Unit 3

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Capacitor (C in Farads)

$$\boxed{i(t) = C\frac{dv(t)}{dt}}$$

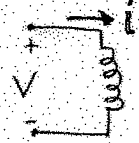
parallel plates dielectric between
V causes i to flow
- stores charge on plates like a small battery

V can not change instantly for C

V constant $\Rightarrow i=0$ $\left(\frac{dv}{dt}=0\right)$

Energy = $\boxed{\frac{1}{2}CV^2}$

Inductor (L in Henry)



$$\boxed{V(t) = L\frac{di(t)}{dt}}$$

i cannot change instantly for L

i constant $\Rightarrow V=0$ $\left(\frac{di}{dt}=0\right)$

Energy = $\boxed{\frac{1}{2}Li^2}$

Cap
Series: $C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}}$
parrallel: $C_{eq} = C_1 + C_2$

L
Series: $L_{eq} = L_1 + L_2$
parrallel: $L_{eq} = \frac{1}{\frac{1}{L_1}+\frac{1}{L_2}}$

General Equation:

$$X(t) = \text{Final} + [\text{Initial} - \text{Final}]\,e^{-(t-t_0)/\tau}$$

t_0: start time at switch (usually set to zero)

procedure:

1. Find Initial value on cap / inductor before switch $(t=0^-)$
   $\{ i_L(t=0^-)$ OR $V_c(t=0^-) \}$
   $\Longrightarrow$ Note: voltage in C cannot change instantaneously
   $V_c(t=0^-) = V_c(t=0^+)$ {V src.}
   $\Longrightarrow$ Note: current in L cannot change instantaneously
   $i_L(t=0^-) = i_L(t=0^+)$ {I src.}
2. Find final value on C/L $(t\to\infty)$
   $\Longrightarrow$ Note: use $V_L(t)=0$ (wire) ; $i_c(t)=0$ (open)
3. Find τ (use switch in <u>FINAL</u> position)
   $\tau = R_{eq}C$ OR $\tau = L/R_{eq}$ where R_{eq} is R seen by element
4. Plug into general equation
   Note: If finding other variables, relate to $V_c(t)$ or $i_L(t)$ eq.

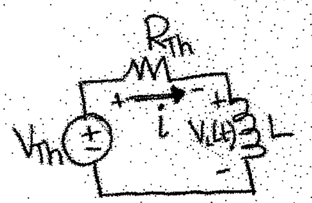
Derivation

Recall #1

$$\int \frac{du}{au+b} = \frac{1}{a}\ln|au+b| + C$$

Recall #2

$$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$$



V-Loop ⇒

$$+V_{Th} - iR_{Th} - V_L(t) = 0$$

$$V_{Th} - iR_{Th} - L\frac{di(t)}{dt} = 0$$

$$L\frac{di(t)}{dt} = +V_{Th} - iR_{Th}$$

$$\frac{di(t)}{dt} = \frac{-(iR_{Th} - V_{Th})}{L}$$

$$\int_{i(t_0)}^{i(t)} \frac{di(t)}{i(t)R_{Th} - V_{Th}} = \int_{t_0}^{t} -\frac{1}{L}\,dt$$

$$\frac{1}{R_{Th}}\left[\ln(R_{Th}i(t) - V_{Th}) - \ln(R_{Th}i(t_0) - V_{Th})\right] = \frac{-(t-t_0)}{L}$$

$$e^{\ln\left[\frac{R_{Th}i(t) - V_{Th}}{R_{Th}i(t_0) - V_{Th}}\right]} = e^{\frac{-(t-t_0)R_{Th}}{L}}$$

$$R_{Th}i(t) - V_{Th} = (R_{Th}i(t_0) - V_{Th})e^{\frac{-(t-t_0)R_{Th}}{L}}$$

$$i(t) = \frac{V_{Th}}{R_{Th}} + \left(i(t_0) - \frac{V_{Th}}{R_{Th}}\right)e^{-\frac{(t-t_0)R_{Th}}{L}}$$

Note: after a long time ($t \to \infty$)

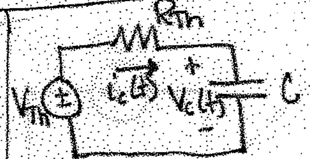
V_{Th} = constant and so $V_L(t\to\infty) = 0$ (wire)

so $i_L(t\to\infty) = \frac{V_{Th}}{R_{Th}}$ (final value)

$i(t_0)$ = initial value

$$\boxed{\tau = L/R_{Th}}$$

$$\therefore\ i(t) = \text{Final} + (\text{initial} - \text{Final})\,e^{-\frac{(t-t_0)}{\tau}}$$



$$+V_{Th} - i_c(t)\cdot R_{Th} - V_c(t) = 0$$

$$V_{Th} - C\frac{dv_c(t)}{dt}\cdot R_{Th} - V_c(t) = 0$$

$$\frac{dv_c(t)}{dt} = \frac{V_{Th} - V_c(t)}{R_{Th}C}$$

$$\int_{V_c(t_0)}^{V_c(t)} \frac{dV_c(t)}{V_c(t) - V_{Th}} = \int_{t_0}^{t} \frac{-1}{R_{Th}C}\,dt$$

$$\ln(V_c(t) - V_{Th}) - \ln(V_c(t_0) - V_{Th}) = \frac{-(t-t_0)}{R_{Th}C}$$

$$e^{\ln\left[\frac{(V_c(t) - V_{Th})}{V_c(t_0) - V_{Th}}\right]} = e^{\frac{-(t-t_0)}{R_{Th}C}}$$

$$V_c(t) - V_{Th} = (V_c(t_0) - V_{Th})e^{\frac{-(t-t_0)}{R_{Th}C}}$$

$$V_c(t) = V_{Th} + (V_c(t_0) - V_{Th})e^{-\frac{(t-t_0)}{R_{Th}C}}$$

Note: After a long time ($t\to\infty$)

V_{Th} = constant and so $i_c(t) = 0$ (open)

so $V_c(t\to\infty) = V_{Th}$ (final value)

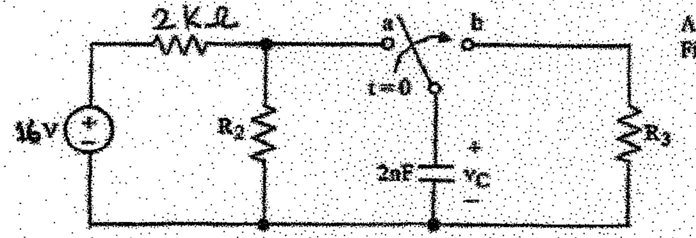
$V_c(t_0)$ = initial value

$$\boxed{\tau = R_{Th}\cdot C}$$

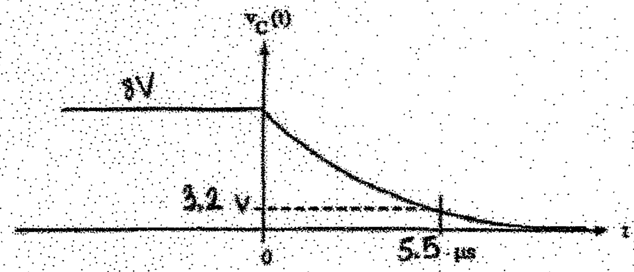
$$\therefore\ V_c(t) = \text{Final} + [\text{initial} - \text{Final}]\,e^{-\frac{(t-t_0)}{\tau}}$$

Homework #5 Examples

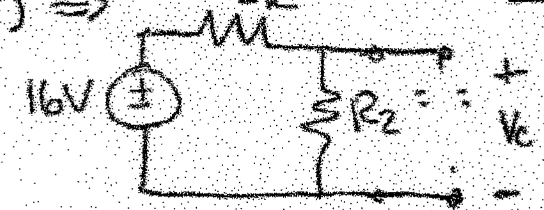
1.



