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Homework #8 Example Dr. Neil Cotter

1.

Give numerical answers to each of the following questions:

pts

a. Rationalize $\dfrac{3+j4}{7-j24}$. Express your answer in rectangular form.

b. Find the rectangular form of $\left[\dfrac{1-\sqrt{2}\,e^{-j45^\circ}}{e^{j30^\circ}}\right]^*$. Note the asterisk that means "conjugate".

c. Given $\omega = 100k$ rad/s, find the following inverse phasor: $P^{-1}\left[j30\sin\left(-53^\circ\right)\right]$

d. Find the magnitude of $\left(2e^{j30^\circ} - j\right)\left(\dfrac{5-j12}{e^{j17^\circ}}\right)$.

e. Find the real part of $\dfrac{1}{e^{-j30^\circ}}$.

soln: a) $\dfrac{3+j4}{7-j24}\,\dfrac{7+j24}{7+j24} = \dfrac{21-4(24)+j\,3(24)+j\,4(7)}{7^2+24^2} = \dfrac{-75+j\,100}{25^2} = \boxed{\dfrac{-3+j4}{25}}$

b) $\left[\dfrac{1-\sqrt{2}\,e^{-j45^\circ}}{e^{j30^\circ}}\right]^* = \dfrac{1-\sqrt{2}\,e^{j45^\circ}}{e^{-j30^\circ}}\cdot\dfrac{e^{j30^\circ}}{e^{j30^\circ}} = \left[1-\sqrt{2}\dfrac{(1+j)}{\sqrt{2}}\right]\cdot e^{j30^\circ} = -j\,e^{j30^\circ}$

$= e^{-j90^\circ}e^{j30^\circ} = e^{-j60^\circ} = \boxed{+\frac{1}{2} - j\frac{\sqrt{3}}{2}}$

c) $P^{-1}[j30\sin(-53^\circ)] = P^{-1}[j30\,j\cdot 1\angle -53^\circ = -30\angle -53^\circ = 30\angle 137^\circ]$

$= \boxed{30\cos(100kt + 137^\circ) \text{ or } 30\cos(100kt - 233^\circ)}$

d) $\left|\left(2e^{j30^\circ} - j\right)\dfrac{5-j12}{e^{j17^\circ}}\right| = \left|2e^{j30^\circ}-j\right|\dfrac{|5-j12|}{|e^{j17^\circ}|} = \left|\cancel{2}\left(\dfrac{\sqrt{3}}{\cancel{2}} + \cancel{\dfrac{j}{2}}\right)\right|\cdot\dfrac{13}{1} = \boxed{\sqrt{3}\cdot 13}$

e) $\mathrm{Re}\left[\dfrac{1}{e^{-j30^\circ}}\right] = \mathrm{Re}\left[e^{j30^\circ}\right] = \mathrm{Re}\left[\dfrac{\sqrt{3}}{2} + j\dfrac{1}{2}\right] = \boxed{\dfrac{\sqrt{3}}{2}}$

Homework #8 Examples

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Give numerical answers to each of the following questions:

1. a. Rationalize $\frac{23+j7}{15-j8}$. Express your answer in rectangular form.

   b. Find the polar form of $(2+j3)(3+j2)\div[3+j16]^{*}$. Note the asterisk that means "conjugate".

   c. Find the following phasor: $P\left[-5\sin\left(100t-30^{\circ}\right)\right]$.

   d. Find the magnitude of $\frac{100(3+j4)(4+j3)}{(7+j)(7-j)}$.

   e. Find the imaginary part of $(1+j)e^{-j45^{\circ}}(j2)$.

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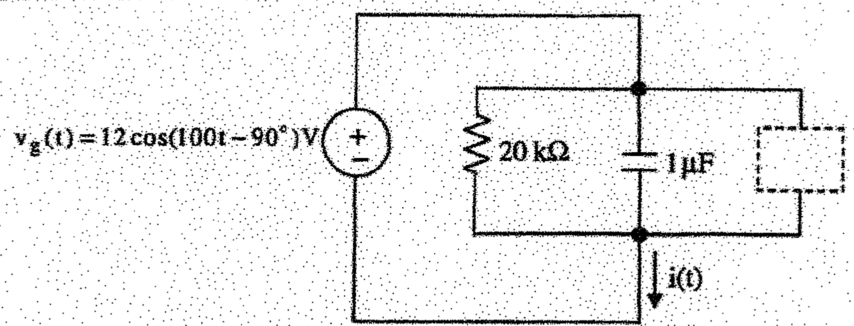
2.



pts

a. Choose an R, an L, or a C to be placed in the dashed-line box to make

$$i(t) = \mathbf{I}_0 \cos(100t - 45°)\,A$$

where $\mathbf{I}_0$ is a real constant. State the value of the component you choose.

b. With your component from (a) in the circuit, calculate the resulting value of $\mathbf{I}_0$.

soln: a) Use conductance: (and phasors) $\mathbf{I} = \mathbf{I}_0 \angle -45° A = V_g \cdot \underbrace{\left(\frac{1}{20k\Omega} + j100 \cdot \frac{1\mu}{\Omega} + \frac{1}{Z_{box}}\right)}_{G_{tot}}$

