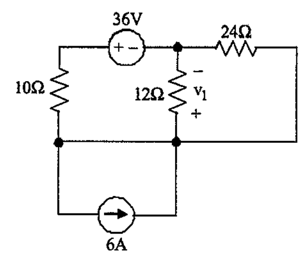
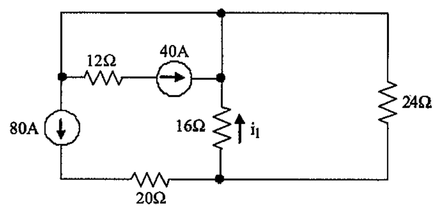
Problem Session #1 Problems:

1. Calculate v_1.



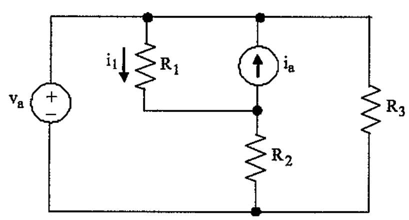
Solution: $v_1 = 16V$

2. Calculate i_1.



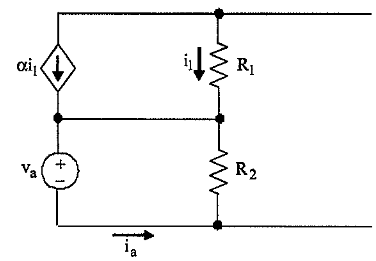
Solution: $i_1 = 48A$

3. Derive an expression for i_1. The expression must not contain more than the circuit parameters v_a, i_a, R_1, R_2, and R_3.



Solution: $i_1 = \frac{v_a}{R_1 + R_2} + i_a \frac{R_2}{R_1 + R_2}$

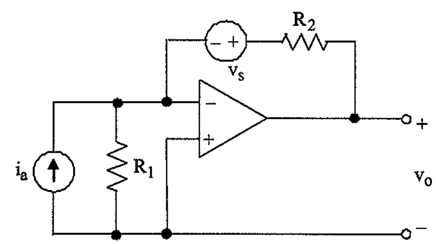
4. Derive an expression for i_a. The expression must not contain more than the circuit parameters α, v_a, R_1, and R_2.



Solution: $i_a = -\frac{v_a}{R_2 \| \frac{R_1}{1+\alpha}}$

Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

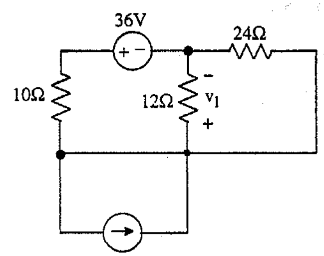
5. The op-amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for v_o in terms of not more than v_s, i_a, R_1, and R_2.



Solution: $v_o = v_s - i_a R_2$

1. a. (5 points)

Calculate v_1.



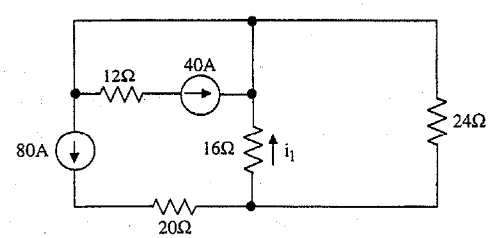
sol'n: The 6A source across the wire may be ignored. Its current flows through the wire but produces no V-drop. Without the 6A src we have a V-divider:

$v_1 = 36V \cdot 12\Omega \| 24\Omega \,/\, 12\Omega\|24\Omega + 10\Omega = 36V \cdot 8\Omega / 18\Omega = 16V$

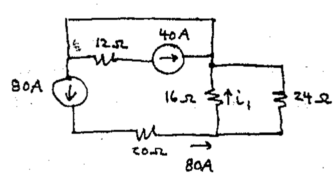
$\boxed{v_1 = 16V}$

b. (5 points)

Calculate i_1.



sol'n: If we redraw the circuit, we see a current divider:

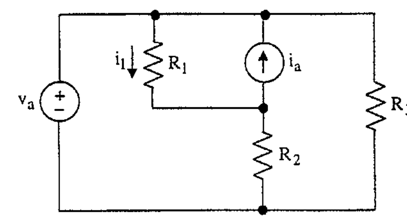


$i_1 = 80A \cdot \frac{24\Omega}{16+24\Omega} = 80A \cdot \frac{24\Omega}{40\Omega}$

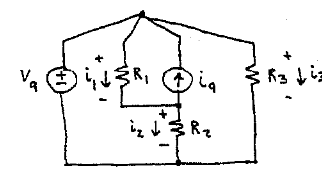
$\boxed{i_1 = 48A}$

2. (30 points)

Derive an expression for i_1. The expression must not contain more than the circuit parameters v_a, i_a, R_1, R_2, and R_3.



sol'n: Redraw with top as one node:



Current sum at top or bottom node? No, because we would have to define a current for source v_a.

Current at center node: $i_a - i_1 + i_2 = 0A$

V-loop around left inner loop: $v_a - i_1R_1 - i_2R_2 = 0V$

No V-loop for other inner loops because we would have to define V-drop for i_a.