After being in position a for a long time, the switch moves to position b at t = 0. Find R_2 and R_3 that give the following plot for $v_C(t)$:



1. $(t=0^-) \Rightarrow$



$$V_c = \frac{16(R_2)}{R_2+2k}$$

$(t=0^+) \Rightarrow$ because cap V can not change instantaneou

$$V_c(t=0^-) = V_c(t=0^+) = \frac{16(R_2)}{R_2+2k}$$

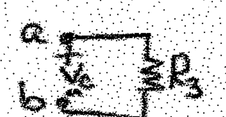
· from graph →

$$V_c(t=0^+) = 8V$$

$$8 = \frac{16(R_2)}{R_2+2k} \Rightarrow 8R_2 + 16k = 16R_2 - 8R_2$$

$$\boxed{R_2 = 2k\,\Omega}$$

$(t=\infty)$:



$V_c = 0$

$\tau = R_3(2n)$

$$V_c(t) = 8 + (0-8)(1-e^{-t/R_3(2n)}) = +8e^{-t/R_3(2n)}$$

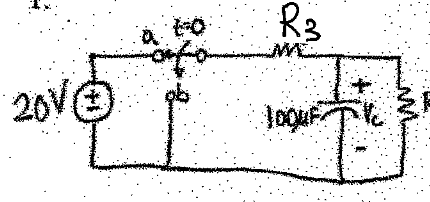
from graph $V_c(5.5\mu) = 3.2V$

$$3.2 = 8e^{-5.5\mu/R_3(2n)}$$

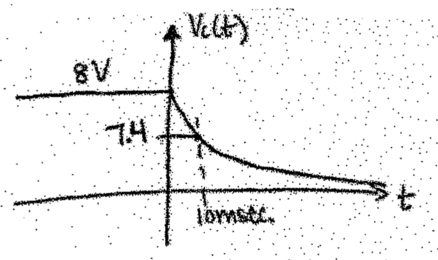
$$\ln\frac{3.2}{8} = -\frac{5.5\mu}{R_3(2n)} \Rightarrow R_3 = \frac{-5.5\mu}{2n\left(\ln\left(\frac{3.2}{8}\right)\right)} \doteq \boxed{3k\,\Omega}$$

Homework #5 Example

1.

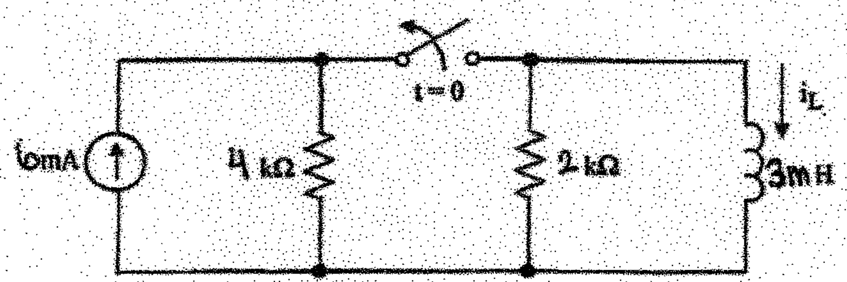


After being in position a for a long time, the switch moves to position b at $t=0$. Find R_2 and R_3 that gives the plot below for $V_c(t)$.



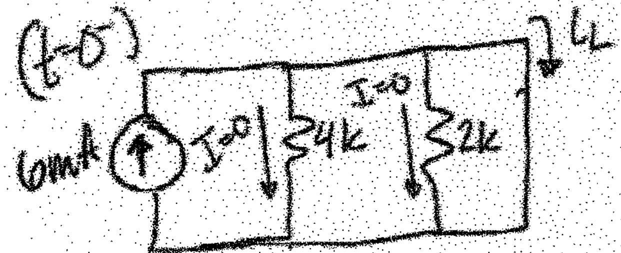
Homework #5 Examples

2.



After being closed for a long time, the switch opens at t = 0. Find $i_L(t)$ for $t > 0$.

$(t=0^-)$



$i_L = 6mA$

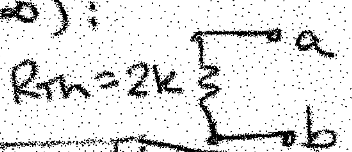
$(t=0^+)$: · inductor current remains the same

$i_L(t=0^-) = i_L(t=0^+) = 6mA \leftarrow$ (initial value)

$(t=\infty)$:

$i_L(t) = 6m + (0-6m)(1-e^{-t/\tau})$

(switch at $t=\infty$):



$\therefore \boxed{i_L(t) = +6m\,e^{-t/3m/2k}\ A}$

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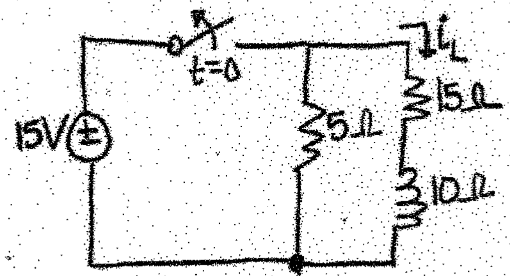
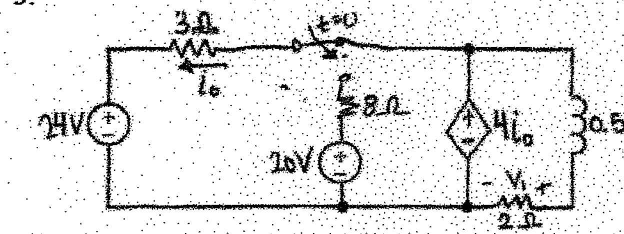
Homework #5 Example

2.



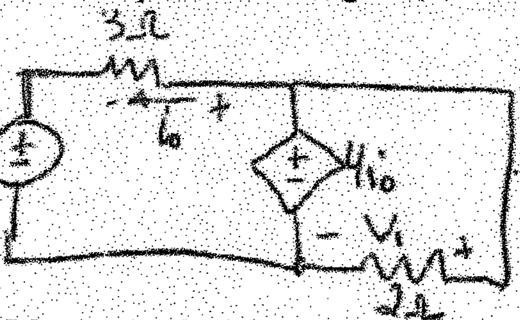
After being closed for a long time, the switch opens at t = 0. Find $i_L(t)$ for t > 0.

3.



After being in the above position for a long time, the switch changes at t=0. Find $v_1(t)$ for $t > 0$.

($t=0^-$):



$+4i_o - 3i_o - 24 = 0$

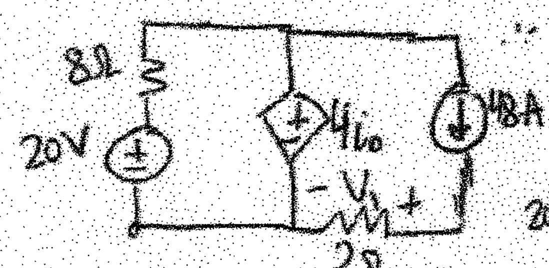
$i_o = +24V/\Omega$

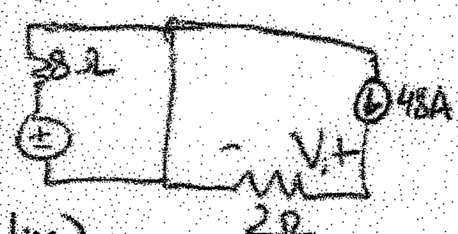
$i_L = \frac{4i_o}{2} = \frac{4(24)}{2} = 48A$

($t=0^+$): Inductor current stays the same ⇒

$i_L(t=0^-) = i_L(t=0^+) = 48A$ ← initial value

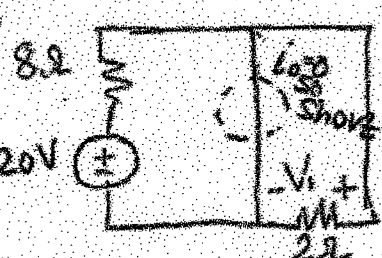
Note that V_1 changes from $t=0^-$ to $t=0^+$.