Note: $\omega = 100$ from $v_g(t)$ where $V_g = 12\angle -90°$ V

We have $\angle \mathbf{I} = \angle V_g + \angle G_{tot}$ from phasor multiplication

$$-45° = -90° + \angle G_{tot}$$

$\therefore \angle G_{tot} = 45°$ or $Re[G_{tot}] = Im[G_{tot}]$

$$G_{tot} = 50\frac{\mu}{\Omega} + j\frac{100\mu}{\Omega} + \frac{1}{Z_{box}}$$

we can choose $\frac{1}{Z_{box}} = 50\frac{\mu}{\Omega}$ $\Rightarrow$ $\boxed{Z_{box} = 20k\Omega \text{ resistor}}$

or $\frac{1}{Z_{box}} = -j\frac{50\mu}{\Omega} = \frac{-j}{\omega L} = \frac{-j}{100 \cdot L}$

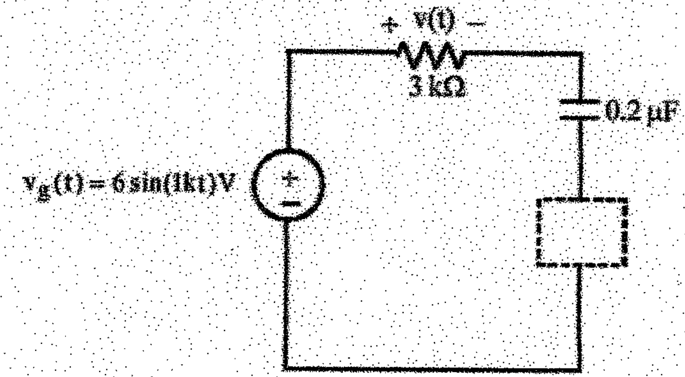
Note: Either answer accepted but 20kΩ R is more sensible. $\Rightarrow$ $\boxed{Z_{box} = 200 \text{ H inductor}}$

b) $\mathbf{I}_0 = |\mathbf{I}| = |V_g| \cdot |G_{tot}| = 12 \cdot \sqrt{2} \cdot 100\mu A = \boxed{\sqrt{2}\ 1.2 mA \text{ for } 20k\Omega\ R}$

or $12 \cdot \sqrt{2} \cdot 50\mu A = \boxed{\sqrt{2}\ 600\mu A \text{ for } 200H\ L}$

Homework #8 Examples

2.



Choose an R, an L, or a C to be placed in the dashed-line box to make

$$v(t) = V_o \cos(1kt - 45^\circ)\text{V}$$

where V_o is a real constant. State the value of the component you choose.

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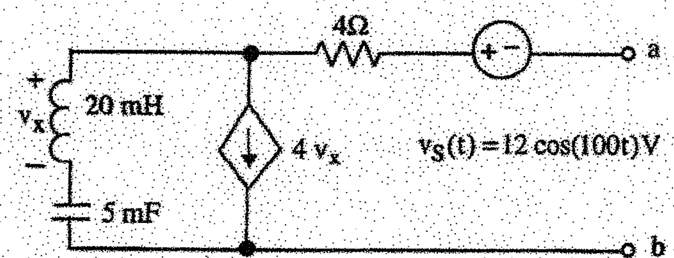
3.



pts

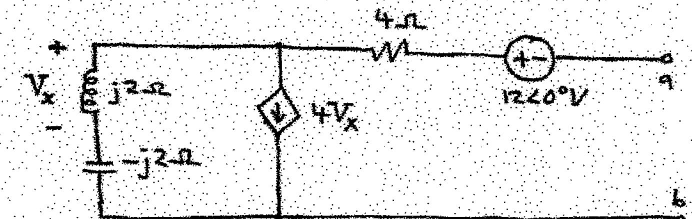
a. Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $v_S(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

b. Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for V_{Th} and the numerical impedance value of z_{Th}.

sol'n: a) $\omega = 100$ from $v_s(t)$ $\quad j\omega L = j\,100 \cdot 20\,m\,\Omega = j2\,\Omega$

$$\frac{-j}{\omega C} = \frac{-j}{100 \cdot 5m}\,\Omega = \frac{-j}{500m} = -j2\,\Omega$$

phasor $V_s \equiv P[12\cos(100t)]\,V = 12\angle 0°\,V$



b) $V_{Th} = V_{a,b}$ with no load.

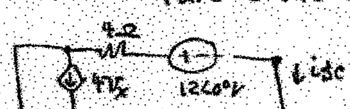
We have $z_L + z_C = 0\,\Omega$ so 0V across L & C together.

Also, no current in 4Ω ⇒ 0V across 4Ω.

Add the −12V for v-src to get $V_{Th} = -12V$

For z_{Th}, short a,b and measure i out of a terminal.

