Ex. ~~Find I_o~~ (Write Node-V eq.) Problem Session #4

The network is redrawn in Fig. 3.17b in order to label the nodes and identify the supernode. Since the network has six nodes, five linear independent equations are needed to determine the unknown node voltages.

The two equations for the supernode are

$$V_1 - V_4 = -6$$

$$\frac{V_1 - 12}{1\text{k}} + \frac{V_1 - V_3}{1\text{k}} + 2I_x + \frac{V_4 - V_3}{1\text{k}} + \frac{V_4}{1\text{k}} + \frac{V_4 - V_5}{1\text{k}} = 0$$

The three remaining equations are

$$V_2 = 12$$

$$V_3 = 2V_x$$

$$\frac{V_5 - V_4}{1\text{k}} + \frac{V_5}{1\text{k}} = 2I_x$$

The equations for the control parameters are

$$V_x = V_1 - 12$$

$$I_x = \frac{V_4}{1\text{k}}$$

Combining these equations yields the following set of equations

$$-2V_1 + 5V_4 - V_5 = -36$$

$$V_1 - V_4 = -6$$

$$-3V_4 + 2V_5 = 0$$

Solving these equations by any convenient means yields

$$V_1 = -38 \text{ V}$$

$$V_4 = -32 \text{ V}$$

$$V_5 = -48 \text{ V}$$

Then, since $V_3 = 2V_x$, $V_3 = -100$ V. I_o is -48 mA. The reader is encouraged to verify that KCL is satisfied at every node.

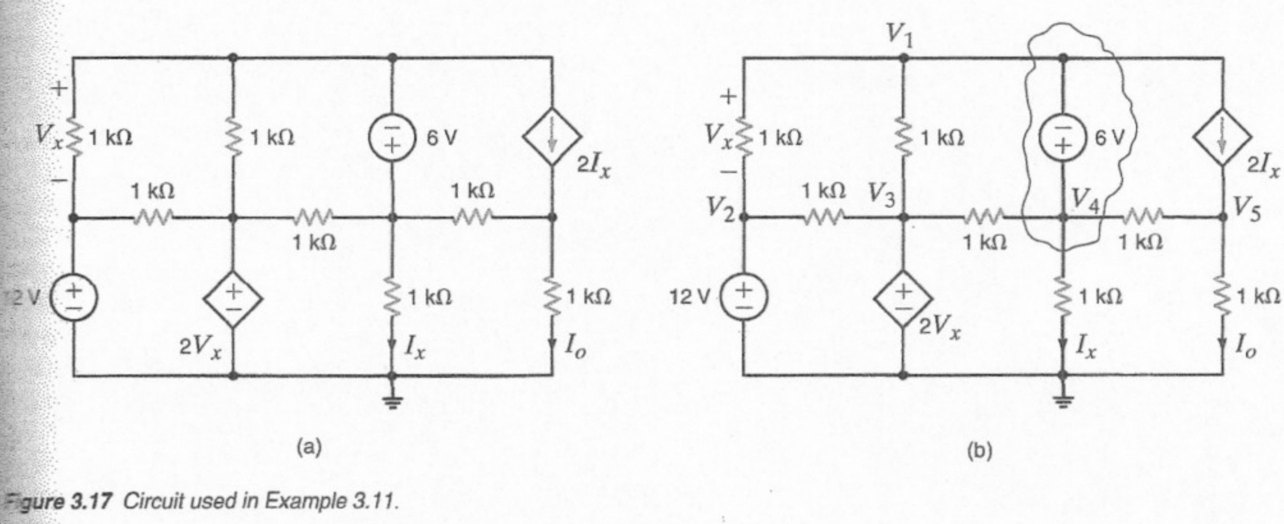


Figure 3.17 Circuit used in Example 3.11.

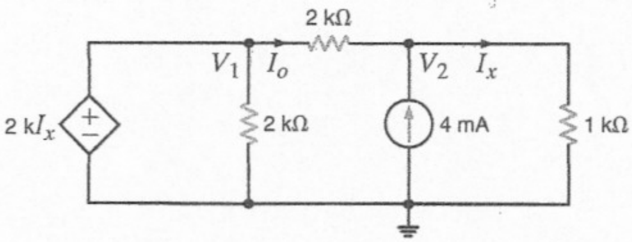


Figure 3.14
Circuit used in Example 3.8.

Example 3.9 — Write eq's

Let us find the current I_o in the network in Fig. 3.15.

