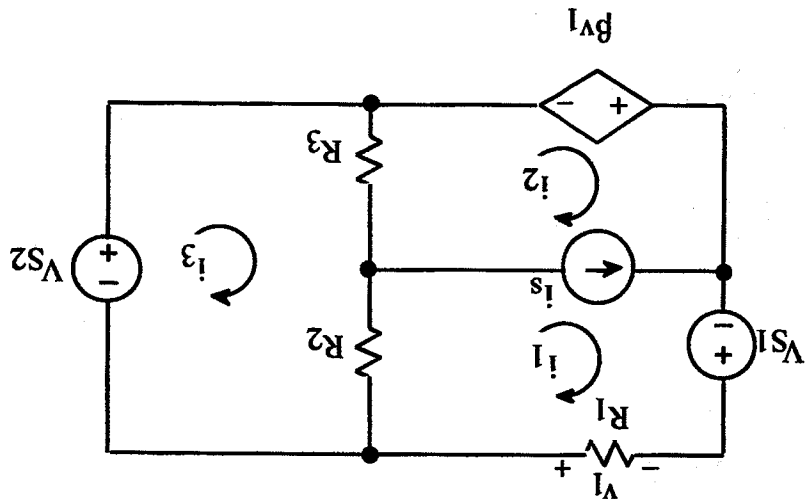
$R_1 = 1 \Omega$ ,  $R_2 = 2 \Omega$ ,  $R_3 = 3 \Omega$ ,  $i_s = 4 A$ ,  $V_{S1} = 5 V$ ,  $V_{S2} = 10 V$ , for example.

We might choose  $\beta = 9$  so we get  $1 + \beta = 10$ , for convenience.

We might also choose to avoid multiplying numbers, and instead try to cancel out terms as in the above solution.

**P.S.:** I found an error in my solution to (a) from the consistency checks performed above. It works!

(c)



We have a supermesh where the  $i_s$  source is located. We go around the outer loop containing  $i_s$ .

$$V_{S1} - i_1 R_1 - (i_1 - i_3) R_2 - (i_2 - i_3) R_3 + \beta v_1 = 0 \text{ V}$$

We observe that we have constraint equation  $v_1 = -i_1 R_1$ . Substitute this constraint into the first equation to eliminate  $v_1$ :

$$V_{S1} - i_1 R_1 - (i_1 - i_3) R_2 - (i_2 - i_3) R_3 - \beta i_1 R_1 = 0 \text{ V}$$

We need one more constraint equation. It arises from the current source we left out of the supermesh equation

$$i_s = i_1 - i_2$$

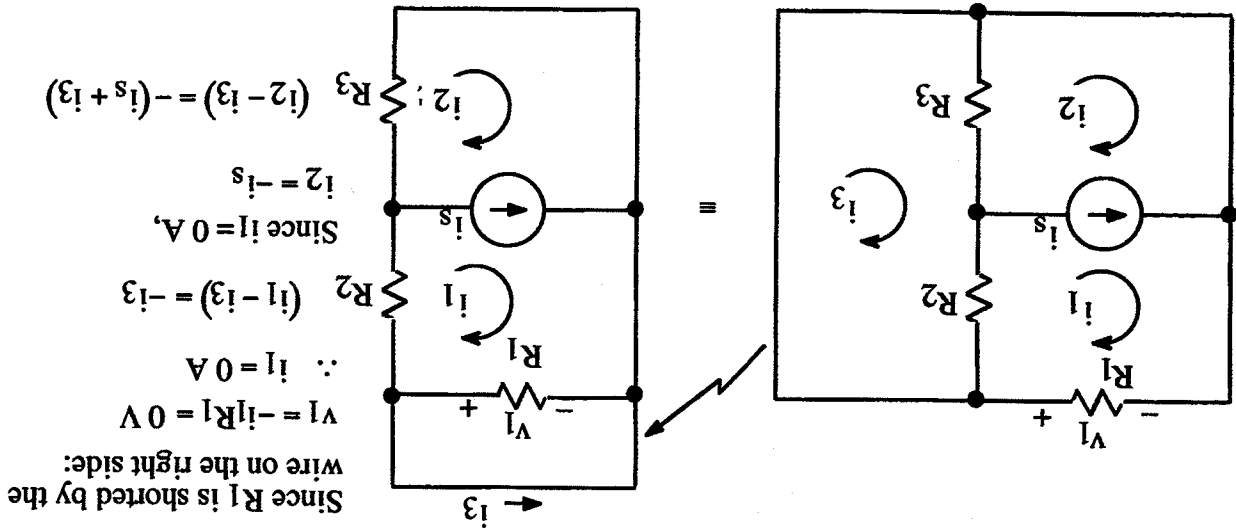
For  $i_3$  mesh loop, we have:

$$V_{S2} - (i_3 - i_2) R_3 - (i_3 - i_1) R_2 = 0 \text{ V}$$

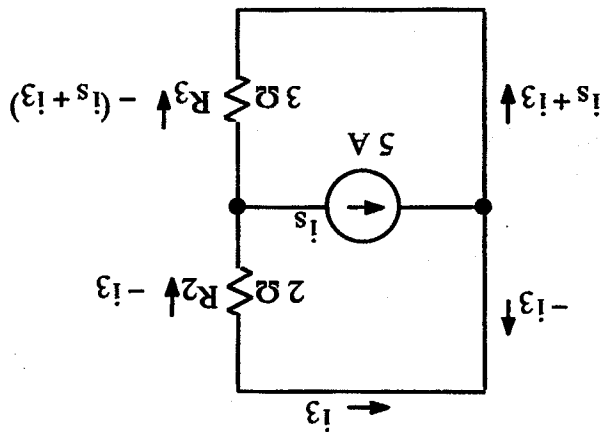
#### 1. (d) Consistency checks.

Check:  $V_{S1} = 0$  and  $V_{S2} = 0$  and  $\beta = 0$  and  $R_1 = 1 \Omega$ ,  $R_2 = 1 \Omega$ ,  $R_3 = 3 \Omega$  and  $i_s = 5 \text{ A}$ .

We may redraw the circuit  $R_1$  shorted by the wire on the right. When drawn this way,  $i_3$  flows up on the right outside edge.



Only  $i_1$  flows through  $R_1$ , and  $i_1 = 0$ . Therefore, we may remove  $R_1$ .



We have a current divider.

$$i_3 = \frac{R_3}{R_2 + R_3} \cdot (-i_s) = \frac{3\ \Omega}{2\ \Omega + 3\ \Omega} (-5\text{ A}) = -3\text{ A}$$

$$i_2 = -i_s = -5\text{ A}, \quad i_1 = 0\text{ A}$$

$$i_1 - i_2 = i_s$$

Plug  $i$ 's and  $R$ 's into our answer to (c):

1)

$$V_{S1} - i_1 R_1 - (i_1 - i_3) R_2 - (i_2 - i_3) R_3 - \beta i_1 R_1 = 0 - 0 - 1 - (0 - (-3)) \cdot 2 - (-5 - (-3)) \cdot 3 - 0 \cdot 0 \cdot 1 = 0 - 0 - 6 + 6 = 0\text{ V}$$

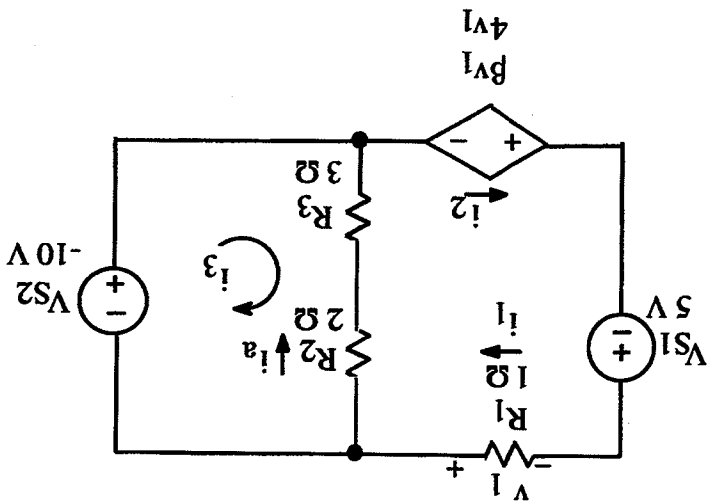
2)

$$V_{S2} - (i_3 - i_2) R_3 - (i_3 - i_1) R_2 = 0 - (-3 - (-5)) \cdot 3 - (-3 - 0) \cdot 2 = 0 - 6 + 6 = 0\text{ V}$$

3)

$$i_s = 5\text{ A} = 0 - -5 = i_1 - i_2$$

Or check:  $i_s = 0$ ,  $R_1 = 1 \Omega$ ,  $R_2 = 2 \Omega$ ,  $R_3 = 3 \Omega$ ,  $V_{S2} = -10 \text{ V}$ ,  $V_{S1} = 5 \text{ V}$ ,  $B = 4$



10 V across  $R_2 + R_3 = 5 \Omega$

$$\therefore i_a = \frac{10 \text{ V}}{5 \Omega} = 2 \text{ A} = i_1 - i_3$$

$$i_1 = i_2 = -\frac{V_1}{R_1}$$

But

$$\beta V_1 + V_{S1} + V_1 + V_{S2} = 0 \text{ V}$$

$$4V_1 + 5 \text{ V} + V_1 + -10 \text{ V} = 0 \text{ V}$$

or

$$5V_1 = 5 \text{ V} \Rightarrow V_1 = 1 \text{ V}$$

$$\therefore i_1 = -\frac{1 \text{ V}}{1 \Omega} = -1 \text{ A}, \quad i_2 = -1 \text{ A}$$

$$i_3 = i_1 - i_a = -1 - 2 = -3 \text{ A}$$

Plug into equations from (c)