$V_1(t=0^+) = 48(2)$ (initial value)

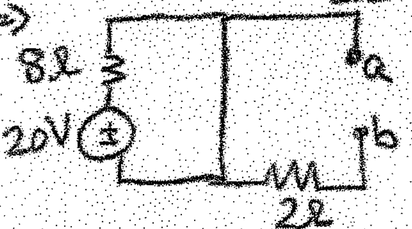
($t=\infty$):



$V_1 = 0$

$i_o = 0$ so Short dependent source

⇒

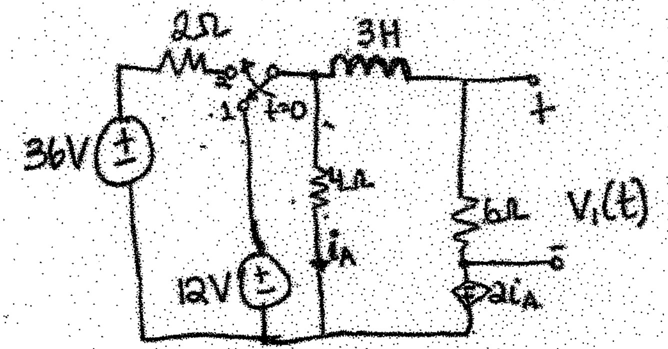


$\frac{L=0.5}{R_{Th}=2} = 0.25$

$V_1(t) = 96 + (0-96)(1-e^{-t/.25}) = \boxed{96e^{-4t}\ V}$

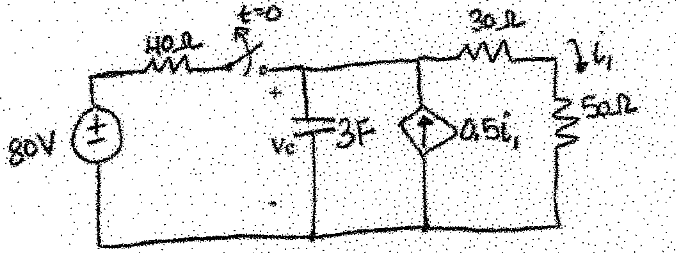
Homework #5 Example

3.



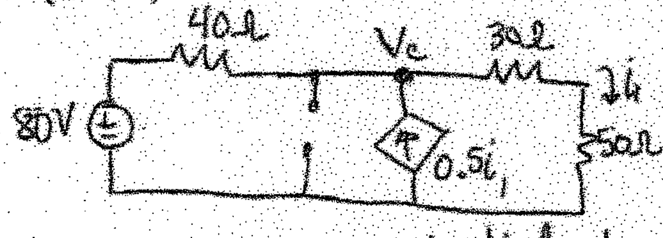
After being at position 1 for a long time, the switch moves to position 2 at t=0. Find $v_1(t)$ for $t > 0$.

4.



After being closed for a long time, the switch opens at t=0. Find Vc(t) for t > 0.

$(t=0^-)$



$$\frac{V_c-80}{40} - 0.5i_1 + \frac{V_c}{80} = 0$$

$$i_1 = \frac{V_c}{80}$$

$$\frac{V_c}{40} - 0.5\left(\frac{V_c}{80}\right) + \frac{V_c}{80} = \frac{80}{40}$$

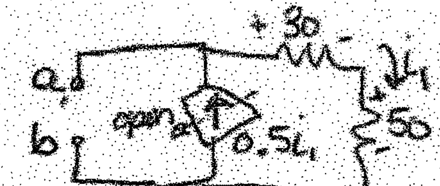
$$V_c\left(\frac{4}{160} - \frac{1}{160} + \frac{2}{160}\right) = \frac{80}{40}$$

$$V_c = 2\,\frac{1}{(5/160)} = \frac{2(160)}{5}$$

initial value

$64V = V_c(t=0^+)$

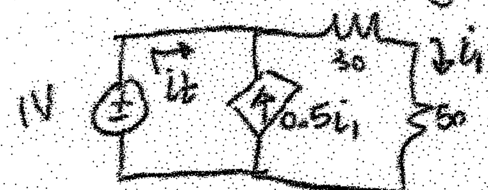
$(t\to\infty)$



$0.5i_1 - i_1 = 0 \Rightarrow i_1 = 0$

$V_c = 0V \leftarrow$ final

. Need to use a test supply since $V_{Th}=0$ and $R_{Th} \neq 0$ or the capacitor will not discharge. It must though since $V_c=0$ as a final value and it started with 64V initially.



$$i_t + 0.5i_1 - i_1 = 0$$

$$i_t = 0.5\,i_1$$

$$i_1 = \frac{1}{80}$$

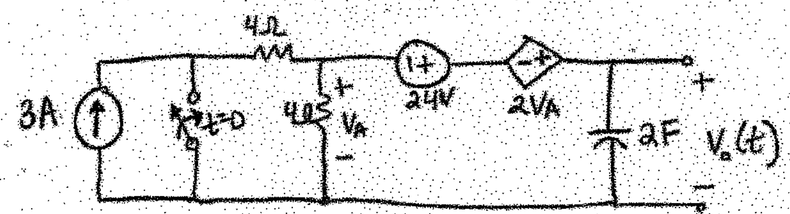
$$i_t = 0.5\left(\frac{1}{80}\right) = \frac{1}{160}$$

$$R_{Th} = \frac{1V}{\frac{1}{160}} = 160\Omega$$

$$\boxed{V_c(t) = 64e^{-t/480}\ V}$$

Homework #5 Example

4.



After being open for a long time, the switch closes at t=0. Find Vo(t) for t > 0.

Examples

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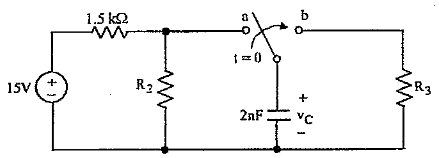
Dr. Cotter

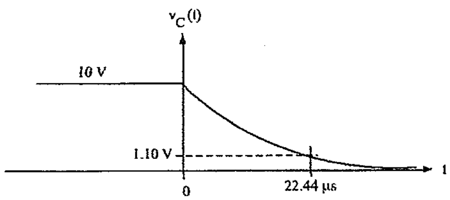
UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

ECE 1000 Spring 2005

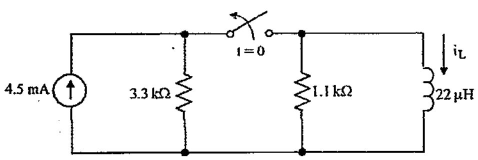
1. {RC}



After being in position a for a long time, the switch moves to position b at $t = 0$. Find R_2 and R_3 that give the following plot for $v_C(t)$:

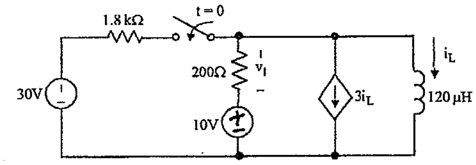


2. {RL}



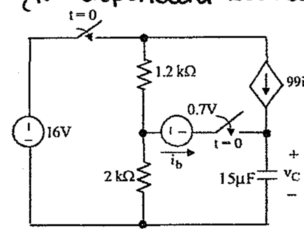
After being closed for a long time, the switch opens at $t = 0$. Find $i_L(t)$ for $t > 0$.

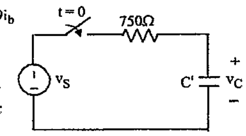
3. {RL, dependent source}



After being open for a long time, the switch closes at $t = 0$. Find $v_1(t)$ for $t > 0$.

4. {RC dependent source}





For both circuits, the switches close at $t = 0$ after being open for a long time.

a. Find $v_C(t)$ for $t > 0$ for the circuit on the left.

b. Find v_S and C' values that make v_C for the circuit on the right equal v_C for the circuit on the left. Assume $v_C(0^-) = 0V$.

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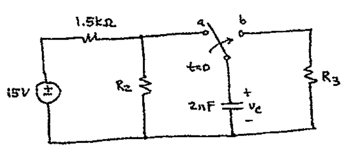
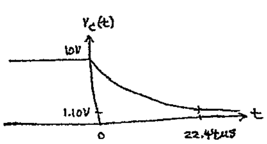
1.



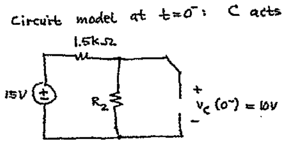


Find R_2 and R_3 to give above plot.

Sol'n: R_2 determines init voltage on C.

From plot, $v_c(t=0^-) = 10V$.

Circuit model at $t=0^-$: C acts like open circuit.



