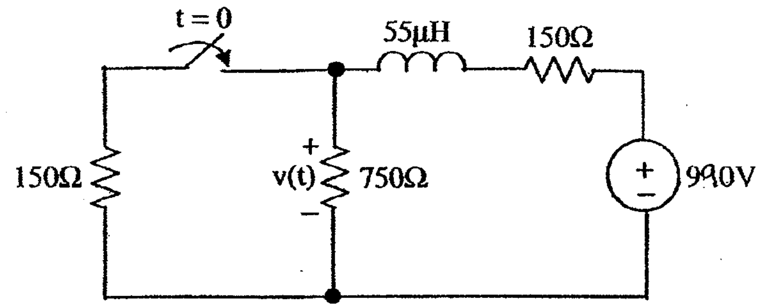
Problem Set #8

1.



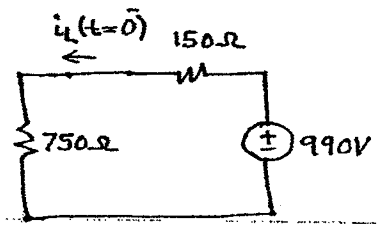
After being open for a long time, the switch is closed at t = 0.

a. Calculate the energy L stored on the inductor at t = 0.

b. Write a numerical expression for $v(t)$, $t > 0$.

soln: a) Energy $w = \frac{1}{2} L i_L^2$ so we find $i_L(t \to \infty)$.

$t = 0^-$: L acts like wire. Switch is open.



$R_{eq} = 150 + 750\,\Omega$

$R_{eq} = 900\,\Omega$

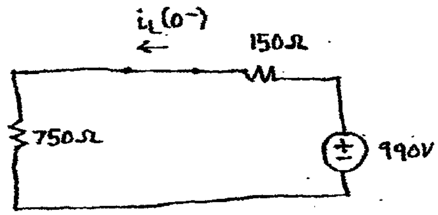
$$i_L(t=0) = \frac{990V}{150\Omega + 750\Omega} = \frac{990V}{900\,\Omega} = 1.1\,A$$

$$w(t=0) = \frac{1}{2}\,55\mu H \cdot (1.1\,A)^2 = 33.275\,\mu J$$

$$\boxed{w \doteq 33.3\ \mu J}$$

Sol'n: 1.b) First, find $i_L(0^-)$.

$t=0^-$: L acts like wire. Switch is open.



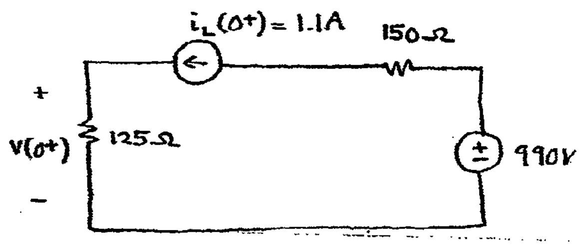
$$i_L(0^-) = \frac{990V}{150+750\Omega} = \frac{990V}{900\Omega} = 1.1\ A$$

Now we find $v(t=0^+)$.

$t=0^+$: $i_L(0^+) = i_L(0^-) = 1.1A$ modeled as i src.

Switch closed.

$v(t)$ is across $150\Omega \| 750\Omega = 125\Omega$



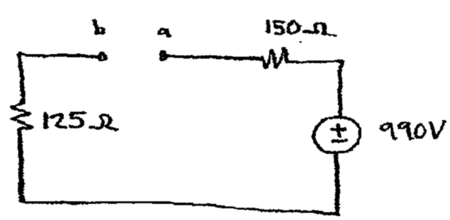
$$v(0^+) = i_L(0^+) \cdot 125\Omega = 1.1A \cdot 125\Omega = 137.5\ V$$

$t \to \infty$:

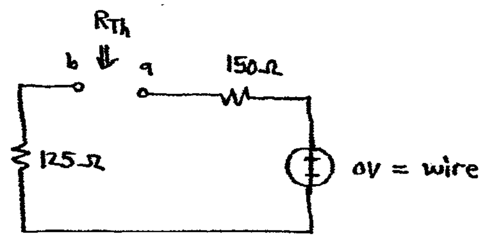
We find $v(t\to\infty)$ from L=wire and $R_{tot} = 150\Omega + 150\Omega \| 750\Omega$

$$v(t\to\infty) = \frac{990V}{R_{tot}} \cdot 125\Omega = \frac{990V}{275}\ 125\Omega = 450\ V$$