1)

$$V_{S1} - i_1 R_1 - (i_1 - i_3) R_2 - (i_2 - i_3) R_3 - \beta i_1 R_1$$

$$= 5 - (-1) \cdot 1 - (-1 - (-3)) \cdot 2 - (-1 - (-3)) \cdot 3 - 4 \cdot (-1) \cdot 1$$

$$= 5 + 1 - 4 - 6 + 4 = 0 \text{ V} \quad \checkmark$$



2)

$$V_{S2} - (i_3 - i_2)R_3 - (i_3 - i_1)R_2 \\ = -10 - (-3 - (-1) \cdot 3 - (-3 - (-1)) \cdot 2 \\ = -10 + 6 + 4 = 0 \text{ V}$$

3)

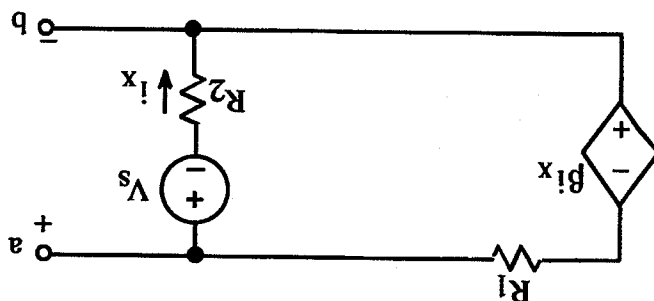
$$i_s = 0, \quad i_1 - i_2 = -1 - (-1) = 0 = i_s$$

Note:

We have to be very careful about consistency checks that change the circuit topology. One helpful fact is that if a mesh loop has one side that is on the edge of the circuit diagram—so only one mesh current flows in it—then we are safe in saying the current in that component is equal to that mesh loop current. Thus, we look for components on the perimeter of the circuit when we do consistency checks.

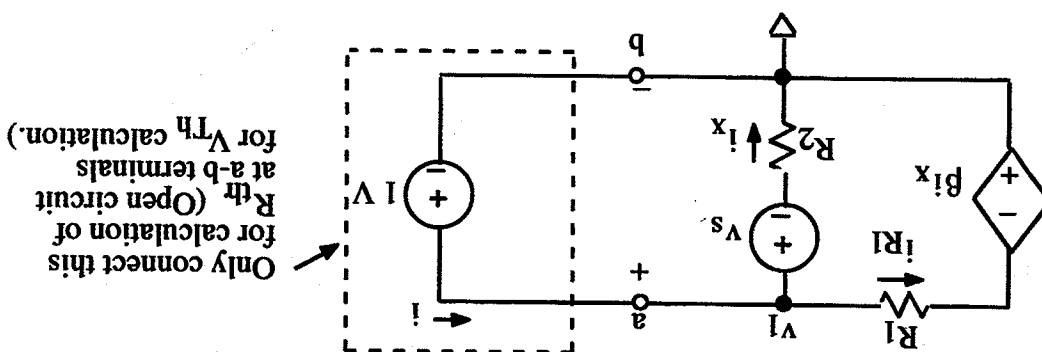


UNIT 2  
PRACTICE EXAM SOLUTION Prob 2



Find the Thevenin's equivalent circuit at terminals a-b. Hint: Use node voltage method to find  $V_{Th}$ .

**Solution:**



First, we find  $V_{Th}$  by node-V method (with nothing connected at a-b). We have one node,  $V_1$ , to write an equation for: ( $V_{Th} = V_1$  no load)

$$\frac{V_1 + \beta i_x}{V_1 - V_s} + \frac{R_1}{R_2} = 0 \text{ A}$$

We have  $i_x = \frac{V_1 - V_s}{R_2}$ . Substitute this into our equation

$$\frac{V_1 + \beta \left( \frac{V_1 - V_s}{R_2} \right)}{V_1 - V_s} + \frac{R_1}{R_2} = 0 \text{ A}$$

or

$$V_1 \left( \frac{1}{\beta} + \frac{R_1 R_2}{\beta} + \frac{R_2}{1} \right) = V_s \left( \frac{1}{\beta} + \frac{R_1 R_2}{\beta} \right)$$

Multiply both sides by  $R_1 R_2$ :

$$V_1 (R_2 + \beta + R_1) = V_s (R_1 + \beta)$$

$$V_{Th} = V_1 = V_s \frac{R_1 + \beta}{R_2 + \beta + R_1}$$

To find  $R_{Th}$ , we have two possible approaches:

1. Find current in wire shorting a to b. Then use  $R_{Th} = \frac{V_{Th}}{I_{ab}}$ .

Then use  $R_{Th} = \frac{V}{I}$  where  $I$  = current flowing into "a" terminal.

We'll use the second approach. Set  $V_s = 0V$ . Connect 1 V source to a-b. Then

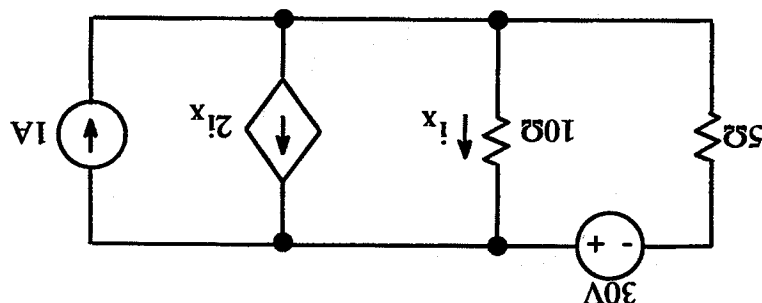
$$I_x = \frac{1V}{R_2}, \quad I_{R_1} = \frac{1V + \beta I_x}{R_1} = \frac{1V + \beta \cdot \frac{1V}{R_2}}{\frac{1}{\beta} + \frac{R_1 R_2}{\beta}} = 1V \left( \frac{1}{\beta} + \frac{R_1 R_2}{\beta} \right)$$

$$I = I_x + I_{R_1} = 1V \left( \frac{1}{R_2} + \frac{1}{\beta} + \frac{R_1 R_2}{\beta} \right) = 1V \left( \frac{1}{R_1 + R_2 + \beta} \right)$$

$$R_{Th} = \frac{1V}{I} = \frac{1}{\frac{1}{R_2} + \frac{1}{\beta} + \frac{R_1 R_2}{\beta}} = R_1 R_2 \parallel R_2 \parallel \frac{\beta}{R_1 R_2} = \frac{\beta}{\frac{R_1 R_2}{R_1 + R_2 + \beta}}$$

## UNIT 2

## PRACTICE EXAM SOLUTION Prob 3



Calculate the power furnished or absorbed by the 30V voltage source, and state whether it is furnished or absorbed.

**Solution:**

Because there are several current sources, the node-V method is a good choice here. For node  $v_1$ :

$$\frac{v_1 - 30\text{V}}{5\Omega} + \frac{v_1}{10\Omega} - 2i_x + 1\text{A} = 0\text{A}$$

Our constraint equation:  $i_x = -\frac{v_1}{10\Omega}$ . Substitute this into the node  $v_1$  equation:

$$\frac{v_1 - 30\text{V}}{5\Omega} + \frac{v_1}{10\Omega} + \frac{2v_1}{10\Omega} + 1\text{A} = 0\text{A}$$

OR

$$v_1 \left( \frac{1}{5\Omega} + \frac{1}{10\Omega} + \frac{2}{10\Omega} \right) = \frac{30\text{V}}{5\Omega} - 1\text{A} = 5\text{A}$$

OR

$$v_1 = 5\text{A} \cdot 5\Omega \parallel \frac{10\Omega}{3} = 5\text{A} \cdot 5\Omega \cdot \frac{1}{\frac{2}{3}} = 10\text{V}$$

OR

$$v_1 = 25\text{V} \cdot \frac{2/3}{5/3} = 10\text{V}$$

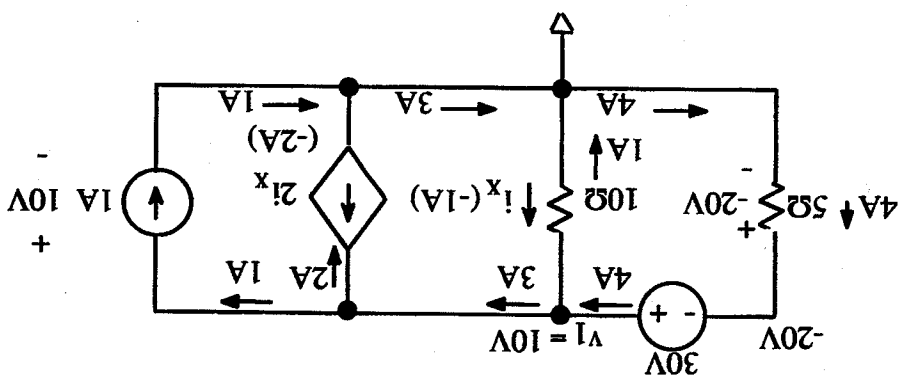
Current  $i$  flowing from + to - in 30 V source:

$$i = \frac{v_1 - 30 \text{ V}}{10 \text{ V} - 30 \text{ V}} = \frac{5 \Omega}{5 \Omega} = -4 \text{ A}$$

$$\text{Power} = i \cdot v = -4 \text{ A} \cdot 30 \text{ V} = -120 \text{ W} < 0 \Rightarrow \text{pwr furnished}$$

Power 120 W furnished (or delivered)

Check: Calculate all i's and v's and verify Kirchhoff's laws.



