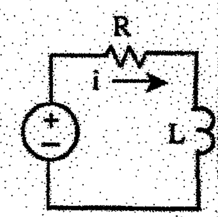
## TIME DOMAIN

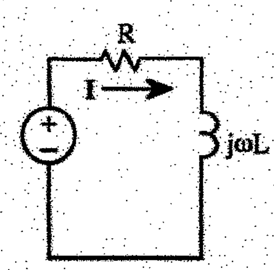
$v_g = v_m \cos(\omega t + \phi)$




phasor transform →

## FREQUENCY DOMAIN

$\mathbf{V}_g = v_m e^{j\phi}$




Frequency-domain representation of the circuit (transformed circuit)

↓

Apply circuit laws to obtain algebraic equations:

$$\mathbf{V}_g = R\,\mathbf{I} + j\omega L\,\mathbf{I}$$

↓

Solve for the desired quantities:

$$\mathbf{I} = \frac{\mathbf{V}_g}{R + j\omega L}$$

← Inverse phasor transform

$$i = \frac{v_m \cos(\omega t + \phi - \psi)}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\psi = \tan^{-1}(\omega L/R)$$

Time-domain expression for the desired quantities

148

**EX:** If $f(t) = 2\sin(\omega t + \pi/3)$ find $\mathbf{P}[f(t)]$, (i.e., find the phasor)

**ANS:** $\mathbf{P}[f(t)] = \mathbf{F} = 2e^{-j\pi/6} = 2\angle\pi/6$

**SOL'N:** If we have a cosine, we use the standard identity for phasors:

$$\mathbf{P}[A\cos(\omega t + \phi)] = Ae^{j\phi} = A\angle\phi$$

For a sine, we multiply the standard identity by $-j$ (which is the phasor for a sine of magnitude one and zero phase shift):

$$\mathbf{P}[\sin(\omega t)] = -j = 1\angle -90°$$

Thus, we have

$$\mathbf{P}[f(t)] = \mathbf{F} = -2je^{j\pi/3}.$$

The above is mathematically correct and works properly in solving problems, but we will apply identities to express the answer in standard form:

$$-1 = e^{j180°} = e^{-j180°} = e^{j\pi} = e^{-j\pi}.$$

**NOTE:** (We use whichever of +180° or –180° is most convenient.)

$$j = e^{j90°} = e^{j\pi/2}.$$

Applying the identities:

$$\mathbf{F} = -2je^{j\pi/3} = 2e^{-j\pi}e^{j\pi/2}e^{j\pi/3} = 2e^{-j\pi/6} = 2\angle -\pi/6.$$

**EX:** If $\mathbf{F} = (2.5 + j3.2)$ find $\mathbf{P}^{-1}[\mathbf{F}]$, (i.e., find the inverse phasor)

**ANS:** $\mathbf{P}^{-1}[\mathbf{F}] = 4.06\cos(\omega t + 52°)$

**SOL'N:** We convert to polar form:

$$2.5 + j3.2 = \sqrt{2.5^2 + 3.2^2}\, e^{j\tan^{-1}\left(\frac{3.2}{2.5}\right)} \approx 4.06e^{j52°}$$

Now use the standard inverse phasor identity:

$$\mathbf{P}^{-1}[Ae^{j\phi}] = A\cos(\omega t + \phi)$$

**NOTE:** There is no math to do here—we just substitute the values of $A$ and $\phi$ into the cos( ).

**NOTE:** We don't know the value of $\omega$ for this problem. Thus, we just use a symbolic variable for $\omega$. The value of $\omega$ is *not* part of a phasor. (The value of $\omega$ must be kept track of separately.)

Using the identity gives the answer:

$$\mathbf{P}^{-1}[\mathbf{F}] = 4.06\cos(\omega t + 52°)$$

**NOTE:** Mathematically, it is also correct to invert the given phasor in two pieces, with the real part giving a cosine term having no phase shift and the imaginary part giving a (negative) sine term having no phase shift:

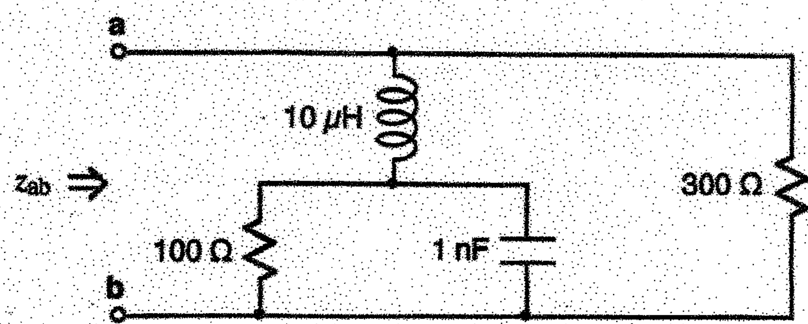
$$\mathbf{P}^{-1}[2.5 + j3.2] = 2.5\cos(\omega t) - 3.2\sin(\omega t).$$

Although this answer is correct, it is usually easier to visualize a single sinusoid with a phase shift. The sum of the cos and sin terms is equal to the single cos with a phase shift given above. This follows from the observation that the sum of any number of sinusoids of the same frequency may be expressed as a single sinusoid of that frequency. (The challenging part is determining the magnitude and phase shift of the single sinusoid.)

(150) - (152)



**Ex:**

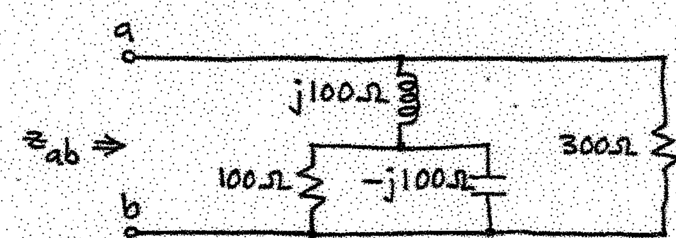



**Given ω = 10 M rad/s, find $z_{ab}$.**

Sol'n: $z_L = j\omega L = j\,10\text{M r/s}\cdot 10\mu\text{H} = j100\,\Omega$

$$z_C = \frac{-j}{\omega C} = \frac{-j}{10\text{M r/s}\cdot 1\text{nF}} = \frac{-j}{10\text{m}}\,\Omega = -j100\,\Omega$$

frequency domain (or s-domain) model:




We first consider the R and C in parallel.