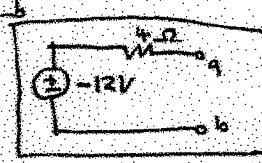
Circuit model:



4V_x irrelevant

$$i_{sc} = \frac{-12V}{4\Omega} = -3A$$

$$z_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{-12V}{-3A} = 4\,\Omega$$



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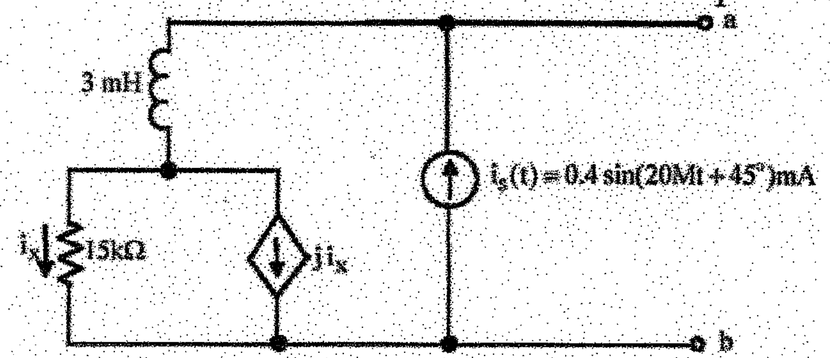
Homework #8 Examples

3.



Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for V_{Th} and the numerical impedance value of z_{Th}.

Example

EX: Give numerical answers to each of the following questions:

a) Rationalize $\frac{-25j}{3-4j}$. Express your answer in rectangular form.

b) Find the rectangular form of $\left[\frac{(1+j)}{e^{j30^\circ}}\right]\left[\frac{(1+j)}{e^{-j60^\circ}}\right]^*$. (Note the asterisk that means "conjugate".)

c) Given $\omega = 2\pi$ rad/s, find the following inverse phasor: $P^{-1}\ [10(-0.866-0.5j)]$

d) Find the magnitude of $\frac{\left(4e^{j30^\circ}-\frac{1}{2}j\right)(-1-j)}{\sqrt{2}e^{j10^\circ}}$.

e) Find the real part of $\frac{e^{-2}}{e^{-j45^\circ}}$.

Sol'n: a) $\frac{-25j}{3-4j}\cdot\frac{3+4j}{3+4j}$ multiply top & bottom by complex conjugate of denominator

$$= \frac{(-25j)(4j)+(-25j)(3)}{3^2+4^2}$$

$$= \frac{100-75j}{25}$$

$$= 4-3j$$

b) $\frac{1+j}{e^{j30^\circ}}\left[\frac{1+j}{e^{-j60^\circ}}\right]^* = \frac{1+j}{e^{j30^\circ}}\cdot\frac{1-j}{e^{j60^\circ}} = \frac{1^2+1^2}{e^{j90^\circ}}$

But $e^{j90^\circ} = j$ and $1/j = -j$

$\therefore$ our answer is $-j2$

c) $P^{-1}[10(-0.866-0.5j)]$

$$= P^{-1}\left[10\sqrt{0.866^2+0.5^2}\, e^{j\tan^{-1}\left(\frac{-0.5}{-0.866}\right)}\right]$$

$$= P^{-1}\left[10\cdot 1\, e^{j(-150^\circ)}\right]$$

$$= P^{-1}\left[10e^{-j150^\circ}\right]$$

$$= 10\cos(\omega t-150^\circ)$$

$$= 10\cos(2\pi t-150^\circ)$$

d) $\left|\frac{\left(4e^{j30^\circ}-j\frac{1}{2}\right)(-1-j)}{\sqrt{2}e^{j10^\circ}}\right| = \frac{\left|4e^{j30^\circ}-j\frac{1}{2}\right|\left|-1-j\right|}{\left|\sqrt{2}e^{j10^\circ}\right|}$

Magnitude of product = product of magnitudes

Now use $\left|e^{jx}\right| = 1$ for any real x, and $|a+jb| = \sqrt{a^2+b^2}$.

$$= \frac{\left|4e^{j30^\circ}-\right|\cancel{\sqrt{1^2+1^2}}}{\cancel{\sqrt{2}}} = \left|4\cos 30^\circ + j4\sin 30^\circ - \frac{j}{2}\right|$$

$$= \left|4\frac{\sqrt{3}}{2}+j4\frac{1}{2}-\frac{j}{2}\right| = \left|2\sqrt{3}+j\frac{3}{2}\right|$$

$$= \sqrt{(2\sqrt{3})^2+\left(\frac{3}{2}\right)^2} = \sqrt{4(3)+\frac{9}{4}} = \sqrt{14\frac{1}{4}} \text{ or } \sqrt{\frac{57}{4}} = \frac{\sqrt{57}}{4}$$

e) $\mathrm{Re}\left[\dfrac{e^{-2}}{e^{-j45°}}\right] = \mathrm{Re}\left[\dfrac{e^{-2}}{e^{-j45°}}\dfrac{e^{j45°}}{e^{j45°}}\right]$

$= \mathrm{Re}\left[e^{-2}e^{j45°}\right]$ since $e^{j0°} = 1$ in denominator

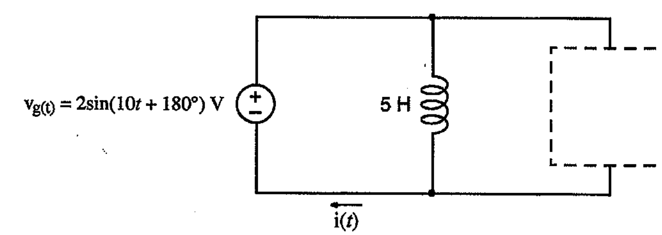
$= e^{-2}\,\mathrm{Re}\left[e^{j45°}\right]$ we can take positive real constants outside.