✓ All sums of currents out of nodes = 0 A

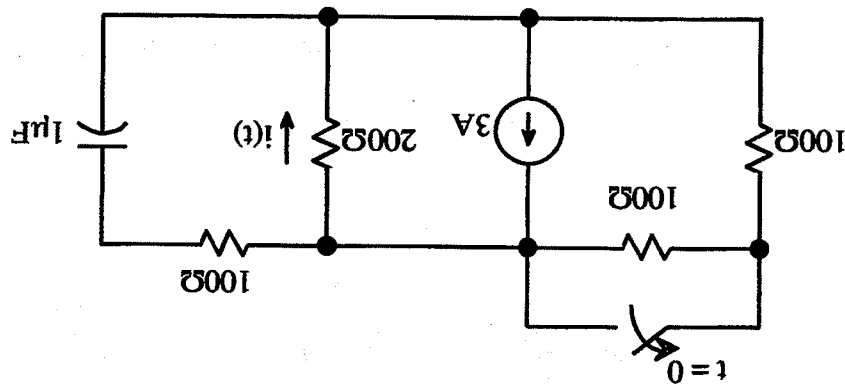
✓ All sums of V-drops around loops = 0 V

To pass the unit exam, you must be able to do the following (using books and notes):

| CONCEPTUAL TOOLS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | Learning Objective                                                                                                                                                                                                                                                                                                                                                                          | Reading                        |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------|
| <p><b>RLC CIRCUITS</b></p> <p><b>C (CAPACITOR) EQUATIONS</b></p> <p><math>i = C dv/dt</math></p> <p>Series capacitors</p> <p>Parallel capacitors</p> <p>Initial conditions</p> <p><math>C = \text{OPEN CIRCUIT}</math></p> <p>CHARGE SHARING</p> <p><math>V \text{ SRC MODEL}</math></p> <p>Final conditions open circuit</p> <p>Energy stored</p> <p>Example 1 (pdf)</p> <p>Example 2 (pdf)</p> <p><b>L (INDUCTOR) EQUATIONS</b></p> <p><math>v = L di/dt</math></p> <p>Series inductors</p> <p>Parallel inductors</p> <p>Initial conditions</p> <p><math>L = \text{WIRE}</math></p> <p>CURRENT DIVISION</p> <p><math>I \text{ SRC MODEL}</math></p> <p>Final conditions wire</p> <p>Energy stored</p> <p>Example 1 (pdf)</p> <p>Example 2 (pdf)</p> | <p>3.1 For a specified current through an inductance, find the voltage across it, and vice versa. For a specified voltage across a capacitance, find the current through it, and vice versa. From the voltages and currents, find energy stored in inductances and capacitances. Find the equivalence of inductances in series and parallel and of capacitances in series and parallel.</p> | <p>Chap 6:<br/>Sec 6.1-6.3</p> |
| <p><b>RLC CIRCUITS</b></p> <p><b>GENERAL RC/RL SOLUTION</b></p> <p>General solution</p> <p>Time const <math>\tau</math> they equiv</p> <p>Solution procedure</p> <p>Example 1 (pdf)</p> <p>Example 2 (pdf)</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | <p>3.2 Find the natural response of any circuit containing just one inductance or one capacitance (or one equivalent inductance or one equivalent capacitance).</p>                                                                                                                                                                                                                         | <p>Chap 7:<br/>Sec 7.1-7.2</p> |
| <p><b>RLC CIRCUITS</b></p> <p><b>GENERAL RC/RL SOLUTION</b></p> <p>General solution</p> <p>Time const <math>\tau</math> they equiv</p> <p>Solution procedure</p> <p>Example 3 (pdf)</p> <p>Example 4 (pdf)</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | <p>3.3 Find the step-function response of any circuit containing just one inductance or one capacitance (or one equivalent inductance or one equivalent capacitance).</p>                                                                                                                                                                                                                   | <p>Chap 7:<br/>Sec 7.3</p>     |

|                                                                                                                                                                                                                                                                                                                                                                      |                                                           |                                                                                                    |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| <b>RLC CIRCUITS</b><br><b>GENERAL RC/RL SOLUTION</b><br>General solution<br>Time const Thev equiv<br>Solution procedure<br>Example 5.1 (pdf)<br>Example 6 (pdf)<br>Example 7 (pdf)                                                                                                                                                                                   | <b>CIRCUITS</b><br><b>MAX POWER XFER</b><br>Example (pdf) | <b>SUPERPOSITION</b><br><b>CIRCUITS</b><br>$V_{DC} + V_{DC}$<br>EXAMPLE 1 (PDF)<br>EXAMPLE 2 (PDF) |
| 3.4 For given RC and RL circuits (containing only one equivalent storage element) give qualitative explanations based on the interpretations that (1) uncharged capacitance looks initially like a short circuit and finally like an open circuit, and (2) inductance with no initial current looks initially like an open circuit and finally like a short circuit. | 3.5 Apply the maximum power transfer theorem.             | 3.6 Apply the principle of superposition.                                                          |
| Chap 7:<br>Sec 7.4                                                                                                                                                                                                                                                                                                                                                   | Chap 4:<br>Sec 4.12                                       | Chap 4:<br>Sec 4.13                                                                                |

1. (30 points)



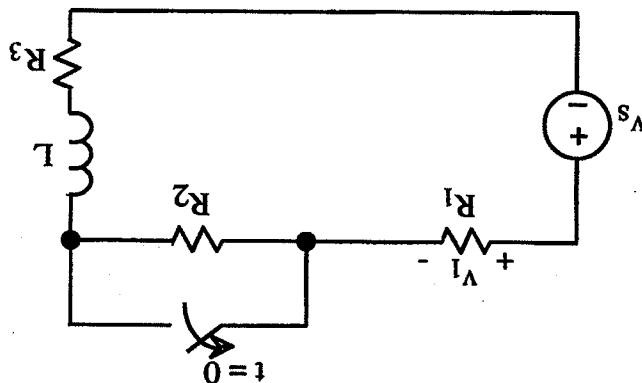
After being closed for a long time, the switch is opened at  $t = 0$ .

a. Write a numerical expression for  $i(t)$ ,  $t > 0$ .

b. Calculate the energy stored in the capacitor at  $t = 0^+$ .

c. Calculate the energy stored in the capacitor as  $t \rightarrow \infty$ .

2. (25 points)

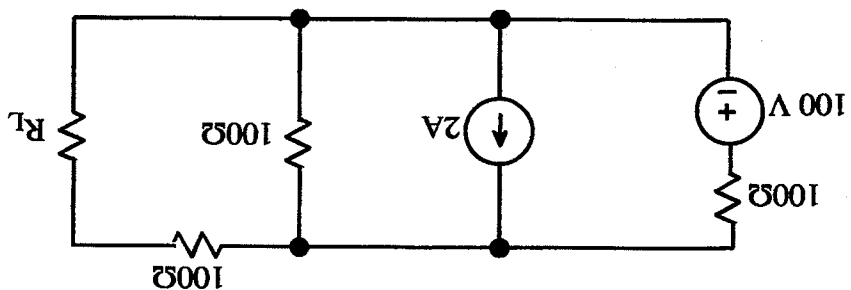


After being closed for a long time, the switch is opened at  $t = 0$ .

a. Write an expression for  $v_1(t)$ ,  $t > 0$ .

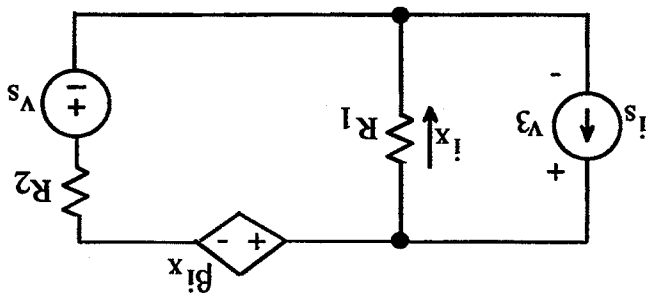
b. Make one consistency check (other than units) on your answer.

3. (20 points)



- Calculate the value of  $R_L$  that would absorb maximum power.
- Calculate that value of maximum power  $R_L$  could absorb.

4. (25 points)



Using superposition, derive an expression for  $v_3$  that contains no circuit quantities other than  $i_s$ ,  $v_s$ ,  $R_1$ ,  $R_2$ , and  $\beta$ .

a) Write a numerical expression for  $i(t)$ ,  $t \geq 0$ .

soln: Any  $i(t)$  or  $v(t)$  will be described by the general form of solution whenever we solve a switching problem:

$$i(t) = i(t=0^+) + [i(t=\infty) - i(t=0^+)] [1 - e^{-t/RC}]$$

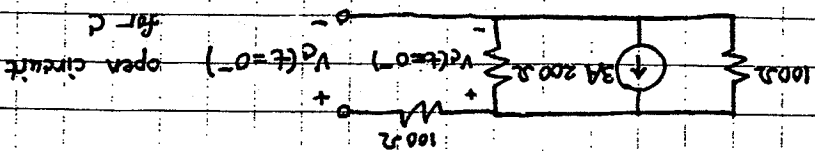
We only need to find  $i(t=0^+)$ ,  $i(t=\infty)$ , and  $RC$ .

$i(t=0^+)$ : To find  $i(t=0^+)$ , we need the initial conditions for the circuit. Since there is a capacitor, we need to find  $v_C(t=0^+)$ . (If there were an inductor, we would need to find  $i_L(t=0^+)$ ).

Since the voltage on the capacitor cannot change instantly,  $v_C(t=0^+) = v_C(t=0^-)$ .

To find  $v_C(t=0^-)$ , we treat  $C$  as an open circuit, (because the switch has been closed for a long time, allowing  $C$  to charge to final value where current no longer flows).

Circuit model:  $t=0^-$  (switch closed)



Since no current flows in the  $100\Omega$  top right,  $v_C(t=0^-)$  also appears across the  $200\Omega$  R. (No current in  $100\Omega \Rightarrow$  no v drop.)