$$100\,\Omega \,\|\, -j100\,\Omega = 100\,\Omega\cdot 1 \,\|\, -j = 100\,\Omega\cdot\frac{-j}{1-j}$$

Rationalizing this expression, we have

$$100\,\Omega \,\|\, -j100\,\Omega = 100\,\Omega\,\frac{-j}{1-j}\cdot\frac{1+j}{1+j}$$

or

$$100\,\Omega \,\|\, -j100\,\Omega = 100\,\Omega \;\; \frac{1-j}{2} = 100\,\Omega \cdot \left(\frac{1}{2} - \frac{j}{2}\right)$$

Now we add $z_L = j100\,\Omega$

$$100\,\Omega \,\|\, -j100\,\Omega + j100\,\Omega = 100\,\Omega \left(\frac{1}{2} - \frac{j}{2} + j\right)$$

$$= 100\,\Omega \left(\frac{1}{2} + \frac{j}{2}\right)$$

To find $z_{ab}$, we use conductance $g_{ab} = \frac{1}{z_{ab}}$:

$$g_{ab} = \frac{1}{100\,\Omega \,\|\, -j100\,\Omega + j100\,\Omega} + \frac{1}{300\,\Omega}$$

$$= \frac{1}{100\,\Omega \left(\frac{1}{2} + \frac{j}{2}\right)} + \frac{1}{100\,\Omega} \cdot \frac{1}{3}$$

$$= \frac{1}{100\,\Omega} \cdot \frac{2}{1+j} \cdot \frac{1-j}{1-j} + \frac{1}{100\,\Omega} \cdot \frac{1}{3}$$

$$= \frac{1}{100\,\Omega} \cdot \frac{2-j2}{2} + \frac{1}{100\,\Omega} \cdot \frac{1}{3}$$

$$= \frac{1}{100\,\Omega} (1-j) + \frac{1}{100\,\Omega} \cdot \frac{1}{3}$$

$$= \frac{1}{100\,\Omega} \left(\frac{4}{3} - j\right)$$

Now we calculate $\frac{1}{g_{ab}} = z_{ab}$.

$$z_{ab} = \frac{1}{g_{ab}} = 100\,\Omega \frac{1}{\frac{4}{3} - j}$$

$$= 100\,\Omega \frac{3}{4 - j3}$$

$$= 100\,\Omega \frac{3}{4 - j3} \cdot \frac{4 + j3}{4 + j3}$$

$$= 100\,\Omega \frac{12 + j9}{4^2 + 3^2}$$

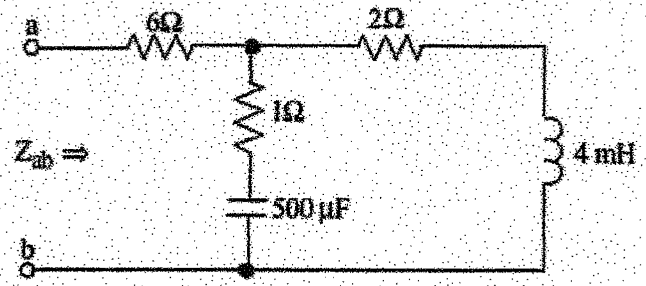
$$= 100\,\Omega \frac{12 + j9}{25}$$

$$= 4 \cdot (12 + j9)$$

$$z_{ab} = 48 + j36$$

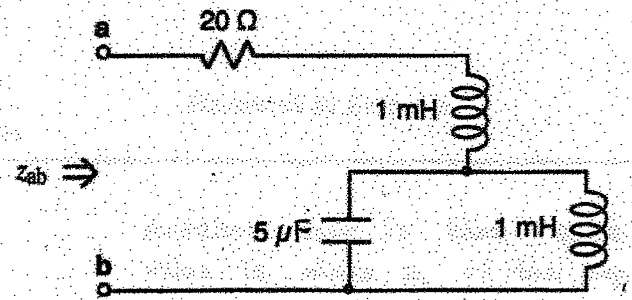
# Homework #7 Examples

1.




Given $\omega = 1k$ rad/sec, find $Z_{ab}$.

**Ex:**




**Find a frequency, ω, that causes $z_{ab}$ to be real, (i.e., imaginary part equals zero).**

Sol'n: $z_{ab} = 20\,\Omega + z_{L1} + z_C \| z_{L2}$

For $z_{ab}$ to be real, we must have

$$z_{L1} + z_C \| z_{L2} = \text{real}$$

One simple sol'n is to let $\omega = 0$ so both L's act like wires and C acts like open circuit.

Other potential sol'ns are $\omega = \infty$, (so L's act like opens, resulting in $z_{ab} = \infty$), and $\omega$ = frequency where $z_C = -z_{L2}$, (so C and L in parallel have equal but opposite impedances).

The latter case, where $z_C = -z_L$ gives the interesting result that $z_C \| z_L = \frac{L/C}{0} = \infty$

This means $z_{ab} = \infty\,\Omega$. In this case, (unlike $\omega \to \infty$), $z_{ab} \to \infty$ along real axis as $z_C \| z_L \to \infty$.

or $\omega = 20K$ r/s

Another sol'n is that $z_C \| z_L$ has a value is minus $z_L$ of the top inductor.

In that case, $z_L + z_C \| z_L = 0$ and $z_{ab} = 0 =$ wire.

$$z_L = j\omega L$$

$$z_C \| z_L = \frac{-j}{\omega C} \| j\omega L = \frac{\frac{-j}{\omega C} \cdot j\omega L}{\frac{-j}{\omega C} + j\omega L} = \frac{L/C}{j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= -j \frac{L/C}{\omega L - \frac{1}{\omega C}}$$

Thus, we want $j\omega L - \dfrac{jL/C}{\omega L - \frac{1}{\omega C}} = 0$

or $\quad \omega L = \dfrac{L/C}{\omega L - \frac{1}{\omega C}}$

or $\quad \omega L \left( \omega L - \frac{1}{\omega C} \right) = L/C$

or $\quad \omega^2 L^2 - \dfrac{L}{C} = \dfrac{L}{C}$

or $\quad \omega^2 L^2 = \dfrac{2L}{C} \quad$ or $\quad \omega^2 = \dfrac{2}{LC}$

