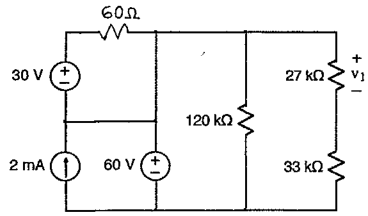
problem Set #2

PS2

Ex:



a) Calculate v_1.

sol'n: We have voltage divider consisting of the 60V src and the 27kΩ and 33kΩ resistors.

$$v_1 = 60V \cdot \frac{27k\Omega}{27k\Omega + 33k\Omega}$$

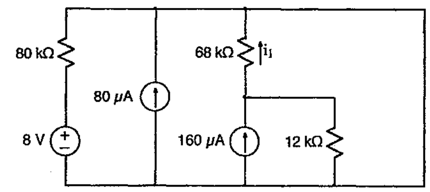
$$= 60V \frac{27k\Omega}{60k\Omega}$$

$$v_1 = 27V$$

Note: Here, we have several circuits in parallel across the 60V src: the 2mA src, the 120kΩ resistor, and the 27kΩ and 33kΩ resistors. We can solve each of these circuits separately, as though they each had their own 60V src.

The 30V src and 60Ω resistor are isolated by wires from the rest of the circuit

Ex:



b) Calculate i_1.

sol'n: We have a current divider consisting of the 160 μA src and the 68kΩ and 12 kΩ resistors.

$$i_1 = 160\,\mu A \cdot \frac{12\,k\Omega}{12k\Omega + 68k\Omega}$$

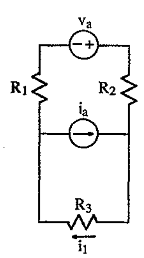
$$= 160\,\mu A \cdot \frac{12k\Omega}{80k\Omega}$$

$$i_1 = 24\,\mu A$$

Note: The 12kΩ and 68kΩ resistors are parallel since they connected by wires at either end.

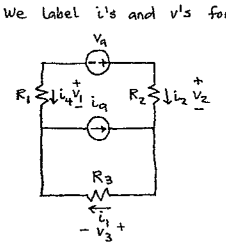
We know the total current flowing into the 12kΩ and 68kΩ resistors, and the resistors have the same voltage across them.

Ex:



Derive an expression for i_1. The expression must not contain more than the circuit parameters v_a, i_a, R_1, R_2, and R_3.

sol'n: We label i's and v's for R's.



We have a v-loop around the outside. (Other loops would include i-src, i_a.)

$$+v_1 + v_a - v_2 - v_3 = 0V$$

We could write an i-sum eq'n for either the node on the left or right. We only need one node. The other node is redundant.

If we use the node on the left, our eq'n is

$$-i_4 + i_a - i_1 = 0A$$

We look for components in series carrying the same current. R_1 and R_2 are in series.

$$i_4 = -i_2$$

From Ohm's law, we have

$$v_1 = i_4 R_1 = -i_2 R_1$$

$$v_2 = i_2 R_2$$

$$v_3 = i_1 R_3$$

Substituting Ohm's law and $i_4 = -i_2$ into our v-loop and i-sum eq'ns gives

$$-i_2 R_1 + v_a - i_2 R_2 - i_1 R_3 = 0V$$

$$i_2 + i_a - i_1 = 0A$$

Solving the second eq'n for i_2 gives

$$i_2 = i_1 - i_a$$

Substituting for i_2 in the first eq'n gives

$$-(i_1 - i_q)(R_1 + R_2) + v_q - i_1 R_3 = 0V$$

or $$-i_1(R_1 + R_2 + R_3) = -i_q(R_1 + R_2) - v_q$$

or $$i_1 = \frac{i_q(R_1 + R_2) + v_q}{R_1 + R_2 + R_3}$$

Consistency checks:

Set $i_q = 0A$. $i_1 = \dfrac{v_q}{R_1 + R_2 + R_3}$

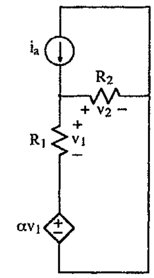
Eq'n gives $i_1 = \dfrac{\cancel{0(R_1 + R_2)} + v_q}{R_1 + R_2 + R_3}$ ✓

Set $v_q = 0V$. $i_1 = \dfrac{i_q(R_1 + R_2)}{R_1 + R_2 + R_3}$

(current divider)

Eq'n gives $i_1 = \dfrac{i_q(R_1 + R_2) + 0V}{R_1 + R_2 + R_3}$ ✓

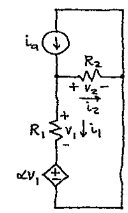
EX:



a) Derive an expression for v_2. The expression must not contain more than the circuit parameters α, i_a, R_1, and R_2. **Note:** $\alpha > 0$.

b) Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

sol'n: a) Label resistor currents.