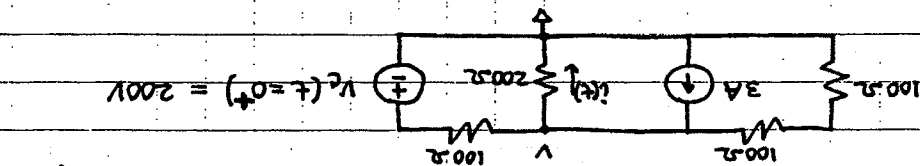
1.a) (cont)

We have  $v_c(t=0^-) = 3A \cdot 100\Omega \parallel 200\Omega = 3A \cdot 100\Omega \cdot \frac{1}{2}$

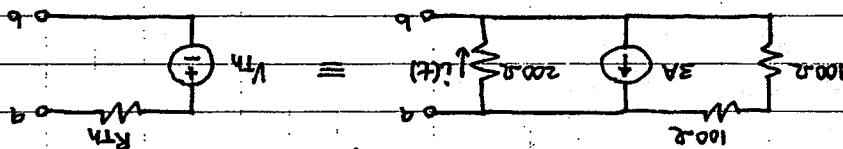
$$v_c(t=0^-) = 3A \cdot 100\Omega \cdot \frac{1}{2} = 200V$$

$$v_c(t=0^+) = v_c(t=0^-) \quad (v_c \text{ cannot change instantly})$$

For  $t=0^+$  we use a v-source to represent  $C$ , and we find  $i(t=0^+)$ :

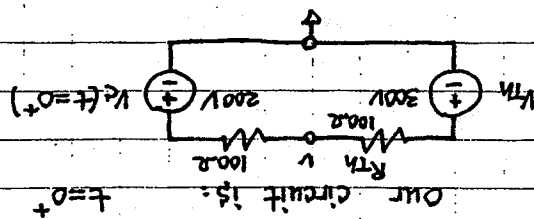
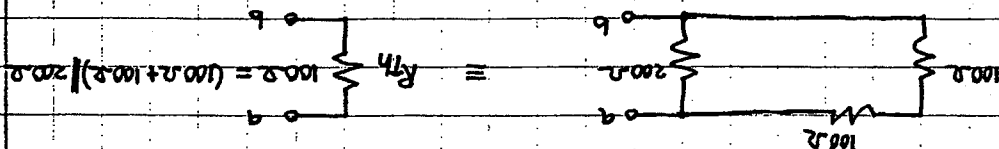


We can find  $i(t)$  by any method we prefer. My choice is to take a Thevenin equivalent of the left side of the circuit, (thru the  $200\Omega R$ ).



$$V_{Th} = \text{open circuit } v = 3A \cdot (100\Omega + 100\Omega) \parallel 200\Omega = 3A \cdot 100\Omega = 300V$$

$R_{Th}$  = resistance seen when looking into terminals a-b with independent 3A source set to zero. (This is the same as connecting a 1V source at a-b, measuring current into 'a' and taking  $R_{Th} = 1V / i_{1V}$ . This simpler approach works whenever there are no dependent sources.)



Our circuit is:

$t=0^+$

We see that  $v = 250V$ , (the two  $100\Omega R$ 's divide the 100V drop into two equal v-drops of 50V each).

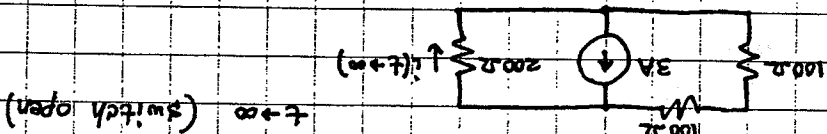
1.a) (cont) Finally, we have  $i(t=0^+) = \frac{250V}{200\Omega} = 1.25A$

Note: I used a Thevenin equivalent that had  $i(t)$  transformed inside it, but I used it to find the output voltage,  $v$ , of the Thevenin equivalent. Then I found  $i(t=0^+)$  from the voltage. What I cannot do is try to find  $i(t)$  somewhere inside the Thevenin equivalent.

Note: We could use a mesh current, or node-v, or superposition method to find  $i(t)$ . User's choice. Superposition might be a good choice for finding  $i(t=0^+)$ .

$i(t \rightarrow \infty)$ : The capacitor will be charged and acting like

an open circuit. Again there will be no current thru the  $100\Omega$  top right, and  $v(t \rightarrow \infty)$  will appear across the  $200\Omega R$ .



We have a current divider.  $i(t \rightarrow \infty) = 3A \cdot \frac{100\Omega + 100\Omega}{100\Omega + 100\Omega + 200\Omega}$

$$i(t \rightarrow \infty) = 1.5A$$

RC: Use the Thevenin equivalent found earlier for  $t=0^+$  so

we have simple RC circuit. Then  $R = R_{th} + 100\Omega = 200\Omega$ .

$$R_C = 200\Omega \cdot 1\mu F = 200\mu s.$$

$$\therefore i(t=0^+) = 1.25A + (1.5 - 1.25A)(1 - e^{-t/200\mu s}) = 1.5A - 0.25A \cdot e^{-t/200\mu s}$$

1.b) Calculate energy stored in C at  $t=0^+$ .

Found in soln to 1.a)

$$\text{sol'n: } w = \frac{1}{2} C v_C^2(t=0^+) = \frac{1}{2} \cdot 1\mu F \cdot (200V)^2 = 0.02 J \text{ or } 20 mJ$$

2.b)

Make one consistency check (other than units).

Sol'n: check: If  $V_s = 0$ ,  $L$  always discharged so  $v_L(t \geq 0) = 0$

Our answer is multiplied by  $V_s$  so we get  $v_L(t \geq 0) = 0$  if  $V_s = 0$ . ✓

check: If  $R_1 = 0$ , then  $v_L = 0$  for short of  $0.2$ .

Our answer gives  $v_L(t \geq 0) = V_s \cdot \left\{ \frac{0}{0 + R_3} + \left[ \frac{0}{0 + R_3} - \frac{0}{0 + R_3} \right] e^{-\frac{t}{L/(R_1 + R_3)}} \right\}$

Note: We have to be sure we don't get a/o (which we have verified above, assuming  $R_1, R_3 \neq 0$ ).

check: If  $R_2 = 0$ , then open switch doesn't add any  $R$ , and  $L$  continues to act like wire, and  $v_L = V_s \cdot R_1 / (R_1 + R_3)$

Our answer gives  $v_L(t \geq 0) = V_s \cdot \left\{ \frac{R_1}{R_1 + R_3} + \left[ \frac{R_1}{R_1 + R_3} - \frac{R_1}{R_1 + R_3} \right] e^{-\frac{t}{L/(R_1 + R_3)}} \right\}$

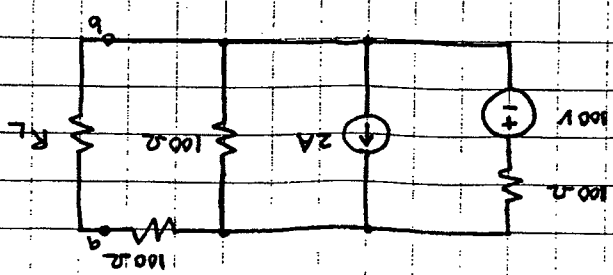
check: If  $L = 0$ H, then we just have wire for  $L$ . Then  $v_L = V_s \cdot \frac{R_1}{R_1 + R_3}$  for  $t \geq 0$ .

Our answer gives  $v_L(t \geq 0) = V_s \cdot \left\{ \frac{R_1}{R_1 + R_3} + \left[ \frac{R_1}{R_1 + R_3} - \frac{R_1}{R_1 + R_3} \right] e^{-\frac{t}{L/(R_1 + R_3)}} \right\}$

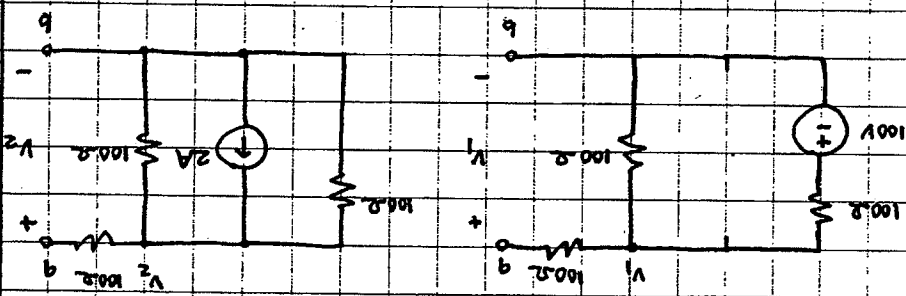
Now,  $e^{-t/0} = e^{-\infty} = 0$ .

$\therefore v_L(t \geq 0) = V_s \cdot \left\{ \frac{R_1}{R_1 + R_3} + \left[ \frac{R_1}{R_1 + R_3} - \frac{R_1}{R_1 + R_3} \right] e^{-\frac{t}{L/(R_1 + R_3)}} \right\} = V_s \cdot \frac{R_1}{R_1 + R_3}$  ✓

a) Calculate the value of  $R_L$  that would absorb maximum power.



Soln: Transform into Thevenin equivalent of circuit left of  $R_L$ . Use superposition the find  $V_{TH}$ .

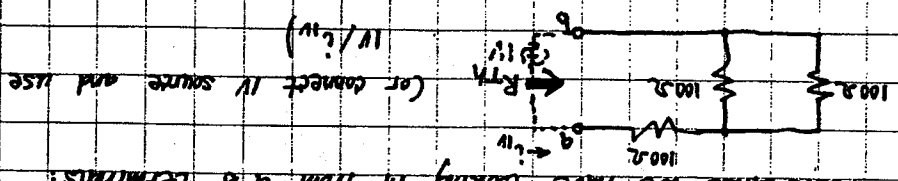


In both cases, we have no current thru  $100\Omega$  top right. So no V-drop across it.