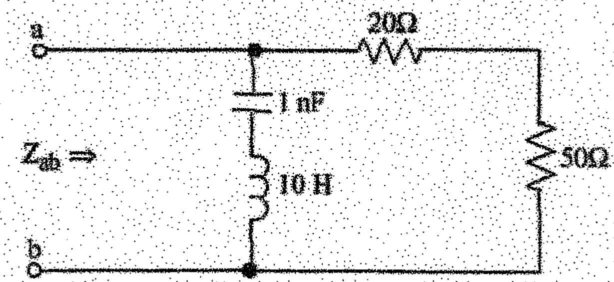
or $\quad \omega = \sqrt{\dfrac{2}{LC}} \quad$ or $\quad \omega = \sqrt{\dfrac{2}{5\mu F \cdot 1mH}}$

or $\quad \omega = \sqrt{\dfrac{2}{5} G}$ r/s $= \sqrt{400M}$ r/s

or $\quad \omega = 20k$ r/s

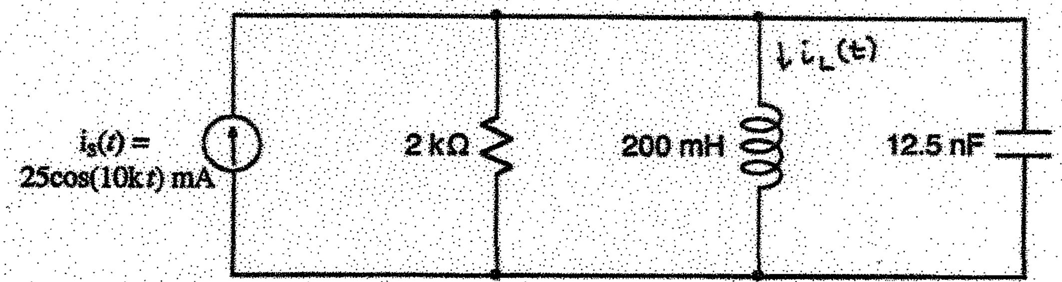
# Homework #7 Examples

2.




Find a frequency, $\omega$, that causes $Z_{ab}$ to be real (i.e. imaginary part equals zero). $\omega \neq 0$ or $\omega \neq \infty$.

**Ex:**




a) Find the phasor value for $i_s(t)$.

b) Draw the frequency-domain circuit diagram, including the phasor value for $i_s(t)$ and impedance values for components.

c) Find the phasor value for $i_L(t)$.

Sol'n: a) The phasor for $A\cos(\omega t + \phi)$ is $Ae^{j\phi}$.

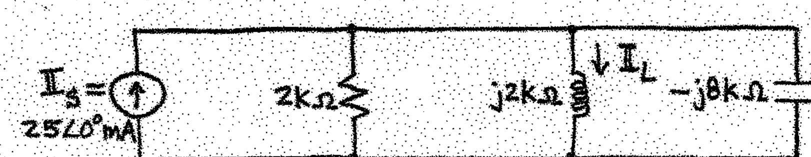
$$\therefore \mathbb{I}_s = 25\, e^{j0^\circ} \text{ mA or } 25\angle 0^\circ \text{ mA}$$

b) From $i_s(t)$, we see that $\omega = 10\text{k rad/s}$.

Impedance $Z_L = j\omega L = j\,10\text{k}\; 200\text{mH} = j2\text{k}\Omega$

$$Z_C = \frac{-j}{\omega C} = \frac{-j}{10\text{k}\; 12.5\text{nF}} = \frac{-j\,\Omega}{125\,\mu}$$

$$Z_C = \frac{-j\,8\text{k}\,\Omega}{8\text{k}\cdot 125\,\mu} = -j8\text{k}\Omega$$



c) The value for $I_L$ is given by the current divider formula:

$$I_L = I_s \cdot \frac{R \| Z_C}{R \| Z_C + Z_L}$$

$$= I_s \frac{1}{1 + \frac{Z_L}{R \| Z_C}}$$

$$= I_s \frac{1}{1 + Z_L\left(\frac{1}{R} + \frac{1}{Z_C}\right)}$$

$$= 25\angle 0^\circ \text{ mA} \frac{1}{1 + j2k\Omega\left(\frac{1}{2k\Omega} + \frac{1}{-j8k\Omega}\right)}$$

$$= 25\angle 0^\circ \text{ mA} \frac{4}{4} \frac{1}{1 + j - \frac{1}{4}}$$

$$= 25\angle 0^\circ \text{ mA} \frac{4}{3 + j4}$$

$$= 25\angle 0^\circ \text{ mA} \frac{4}{3 + j4} \cdot \frac{3 - j4}{3 - j4}$$

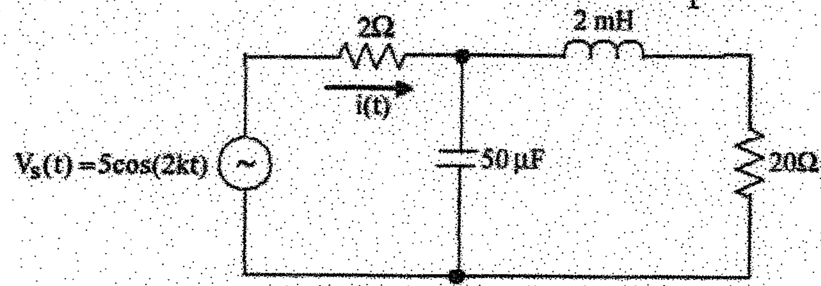
$$= \cancel{25}\angle 0^\circ \text{ mA} \frac{12 - j16}{\cancel{3^2 + 4^2}}$$

$$= 1\angle 0^\circ \text{ mA} \cdot 20\angle -53.1^\circ$$

$$I_L = 20 \text{ mA} \angle -53.1^\circ$$

# Homework #7 Examples

3.