SOLUTION This circuit contains both an independent voltage source and a voltage-controlled voltage source. Note that $V_3 = 6$ V, $V_2 = V_x$, and a supernode exists between the nodes labeled V_1 and V_2.

Applying KCL to the supernode, we obtain

$$\frac{V_1 - V_3}{6\text{k}} + \frac{V_1}{12\text{k}} + \frac{V_2}{6\text{k}} + \frac{V_2 - V_3}{12\text{k}} = 0$$

where the constraint equation for the supernode is

$$V_1 - V_2 = 2V_x$$

The final equation is

$$V_3 = 6$$

Solving these equations, we find that

$$V_1 = \frac{9}{2}\text{ V}$$

and, hence,

$$I_o = \frac{V_1}{12\text{k}} = \frac{3}{8}\text{ mA}$$

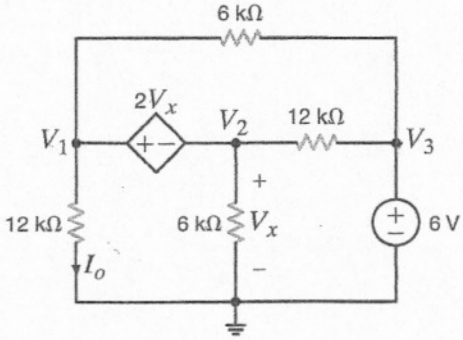


Figure 3.15
Circuit used in Example 3.9.

Finally, let us consider two additional circuits that, for purposes of comparison, we will examine using more than one method.

Example 3.10

Let us find V_o in the network in Fig. 3.16a. Note that the circuit contains two voltage sources, one of which is a controlled source, and two independent current sources. The circuit is redrawn in Fig. 3.16b in order to label the nodes and identify the supernode surrounding the controlled source. Because of the presence of the independent voltage source, the voltage at node 4 is known to be 4 V. We will use this knowledge in writing the node equations for the network.

Example 3.20

(write eq.)

At this point we will again examine the circuit in Example 3.10 and analyze it using loop equations. Recall that because the network has two voltage sources, the nodal analysis was somewhat simplified. In a similar manner, the presence of the current sources should simplify a loop analysis.

Clearly, the network has four loops, and thus four linearly independent equations are required to determine the loop currents. The network is redrawn in Fig. 3.28 where the loop currents are specified. Note that we have drawn one current through each of the independent current sources. This choice of currents simplifies the analysis since two of the four equations are

$$I_1 = 2/\text{k}$$
$$I_3 = -2/\text{k}$$

The two remaining KVL equations for loop currents I_2 and I_4 are

$$-2V_x + 1\text{k}I_2 + (I_2 - I_3)1\text{k} = 0$$
$$(I_4 + I_3 - I_1)1\text{k} - 2V_x + 1\text{k}I_4 + 4 = 0$$

where

$$V_x = 1\text{k}(I_1 - I_3 - I_4)$$

Substituting the equations for I_1 and I_3 into the two KVL equations yields

$$2\text{k}I_2 + 2\text{k}I_4 = 6$$
$$4\text{k}I_4 = 8$$

Solving these equations for I_2 and I_4, we obtain

$$I_4 = 2 \text{ mA}$$
$$I_2 = 1 \text{ mA}$$

and thus

$$V_o = 1\text{V}$$

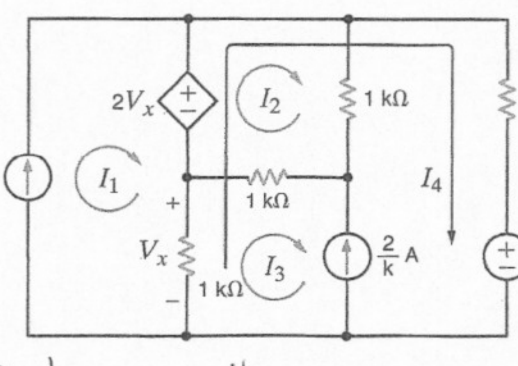


Figure 3.28
Circuit used in Example 3.20.

Example 3.21

★ -write eq.

Let us once again consider Example 3.11. In this case we will examine the network using loop analysis. Although there are four sources, two of which are dependent, only one of them is a current source. Thus, from the outset we expect that a loop analysis will be more difficult than a nodal analysis. Clearly, the circuit contains six loops. Thus, six linearly independent equations are needed to solve for all the unknown currents.