We have a v-loop on the bottom:

$$+\alpha v_1 + v_1 - v_2 = 0V$$

All other loops would pass thru a current src.

We can write i-sum eq'ns for the node on the left or right but we only need one of them.

For the left node, our i-sum eq'n is

$$-i_q + i_2 + i_1 = 0A$$

We have no components in series carrying the same current, (other than a v-src and R).

Last, we write Ohm's law eq'ns for R_1 and R_2:

$$v_1 = i_1 R_1$$

$$v_2 = i_2 R_2$$

Substituting the Ohm's law expressions into the v-loop eq'n gives

$$\alpha i_1 R_1 + i_1 R_1 - i_2 R_2 = 0V$$

or $$i_1(\alpha + 1) R_1 - i_2 R_2 = 0V$$

Now we solve our i-sum eq'n for i_1. (This is convenient since we will retain i_2 and be able to write an eq'n for v_2.)

$$i_1 = i_q - i_2$$

Substituting into our last v-loop eq'n, we have

$$(i_q - i_2)(\alpha + 1) R_1 - i_2 R_2 = 0V$$

We simplify and solve for i_2:

$$i_q(\alpha + 1) R_1 = i_2[(\alpha + 1) R_1 + R_2]$$

or $$i_2 = i_q \frac{(\alpha + 1) R_1}{(\alpha + 1) R_1 + R_2}$$

b) Many consistency checks are possible.

ex: Set $\alpha = 0$ so dependent v-src becomes a wire. Circuit becomes i-divider.

$$i_2 = i_q \frac{R_1}{R_1 + R_2}$$

Plug $\alpha = 0$ into answer from (a):

$$i_2 = i_q \frac{(0+1) R_1}{(0+1) R_1 + R_2} = i_q \frac{R_1}{R_1 + R_2}$$ ✓

(We get the same answer.)

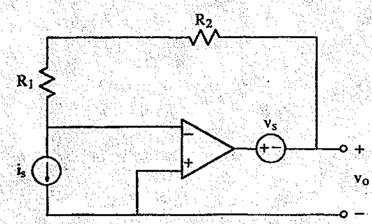
ex: Set $R_2 = 0\Omega$ (a wire). All current will flow thru the wire since it is the path of least resistance.

$$\therefore i_2 = i_q$$

Plug $R_2 = 0\Omega$ into answer from (a):

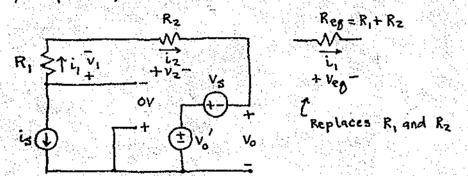
$$i_2 = i_q \frac{(\alpha + 1) R_1}{(\alpha + 1) R_1 + 0\Omega} = i_q$$ ✓ (same answer)

EX:



The op-amp operates in the linear mode. Using an appropriate model of the op-amp, derive an expression for v_o in terms of not more than v_s, i_s, R_1, and R_2.

sol'n: We assume a 0V drop across the + and − inputs of the op-amp, and we replace the op-amp output with a src called v_o'.



We may combine R_1 and R_2 into one resistor: $R_{eg} = R_1 + R_2$.

R_{eg} carries current i_1 and has v-drop $v_{eg} = i_1 R_{eg}$.

We now look for v-loops, including those passing thru the 0V drop across the op-amp inputs.

The only v-loop not containing an i-src is on the right side.

$$-0V - v_{eg} + v_s - v_o' = 0V$$

If we ignore dangling wires, we have no nodes where three or more wires are connected.

Thus, we have no i-sum eq'ns for nodes.

We look for components in series that carry the same current.

$$i_1 = -i_s$$

Finally, we use Ohm's law for R_{eg}:

$$v_{eg} = i_1 R_{eg}$$

Since $i_1 = -i_s$, we have

$$v_{eg} = -i_s R_{eg}.$$

Using our earlier v-loop eq'n, we can solve for v_o':

$$v_o' = -v_{eg} + v_s = i_s R_{eg} + v_s$$

We use a v-loop to find v_o from v_o':

$$+v_o' - v_s - v_o = 0V$$

or $v_o = v_o' - v_s$

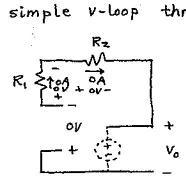
or $v_o = i_s R_{eg} + v_s - v_s$

or $v_o = i_s R_{eg}$

Consistency check:

Let $i_s = 0A$. Then no current flows in R_1 and R_2. Thus, there is no v-drop across R_1 and R_2.

We have a simple v-loop thru v_o:



$$-0V - 0V - 0V - v_o = 0V$$

Thus $v_o = 0V$.

Plugging $i_s = 0A$ into $v_o = i_s R_{eg}$ gives $v_o = 0$ ✓