We find time constant $\frac{L}{R_{Th}}$ from Thevenin equivalent of circuit where L is connected



Sol'n: 1.b) cont. We turn off the 990V src and look into the a,b terminals to find R_{Th}.



$$R_{Th} = 125\Omega + 150\Omega = 275\Omega$$

$$\frac{L}{R_{Th}} = \frac{55\mu H}{275\Omega} = 0.2\ \mu s = 200\ ns$$

Now plug all values into general sol'n:

$$v(t>0) = v(t\to\infty) + [v(0^+) - v(t\to\infty)]\, e^{-t/L/R_{Th}}$$

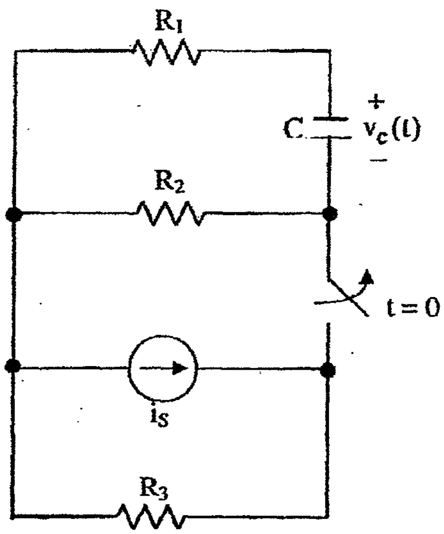
$$v(t>0) = 450V + [137.5 - 450V]\, e^{-t/200ns}$$

$$\boxed{v(t>0) = 450V - 312.5V\, e^{-t/200ns}}$$

HW #6 Cont.

Su 05

2.



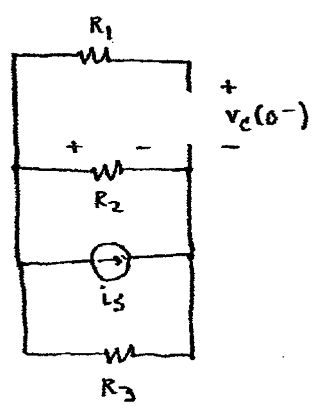
After being closed for a long time, the switch is opened at t = 0.

a. Write an expression for $v_C(t = 0^+)$.

b. Write an expression for $v_C(t > 0)$.

Sol'n: a) $v_c(t=0^+) = v_c(0^-)$

$t=0^-$: C acts like open circuit.
Switch is closed.



No current thru R_1
∴ no v-drop across R_1

It follows that $v_c(0^-)$ equals the v-drop across R_2, measured as shown.

Thus, $v_c(0^-) = -i_s \cdot R_2 \| R_3$.

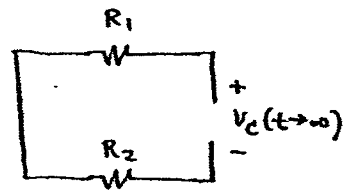
$$\boxed{v_c(t=0^+) = -i_s \, R_2 \| R_3}$$

soln: 2.b) We have $v_c(0^+) = -i_s \cdot R_2 \| R_3$ from 2(a).

Find $v_c(t\to\infty)$ and $R_{Th}C$ to finish soln.

$t\to\infty$: C acts like open circuit.

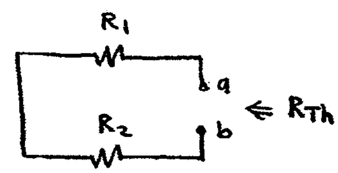
Switch is open. $\therefore$ i_s and R_3 not in circuit



C will discharge to 0V thru R_1 and R_2.

Thus, $v_c(t\to\infty) = 0V$

we find R_{Th} from circuit where C connected.



we see $R_{Th} = R_1 + R_2$.

Plug values into general solution:

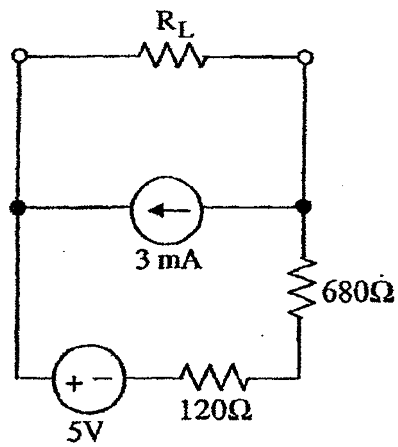
$$v_c(t>0) = v_c(t\to\infty) + \left[v_c(0^+) - v_c(t\to\infty)\right] e^{-t/R_{Th}C}$$

$$v_c(t>0) = 0V + \left[-i_s \cdot R_2 \| R_3 - 0V\right] e^{-t/(R_1+R_2)C}$$

$$\boxed{v_c(t>0) = -i_s \cdot R_2 \| R_3 \; e^{-t/(R_1+R_2)C}}$$

HW #6 Cont. Su 05

3.