The network is redrawn in Fig. 3.29 where the loops are specified. The six KVL equations that describe the network are

$$1\text{k}I_1 + 1\text{k}(I_1 - I_2) + 1\text{k}(I_1 - I_4) = 0$$
$$1\text{k}(I_2 - I_1) - 6 + 1\text{k}(I_2 - I_5) = 0$$
$$I_3 = 2I_x$$

$$-12 + 1\text{k}(I_4 - I_1) + 2V_x = 0$$
$$-2V_x + 1\text{k}(I_5 - I_2) + 1\text{k}(I_5 - I_o) = 0$$
$$1\text{k}(I_o - I_5) + 1\text{k}(I_o - I_3) + 1\text{k}I_o = 0$$

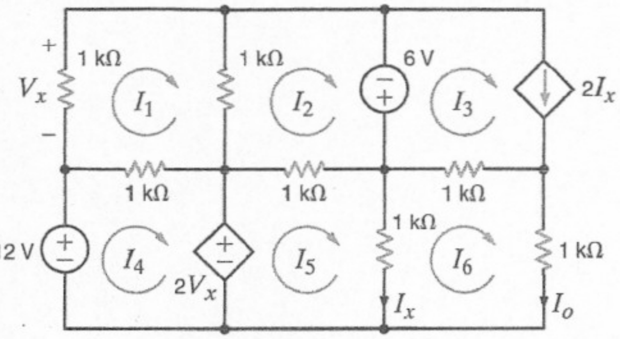


Figure 3.29
Circuit used in Example 3.21.

And the control variables for the two dependent sources are

$$V_x = -1\text{k}I_1$$
$$I_x = I_5 - I_o$$

Substituting the control parameters into the six KVL equations yields

$$\begin{array}{ccccccl}
3I_1 & -I_2 & 0 & -I_4 & 0 & 0 & = 0 \\
-I_1 & +2I_2 & 0 & 0 & -I_5 & 0 & = 6/\text{k} \\
0 & 0 & I_3 & 0 & -2I_5 & +2I_o & = 0 \\
-3I_1 & 0 & 0 & +I_4 & 0 & 0 & = 12/\text{k} \\
2I_1 & -I_2 & 0 & 0 & +2I_5 & -I_o & = 0 \\
0 & 0 & 0 & 0 & -3I_5 & +5I_o & = 0
\end{array}$$

which can be written in matrix form as

$$\begin{bmatrix} 3 & -1 & 0 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -2 & 2 \\ -3 & 0 & 0 & 1 & 0 & 0 \\ 2 & -1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -3 & 5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_o \end{bmatrix} = \begin{bmatrix} 0 \\ 6/\text{k} \\ 0 \\ 12/\text{k} \\ 0 \\ 0 \end{bmatrix}$$

Although these six linearly independent simultaneous equations can be solved by any convenient method, we will employ a MATLAB solution. As the results listed below indicate, the current I_o is −48 mA.

```
>> R = [3 -1 0 -1 0 0 ; -1 2 0 0 -1 0 ; 0 0 1 0 -2 2 ;
-3 0 0 1 0 0 ; 2 -1 0 0 2 -1 ; 0 0 0 0 -3 5]

   R =

       3    -1     0    -1     0     0
      -1     2     0     0    -1     0
       0     0     1     0    -2     2
      -3     0     0     1     0     0
       2    -1     0     0     2    -1
       0     0     0     0    -3     5

>> V = [0; 0.006; 0; 0.012; 0; 0]
```

$V_{Th} = I_1 (2K)$

$\frac{V_1 - 2000 I_x}{2K} + \frac{V_1}{1K} + \frac{V_1}{5K} = 0$

$V_1\left(\frac{1}{2K} + \frac{1}{1K} + \frac{1}{5K}\right) = \frac{2{,}000 I_x}{2K}$

$I_x = \frac{V_1}{1K}$

$V_1 = 0$

$\therefore$ No Voltage, but it still may have a R depending on what is applied between ~~a~~ a-b. Can not use $R_{Th} = \frac{V_{Th}}{I_{SC}}$!

Solving the equations for V_x yields $V_x = 3/7$ V. Knowing V_x, we can compute the currents I_1, I_2, and I_3. Their values are

$$I_1 = \frac{V_x}{1\text{k}} = \frac{3}{7}\text{ mA}$$

$$I_2 = \frac{1 - 2V_x}{1\text{k}} = \frac{1}{7}\text{ mA}$$

$$I_3 = \frac{1}{2\text{k}} = \frac{1}{2}\text{ mA}$$

Therefore,

$$I_o = I_1 + I_2 + I_3 = \frac{15}{14}\text{ mA}$$

and

$$R_{\text{Th}} = \frac{1}{I_o} = \frac{14}{15}\text{ k}\Omega$$

Example 5.10 - Find Thevenin Equivalent

~~Let us determine R_{Th}~~ at the terminals *A-B* for the network in Fig. 5.12a.