$= e^{-2}\,\mathrm{Re}\left[\cos 45° + j\sin 45°\right]$

$= e^{-2}\,\mathrm{Re}\left[\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right]$

$= e^{-2}\cdot\frac{1}{\sqrt{2}}$ since $\mathrm{Re}[a+jb] = a$

Ex:



a) Choose an R, an L, or a C to be placed in the dashed-line box to make

$$i(t) = I_0\cos(10t + 45°)\text{ A}$$

where I_0 is a real constant. State the value of the component you choose.

b) With your component in the circuit, calculate the resulting value of I_0.

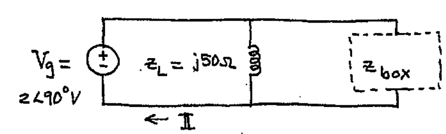
Sol'n: a) Transform to frequency domain.

$V_g = -j2\,e^{j180°}$ V since $P[\sin(\omega t)] = -j$

$V_g = +j2\text{ V} = 2\angle 90°$ V

$Z_L = j\omega L = j\cdot 10\text{ r/s}\cdot 5\text{ H} = j\,50\,\Omega$

Note: $\omega = 10$ r/s from $v_g(t)$.



We have $I = \dfrac{V_g}{Z_L \parallel Z_{box}}$.

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In terms of angles:

$\angle I = 45^\circ$ from $i(t) = I_o \cos(10t + 45^\circ)$ A

$\angle I = 45^\circ = \angle V_g - \angle(z_L \| z_{box})$

or $45^\circ = 90^\circ - \angle(z_L \| z_{box})$

Thus, $\angle(z_L \| z_{box}) = 45^\circ$.

For parallel components, it is easier to use conductance $= \frac{1}{z} \equiv g$

$$g_L = \frac{1}{z_L} \qquad g_{box} = \frac{1}{z_{box}}$$

$$\angle(z_L \| z_{box}) = -\angle\left(\frac{1}{z_L \| z_{box}}\right) = -\angle\left(\frac{1}{z_L} + \frac{1}{z_{box}}\right)$$

or

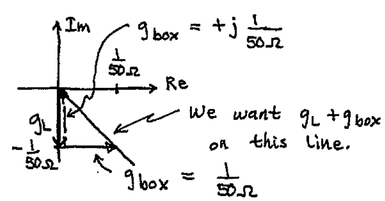
$$\angle\left(\frac{1}{z_L} + \frac{1}{z_{box}}\right) = -\angle(z_L \| z_{box}) = -45^\circ$$

or

$$\angle(g_L + g_{box}) = -45^\circ$$

Now we use a plot:

$$g_L = \frac{1}{j50\Omega} = -j \cdot \frac{1}{50\Omega}$$



ECE 1270
Sp 06
HOMEWORK #8 Solution Prob 2, 3 (cont.)
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We have two possible solutions:

1) $g_{box} = j\frac{1}{50\Omega} \Rightarrow z_{box} = \frac{50\Omega}{j} = -j50\Omega$

Need capacitor: $z_C = \frac{-j}{\omega C} = \frac{-j}{10 \cdot C} = -j50\Omega$

$\Rightarrow C = \frac{1}{10 \cdot 50}$ F $= 2$ mF

or 2) $g_{box} = \frac{1}{50\Omega} \Rightarrow z_{box} = 50\Omega$ resistor

Either soln is technically correct, but solution (1) gives $g_L + g_{box} = 0 \Rightarrow z = \infty$.

Thus, no current flows for this soln.

b) Now use magnitude.

$$|I| = \left|\frac{V_g}{z_L \| z_{box}}\right| = \frac{|V_g|}{|z_L \| z_{box}|}$$

or $I_o = |V_g| \cdot |g_L + g_{box}|$

$I_o = 2 \cdot |g_L + g_{box}|$

From diagram used to find $g_L + g_{box}$ on -45° line, use magnitude of vector for $g_L + g_{box}$.