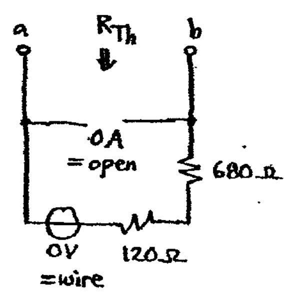


a. Calculate the value of R_L that would absorb maximum power.

b. Calculate that value of maximum power R_L could absorb.

sol'n: a) $R_L = R_{Th}$ for max pwr xfer

Remove R_L and find R_{Th} by turning off independent src's and look into terminals where R_L connected.



$R_{Th} = 680\Omega + 120\Omega$

$R_{Th} = 800\Omega$

$$\boxed{R_L = 800\,\Omega}$$

HW #6 Cont.

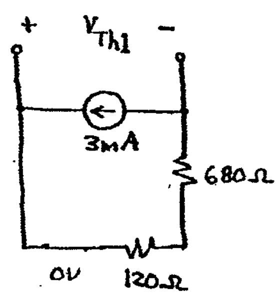
Su 05

soln: 3.b) max pwr $= \frac{V_{Th}^2}{4R_{Th}}$

Find V_{Th} as voltage where R_L connected when R_L is removed.

Use superposition, (or node-voltage or etc.)

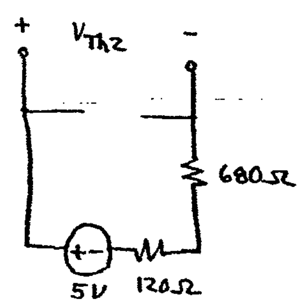
case I: 3 mA on, 5V off = wire



$V_{Th1} = 3\,mA \cdot (680\Omega + 120\Omega)$

$V_{Th1} = 2.4V$

case II: 3 mA off, 5V on = open



No current flows.
∴ no V-drop across R's

$V_{Th2} = 5V$

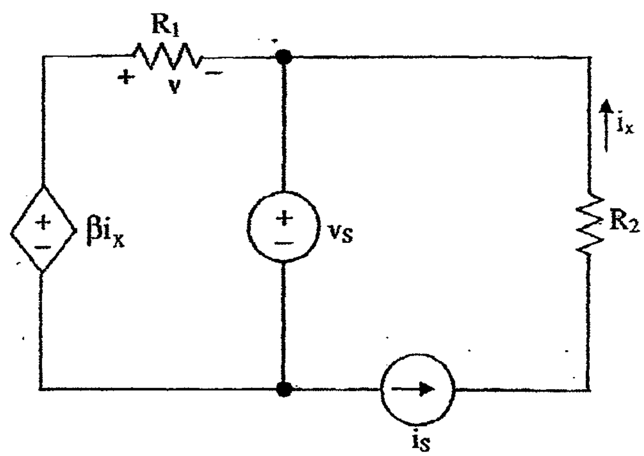
$V_{Th} = V_{Th1} + V_{Th2} = 2.4V + 5V = 7.4V$

max pwr $= \frac{(7.4)^2}{4 \cdot 800\Omega} \doteq 17.1\,mW$

$\boxed{\text{max pwr} \doteq 17.1\ mW}$

HW #6 Cont. Su 05

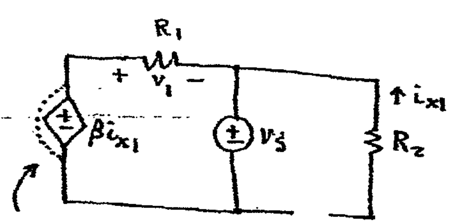
4.