SOLUTION Our approach to this problem will be to apply a 1-mA current source at the terminals *A-B* and compute the terminal voltage V_2 as shown in Fig. 5.12b. Then $R_{\text{Th}} = V_2/0.001$.

The node equations for the network are

$$\frac{V_1 - 2000I_x}{2\text{k}} + \frac{V_1}{1\text{k}} + \frac{V_1 - V_2}{3\text{k}} = 0$$

$$\frac{V_2 - V_1}{3\text{k}} + \frac{V_2}{2\text{k}} = 1 \times 10^{-3}$$

and

$$I_x = \frac{V_1}{1\text{k}}$$

Solving these equations yields

$$V_2 = \frac{10}{7}\text{ V} = \cancel{V_{Th}}$$

and hence,

$$R_{\text{Th}} = \frac{V_2}{1 \times 10^{-3}} = \frac{10}{7}\text{ k}\Omega$$

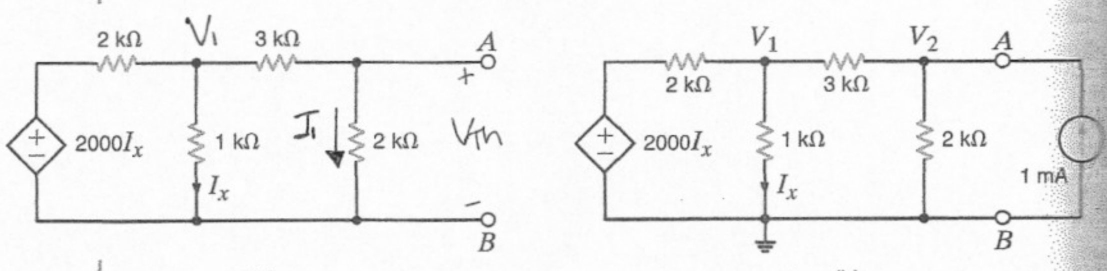
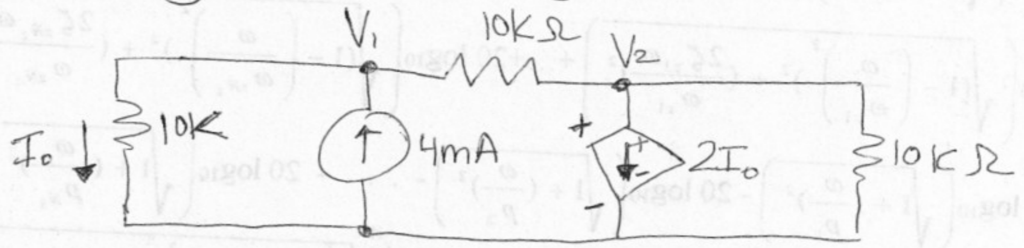


Figure 5.12 Networks used in Example 5.10.

EX

Find power in $2I_o$ dependent src. Is it absorbing or supplying?



① $\dfrac{V_1}{10K} - 4m + \dfrac{(V_1 - V_2)}{10K} = 0$

② $\dfrac{(V_2 - V_1)}{10K} + 2I_o + \dfrac{V_2}{10K} = 0$

$I_o = \dfrac{V_1}{10K}$ {plug into ②}

$\therefore \dfrac{(V_2 - V_1)}{10K} + \dfrac{2V_1}{10K} + \dfrac{V_2}{10K} = 0$

$(V_2 - V_1) + 2V_1 + V_2 = 0$

$2V_2 + V_1 = 0 \Rightarrow V_1 = -2V_2$ {plug into ①}

$\dfrac{-2V_2}{10K} - 4m + \dfrac{-2V_2}{10K} - \dfrac{V_2}{10K} = 0$

$\dfrac{-5V_2}{10K} = +4m$

$V_2 = \dfrac{-4m(10K)}{5} = -8V \quad \therefore V_1 = -2(-8) = +16V$

power $= I * V = (2I_o) * V_2 = 2\left(\dfrac{V_1}{10K}\right)(-8) = 2\left(\dfrac{+16}{10K}\right)(-8)$

$= \boxed{-25.6mW}$

absorbing