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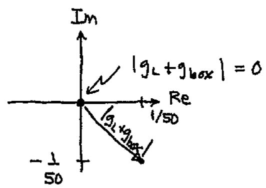


For sol'n (1), $|g_L + g_{box}| = 0 \quad \Rightarrow I_o = 0\text{ A}$

For sol'n (2), $|g_L + g_{box}| = \left| \frac{1}{50\Omega} - j\frac{1}{50\Omega} \right|$

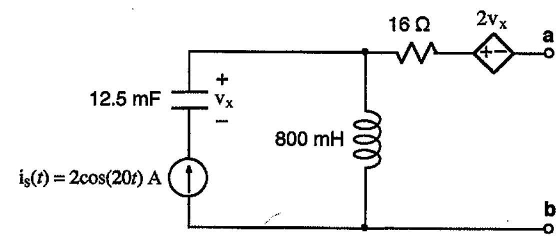
$$= \left| \frac{1}{50\Omega}(1-j) \right|$$

$$= \frac{1}{50\Omega}\sqrt{1^2+1^2}$$

$$= \frac{\sqrt{2}}{50\Omega}$$

$$\Rightarrow I_o = \frac{2 \cdot \sqrt{2}\text{ A}}{50} = \frac{\sqrt{2}}{25}\text{ A}$$

Ex:



a) Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $i_s(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

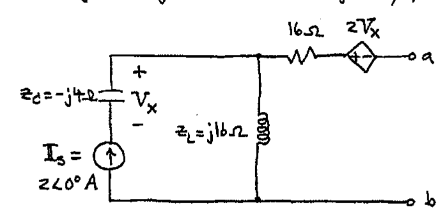
b) Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for $\mathbf{V_{Th}}$ and the numerical impedance value of z_{Th}.

Sol'n: a) $\omega = 20$ r/s from $i_s(t)$

$$\mathbb{I}_s = 2\angle 0°\text{ A}$$

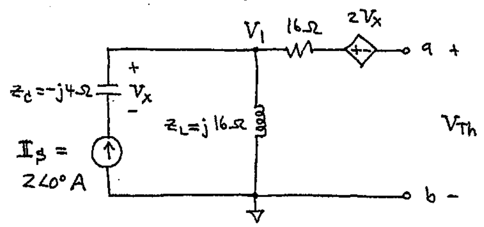
$$z_C = -\frac{j}{\omega C} = \frac{-j}{20\text{r/s}\cdot 12.5\text{mF}} = \frac{-j}{250\text{m}}\Omega = -j4\,\Omega$$

$$z_L = j\omega L = j\,20\text{r/s}\cdot 800\text{mH} = j\,16\cancel{\text{K}}\cancel{\text{m}}\Omega = j16\,\Omega$$



b) $V_{Th} = V_{ab}$ open circuit.

Use node voltage V_1:



V_1 (from $I_s \cdot z_L$) $= 2\angle 0° A \cdot j16\Omega = j32\,V$

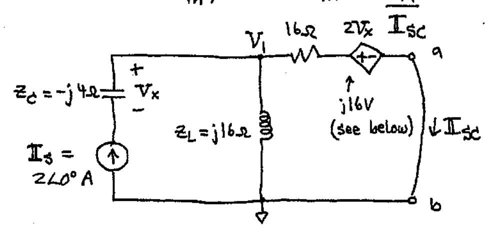
We've 0V across 16Ω since no current flows.

Thus, $V_{Th} = V_1 - 2V_x$.

$V_x = -I_s z_c = -2\angle 0° A \cdot (-j4\Omega) = j8\,V$

So $V_{Th} = j32V - 2(j8V) = j16V$ or $16\angle{-90°}\,V$

To find z_{Th}, use $z_{Th} = \dfrac{V_{Th}}{I_{sc}}$

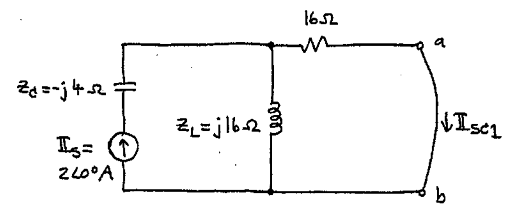


Since C is in series with current source, we have $V_x = -I_s z_c = j8V$ as before.

Thus, $2V_x = j16V$. We can now treat the dependent source as an independent source of j16V.

Now we use superposition to find I_{sc}:

case I: I_s on, $2V_x$ off



This is a current divider.

$$I_{sc1} = I_s \cdot \frac{z_L}{z_L + 16\Omega} = \frac{j16\Omega\, I_s}{j16\Omega + 16\Omega} = \frac{j\, I_s}{1+j}$$

$$= 2\angle 0° A \cdot \frac{j}{1+j}\,\frac{1-j}{1-j} = 2\angle 0° A\,\frac{1+j}{1^2+1^2} = 1+j\ A$$

$I_{sc1} = 1+j\ A$

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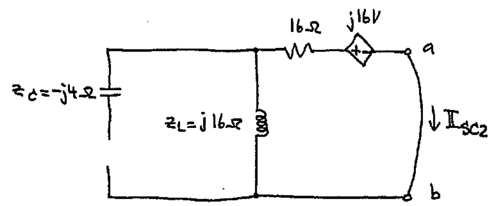
Case II: I_s off, $2V_x$ on



We have a V-loop on the right:

$$I_{sc2} = \frac{-j16V}{16\Omega + Z_L} = \frac{-j16V}{16\Omega + j16\Omega} = \frac{-j}{1+j}\ A$$

$$" = \frac{-j}{1+j}\,\frac{1-j}{1-j}\ A = \frac{-1-j}{1^2+1^2} = \frac{-1-j}{2}$$