Using superposition, derive an expression for v that contains no circuit quantities other than i_s, v_S, R_1, R_2, and β, where $\beta > 0$.

sol'n: Turn on one independent src at a time.
(Keep dependent sources on at all times.)

case I: v_s on, i_s off = open



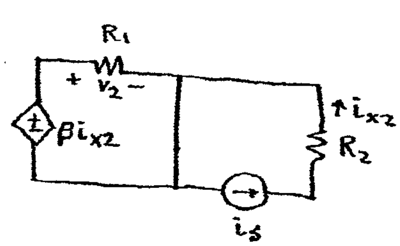
acts like wire

$i_{x1} = 0\,A$ because of open circuit

$\beta i_{x1} = 0V$ = wire

$v_1 = -v_s$ from v loop on left side

case II: v_s off, i_s on
= wire



$i_{x2} = i_s$
$\beta i_{x2} = \beta i_s$

$v_2 = \beta i_{x2} = \beta i_s$ from v loop on left side

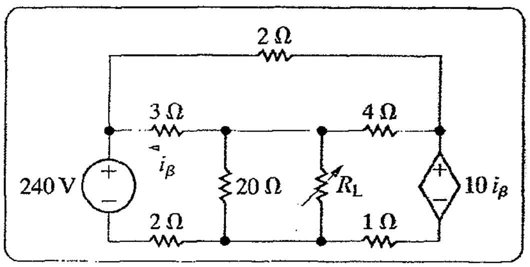
$$\boxed{v = v_1 + v_2 = -v_s + \beta i_s}$$

4.76 The variable resistor (R_L) in the circuit in Fig. P4.76 is adjusted for maximum power transfer to R_L.

a) Find the numerical value of R_L.

b) Find the maximum power transferred to R_L.

Figure P4.76

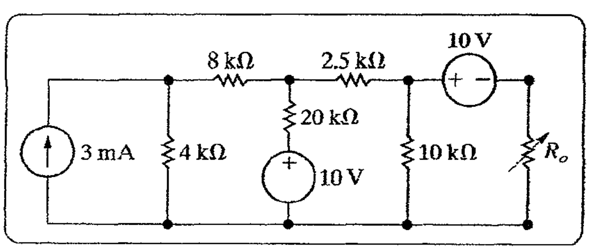


4.77 The variable resistor in the circuit in Fig. P4.77 is adjusted for maximum power transfer to R_o.

a) Find the value of R_o.

b) Find the maximum power that can be delivered to R_o.

Figure P4.77



4.78 What percentage of the total power developed in the circuit in Fig. P4.77 is delivered to R_o when R_o is set for maximum power transfer?

4.79 A variable resistor R_o is connected across the terminals a,b in the circuit in Fig. P4.68. The variable resistor is adjusted until maximum power is transferred to R_o.

a) Find the value of R_o.

b) Find the maximum power delivered to R_o.

c) Find the percentage of the total power developed in the circuit that is delivered to R_o.

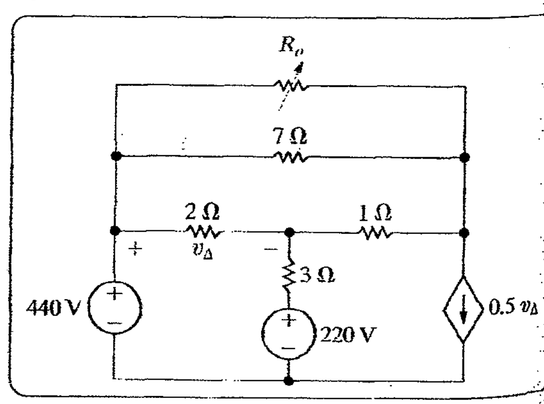
4.80 a) Calculate the power delivered for each value of R_o used in Problem 4.67.

b) Plot the power delivered to R_o versus the resistance R_o.

c) At what value of R_o is the power delivered to R_o a maximum?

4.81 The variable resistor (R_o) in the circuit in Fig. P4.81 is adjusted for maximum power transfer to R_o. What percentage of the total power developed in the circuit is delivered to R_o?

Figure P4.81



$$-240 + 3(i_1 - i_2) + 2i_1 = 0$$
$$2i_2 + 4(i_2 - i_3) + 3(i_2 - i_1) = 0$$
$$10i_\beta + 1i_3 + 4(i_3 - i_2) = 0$$

The dependent source constraint equation is:

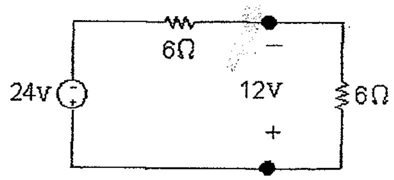
$i_\beta = i_2 - i_1$

Place these equations in standard form:

$$i_1(3+2) + i_2(-3) + i_3(0) + i_\beta(0) = 240$$
$$i_1(-3) + i_2(2+4+3) + i_3(-4) + i_\beta(0) = 0$$
$$i_1(0) + i_2(-4) + i_3(4+1) + i_\beta(10) = 0$$
$$i_1(1) + i_2(-1) + i_3(0) + i_\beta(1) = 0$$