Use v-divider. $v_c(0^-) = 15V \cdot \frac{R_2}{R_2 + 1.5k\Omega} = 10V$

$\therefore \frac{R_2}{R_2 + 1.5k\Omega} = \frac{2}{3}$ or $3R_2 = 2(R_2 + 1.5k\Omega)$

$\boxed{R_2 = 3k\Omega}$

R_3 determines the time constant $= R_3C$

Using the general sol'n for $v_c(t)$ we have:

ECE 1000

Sol'n: 1. cont.

$$v_c(t>0) = \underbrace{v_c(t\to\infty)}_{0V} + [\underbrace{v_c(t=0^+)}_{v_c(0^-) = 10V} - \underbrace{v_c(t\to\infty)}_{0V}]\, e^{-t/R_3C}$$

$v_c(t>0) = 10V\, e^{-t/R_3C}$ where $C = 2nF$

From the plot, we have $v_c(22.44\mu s) = 1.10V$.

$\therefore 1.10V = 10V\, e^{-22.44\mu s/R_3C}$

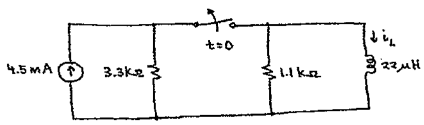
Solve for R_3:

$$\ln \frac{1.10V}{10V} = -22.44\mu s / R_3C$$

$$R_3 = \frac{-22.44\mu s/2nF}{\ln \frac{1.10V}{10V}}$$

$\boxed{R_3 \doteq 5.1k\Omega}$

2.



Find $i_L(t)$ for $t>0$.

Sol'n: We use general sol'n:

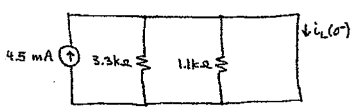
$$i_L(t>0) = i_L(t\to\infty) + \left[i_L(t=0^+) - i_L(t\to\infty)\right] e^{-t/L/R_{Th}}$$

To find $i_L(t=0^+)$, we start at $t=0^-$.

$t=0^-$: Treat L as wire when circuit has been sitting for a long time, (so $\frac{di_L}{dt}=0 \Rightarrow v_L = L\frac{di_L}{dt}=0$).

Find $i_L(0^-)$ because $i_L(0^+) = i_L(0^-)$.

Switch is closed at $t=0^-$.



All the current will flow thru the wire.

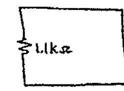
$i_L(0^-) = 4.5$ mA

Since the energy stored by the inductor cannot change instantly, $i_L(0^+) = i_L(0^-)$.

$\therefore\ i_L(0^+) = 4.5$ mA

Sol'n: 2. cont. To find $i_L(t\to\infty)$, we again treat L as wire.

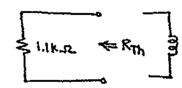
$t\to\infty$: Switch open. 4.5mA and 3.3kΩ not connected.



No power source so current = 0A.

$i_L(t\to\infty) = 0$ A

The time constant is $\frac{L}{R_{Th}}$ where R_{Th} is from the Thevenin equivalent of circuit (for t>0) where L is connected.



$R_{Th} = 1.1$ kΩ

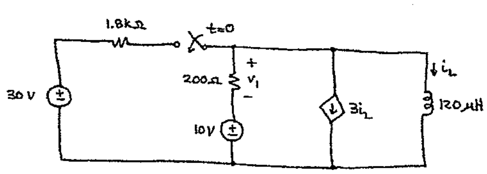
$$\frac{L}{R_{Th}} = \frac{22\mu H}{1.1k\Omega} = 20 \text{ ns}$$

Plug quantities into general sol'n:

$$i_L(t>0) = 0A + \left[4.5mA - 0A\right] e^{-t/20ns}$$

or $\boxed{i_L(t>0) = 4.5mA\, e^{-t/20ns}}$

3.

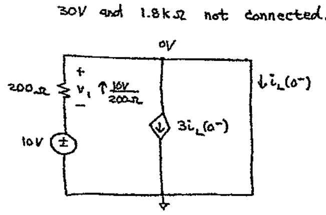


Find $v_1(t)$ for $t>0$.

Sol'n: Find i_L for $t=0^-$.

$t=0^-$: Switch is open. L = wire.

30V and 1.8kΩ not connected.



Because L = wire, we have 10V across 200Ω (so 10V src + $V_{200\Omega} = 0$ V).

Thus, current $\frac{10V}{200\Omega}$ flows upward thru 200Ω. This current equals $3i_L(0^-) + i_L(0^-)$.

$$\therefore\ \frac{10V}{200\Omega} = 3i_L(0^-) + i_L(0^-) = 4i_L(0^-)$$

$$50mA = 4i_L(0^-)$$

$$i_L(0^-) = \frac{50mA}{4} = 12.5 \text{ mA}$$

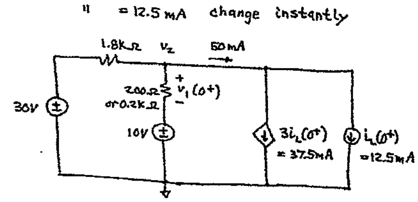
Sol'n: 3. cont. Now find $v_1(t=0^+)$.

$t=0^+$: Switch is closed.

L = current source at $t=0^+$

$i_L(0^+) = i_L(0^-)$ since i_L doesn't change instantly

" = 12.5 mA



Use node voltage to find v_2.

$$\frac{V_2 - 30V}{1.8k\Omega} + \frac{V_2 - 10V}{0.2k\Omega} + 50mA = 0A$$

$$V_2\left(\frac{1}{1.8k\Omega} + \frac{1}{0.2k\Omega}\right) = \frac{30V}{1.8k\Omega} + \underbrace{\frac{10V}{0.2k\Omega}}_{50mA} - \cancel{50mA}$$

$$V_2\left(\frac{1}{1.8k\Omega} + \frac{1}{0.2k\Omega}\right)\cdot 1.8k\Omega = 30V$$

$$V_2(1+9) = 30V$$

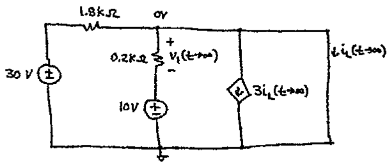
$$V_2 = \frac{30V}{10} = 3V$$

$$10V + v_1(0^+) = V_2 = 3V$$

$$v_1(0^+) = -7V$$

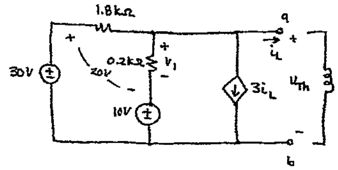
sol'n: 3. cont. Now find $v_1(t\to\infty)$.

$t\to\infty$: Switch is closed.
L = wire



The voltage across 10V src and 0.2kΩ is 0V. ∴ $v_1(t\to\infty) = -10V$

Now find time constant $\frac{L}{R_{Th}}$. R_{Th} is for circuit where L is connected.



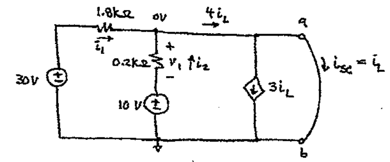
$V_{Th} = V_{a,b\ open\ circ}$

$i_L = 0$ so we treat $3i_L$ src as open circuit.
We could use node voltage method to find $v_{a,b}$, but a simpler approach is to observe that we have 20V across 1.8kΩ and 0.2kΩ in series.

soln: 3. cont. Using a voltage divider, $v_1 = 20V \cdot \frac{0.2k\Omega}{0.2k + 1.8k\Omega} = 2V$.

∴ $V_{Th} = V_{ab\ open\ circ} = 10V + v_1 = 12V$

Now short a, b terminals to find i_{sc}.



This is the same circuit as for $t\to\infty$. Using the value of 0V on the top node, we can find i_1 and i_2:

$i_1 = \frac{30V}{1.8k\Omega}$ $\quad i_2 = \frac{10V}{0.2k\Omega}$

Summing currents, we have

$-i_1 - i_2 + 4i_L = 0A$

or $i_L = i_{sc} = \frac{i_1 + i_2}{4} = \frac{\frac{30V}{1.8k\Omega} + \frac{10V}{0.2k\Omega}}{4}$

$i_{sc} = \frac{\frac{30V}{1.8k\Omega} + \frac{9\cdot 10V}{1.8k\Omega}}{4} = \frac{120V}{4(1.8k\Omega)}$

$i_{sc} = \frac{30V}{1.8k\Omega}$

$R_{Th} = V_{Th}/i_{sc} = \frac{12V}{30V/1.8k\Omega} = 1.8k\Omega\,\frac{12V}{30V}$

$R_{Th} = 1.8k\Omega\cdot\frac{2}{5} = \frac{3.6k\Omega}{5} = 720\Omega$

sol'n: 3. cont. Our time constant is $\frac{L}{R_{Th}} = \frac{120\mu H}{720\Omega} = \frac{1}{6}\mu s$.