$V_1$  from V-divider:  
 $V_1 = \frac{100V \cdot 100\Omega}{100\Omega + 100\Omega} = 50V$   
 $V_2 = 2A \cdot 100\Omega \parallel 100\Omega = 2A \cdot 50\Omega = 100V$

$V_{TH} = V_1 + V_2 = 150V$

(independent)  
 For  $R_{TH}$ , we turn 1 sources to zero and see what resistance we have looking in from a-b terminals.

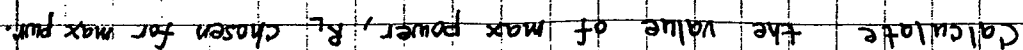


$R_{TH} = 100\Omega + 100\Omega \parallel 100\Omega = 150\Omega$

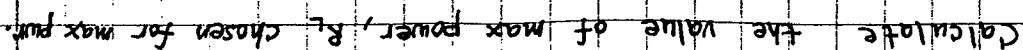
Max power xfer when  $R_L = R_{TH} = 150\Omega$

Calculate the value of max power,  $R_L$  chosen for max. pow.

Calculate the value of max power,  $R_L$  chosen for max. pow.



Calculate the value of max power,  $R_L$  chosen for max. pow.



Calculate the value of max power,  $R_L$  chosen for max. pow.

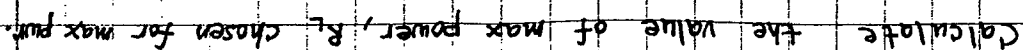
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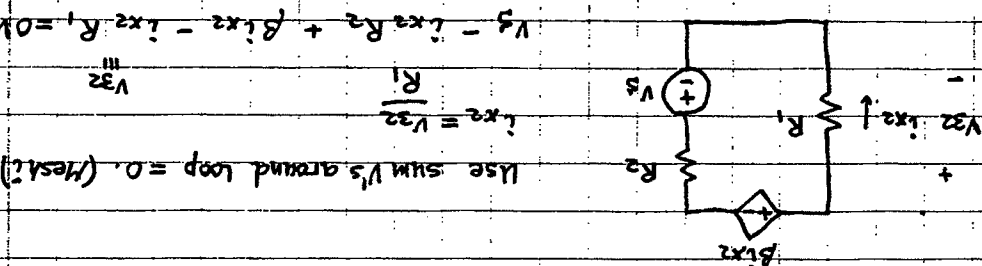


Calculate the value of max power,  $R_L$  chosen for max. pow.

Calculate the value of max power,  $R_L$  chosen for max. pow.

Calculate the value of max power,  $R_L$  chosen for max. pow.

4. (cont) Circuit 2:  $V_S \neq 0, I_S = 0$ . Add a "2" subscript to  $I_S$  &  $V_S$ .



$$\text{or } V_S - V_{32} \frac{R_1}{R_2} + \beta V_{32} \frac{R_1}{R_2} - V_{32} = 0$$

$$V_S = V_{32} \left( \frac{R_1}{R_2} + 1 - \beta \frac{R_1}{R_2} \right) \quad V_{32} = \frac{V_S}{\frac{R_1}{R_2} + 1 - \beta \frac{R_1}{R_2}}$$

$$\text{Now } V_3 = V_{31} + V_{32}$$

$$\text{eqn 1} \quad \text{or } V_3 = \left( i_S + \frac{V_S}{R_2} \right) / \left( \frac{1}{R_1} + 1 - \beta \frac{R_1}{R_2} \right) \quad \text{eqn 2} \quad V_3 = \left( i_S + \frac{V_S}{R_2} \right) (R_1 \| R_2 \| -\beta R_1 R_2)$$

$$\text{eqn 2} \quad V_3 = \left( i_S + \frac{V_S}{R_2} \right) (R_1 \| R_2 \| -\beta R_1 R_2) \quad \text{eqn 3} \quad V_3 = (i_S R_2 + V_S) / \left( \frac{R_1}{R_2} + 1 - \beta \frac{R_1}{R_2} \right) = \frac{R_2 + (-R_2)}{R_2 + (-R_2)} = \frac{R_2 - R_2}{R_2 - R_2}$$

Consistency Checks: 1)  $\beta = 0, V_S = 0 \Rightarrow V_3 = i_S \cdot (R_1 \| R_2)$

$$\text{eqn 2 gives } V_3 = \left( i_S + \frac{0}{R_2} \right) (R_1 \| R_2 \| -\beta R_1 R_2) = i_S \cdot (R_1 \| R_2) \quad \checkmark$$

$$2) R_1 = \infty \Rightarrow V_3 = V_S + i_S R_2$$

$$\text{eqn 2 gives } V_3 = \left( i_S + \frac{V_S}{R_2} \right) (0 \| R_2 \| -\beta R_1 R_2) = \left( i_S + \frac{V_S}{R_2} \right) R_2 \quad \checkmark$$

$$3) i_S = 0, \beta = 0 \Rightarrow V_3 = V_S \quad \frac{R_1}{R_1 + R_2}$$

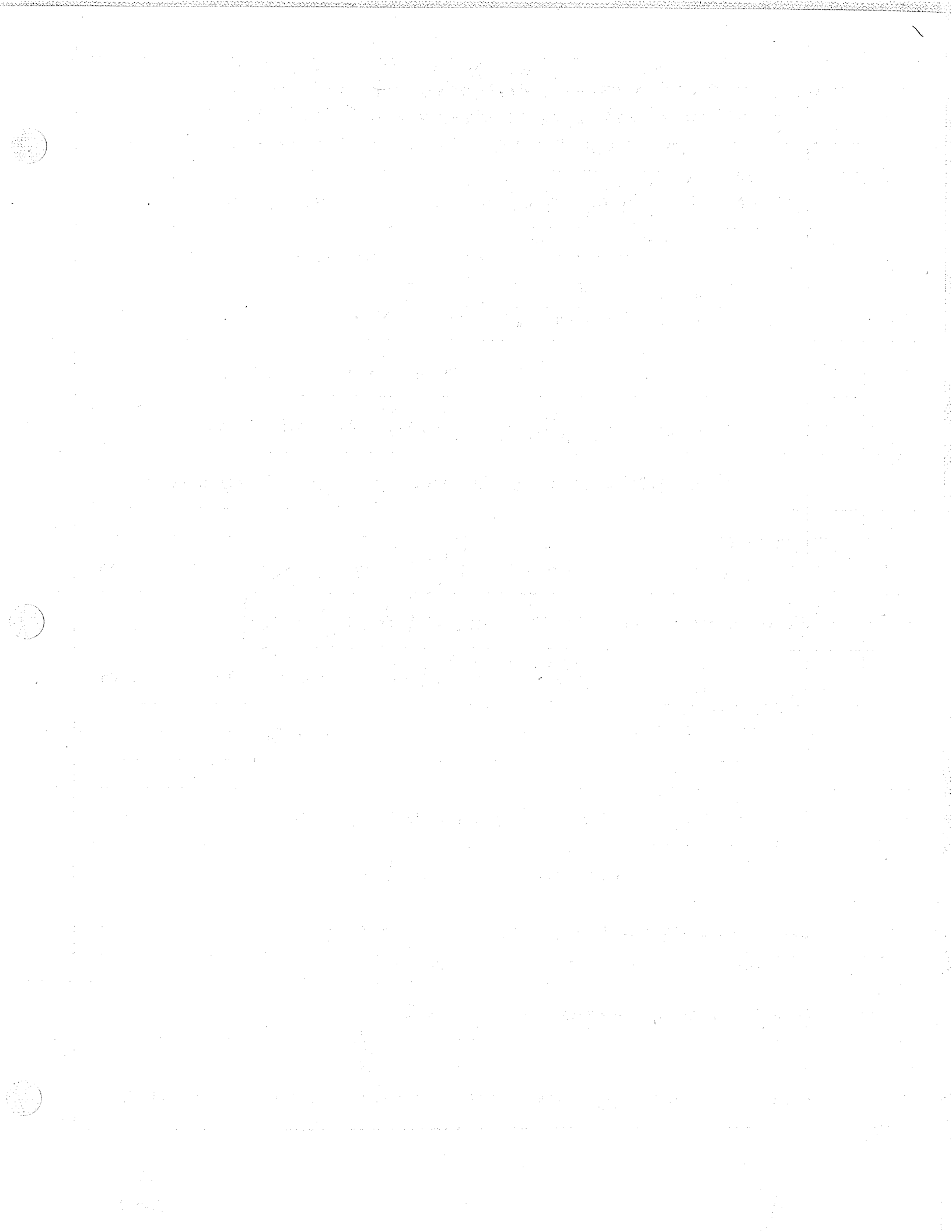
$$\text{eqn 2 gives } V_3 = \left( 0 + \frac{V_S}{R_2} \right) (R_1 \| R_2 \| -\beta R_1 R_2) = \frac{V_S R_1 R_2}{R_1 + R_2} \quad \checkmark$$

$$4) R_1 = 1 \Omega, R_2 = 2 \Omega, V_S = 3V, i_S = 4A, V_3 = 4V, \beta = \frac{1}{2}$$

$$\beta i_{x2} = \frac{1}{2} V, V_{32} = V_3 - (i_S + \beta i_{x2}) R_2 = 4 - (4 + \frac{1}{2} \cdot 2) \cdot 2 = 4 - 10 = -6V \Rightarrow i_{32} = \frac{-6}{2} = -3A$$

$$i_S = i_{x2} + i_{32} = 4 + (-3) = 1A \Rightarrow i_{x2} = 1A$$

$$V_3 = i_{x2} R_2 + V_{32} = 1 \cdot 2 + (-6) = -4V$$



To pass the unit exam, you must be able to do the following (using books and notes):