Next larger loop is R_1, R_2, R_3: $i_2R_2 + i_1R_1 - i_3R_3 = 0V$

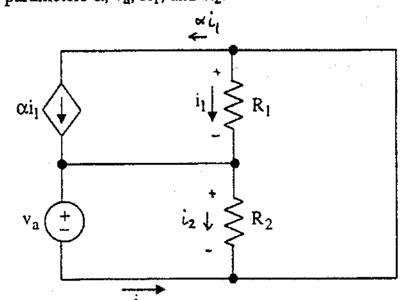
Now we have 3 eq'ns in 3 unknowns, and we want to find i_1. We observe, however, that the first two eq'ns have only two unknowns. So we don't actually need the 3rd eq'n. Use 1st eq'n to find $i_2 = i_1 - i_a$.

Substitute into 2nd eq'n: $v_a - i_1R_1 - (i_1 - i_a)R_2 = 0V$

or $i_1(-R_1 - R_2) = -v_a - i_aR_2$ or $\boxed{i_1 = \frac{v_a + i_aR_2}{R_1 + R_2}}$

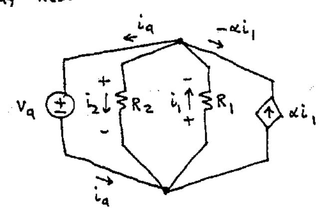
3. (30 points)

a. Derive an expression for i_a. The expression must not contain more than the circuit parameters α, v_a, R_1, and R_2.



b. Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

soln: a) Redraw circuit



No current sums at nodes because of v_a.

V- loop on left: $v_a - i_2 R_2 = 0V \Rightarrow i_2 = \frac{v_a}{R_2}$

V- loop in middle: $i_2 R_2 + i_1 R_1 = 0V \Rightarrow i_1 = -\frac{v_a}{R_1}$, where $i_2 R_2 = \frac{v_a}{R_2} R_2$

we could also just observe that v_a is across R_1 and R_2.

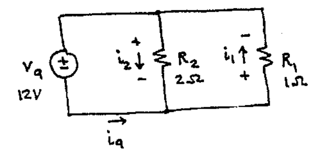
Now that we have found i_1 and i_2, we use a current at top node to find i_a:

$i_a + i_2 - i_1 - \alpha i_1 = 0A$ or $i_a + \frac{v_a}{R_2} + \frac{v_a}{R_1} + \alpha\frac{v_a}{R_1} =$

or $\boxed{i_a = -v_a\left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{\alpha}{R_1}\right)}$ or $i_a = -\frac{v_a}{R_1 \| \frac{R_1}{\alpha} \| R_2}$

soln: 3.b) Many possible answers.

Example: Suppose $\alpha = 0$. Choose other simple values:



we see that i_a is current thru $R_1 \| R_2$ with $R_1 \| R_2$ across $-v_a$.

$R_1 \| R_2 = 1 \| 2\ \Omega = \frac{1 \cdot 2}{1+2} = \frac{2}{3}\Omega$

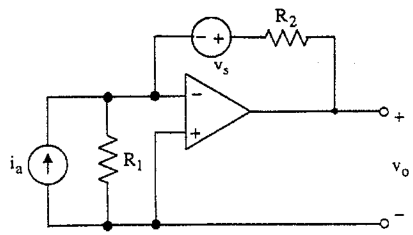
$\therefore i_a = -v_a / R_1 \| R_2 = -\frac{12V}{\frac{2}{3}\Omega} = -12V \cdot \frac{3}{2\Omega} = -18A$

Use formula from (a) with these component values: $i_a = -12V\left(\frac{1}{2\Omega} + \frac{1}{1\Omega} + \frac{0}{1\Omega}\right) = -12V \cdot \frac{3}{2}$

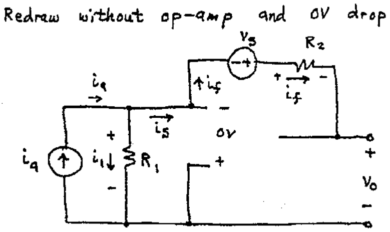
$i_a = -18V$ ✓ agrees with obvious soln for this simple case

4. (30 points)

The op-amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for v_o in terms of not more than v_s, i_a, R_1, and R_2.



sol'n: Redraw without op-amp and 0V drop across + and − inputs:



V-loop on left thru R_1 and 0V drop:

$$i_1 R_1 + 0V = 0V \quad \text{or} \quad i_1 = 0$$

Current sum at node above R_1:

$$-i_a + \underset{0}{\underset{\parallel}{i_1}} + i_s = 0A \quad \text{or} \quad i_s = i_a$$

V-loop on right thru 0V drop, v_s, R_2, and v_o:

$$-0V + v_s - i_f R_2 - v_o = 0V \quad \text{or} \quad i_f = \frac{v_s - v_o}{R_2}$$

Now use $i_s = i_f$.

$$i_a = \frac{v_s - v_o}{R_2}$$

Thus $v_s - v_o = i_a R_2$

or $\boxed{v_o = v_s - i_a R_2}$