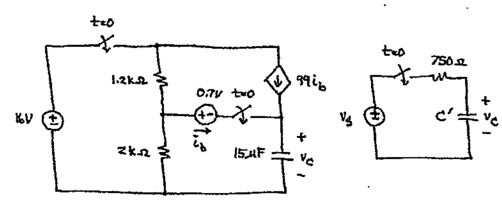
Plug values into general sol'n:

$v_1(t>0) = v_1(t\to\infty) + [v_1(0^+) - v(t\to\infty)]\, e^{-t/L/R_{Th}}$

$v_1(t>0) = -10V + [-7V - -10V]\, e^{-t/\frac{1}{6}\mu s}$

$\boxed{v_1(t>0) = -10V + 3V\, e^{-t/\frac{1}{6}\mu s}}$

4.



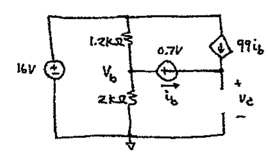
a. Find $v_c(t)$ for $t>0$ for circuit on left

b. Find v_s and C′ values that make circuit on right have same v_c as circuit on left. Assume $v_c(0^-) = 0V$.

sol'n: a) $v_c(0^+) = v_c(0^-) = 0V$ since v_c can't change instantly

Now find $v_c(t\to\infty)$.

$t\to\infty$: C = open
switches closed



If we sum currents at node above C, we have $-i_b - 99i_b = 0A \Rightarrow i_b = 0$.
Thus, $99i_b$ src = open.
∴ $v_c = v_b - 0.7V$

sol'n: 4.a) cont. We use v-divider formula to find v_b:

$$v_b = 16V \cdot \frac{2\,k\Omega}{1.2k\Omega + 2\,k\Omega} = 16V \cdot \frac{2\,k\Omega}{3.2k\Omega}$$

$$v_b = 16V \cdot \frac{20\,k\Omega}{32\,k\Omega} = \frac{1}{2} \cdot 20\,V$$

$$v_b = 10\ V$$

$$v_c(t\to\infty) = 10V - 0.7V \doteq 9.3\,V$$

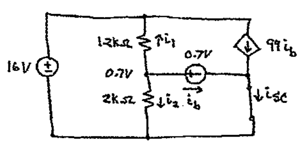
Now we find $R_{Th}C$. First, we find

V_{Th} = V across terminals where C connected but with C removed

But this is the same as $v_c(t\to\infty)$.

$$V_{Th} = 9.3V$$

Now short the terminals where C connected and find i_{sc}.



From current summation at node above C, $i_{sc} = i_b + 99 i_b = 100\,i_b$.
Because of 0.7V src, we 0.7V at center node. We get i_b from current summation at this node.

sol'n: 4.a) cont.

$$i_1 = \frac{0.7V - 16V}{1.2k\Omega} = -\frac{15.3\,V}{1.2k\Omega}$$

$$i_2 = \frac{0.7V}{2k\Omega}$$

$$i_1 + i_2 + i_b = 0A$$

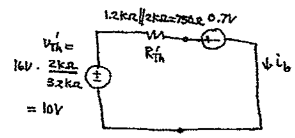
$$i_b = -(i_1 + i_2) = \frac{15.3V}{1.2k\Omega} - \frac{0.7V}{2k\Omega}$$

$$i_{sc} = 100\,i_b = 100\left(\frac{15.3V}{1.2k\Omega} - \frac{0.7V}{2k\Omega}\right)$$

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{9.3\,V}{100\left(\frac{15.3V}{1.2k\Omega} - \frac{0.7V}{2k\Omega}\right)}$$

$$R_{Th} = 7.5\Omega$$

This worked, but a better approach is to use a Thevenin equiv of 16V, 1.2kΩ, and 2kΩ.



We have $i_b = \frac{V'_{Th} - 0.7V}{R'_{Th}} = \frac{9.3V}{750\Omega}$

$$i_{sc} = 100\,i_b = \frac{9.3V}{750\Omega} \cdot 100$$

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{9.3V}{\frac{9.3V \cdot 100}{750\Omega}} = \frac{750\Omega}{100} = 7.5\Omega$$

sol'n: 4a) cont. Plug values into general sol'n.

$$v_c(t>0) = \underset{9.3V}{v_c(t\to\infty)} + \left[\underset{0}{v_c(0^+)} - \underset{9.3V}{v_c(t\to\infty)}\right] e^{-t/R_{Th}C}$$

$$R_{Th}C = 7.5\Omega \cdot 15\mu F = 112.5\,\mu s$$

$$\boxed{v_c(t>0) = 9.3V\left[1 - e^{-t/112.5\mu s}\right]}$$

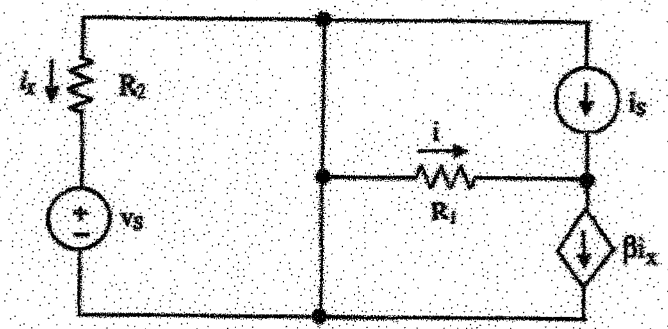
4.b) For circuit on right, we have $v_c(0^+) = v_c(0^-) = 0V$ (given).
We also have $v_c(t\to\infty) = v_s$ and $RC' = 750\Omega \cdot C'$.
To make the v_c's the same for the two circuits, we use $\boxed{v_s = 9.3V}$ and

$$C' = \frac{RC}{750\Omega} = \frac{7.5\Omega \cdot 15\mu F}{750\Omega} = 150\ nF$$

$$\boxed{C' = 150\,nF}$$

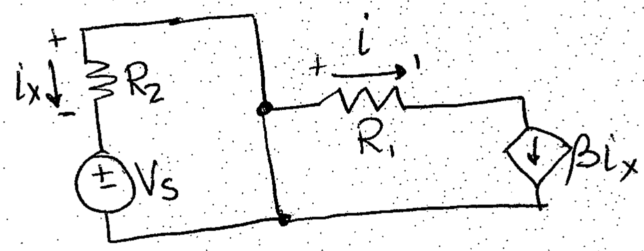
Superposition Example

- if V=0, treat as a wire.
- if I=0, treat as an open.



Using superposition, derive an expression for i that contains no circuit quantities other than i_s, v_S, R_1, R_2, and β.

- Turn on only 1 independent source, all others off.
  ⟨Dependent sources always stay in circuit⟩
- Redraw circuit (V_s on, is off (open))

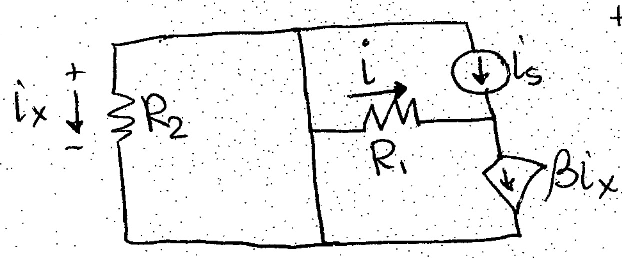


V-loop: $+V_s + i_x R_2 = 0$

$$i_x = -\frac{V_s}{R_2}$$

$$i = \beta i_x = -\frac{\beta V_s}{R_2}$$

- Redraw circuit (V_s off (wire), is on)



$+i_x R_2 = 0 \Rightarrow i_x = 0$

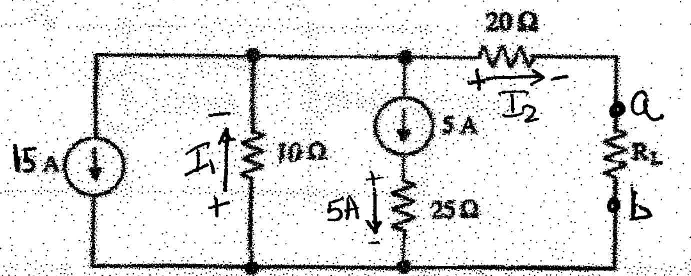
$\beta i_x = 0 \rightarrow$ open

$i = -i_s$

- Sum results of partials for total:

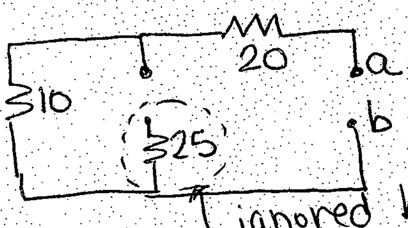
$$\boxed{i = -\frac{\beta V_s}{R_2} - i_s}$$

Max Power Example



Calculate the value of R_L that would absorb maximum power, and calculate that value of maximum power R_L could absorb.