| CONCEPTUAL TOOLS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | Learning Objective                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Reading                        |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------|
| <p><b>COMPLEX ANALYSIS</b><br/>EXPLANATIONS OF!<br/>CONVENIENT EXAMPLES<br/>BASIC MATH<br/>Addition and subtraction<br/>EXAMPLE (PDF)<br/>Multiplication<br/>Rationalization<br/>EXAMPLE 1 (PDF)<br/>EXAMPLE 2 (PDF)<br/>Conjugate<br/>DEFINITION (PDF)<br/>EXAMPLE 1 (PDF)<br/>EXAMPLE 2 (PDF)<br/>EXAMPLE 3 (PDF)<br/>Magnitude<br/>EXAMPLE 1 (PDF)<br/>Phase<br/>Re[]<br/>Im[]<br/>Roots and Powers<br/>NTH ROOTS<br/>example   (pdf)<br/>NTH ROOTS OF UNITY<br/>POWERS<br/>example   (pdf)<br/>RECT AND POLAR FORMS<br/>Euler's formula (complex exp)<br/>Polar form<br/>Rect-&gt;polar xform triangle<br/>EXAMPLE 1 (PDF)<br/>EXAMPLE 2 (PDF)<br/>Example   (pdf)</p> | <p>4.1 Perform these operations on complex numbers:<br/>a. Multiply, divide, add, and subtract complex numbers.<br/>b. Find the complex conjugate of any complex number.<br/>c. Rationalize the denominator of a fraction of complex numbers.<br/>d. Convert from polar form to rectangular form and vice versa.<br/>e. Find the real part of any complex number.<br/>f. Find the absolute value (i.e., magnitude) of any complex number.<br/>g. Find the nth root or power of any complex number.</p> | <p>App B</p>                   |
| <p><b>COMPLEX ANALYSIS</b><br/>PHASORS<br/>Tutorial   (pdf)<br/>Rotating stick shadow<br/>Identities<br/>Phasor math<br/>Phasor-&gt;inv-phasor xform<br/>EXAMPLE 1 (PDF)<br/>EXAMPLE 2 (CONT) (PDF)<br/>EXAMPLE 2 (PDF)</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                | <p>4.2 Take the phasor transform of a sinusoidal function of time and inverse phasor transform of a phasor.</p>                                                                                                                                                                                                                                                                                                                                                                                        | <p>Chap 9:<br/>Sec 9.1-9.3</p> |

\* The material in this handout is based extensively on concepts developed by C. H. Durney, Professor Emeritus of the University of Utah.

|     |                                                                                                                                                                                    |                     |
|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|
| 4.3 | Transform circuits to the frequency domain and apply the concept of impedance in the frequency domain. This includes finding the equivalent impedance of combinations of elements. | CHAP 9: Sec 9.4,9.6 |
| 4.4 | Apply Kirchhoff's laws in the frequency domain.                                                                                                                                    | CHAP 9: Sec 9.5     |
| 4.5 | Apply the node-voltage method in the frequency domain.                                                                                                                             | CHAP 9: Sec 9.8     |
| 4.6 | Apply the mesh-current method in the frequency domain.                                                                                                                             | CHAP 9: Sec 9.9     |
| 4.7 | In the frequency domain, transform sources and find Thevenin and Norton equivalent circuits.                                                                                       | CHAP 9: Sec 9.7     |
| 4.8 | Apply the principle of superposition in the frequency domain.                                                                                                                      |                     |
| 4.9 | Draw appropriate phasor diagrams and use them in analyzing and designing circuits.                                                                                                 | CHAP 9: Sec 9.12    |

1. (20 points)

Evaluate the following:

$$a. \left[ \frac{20}{\cos 60^\circ e^{(2-j2)16}} \right]^* =$$

$$b. \text{The polar form of } \frac{(2+j1.5)}{4e^{j45^\circ}} =$$

$$c. \text{The rectangular form of } \frac{16e^{j30^\circ}}{4e^{j45^\circ}} =$$

$$f. \mathbf{P}^{-1} \left[ \frac{5e^{j45^\circ}}{(3+j4)} \right] =$$

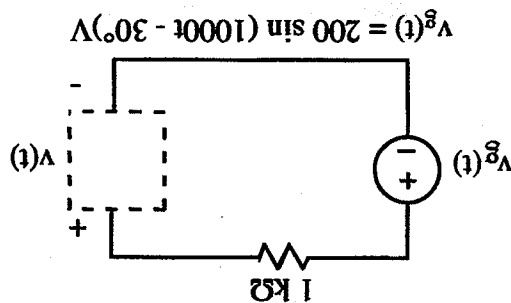
$$e. \mathbf{P}[5 \sin(2\pi 10^5 t - 20^\circ)] =$$

$$d. \operatorname{Re} \left[ \frac{e^{j30^\circ}}{j3(6-j7)} \right] =$$

$$h. \text{Rationalize } \frac{(1-j)}{(5-j4)}$$

$$g. \text{Absolute magnitude of } \left[ \frac{3e^{j\frac{\pi}{2}}}{5-j12} \right]$$

2. (30 points)

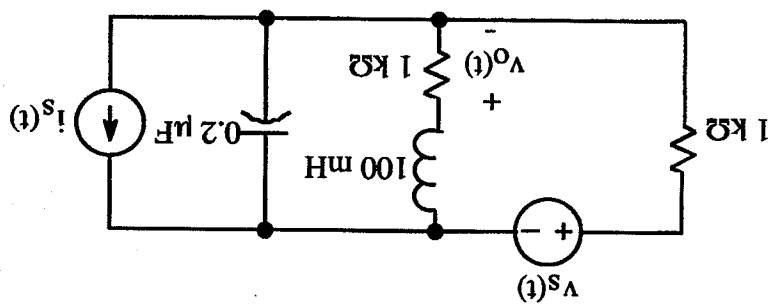


Choose an R, an L, or a C to be placed in the dashed-line box to make

$$v(t) = V_n \sin(1000t) \text{ V}$$

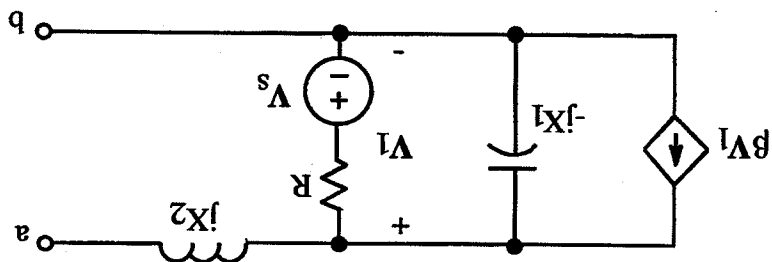
where  $V_n$  is a real constant. State the value of the component you choose. With your component in the circuit, calculate the resulting value of  $V_n$ .

3. (25 points)



- Find a numerical, time-domain expression for  $v_o(t)$ .
- Show  $V_o$  on a phasor diagram.

4. (25 points)



Construct a frequency-domain Thevenin's equivalent circuit at the terminals a-b.

Evaluate:

$$1. a) \left[ \frac{20}{\cos 60^\circ e^{2-j2}} \right] = \left[ \frac{20}{\cos 60^\circ e^{2+j2}} \right] \text{ change every } j \text{ to } -j$$

$$= \frac{1}{2} e^{j2} \cdot \frac{16}{2} = \frac{5}{2} e^{j2} = \frac{5}{2} (\cos 2 + j \sin 2)$$

$$b) \text{ Polar form of } 2 + j1.5 = \sqrt{2^2 + 1.5^2} e^{j \tan^{-1} 1.5/2}$$

$$= \frac{2.5}{2} e^{j45^\circ} = 0.625 e^{j(37^\circ - 45^\circ)} = 0.625 e^{-j8^\circ}$$

$$c) \text{ Rectangular form } \frac{16 e^{j30^\circ}}{4 e^{j45^\circ}} = 4 e^{j(30^\circ - 45^\circ)} = 4 e^{-j15^\circ}$$

$$= 4 \cos 15^\circ - j4 \sin 15^\circ = 3.86 - j1.04$$

$$d) \operatorname{Re} \left[ \frac{j3(6-j7)}{e^{j30^\circ}} \right] = \operatorname{Re} [j3(6-j7) e^{-j30^\circ}] = \operatorname{Re} [(21+j18) e^{-j30^\circ}]$$

$$= \operatorname{Re} \left[ (21+j18)(\cos 30^\circ - j \sin 30^\circ) \right] = \operatorname{Re} \left[ \left( \frac{15}{2} - j\frac{1}{2} \right) \right]$$

$$= \operatorname{Re} \left[ \frac{2}{2} \sqrt{3} + j\frac{18}{2} - j\frac{2}{2} + j\frac{18}{2} \right] = \frac{2}{2} \sqrt{3} + 9$$

$$e) P [5 \sin (2\pi 10^5 t - 20^\circ)] = 5 \angle -90^\circ - 20^\circ = 5 \angle -110^\circ$$

sin adds  $-90^\circ$ 

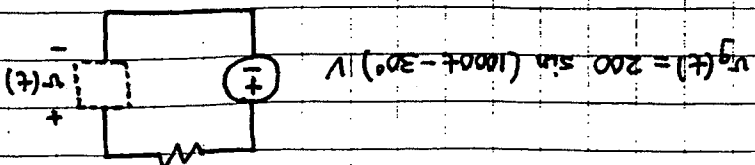
$$f) P^{-1} \left[ \frac{5 e^{j45^\circ}}{3+j4} \right] = P^{-1} \left[ \frac{5 e^{j45^\circ}}{\sqrt{3^2+4^2} e^{j \tan^{-1} 4/3}} \right] = P^{-1} \left[ \frac{5 e^{j45^\circ}}{5 e^{j53^\circ}} \right]$$

$$= P^{-1} [e^{j45^\circ - 53^\circ}] = P^{-1} [e^{-j8^\circ}] = P^{-1} [1 \angle -8^\circ] = 1 \cos (\omega t - 8^\circ)$$

$$g) \text{ Abs mag of } \frac{j\frac{1}{2}}{3e^{j\frac{\pi}{2}}} = \left| \frac{5-j12}{3e^{j\frac{\pi}{2}}} \right| = \frac{|5-j12|}{3} = \frac{\sqrt{5^2+12^2}}{3} = \frac{13}{3}$$

$$h) \text{ Rationalize } \frac{5-j4}{5-j4} = \frac{5-j4}{5-j4} \cdot \frac{1+j}{1+j} = \frac{5+4+j5-j4}{1^2+1^2} = \frac{9+j1}{2}$$

2.