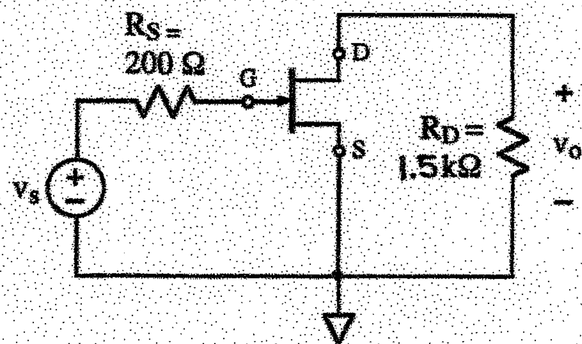


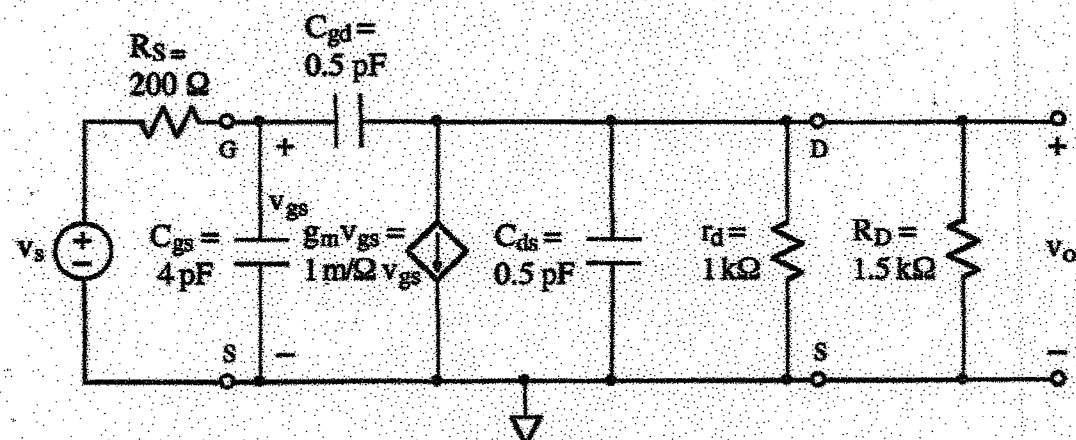

a. Find the phasor value for $V_s(t)$.

b. Draw the frequency-domain circuit diagram, including the phasor value for $V_s(t)$ and impedance values for components.

b. Find the phasor value for i(t).

**EX:**




$v_s(t) = 2 \cos(10kt)$ V

The above circuit diagrams show a common-source JFET amplifier and its high-frequency equivalent circuit. Find $v_o(t)$.

Sol'n: In this practical circuit, we have circuit values that allow us to make simplifying approximations.

We first calculate impedance values.

$\omega = 10k$ r/s from $v_s(t) = 2 \cos(10kt)$ V

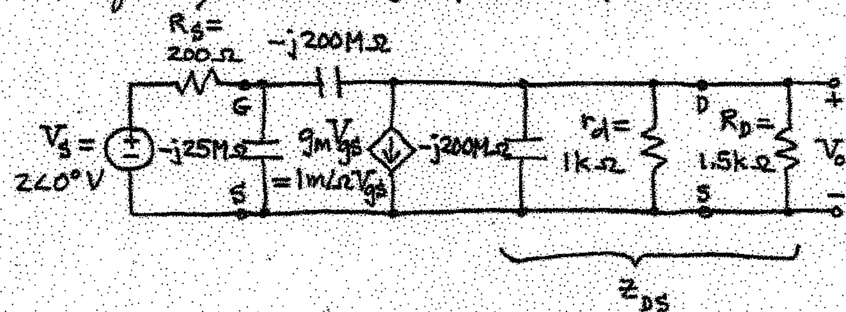
$$Z_{C_{gs}} = \frac{-j}{\omega C_{gs}} = \frac{-j\ \Omega}{10k \cdot 4p} = -j\,25M\,\Omega$$

$$Z_{C_{gd}} = \frac{-j}{\omega C_{gd}} = \frac{-j}{10k\,\frac{1}{2}p} = -j\,200M\,\Omega$$

$$Z_{C_{ds}} = \frac{-j}{\omega C_{ds}} = \frac{-j}{10k\,\frac{1}{2}p} = -j\,200M\,\Omega$$

The phasor for $v_s(t)$ is $V_s = 2\angle 0°$ V.

Frequency domain (or s-domain) model:




$$Z_{DS} = -j\,200M \,\|\, 1k\Omega \,\|\, 1.5k\Omega$$

Starting with $1k\Omega \,\|\, 1.5k\Omega$ we have

$$1k\Omega \,\|\, 1.5k\Omega = 500\Omega \cdot 2\|3 = 500 \cdot \frac{6}{5}$$

$$= 600\Omega$$

Thus, $Z_{DS} = -j\,200M \,\|\, 600\Omega = \dfrac{1}{\dfrac{1}{600} - \dfrac{1}{j200M}}$

Using $-\frac{1}{j} = j$ and rationalizing gives

$$z_{DS} = \frac{1}{\frac{1}{600} + \frac{j}{200M}} \cdot \frac{\frac{1}{600} - \frac{j}{200M}}{\frac{1}{600} - \frac{j}{200M}} \ \Omega$$

$$= \frac{\frac{1}{600} - \frac{j}{200M}}{\left(\frac{1}{600}\right)^2 + \left(\frac{1}{200M}\right)^2} \ \Omega$$

$$\approx \frac{\frac{1}{600} - \frac{j}{200M}}{\left(\frac{1}{600}\right)^2} \ \Omega \quad \text{Since } \frac{1}{200M} \ll \frac{1}{600}$$

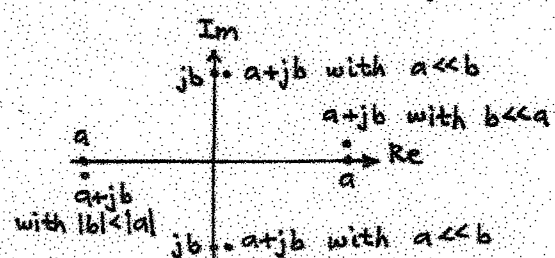
$$z_{DS} \approx \frac{\frac{1}{600}}{\left(\frac{1}{600}\right)^2} \ \Omega = 600 \ \Omega$$

In retrospect, we could have made the approximation that $-j200M \| 600\,\Omega \approx 600\,\Omega$.