- Maximum power is absorbed in element when R_L value $= R_{Th}$ (equivalent resistance seen by R_L)
- Independent sources only in circuit. $R_{Th} = R_{eq}$.
  Set $V=0$, $I=0$ and redraw ⇒

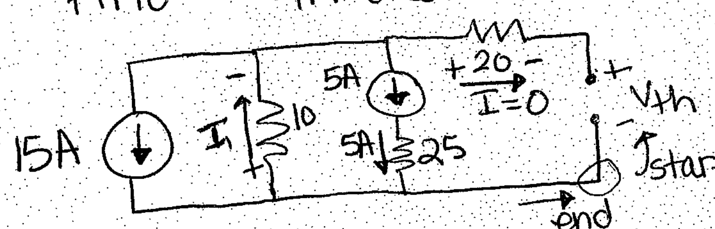


path from a back to b

$20 + 10 = 30 = R_{th}$

$\therefore \boxed{R_L = 30\Omega}$

- $\text{power} = \dfrac{(V_{th})^2}{4(R_{th})}$
- Find V_{th} (redraw circuit)



$+V_{th} + 0 + I_1(10) = 0$

$-15 + I_1 - 5 - 0 = 0$

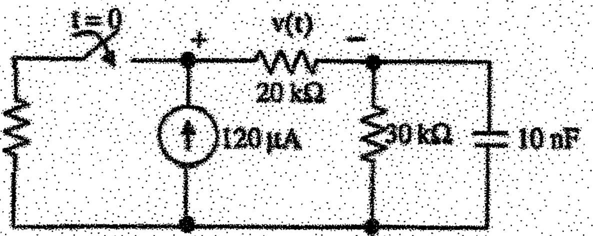
$I_1 = 20$

$\therefore V_{th} = -20(10) = -200$

$\text{power} = \dfrac{(-200)^2}{4(30)} = \boxed{333\,W}$

Energy Example

(111)



Cap ⇒ becomes open after a long time

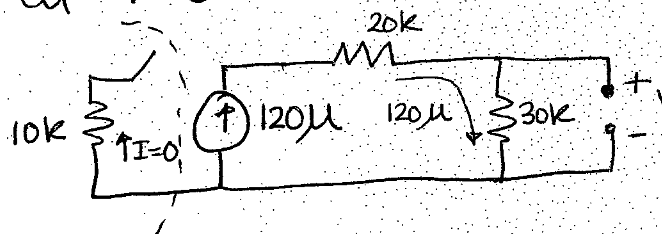
- Stays V value at $t=0^+$ (V src)

$\tau = R_{eq} \cdot C$

After being open for a long time, the switch is closed at t = 0. Calculate the energy stored on the capacitor at $t \to \infty$

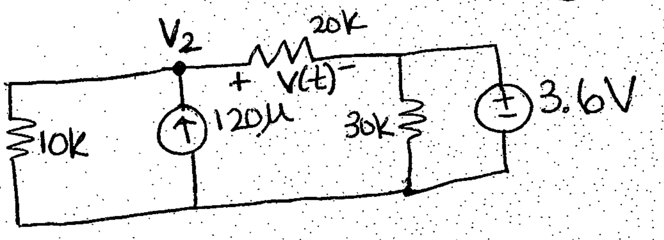
Write a numerical expression for $v(t)$, $t > 0$.

at $t = 0^-$: (redraw circuit – cap acts as open)



$V_c = 120\mu(30k) = 3.6V$

at $t = 0^+$: (redraw circuit – voltage on cap cannot change instantaneously : use v source)



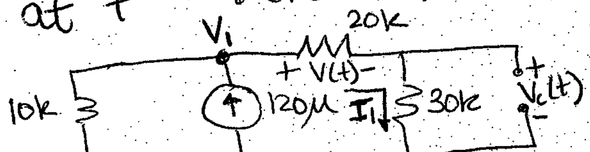
node-voltage: $\frac{V_2}{10k} - 120\mu + \frac{V_2 - 3.6}{20k} = 0$

$V_2\left(\frac{1}{10k} + \frac{1}{20k}\right) = +120\mu + \frac{3.6}{20k}$

$V_2 = \frac{300\mu}{150\mu} = 2V$

$V(t)_{t=0^+} = V_2 - 3.6 = -1.6V$ [Initial value]

at $t = \infty$: (redraw, cap open)



$\tau \Rightarrow 30k \| (20k + 10k) = 15k = R_{eq}$

$V(t)_{t=\infty} = I_1(20k)$ | $V_c(t)_{t\to\infty} = I_1(30k) = \frac{30k}{50k}$

node-voltage:

$\frac{V_1}{10k} - 120\mu + \frac{V_1}{50k} = 0 \Rightarrow V_1\left(\frac{1}{10k} + \frac{1}{50k}\right) = 120\mu \Rightarrow V_1 = 1V$

$I_1 = \frac{V_1}{50k} = \frac{1}{50k} \Rightarrow V(t)_{t=\infty} = \frac{20k}{50k} = 0.4V$ [Final value]

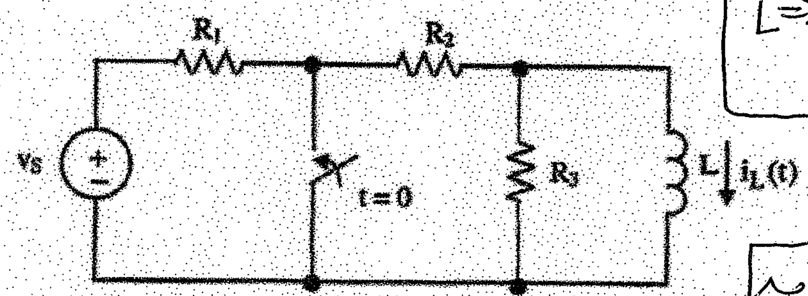
for $t > 0$, $V(t) = \underset{\text{(Final)}}{0.4} + \underset{\text{(Initial)} - \text{(Final)}}{(-1.6 - 0.4)}\, e^{-t/\underbrace{(15k \cdot 10n)}_{\tau}}$

$\boxed{V(t) = 0.4 - 2e^{-t/150\mu sec}\ V}$

$w_c(t \to \infty) = \frac{1}{2}(C)(V_c(t \to \infty))^2$

$w_c(t \to \infty) = \frac{1}{2}(10n)\left(\frac{3}{5}\right)^2 = \boxed{1.8nJ}$

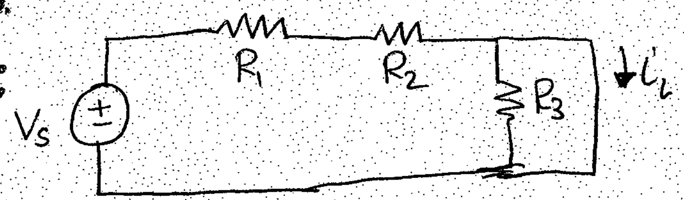
Switch RL Example



$L \Rightarrow$ becomes <u>wire</u> after a long time.
· I stays same at $t=0^+$ (Isrc)
$\tau = L/R_{eq}$