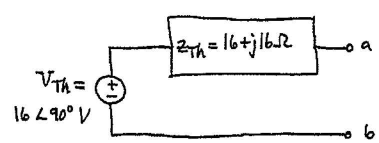
Sum the results: $I_{sc} = I_{sc1} + I_{sc2}$

$$= 1+j\ A + \frac{-1-j}{2}\ A$$

$$I_{sc} = \frac{1+j}{2}\ A \text{ or } \frac{1}{\sqrt{2}}\angle 45^\circ A$$

$$Z_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{j16V}{\frac{1+j}{2}} = \frac{16\angle 90^\circ V}{\frac{1}{\sqrt{2}}\angle 45^\circ A} = 16\sqrt{2}\angle 45^\circ \Omega$$

$$Z_{Th} = 16 + j16\ \Omega$$



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Ex: Give numerical answers to each of the following questions:

a) Rationalize $\frac{120-j22}{-11+j60}$. Express your answer in rectangular form.

b) Find the polar form of $j(1+j)^* e^{j30°}$. (Note the asterisk that means "conjugate".)

c) Find the following phasor: P[−7cos(49t + 135°)].

d) Find the magnitude of $\left(\frac{24+j7}{3-j4}\right)\left(\frac{-1}{e^{j10°}}\right)$.

e) Find the imaginary part of $\frac{e^{j45°}}{e^{-j225°}}$.

Soln: a) $\frac{120-j22}{-11+j60} \cdot \frac{-11-j60}{-11-j60} = \frac{2(60-j11)(-1)(11+j60)}{11^2+60^2}$

$$= \frac{-2\left[60(11)+11(60)+j3600-j121\right]}{61^2}$$

$$= -\frac{1320-j6958}{61^2}$$

$$= -\frac{1320}{3721} - \frac{j6958}{3721}$$

$$\approx -0.7 \quad -j1.870$$

b) $j(1+j)^* e^{j30°} = j(1-j)e^{j30°}$

$$= e^{j90}\sqrt{2}\,e^{-j45°}e^{j30°}$$

$$= \sqrt{2}\,e^{j75°}$$

c) $P[-7\cos(49t+135°)] = -7\angle 135°$

$$= 7\angle 135° \pm 180°$$

$$= 7\angle 315° \text{ or } 7\angle -45°$$

d) $\left|\frac{24+j7}{3-j4}\right|\left|\frac{-1}{e^{j10°}}\right| = \frac{|24+j7|}{|3-j4|}\cdot\frac{|-1|}{|e^{j10°}|}$

$$= \frac{\sqrt{24^2+7^2}}{\sqrt{3^2+4^2}} \cdot \frac{1}{1}$$

$$= \frac{25}{5}$$

$$= 5$$

e) $\text{Im}\left[\frac{e^{j45°}}{e^{-j225°}}\right] = \text{Im}\left[e^{j(45° - -225°)}\right]$

$$= \text{Im}\left[e^{j270°}\right]$$

$$= \text{Im}\left[-j\right]$$

$$= -1$$

HW 8 $\boxed{\text{Example}}$

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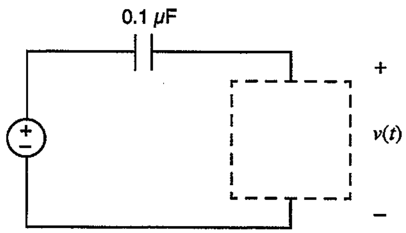
Ex:



$v_{g(t)} = 10\cos(50kt - 135°)$ V

a) Choose an R, an L, or a C to be placed in the dashed-line box to make
$v(t) = V_o \sin(50kt)$
where V_o is a positive real constant (with units of Volts). State the value of the component you choose.

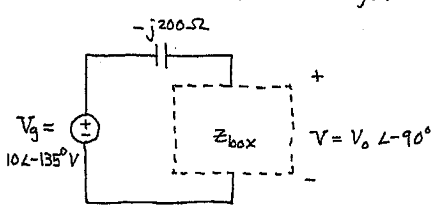
b) With your component from (a) in the circuit, calculate the resulting value of V_o.

Sol'n: a) We first transform the circuit to the frequency domain.

$V_g = 10\angle{-135°}\ V \qquad V = V_o \angle{-90°}$ since $P[\sin\omega t] = -j$ or $1\angle{-90°}$

$$z_C = \frac{-j}{\omega C} = \frac{-j}{50K \cdot 0.1\mu F} = \frac{-j}{5m}\,\Omega = -j200\,\Omega$$

Note: $\omega = 50\text{ k r/s}$ from $v_g(t)$ and $v(t)$.



Now we consider phase relationships.

$$V = V_g \cdot \frac{z_{box}}{z_{box} - j200\Omega} \quad \text{from V-divider}$$

$$\angle V = \angle V_g + \angle z_{box} - \angle(z_{box} - j200\Omega)$$
$$\parallel$$
$$-90° = -135° + \angle z_{box} - \angle(z_{box} - j200\Omega)$$

Thus, $\angle z_{box} - \angle(z_{box} - j200\Omega) = 45°$.

Consider possible contents of z_{box}.

If $z_{box} = j\omega L$ or $\frac{-j}{\omega C}$, then all z values in the circuit are pure imaginary.