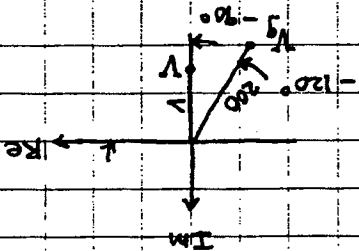


Choose  $R$ ,  $L$ , or  $C$  in dashed box to make  $v(t) = V_n \sin(1000t)$  V where  $V_n$  is real constant.  
State component value and resulting value of  $V_n$ .

sol'n:  $V_g = 200 \angle -90^\circ - 30^\circ \text{ V} = 200 \angle -120^\circ \text{ V}$  (phasor)

$V = P[V(t)] = V_n \angle -90^\circ \text{ V}$  (phasor)

Phasor diagram:



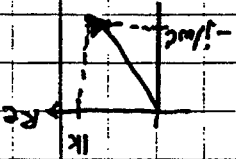
$V = V_g \cdot z$  where  $z$  is impedance of dashed box

From phasor diagram, we need a phase shift of  $+30^\circ$  to get from  $-120^\circ$  for  $V_g$  to  $-90^\circ$  for  $V$ .

$\therefore z$  must give  $\angle +30^\circ$

Consider  $z = R$   
 $z = \text{real \#} = \frac{\text{real \#} + j0}{1 + j0}$   
 $\therefore \angle = 0^\circ$   
 So  $R$  won't work

Consider  $z = -\frac{j}{\omega C}$   
 $\angle -j/\omega C = \angle -j/\omega C - \angle(1k-j/\omega C)$   
 $\angle -j/\omega C$  will be in range  $-90^\circ$  to  $0^\circ$ .



$\therefore \angle z = -90^\circ - (-90^\circ \text{ to } 0^\circ) = \angle -90^\circ \text{ to } 0^\circ \neq 30^\circ$   
 range of possible  $\angle$  range pass

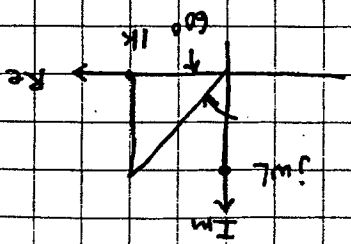
2. (cont)

 $\therefore$  C won't work

$$\text{Consider } z = j\omega L \quad \angle \frac{j\omega L}{1k + j\omega L} = \angle j\omega L - \angle (1k + j\omega L)$$

$$\angle j\omega L = 90^\circ$$

$$\therefore \text{ we need } \angle 1k + j\omega L = 60^\circ \text{ to get } \angle j\omega L - \angle 1k + j\omega L = 30^\circ \text{ as desired.}$$



On phasor diagram,  $1k\Omega$  is real part, and  $j\omega L$  is imaginary part

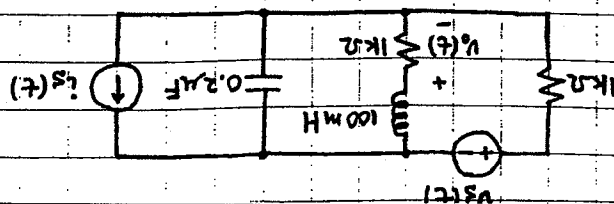
$$\angle 1k + j\omega L = \tan^{-1} \frac{j\omega L}{1k} = 50^\circ \text{ desired}$$

$$\therefore \frac{j\omega L}{1k} = \tan 60^\circ = \sqrt{3}$$

$$\omega L = \sqrt{3} \cdot 1k$$

$$\omega = 1k \text{ so } L = \sqrt{3} H.$$

3.



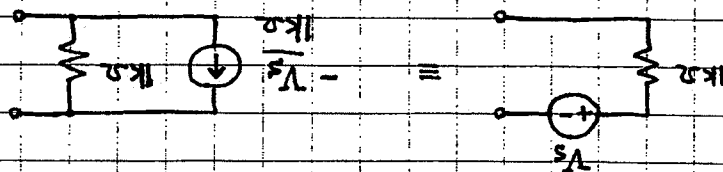
$$v_s(t) = 100 \cos(10^4 t) \text{ V}$$

$$i_s(t) = \sin(10^4 t) \text{ A}$$

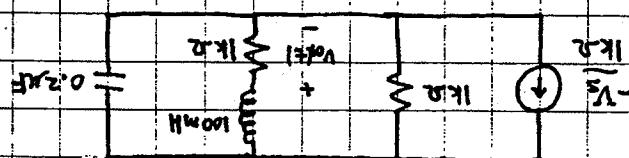
- a) Find a numerical, time-domain expression for  $v_o(t)$ .
- sol'n: I'll use superposition. Other approaches such as node-v or mesh current would work, too.

circuit 1:  $V_s = 100 \angle 0^\circ \text{ V}$  set  $I_s = 0 \text{ A}$

Convert  $V_s$  and  $1k\Omega$  on left to Norton equivalent:

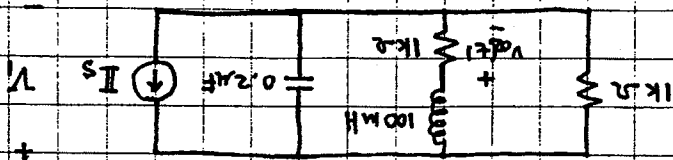


Our circuit 1 is now:



circuit 2:  $V_s = 0 \text{ V}$  and  $I_s = 1 \angle -90^\circ \text{ A}$  (phasor for  $i_s$ )

Keeping the Norton equivalent, our circuit 2 is:



Clearly, we can sum the current sources from circuits 1 & 2 to find  $V_o$  drop from top rail to bottom rail.

3a(cont)

$$V_1 = \left( -\frac{V_s}{k\Omega} + I_s \right) \| (k\Omega \| (j\omega L + k\Omega)) \| \frac{\omega C}{-j}$$

$$j\omega L = j \cdot 10^{-4} \cdot 100 \text{ m} = j k\Omega$$

$$\frac{-j}{\omega C} = \frac{-j}{10^{-4} \cdot 0.2 \mu} = \frac{-j}{2 \text{ m}} = -j \cdot k\Omega$$

$$-\frac{V_s}{k\Omega} + I_s = \frac{-100 \angle 0^\circ \text{ V}}{k\Omega} + 1 \angle -90^\circ \text{ A}$$

$$= -0.1 \angle 0^\circ \text{ A} + 1 \angle -90^\circ \text{ A}$$

$$= -0.1 - j$$

$$= \sqrt{0.1^2 + 1^2} \angle \tan^{-1} \frac{-1}{-0.1}$$

$$= 1.005 \angle -95.7^\circ$$

$$k\Omega \| (j k\Omega + k\Omega) \| \left( -\frac{j}{k\Omega} \right) = k\Omega$$

$$= \frac{k\Omega}{1 + \frac{j}{k\Omega} + 2j} = \frac{k\Omega}{\frac{2}{3} + j\frac{2}{3}} = \frac{3}{2} \cdot \frac{k\Omega}{1+j}$$

$$= \frac{3}{2} k\Omega \cdot \frac{1-j}{(1-j)} = k\Omega \cdot \frac{1-j}{1-j}$$

$$V_1 = 1.005 k\Omega \angle -95.7^\circ \cdot \frac{1-j}{3}$$

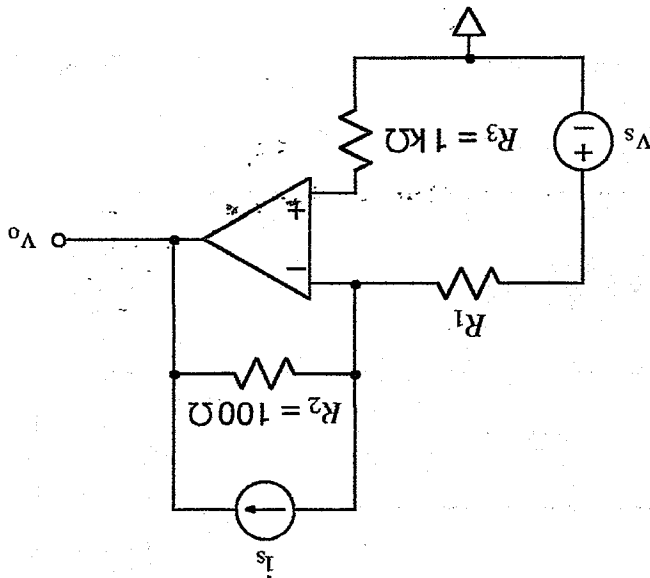
V<sub>o</sub> from V-divider:

$$V_o = V_1 \cdot \frac{k\Omega}{k\Omega + j\omega L} = V_1 \cdot \frac{k\Omega}{k\Omega + j k\Omega} = V_1 \cdot \frac{1}{1+j} = V_1 \cdot \frac{1-j}{2}$$

$$V_o = 1.005 k\Omega \angle -95.7^\circ \cdot \left( \frac{1-j}{3} \right) \cdot \frac{1-j}{2} = -j2 \cdot \frac{6}{3} = -j2 \cdot 2 = -j4 = 4 \angle -90^\circ$$



1.