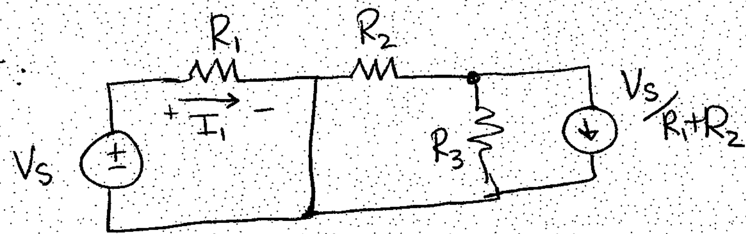
After being open for a long time, the switch is <u>closed</u> at $t = 0$. Write an expression for $i_L(t)$, $t > 0$.

at $t=0^-$:



$$i_L = \frac{V_s}{R_1+R_2}$$

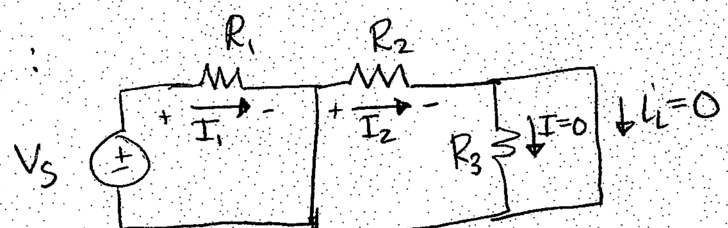
at $t=0^+$:



$$i_L(t=0^+) = \frac{V_s}{R_1+R_2}$$

[Initial Value]

at $t=\infty$:



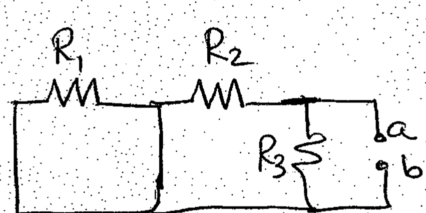
$$+I_2 R_2 = 0$$
$$I_2 = 0$$
$$I_2 = i_L = 0$$

[Final Value]

$t>0$

$$i_L(t) = 0 + \left[\left(\frac{V_s}{R_1+R_2}\right) - 0\right] e^{-t/\tau}$$

$\tau \Rightarrow R_{eq} = R_3 \| R_2$ (redraw)



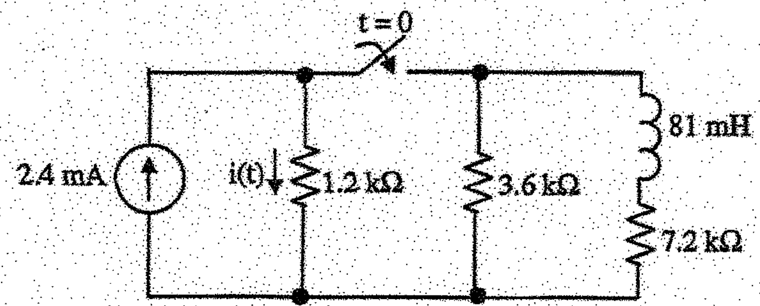
$$\boxed{t>0 \quad i_L(t) = \left(\frac{V_s}{R_1+R_2}\right) e^{-t/\frac{L}{(R_2 \| R_3)}}}$$

HW #6 Examples

Dr. Cotter

(113)

1.



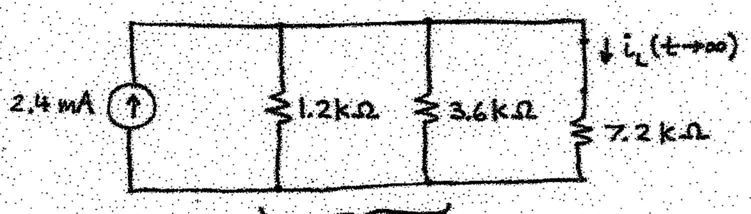
After being open for a long time, the switch is <u>closed</u> at $t = 0$.

Calculate the energy stored on the inductor at $t \to \infty$.

2. Write a numerical expression for $i(t)$, $t > 0$.

Sol'n: 1. For $t \to \infty$, L acts like wire, switch is closed.

Find $i_L(t \to \infty)$. Energy, $w = \frac{1}{2} L i_L^2$.



$$R_{eq} = 1.2\,k\Omega \parallel 3.6\,k\Omega = 1.2\,k\Omega \cdot 1 \parallel 3 = 1.2\,k\Omega \cdot \frac{3}{4}$$

$$R_{eq} = 0.9\,k\Omega$$

We have current divider.

$$i_L(t\to\infty) = 2.4\,mA \cdot \frac{R_{eq}}{R_{eq} + 7.2\,k\Omega} = 2.4\,mA\,\frac{0.9\,k\Omega}{8.1\,k\Omega}$$

$$i_L(t\to\infty) = \frac{2.4\,mA}{9} = \frac{24}{90}\,mA = \frac{12}{45}\,mA = \frac{4}{15}\,mA$$

$$w = \frac{1}{2}\,81\,mH \left(\frac{4}{15}\right)^2 mA^2 = \frac{1}{2}\,81 \cdot \frac{16}{25 \cdot 9}\,nJ$$

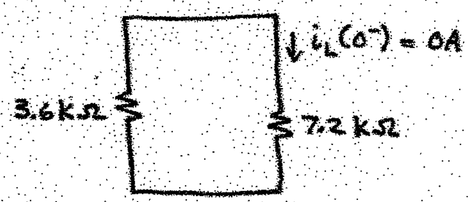
$$w = \frac{9(8)}{25}\,nJ = \frac{72 \cdot 40}{25 \cdot 40}\,nJ = 2.88\,nJ$$

$$\boxed{w_L(t\to\infty) = 2.88\,nJ}$$

sol'n: 2. We want to find $i(t=0^+)$. We start by finding $i_L(t=0^-)$ so we will know $i_L(t=0^+)$, since $i_L(t=0^+) = i_L(t=0^-)$.

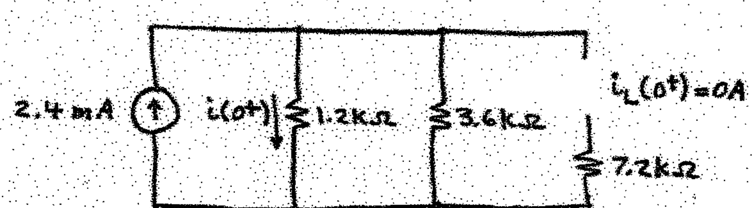
$t=0^-$: L acts like wire, switch is open.

No pwr src for L, so L discharges to give $i_L(0^-) = 0A$.



$t=0^+$: L acts like current src of value $i_L(0^+) = i_L(0^-) = 0A =$ open circuit

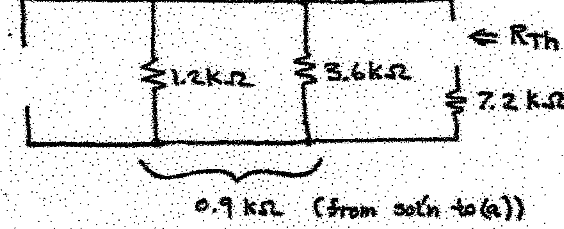
Switch is closed.



This is a current divider.

$$i(0^+) = 2.4mA \cdot \frac{3.6k\Omega}{1.2k\Omega + 3.6k\Omega} = 1.8mA$$

For time constant $\frac{L}{R_{Th}}$, we look into the terminals where L is connected. We turn off independent 2.4mA src which becomes open circuit.



sol'n: 2. cont. $R_{Th} = 7.2k\Omega + 0.9k\Omega = 8.1k\Omega$

Our time constant is $\frac{L}{R_{Th}} = \frac{81\text{ mH}}{8.1k\Omega} = 10\,\mu s$.

Find $i(t\to\infty)$ [see below].

Plug values into general solution.