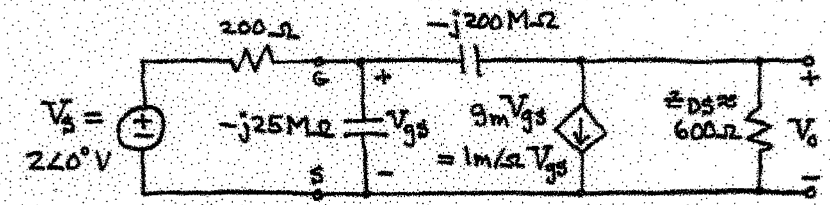
We may make this approximation despite the j in one of the quantities. In general, we may make the following approximations of complex values:

$$a + jb \approx a \quad \text{when} \quad |b| \ll |a|$$

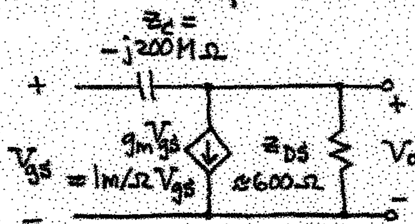
$$a + jb \approx jb \quad \text{when} \quad |a| \ll |b|$$



With our $Z_{DS}$ value, we have a simplified model:




We now analyze the dependent source so we can replace it with an impedance, $Z_{eq}$.




$$Z_{eq} = \frac{V_o}{g_m V_{gs}}$$ using Ohm's law to write $Z_{eq} = V/I$

Now we find a way to write $V_o$ in terms of $V_{gs}$. We use a V-divider:

$$V_o = V_{gs} \frac{Z_{eq} \| Z_{DS}}{Z_{eq} \| Z_{DS} + Z_C}$$

Substituting for $V_o$ in our $Z_{eq}$ eq'n, we have

$$Z_{eq} = \frac{\cancel{V_{gs}} \dfrac{Z_{eq} \| Z_{DS}}{Z_{eq} \| Z_{DS} + Z_C}}{\cancel{V_{gs}} \cdot g_m}$$

$$Z_{eq} = \frac{1}{g_m} \frac{\dfrac{Z_{eq} Z_{DS}}{Z_{eq} + Z_{DS}}}{\dfrac{Z_{eq} Z_{DS}}{Z_{eq} + Z_{DS}} + Z_C}$$

$$z_{eq} = \frac{1}{g_m} \frac{z_{eq} z_{DS}}{z_{eq} z_{DS} + z_C (z_{eq} + z_{DS})}$$

Dividing top and bottom by $z_{eq}$ gives the following:

$$z_{eq} = \frac{1}{g_m} \frac{z_{DS}}{z_{DS} + z_C + \frac{z_C z_{DS}}{z_{eq}}}$$

$$z_{eq} \left( z_{DS} + z_C + \frac{z_C z_{DS}}{z_{eq}} \right) = \frac{1}{g_m} z_{DS}$$

$$z_{eq} (z_{DS} + z_C) + z_C z_{DS} = \frac{1}{g_m} z_{DS}$$

$$z_{eq} \frac{(z_{DS} + z_C)}{z_{DS}} = \frac{\frac{1}{g_m} \cancel{z_{DS}} - z_C \cancel{z_{DS}}}{\cancel{z_{DS}}}$$

$$z_{eq} = \frac{\frac{1}{g_m} - z_C}{1 + \frac{z_C}{z_{DS}}} = \frac{\frac{1\Omega}{1m} - -j200\,M\Omega}{1 + \frac{-j200\,M\Omega}{600\Omega}}$$

$$z_{eq} = \frac{1k + j200M}{1 - j\frac{1}{3}M} \Omega$$

The imaginary parts of the numerator and denominator are much larger than the real parts. Thus, we ignore the real parts.

$$z_{eq} \approx j200\,M\Omega / -j\tfrac{1}{3} M \approx -600\Omega$$

Now we have a problem: $z_{eq} \parallel z_{DS} = -\dfrac{600^2}{0}\,\Omega$.

That means $z_{eq} \parallel z_{DS} = \infty\ \Omega$.

It is a good idea to try a more exact calculation to be sure that $z_{eq} \parallel z_{DS}$ is much larger than $z_c = -j200\,M\Omega$.

We use conductance to simplify calculations.

$$\frac{1}{z_{eq} \parallel z_{DS}} = \frac{1}{z_{eq}} + \frac{1}{z_{DS}} = \frac{1 + \frac{z_c}{z_{DS}}}{\frac{1}{g_m} - z_c} + \frac{1}{z_{DS}}$$

$$= \frac{1}{z_{DS}}\left(\frac{z_{DS} + z_c}{\frac{1}{g_m} - z_c} + 1\right)$$

$$= \frac{1}{z_{DS}}\left(\frac{z_{DS} + z_c}{\frac{1}{g_m} - z_c} + \frac{\frac{1}{g_m} - z_c}{\frac{1}{g_m} - z_c}\right)$$

$$= \frac{1}{z_{DS}}\left(\frac{z_{DS} + \frac{1}{g_m}}{\frac{1}{g_m} - z_c}\right)$$

$$= \frac{g_m + \frac{1}{z_{DS}}}{1 - g_m z_c}$$

$$= \frac{1m + \frac{1}{600}}{1 - 1m(-j200M)}\ /\Omega$$

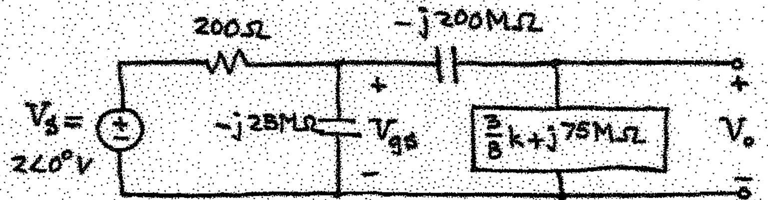
$$z_{eq} \parallel z_{DS} = \frac{1 + j200K}{\frac{1}{1K} + \frac{1}{600}}\ \Omega$$

$$z_{eg} \| z_{DS} = \frac{1 + j\,200\text{k}}{\frac{3+5}{3\text{k}}}\ \Omega = \frac{3}{8}\text{k}\,(1+j200\text{K})\ \Omega$$

$$= \frac{3}{8}\text{k} + j\,75\text{M}\,\Omega$$

We see that the value is <u>smaller</u> than $z_C = -j\,200\ \text{M}\Omega$.