Rail voltage =  $\pm 10$  V

The above circuit operates in linear mode. Derive a symbolic expression for  $v_o$ . The expression must contain not more than the parameters  $i_s$ ,  $v_s$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .

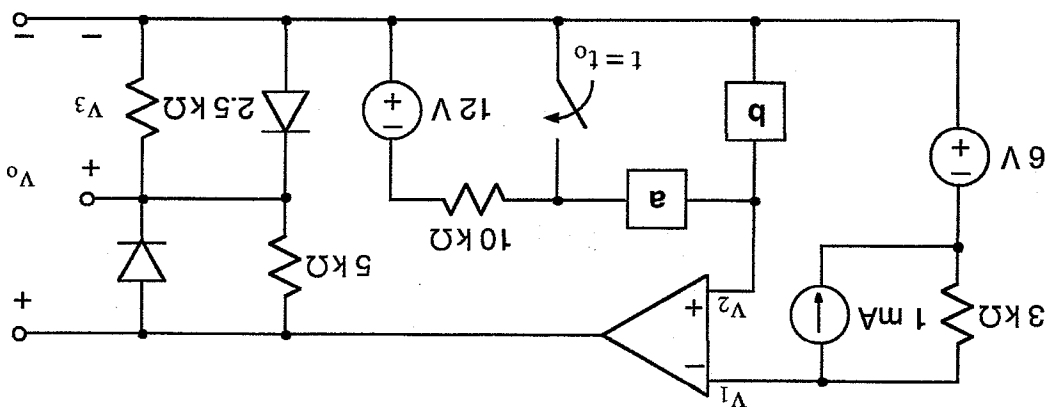
a) If  $i_s = 0$  A, find the value of  $R_1$  that will yield an output voltage of  $v_o = -2$  V when  $v_s = 1$  V.

b) Using the value of  $R_1$  from part (a), find the value of  $i_s$  that will yield  $v_o = 0$  V when  $v_s = 1$  V.

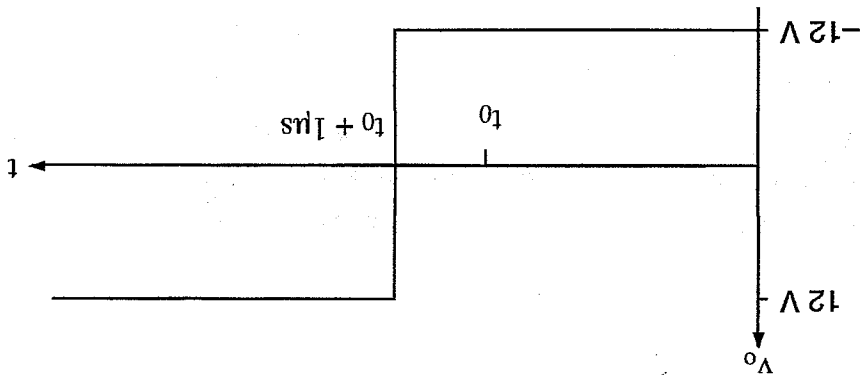
c) Using the value of  $R_1$  from part (a), calculate the input resistance,  $R_{in} = v_s/i_i$ .

d) Using the value of  $R_1$  from part (a), calculate the input resistance,  $R_{in} = v_s/i_i$ .

2.



After being open for a long time, the switch closes at  $t = t_0$ .



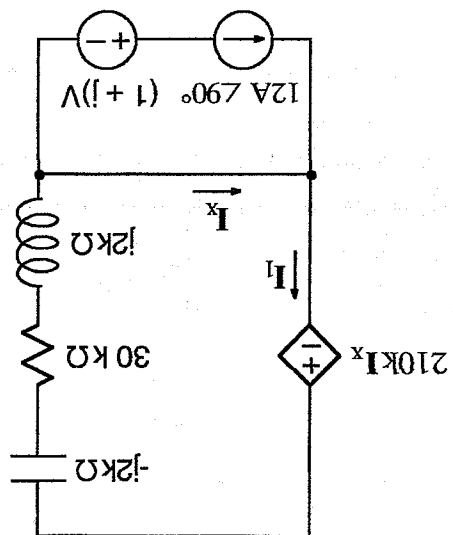
- a) Choose either an R or L to go in box a and either an R or L to go in box b to produce the  $v_o(t)$  shown above. (Note that  $v_o$  stays high forever after  $t_0 + 1 \mu\text{s}$ .) Specify which element goes in each box and its value.

- b) Sketch  $v_1(t)$ , showing numerical values appropriately.

- c) Sketch  $v_2(t)$ , showing numerical values appropriately.

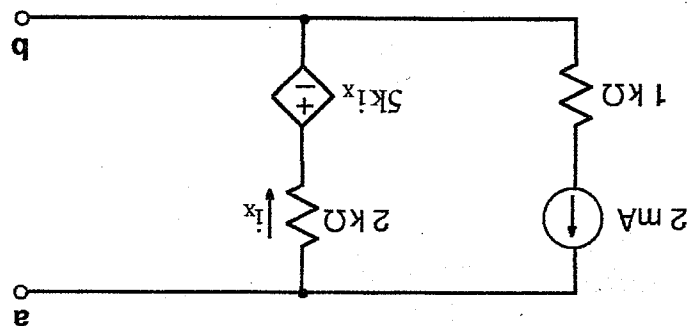
- d) Sketch  $v_3(t)$ . Show numerical values for  $t < t_0$ , for  $t_0 < t < t_0 + 1 \mu\text{s}$ , and for  $t_0 + 1 \mu\text{s} < t$ . Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

Explain your work carefully.



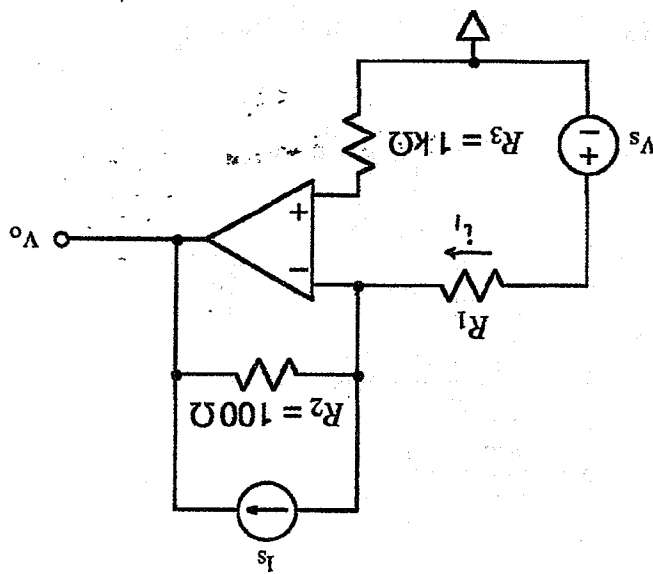
- a) A frequency-domain circuit is shown above. Write the value of phasor  $I_1$  in polar form.
- b) Given  $\omega = 2\text{ k rad/s}$ , write a numerical time-domain expression for  $i_1(t)$ , the inverse phasor of  $I_1$ .

4.



- a) Find the Thevenin equivalent of the above circuit relative to terminals a and b.
- b) If we attach  $R_L$  to terminals a and b, find the value of  $R_L$  that will absorb maximum power.
- c) Calculate the value of that maximum power absorbed by  $R_L$ .

Ex:



- a) The above circuit operates in linear mode. Derive a symbolic expression for  $v_o$ . The expression must contain not more than the parameters  $i_s$ ,  $v_s$ ,  $R_1$ ,  $R_2$ , and  $R_3$ .

- b) If  $i_s = 0$  A, find the value of  $R_1$  that will yield an output voltage of  $v_o = -2$  V when  $v_s = 1$  V.

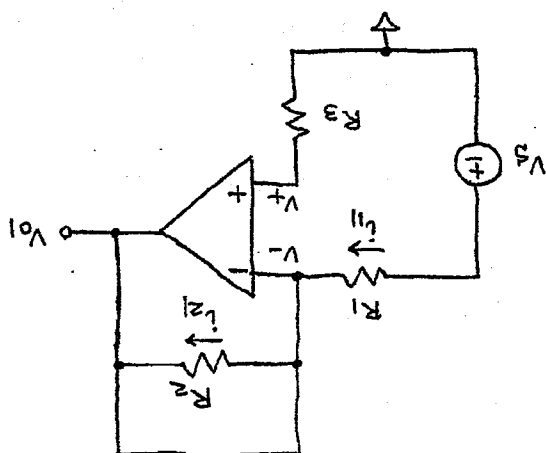
- c) Using the value of  $R_1$  from part (b), find the value of  $i_s$  that will yield  $v_o = 0$  V when  $v_s = 1$  V.

- d) Using the value of  $R_1$  from part (a), calculate the input resistance,  $R_{in} = v_s/i_1$ .

Soln: We use superposition to find  $v_o$ .

We turn on one independent source at a time

Case I:  $v_s$  on,  $i_s$  off



First, we find  $V_+$ :  $V_+ = 0V$  since no current flows into op-amp (or, therefore, thru  $R_3$ )

Second, we set  $V_- = V_+ = 0V$

Third, we find an expression for  $i_{11}$  flowing toward - input from the left. We use  $V_-$  in this expression:

$$i_{11} = \frac{V_+ - V_-}{R_1} = \frac{0}{R_1}$$

Fourth, we find an expression for  $i_{21}$ , the total feedback current, flowing thru  $R_2$ . We use  $V_-$  in this expression:

$$i_{21} = \frac{V_- - V_{O1}}{R_2} = \frac{0 - V_{O1}}{R_2}$$