Solving, $i_1 = 92$ A; $i_2 = 73.33$ A; $i_3 = 96$ A; $i_\beta = -18.67$ A

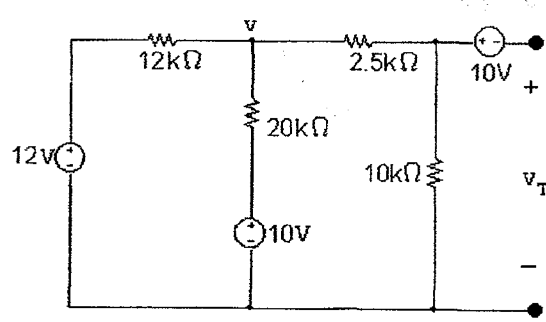
$i_{sc} = i_1 - i_3 = -4$ A; $R_{Th} = \dfrac{V_{Th}}{i_{sc}} = \dfrac{-24}{-4} = 6\,\Omega$



$R_L = R_{Th} = 6\,\Omega$

[b] $p_{max} = \dfrac{12^2}{6} = 24$ W

P 4.77 **[a]**

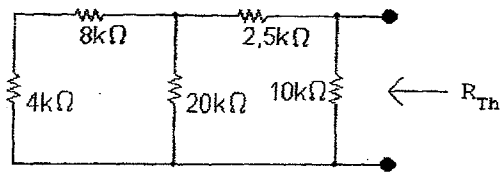


$$\frac{v-12}{12{,}000} + \frac{v-10}{20{,}000} + \frac{v}{12{,}500} = 0$$

Solving, $v = 7.03125$ V

$$v_{10k} = \frac{10{,}000}{12{,}500}(7.03125) = 5.625 \text{ V}$$

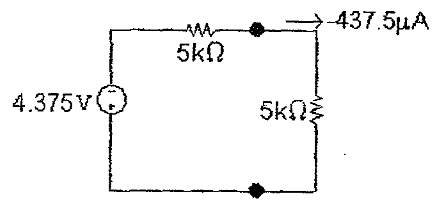
$$\therefore\ V_{Th} = v - 10 = -4.375 \text{ V}$$



$$R_{Th} = [(12{,}000 \| 20{,}000) + 2500] = 5\ \text{k}\Omega$$

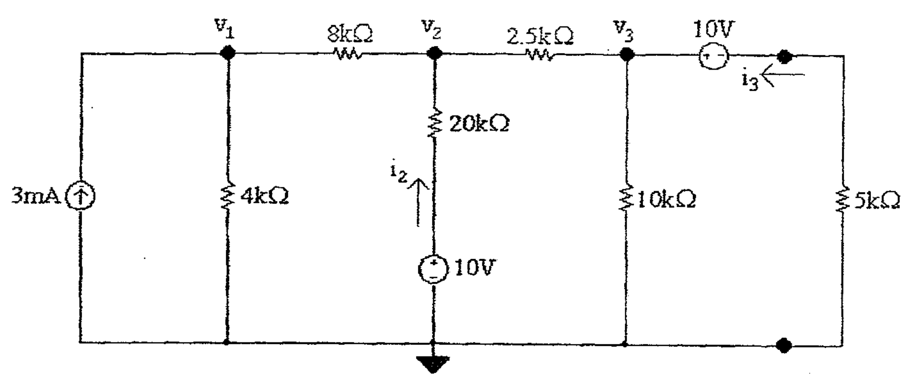
$$R_o = R_{Th} = 5\ \text{k}\Omega$$

[b]



$$p_{max} = (-437.5 \times 10^{-6})^2(5000) = 957.03\,\mu\ \text{W}$$

P 4.78 Write KCL equations at each of the labeled nodes, place them in standard form, and solve:



At v_1: $$-3 \times 10^{-3} + \frac{v_1}{4000} + \frac{v_1 - v_2}{8000} = 0$$

At v_2: $$\frac{v_2 - v_1}{8000} + \frac{v_2 - 10}{20{,}000} + \frac{v_2 - v_3}{2500} = 0$$

At v_3: $$\frac{v_3 - v_2}{2500} + \frac{v_3}{10{,}000} + \frac{v_3 - 10}{5000} = 0$$