Our new, simplified model:




We use V-dividers to find $V_o$.

$$V_{gs} = V_s \cdot \frac{-j25\text{M}\Omega \| (-j200\text{M} + \overbrace{\cancel{\tfrac{3}{8}\text{k}}}^{\text{small}} + j\,75\text{M}\Omega)}{200 + -j25\text{M}\Omega \| (-j200\text{M} + \overbrace{\cancel{\tfrac{3}{8}\text{k}}}^{\text{small}} + j\,75\text{M}\Omega)}$$

$$V_{gs} \doteq \frac{V_s \cdot (-j25\text{M}\Omega \| -j\,125\text{M}\Omega)}{200 - j25\text{M}\Omega \| -j\,125\text{M}\Omega}$$

where $-j25\text{M}\Omega \| -j125\text{M}\Omega = -j25\text{M}\Omega \cdot 1 \| 5$

$$= -j\,25\text{M}\Omega \cdot \frac{5}{6}$$

$$= -j\,\frac{125}{6}\ \text{M}\Omega$$

$$V_{gs} = \frac{V_s \cdot \left(-j\,\frac{125}{6}\text{M}\Omega\right)}{\underbrace{\cancel{200}}_{\text{small}} - j\,\frac{125}{6}\text{M}\Omega} \approx V_s$$

$$V_o = V_{gs} \frac{\overbrace{\tfrac{3}{8}k}^{\text{small}} + j\,75\,M\Omega}{\underbrace{\tfrac{3}{8}k}_{\text{small}} + j\,75\,M\Omega - j\,200\,M\Omega}$$

$$V_o \approx V_{gs} \frac{j\,75\,M\Omega}{-j\,125\,M\Omega} = V_{gs}\left(-\frac{3}{5}\right)$$

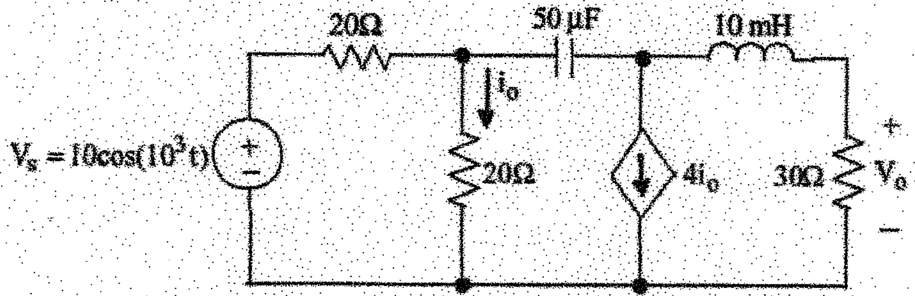
$$V_o \approx 2\angle 0^\circ V \left(-\frac{3}{5}\right) = -\frac{6}{5}\angle 0^\circ V = \frac{6}{5}\angle 180^\circ V$$

Note: *a minus sign is the same as 180° of phase shift.*

$$v_o(t) = \frac{6}{5}\cos(10kt + 180^\circ)\ V$$

170

4.

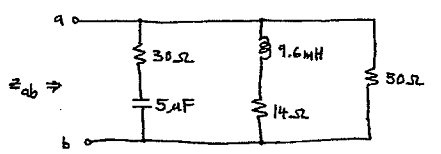



Find $V_o(t)$.

ECE 1000

HW #7 Example

1.



$\omega = 5k$ r/s

Find $Z_{ab}$.

sol'n: Convert C value to impedance $Z_C = \frac{-j}{\omega C}$.

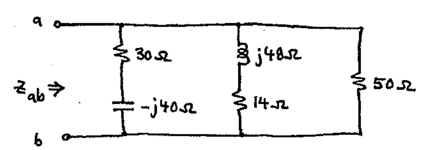
$$Z_C = \frac{-j}{5k\,r/s \cdot 5\mu F} = \frac{-j}{25m}\,\Omega = -j40\,\Omega$$

Convert L value to impedance $Z_L = j\omega L$.

$$Z_L = j\,5k\,r/s \cdot 9.6mH = j\,48\,\Omega$$

R values are the same as $Z_R$ values.

Frequency-domain model:



$$Z_{ab} = (30 - j40) \| (14 + j48) \| 50\ \Omega$$

For parallel $Z$'s, it is often easir to use conductance, $\frac{1}{Z}$.

$$\frac{1}{Z_{ab}} = \frac{1}{30 - j40\,\Omega} + \frac{1}{14 + j48\,\Omega} + \frac{1}{50\,\Omega}$$

sol'n: 1. cont. Rationalize the $\frac{1}{Z}$ values and sum them.

$$\frac{1}{Z_{ab}} = \frac{30 + j40}{30^2 + 40^2\,\Omega} + \frac{14 - j48}{14^2 + 48^2\,\Omega} + \frac{1}{50\,\Omega}$$

$$\frac{1}{Z_{ab}} = \frac{30 + j40}{50^2\,\Omega} + \frac{14 - j48}{50^2\,\Omega} + \frac{50}{50^2\,\Omega}$$

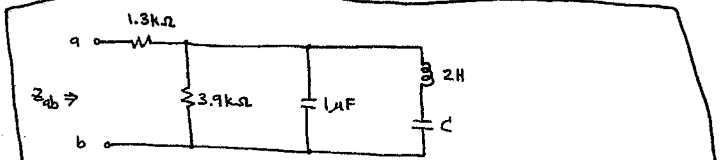
Conveniently, we have the same denominator for every term. (Not typical!)

$$\frac{1}{Z_{ab}} = \frac{30 + 14 + 50 + j(40 - 48)}{50^2\,\Omega} = \frac{94 - j8}{50^2\,\Omega}$$

$$Z_{ab} = \frac{50^2\,\Omega}{94 - j8} = \frac{50^2\,(94 + j8)}{(94 - j8)(94 + j8)} = \frac{50^2\,(94 + j8)}{8900}$$

$$Z_{ab} = \frac{25}{89} \cdot (94 + j8)\,\Omega \doteq 26.4 + j\,2.25\,\Omega$$

2.




Find value of C that makes $Z_{ab}$ real. $\omega = 1k\ r/s$.

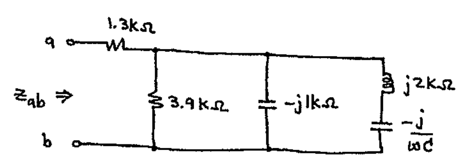
sol'n: First, convert C and L values to Z's.

$$Z_{1\mu F} = \frac{-j}{\omega C} = \frac{-j}{1k\,r/s \cdot 1\mu F} = \frac{-j\,\Omega}{1m} = -j1k\Omega$$

$$Z_{2H} = j\omega L = j\ 1k\,r/s \cdot 2H = j2k\Omega$$

$$Z_C = \frac{-j}{\omega C}$$

Frequency domain model




$$Z_{ab} = 1.3k\Omega + 3.9k\Omega \,\|\, -j1k\Omega \,\|\left(j2k\Omega - \frac{j}{\omega C}\right)$$

If $Z_{ab}$ is to be pure real, we must have that $3.9\,k\Omega \,\|\, -j1k\Omega \,\|\left(j2k\Omega - \frac{j}{\omega C}\right)$ is pure real. This follows from

$Z_{ab} - 1.3k\Omega$ = real # − real # = real #.