Thus, $\angle z_{box} - \angle(z_{box} - j200\Omega)$ would be some multiple of 90°. It follows that z_{box} must be an R value.

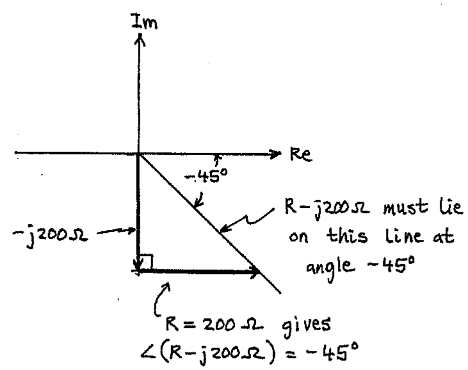
$\therefore$ Let $z_{box} = R$. $\quad \angle R = 0°$

Then $\angle z_{box} - \angle(z_{box} - j200\Omega) = 45°$

$= 0° - \angle(R - j200\Omega)$

or $\angle(R - j200\Omega) = -45°$

Now we can find R graphically.



$$Z_{box} = R = 200\,\Omega$$

b) To find V_o, we use magnitude.

$$V_o = |V| = \left| V_g \cdot \frac{Z_{box}}{Z_{box} - j200\,\Omega} \right|$$

$$= \left| V_g \frac{R}{R - j200\,\Omega} \right|$$

$$= \left| V_g \frac{200\,\Omega}{200\,\Omega - j200\,\Omega} \right|$$

$$= \frac{|10V|\,|200\,\Omega|}{|200\,\Omega - j200\,\Omega|}$$

$$= \frac{10V \cdot 200}{200\,|1-j|}$$

$$= \frac{10V}{|1-j|}$$

$$= \frac{10\,V}{\sqrt{2}}$$

$$V_o = \frac{10}{\sqrt{2}}\,V$$

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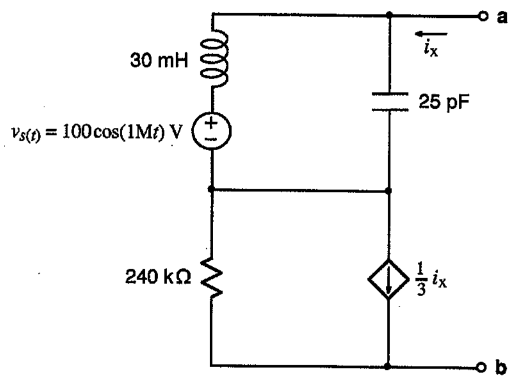
Ex:



a) Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $v_S(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

b) Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for V_{Th} and the numerical impedance value of z_{Th}.

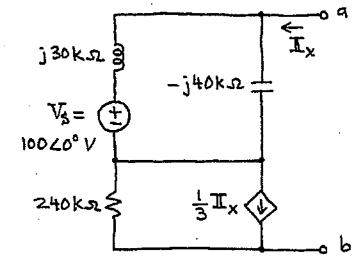
Sol'n: a) First, we find phasors and z values.

$V_s = 100\angle 0°$ V $\qquad \omega = 1M$ r/s

$z_L = j\omega L = j\, 1M\,r/s \cdot 30\,mH = j\,30k\Omega$

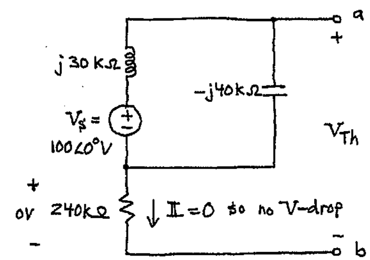
$z_C = \frac{-j}{\omega C} = \frac{-j}{1M\,r/s \cdot 25\,pF} = \frac{-j}{25\mu}\Omega = -j40k\Omega$

Now we can draw the frequency domain model:



b) We first find $V_{Th} = V_{a,b}$ with no load connected across a,b.

In this case, $I_x = 0$ A since a,b = open.



$I_x = 0$ so dependent source disappears.

We have a V-divider:

$$V_{Th} = V_s \frac{-j40k\Omega}{-j40k\Omega + j30k\Omega} = V_s \frac{-j40k\Omega}{-j10k\Omega}$$

$$V_{Th} = 4V_s = 4(100\angle 0°\,V)$$

$$V_{Th} = 400\angle 0°\ V$$

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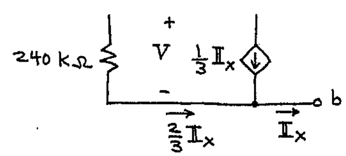
To find Z_{Th}, we replace the dependent source with an equivalent impedance.

We observe that, regardless of what is connected between **a** and **b**, I_x flows <u>out</u> of the **b** terminal, (and <u>into</u> the **a** terminal).

Consider a current summation at the bottom node:



We have current $\frac{2}{3}I_x$ through the 240kΩ.

The voltage across <u>both</u> the 240kΩ and the dependent source, by Ohm's law, will be $V = \frac{2}{3}I_x \cdot 240\,k\Omega$.