We can also show that real $\|$ Z = real if and only if Z is pure real (or ∞, see below)

real $\|$ Z = real $\Rightarrow \frac{1}{\text{real} \| Z} = \frac{1}{\text{real}}$ = real

$\Rightarrow \frac{1}{\text{real}} + \frac{1}{Z}$ = real $\Rightarrow \frac{1}{Z}$ = real $\Rightarrow$ Z = real or ∞

sol'n: 2. cont.

Thus, $-j1k\Omega\left\|\left(j2k\Omega - \frac{j}{\omega C}\right)\right.$ = real

But this means $j \cdot -1k\Omega \left\|\left(2k - \frac{1}{\omega C}\right)\right.$ = real.

Since the left side is of form j · real, the left side is pure imaginary <u>unless it is zero</u>.

$$\therefore\ -1k\Omega \left\|\left(2k - \frac{1}{\omega C}\right)\right. = 0$$

$$\text{or}\quad \frac{-1k\Omega \cdot \left(2k\Omega - \frac{1}{\omega C}\right)}{-1k\Omega + 2k\Omega - \frac{1}{\omega C}} = 0$$

It follows that $2k\Omega - \frac{1}{\omega C} = 0$

or $\frac{1}{\omega C} = 2k\Omega$ or $\omega C = \frac{1}{2k\Omega}$

or $C = \frac{1}{\omega \cdot 2k\Omega}$ or $C = \frac{1}{1k \cdot 2k}F$

$\boxed{C = 0.5\,\mu F}$ $Z_{ab} = 1.3k\Omega$

Earlier, we noted that if

$-1k\Omega \left\|\left(2k\Omega - \frac{1}{\omega C}\right)\right. = \infty\,\Omega$ we also have a sol'n.

We achieve this if the denominator of the parallel impedance becomes zero.

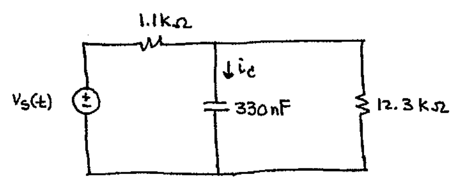
$$-1k\Omega + 2k\Omega - \frac{1}{\omega C} = 0$$

or $\frac{1}{\omega C} = 1k\Omega$ or $\omega C = \frac{1}{1k\Omega}$

or $C = \frac{1}{\omega \cdot 1k\Omega} = \frac{1}{1k \cdot 1k\Omega} = 1\,\mu F$

$\boxed{C = 1\,\mu F}$ $Z_{ab} = 1.3k\Omega + 3.9k\Omega$

3.




$$V_s(t) = 15\cos(3kt)\ V$$

a. Find phasor for $V_s(t)$.

b. Draw frequency domain circuit diagram.

c. Find phasor for $i_c(t)$.

d. Find $i_c(t)$.

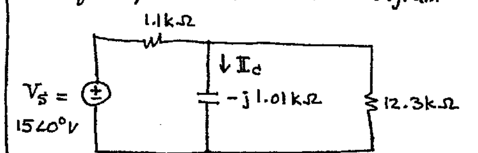
sol'n: a) $P[V_s(t)] = P[15\cos(3kt)\,V] = \boxed{15\angle 0°\ V \equiv \mathbf{V}_s}$

b) From $V_s(t)$, $\omega = 3k$ r/s.

Impedance $Z_c = \dfrac{-j}{\omega C} = \dfrac{-j}{3k\ 330n}\,\Omega = \dfrac{-j}{990}\,\text{M}\Omega$

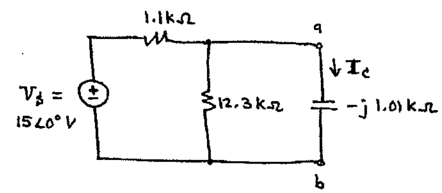
$$Z_c = -j\,1.01\,\text{k}\Omega \approx -j\,1\text{k}\Omega$$

Frequency-domain circuit diagram




c) We could use the node-voltage method or mesh currents, but we can rearrange the circuit and use a Thevenin equivalent involving only R's. This reduces the number of calculations with complex quantities.

sol'n: 3.c)cont.



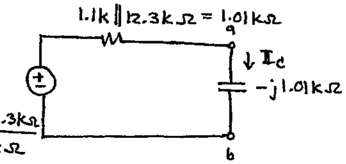

We take the Thevenin equivalent of the circuit to the left of the a, b terminals.

Notice that the above circuit is the same as the original since the 12.3kΩ and −j1.01kΩ are still in parallel.

Using Thevenin equivalent, we have

1.1k ‖ 12.3kΩ = 1.01kΩ

$\mathbf{V}_{Th} = \dfrac{15\angle 0° V \cdot 12.3k\Omega}{12.3k + 1.1k\Omega} \doteq 13.8V\angle 0°$




Now we can calculate $\mathbf{I}_c$ from $\mathbf{V}_{Th}/Z_{Tot}$:

$$\mathbf{I}_c = \frac{13.8\angle 0° V}{1.01k - j1.01k\Omega} = \frac{13.8\angle 0° V}{1.01k\Omega\,(1-j)}\,\frac{(1+j)}{(1+j)}$$

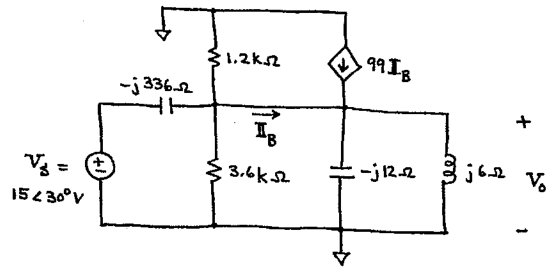
$$= \frac{13.8\angle 0° V}{1.01k\Omega \cdot 2}(1+j) = 6.83\,mA \cdot \sqrt{2}\angle 45°$$

$$\boxed{\mathbf{I}_c = 9.66\angle 45°\ mA}$$

d) $i_c(t) = P^{-1}[\mathbf{I}_c] = P^{-1}[9.66\angle 45°\ mA]$

$$\boxed{i_c(t) = 9.66\cos(3kt + 45°)\ mA}$$

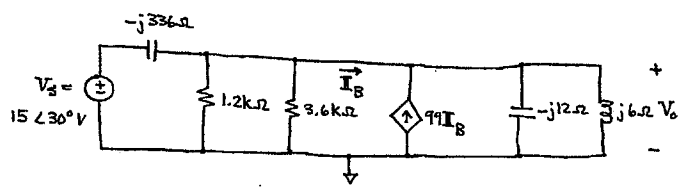
4.




$\omega = 10k$ r/s

Find $V_o(t)$.

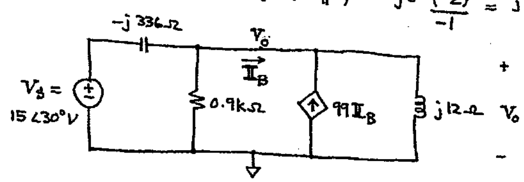
Sol'n: Redraw circuit, being careful to preserve $I_B$.




Combine parallel z's, again preserving $I_B$.

$1.2k\Omega \| 3.6k\Omega = 1.2k\Omega \cdot 1\|3 = 12k\Omega \cdot \frac{3}{4} = 0.9k\Omega$

$-j12\Omega \| j6\Omega = j6(-2\|1) = j6 \cdot \frac{(-2)}{-1} = j12\Omega$




If we try to use node-voltage to find $V_o$, we must express $I_B$ in terms of $V_o$.

We use a current sum at the node above the 99$I_B$ src.

Sol'n: 4. cont.

$$-I_B - 99I_B + \frac{V_o}{j12\Omega} = 0A$$

$$100\, I_B = \frac{V_o}{j12\Omega}$$

$$I_B = \frac{V_o}{j1.2k\Omega}$$

Now we can write node-voltage eq'n for $V_o$:

$$\frac{V_o - 15\angle 30°V}{-j336\Omega} + \frac{V_o}{0.9k\Omega} - \frac{99\,V_o}{j1.2k\Omega} + \frac{V_o}{j12\Omega} = 0A$$

$$V_o\left(\frac{1}{-j336\Omega} + \frac{1}{0.9k\Omega} - \frac{99}{j1.2k\Omega} + \frac{100}{j1.2k\Omega}\right) = \frac{-15\angle 30°V}{j336\Omega}$$

$$V_o\left(\frac{j}{336\Omega} + \frac{1}{0.9k\Omega} + \frac{-j}{1.2k\Omega}\right) = \frac{j15\angle 30°V}{336\Omega}$$

$$V_o = j15\angle 30°V\,\frac{(-j336\Omega \| 0.9k\Omega \| j1.2k\Omega)}{336\Omega}$$

$$-j336\Omega \| j1.2k\Omega = j12 \cdot 28\|100 = j48 \cdot -7\|25\ \Omega$$

$$" = j\overset{8}{\cancel{48}}\frac{(-175)}{18\ 3} = -j\frac{1400}{3}\Omega$$

$$0.9k\Omega \| -j\frac{1400}{3}\Omega = 100 \cdot 9\| -j\frac{14}{3}\ \Omega$$

$$" = 100\,\frac{(-j42)}{9 - j\frac{14}{3}} = 100\,\frac{(-j42)\cdot 3}{27 - j14}$$

$$" = \frac{-j4(25)\,6(7)\cdot 3}{27 - j14} = -j\frac{\overset{336}{\overbrace{28\cdot 12}}\cdot 25\cdot 3/2}{27 - j14}$$

$$\therefore V_o = \frac{\cancel{j}15\angle 30°V\,(\cancel{-j})\,25\cdot 3/2}{27 - j14} \cdot \frac{27 + j14}{27 + j14}$$

soln: 4. cont.

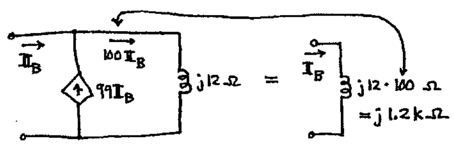
$$V_o = \frac{15\angle 30^\circ (\cancel{25})(3/2)}{\cancel{925}\; 37} \cdot \underbrace{(27+j14)}_{30.4\angle 27.4^\circ}\ V$$

$$V_o \doteq \frac{15(3)\ 30.4}{2\cdot 37} \angle 30^\circ + 27.4^\circ\ V$$

$$V_o \doteq 18.5 \angle 57.4^\circ\ V$$

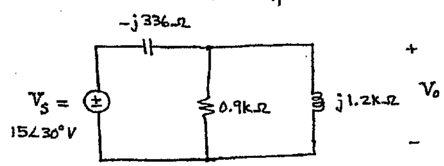
$$\boxed{v_o(t) \doteq 18.5 \cos(10kt + 57.4^\circ)\ V}$$

An alternate approach is to use the idea of impedance multiplication.




The current thru the $j12\Omega$ causes a voltage drop equivalent to $I_B$ flowing thru $j12\Omega \cdot 100$.

Now our circuit is simpler:




$$V_o = \frac{V_S \cdot 0.9k \parallel j1.2k\Omega}{(0.9k \parallel j1.2k\Omega) - j336\Omega}$$

$$0.9k \parallel j1.2k\Omega = 0.3k\Omega \cdot 3 \parallel j4 = 0.3k\Omega \cdot \frac{j12}{3+j4}$$

soln: 4. cont.

$$0.9k \parallel j1.2k\Omega = \cancel{300}^{12} \cdot \frac{j12}{\cancel{3^2+4^2}} (3-j4)\Omega = j144(3-j4)\Omega$$

$$'' = 576 + j432$$

$$V_o = \frac{15\angle 30^\circ V \cdot (576+j432)\Omega}{576 + j432 - j336\ \Omega}$$

$$= \frac{15\angle 30^\circ V\ (576+j432)}{576 + j96}$$

$$= \frac{15\angle 30^\circ V \cdot 720\angle 36.9^\circ}{584\angle 9.5^\circ}$$

$$= \frac{15(720)}{584} \angle 30^\circ + 36.9^\circ - 9.5^\circ\ V$$

$$V_o = 18.5 \angle 57.4^\circ\ V$$

$$\boxed{v_o(t) = 18.5 \cos(10kt + 57.4^\circ)\ V}$$ as before