Thus, the equivalent impendance for the dependent source is found by using Ohm's law:

$$Z_{eq} = \frac{V}{\frac{1}{3}I_x} = \frac{\frac{2}{3}\cancel{I_x}\cdot 240\,k\Omega}{\frac{1}{3}\cancel{I_x}}$$

$$= 2(240\,k\Omega)$$

$$Z_{eq} = 480\,k\Omega$$

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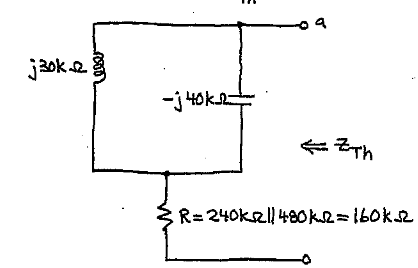
Now we combine the 240kΩ and 480kΩ.

$$R = 240\,k\Omega \parallel 480\,k\Omega = 240\,k\Omega \cdot 1\parallel 2$$

$$R = 240\,k\Omega \cdot \frac{2}{3} = 160\,k\Omega$$

Finally, we turn off the V_s source and look into the circuit from the a,b terminals to find Z_{Th}:



$$Z_{Th} = j30\,k\Omega \parallel -j40\,k\Omega + 160\,k\Omega$$

$$= j10\,k\Omega \cdot 3\parallel -4 + 160\,k\Omega$$

$$= j10\,k\Omega \cdot \frac{-12}{-1} + 160\,k\Omega$$

$$= j120\,k\Omega + 160\,k\Omega$$

or $$Z_{Th} = 160\,k\Omega + j120\,k\Omega$$

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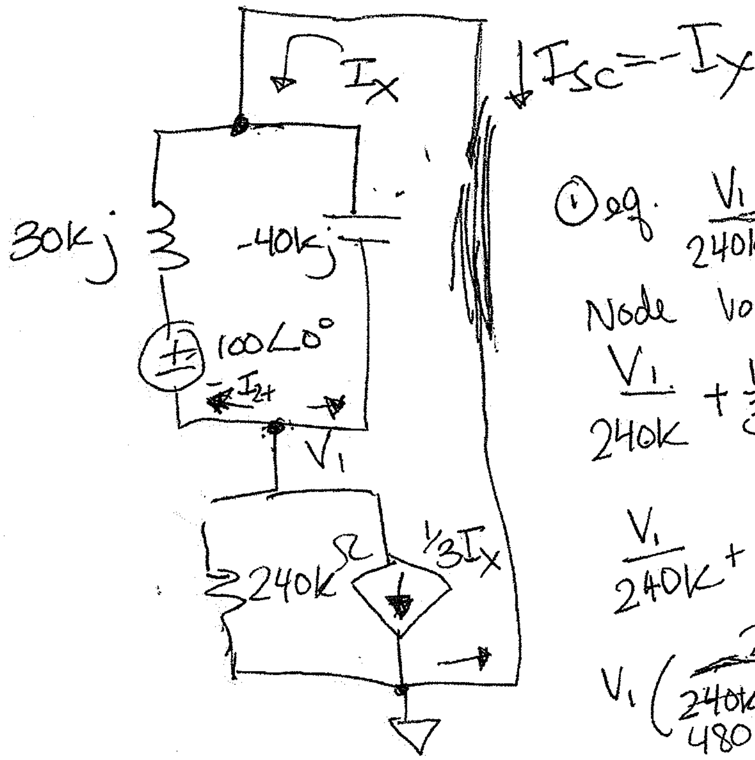
(189)

3. (b)

Different methodology: $+V_1 + 100 \mp 30kj\,(I_2) = 0$

$$I_2 = \frac{V_1 + 100}{30kj}$$

$$2m\angle 143^\circ = \frac{V_1}{2(80k)} = I_x = -I_{sc}$$



① eq. $\frac{V_1}{240k} + \tfrac{1}{3}I_x - I_x = 0$ (current sum)

Node Voltage:

$$\frac{V_1}{240k} + \frac{1}{3}I_x + \frac{V_1}{-40kj} + \frac{V_1 + 100}{30kj} = 0$$

$$\frac{V_1}{240k} + \frac{1}{3}\left[\frac{V_1}{2(80k)}\right] - \frac{V_1}{40kj} + \frac{V_1}{30kj} = -\frac{1\cancel{0}\cancel{0}}{3\cancel{0}kj}$$

$3{,}00\cancel{0}$

$$V_1\left(\frac{2}{\cancel{240k}\ 480k} + \frac{1}{480k} + \frac{12j}{480k} + \frac{-16j}{480k}\right) = +\frac{j}{300}$$

$$V_1\left(\frac{3-4j}{\cancel{480k}}\right) = \frac{\cancel{j}}{300}\,\frac{(480k)\angle 90^\circ + 53^\circ}{5\angle \cancel{-53^\circ}}$$

$$V_1 = 320\angle 143$$

$V_{Th} = 400\angle 0^\circ \Rightarrow Z_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{400\angle 0^\circ}{-2m\angle 143^\circ} = 200k\angle -143^\circ$

$$\boxed{Z_{Th} = (160k + 120kj)\,\Omega}$$