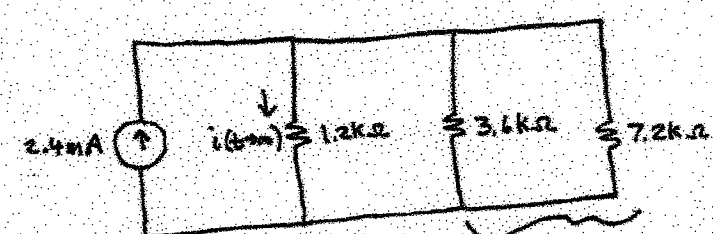
$$i(t>0) = i(t\to\infty) + \left[i(0^+) - i(t\to\infty)\right] e^{-t/\frac{L}{R_{Th}}}$$

$$i(t>0) = 1.6\text{ mA} + \left[1.8\text{mA} - 1.6\text{mA}\right] e^{-t/10\mu s}$$

$$\boxed{i(t>0) = 1.6\text{mA} + 0.2\text{mA}\, e^{-t/10\mu s}}$$

$t\to\infty$: L acts like wire, switch closed

Find $i(t\to\infty)$ (not i_L).

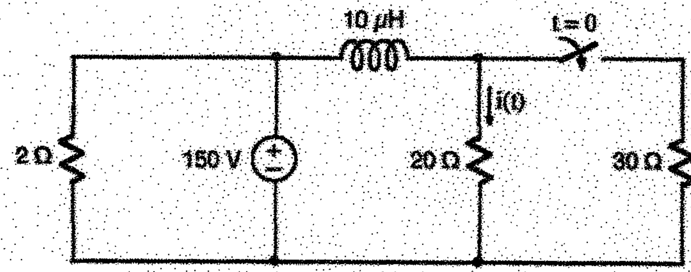


$$3.6k \parallel 7.2k\Omega = 3.6k\Omega \cdot 1 \parallel 2$$
$$\quad\text{"} \quad = 3.6k\Omega \cdot \frac{2}{3}$$
$$\quad\text{"} \quad = 2.4k\Omega$$

i-divider: $i(t\to\infty) = 2.4\text{ mA} \cdot \frac{2.4k\Omega}{2.4k + 1.2k\Omega} = 1.6\text{ mA}$

Homework #6/Review Examples

1.



After being closed for a long time, the switch closes at $t = 0$.

Calculate the energy stored on the inductor as $t \to \infty$.

find $i(t)$ for $t > 0$.

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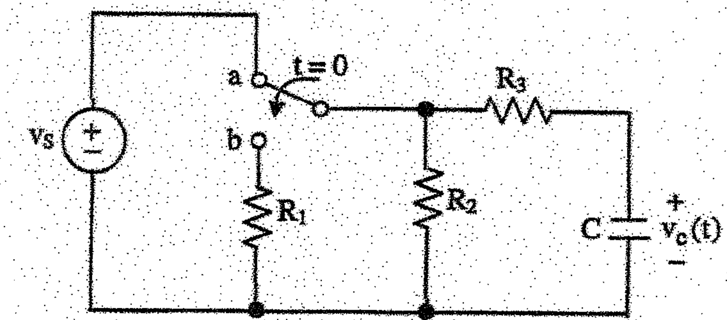
3.



After being at position a for a long time, the switch moves to position b at time $t=0$.

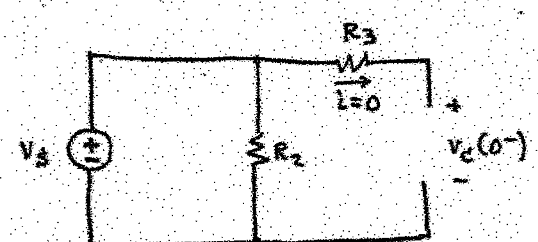
a. Write an expression for $v_c(t = 0^+)$.

b. Write an expression for $v_c(t)$, $t > 0$.

soln: a) $v_c(t=0^+) = v_c(t=0^-)$

At $t=0^-$, switch is in position a.

C acts like open circuit.



No current in R_3 means no v-drop for R_3.

$\therefore v_c(0^-) = V_s$

$$\boxed{v_c(0^+) = V_s}$$

b) Find $v_c(t\to\infty)$ and $R_{Th}C$ = time constant.

$t\to\infty$: Switch is in position b. No pwr src.

C discharges thru R's.

$\therefore v_c(t\to\infty) = 0\,V$

R_{Th} is resistance seen looking into terminals where C connected with switch in position b.

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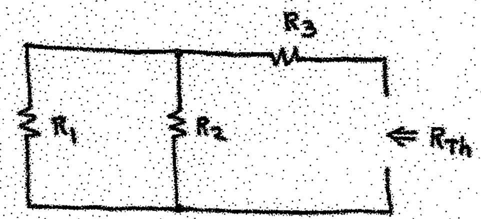
sol'n: 3.b) cont.



$$R_{Th} = (R_1 \| R_2) + R_3$$

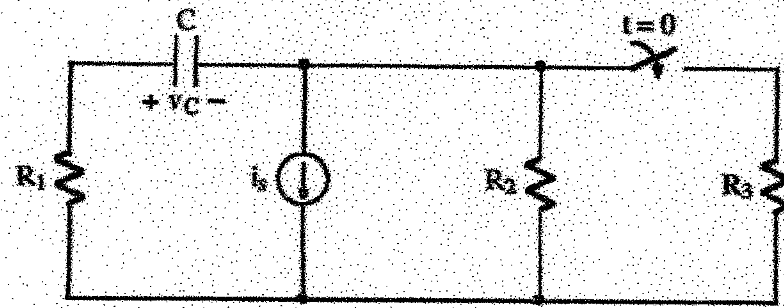
Now plug values into general sol'n.

$$v_c(t>0) = v_c(t\to\infty) + \left[v_c(0^+) - v_c(t\to\infty)\right] e^{-t/R_{Th}C}$$

$$v_c(t>0) = 0V + \left[V_s - 0V\right] e^{-t/(R_1 \| R_2 + R_3)C}$$

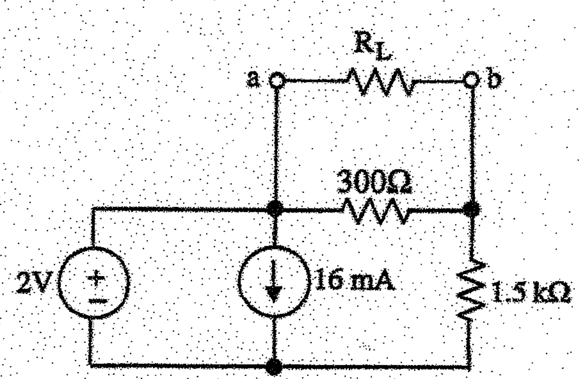
$$\boxed{v_c(t>0) = V_s\, e^{-t/(R_1 \| R_2 + R_3)C}}$$

2.



After being open for a long time, the switch closes at $t = 0$. Write an expression for $v_c(t \geq 0)$ in terms of R_1, R_2, R_3, i_s, and C.

4.

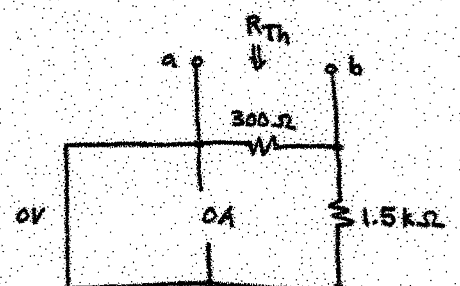


a. Calculate the value of R_L that would absorb maximum power.

b. Calculate that value of maximum power R_L could absorb.

sol'n: a) $R_L = R_{Th}$ yields max pwr transfer

We find R_{Th} by removing R_L, turning independent src's off, and seeing what resistance we have looking into terminals a,b.



$$R_{Th} = 300\,\Omega \parallel 1.5\,k\Omega = 300\,\Omega \cdot 1 \| 5 = 300\,\Omega \cdot \frac{5}{6}$$

$$R_{Th} = 250\,\Omega$$

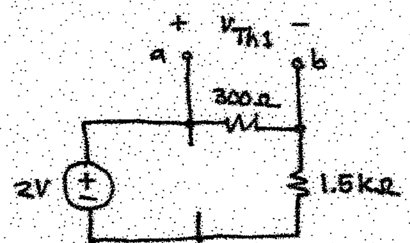
$$\therefore \boxed{R_L = 250\,\Omega}$$

soln: 4.b) Find Thevenin equivalent of circuit where R_L connected.

$V_{Th} = V_{a,b}$ with R_L removed.

Use superposition (or other method such as node-voltage) to find V_{Th}.

case I: 2V src on, 16 mA src off (open circuit)

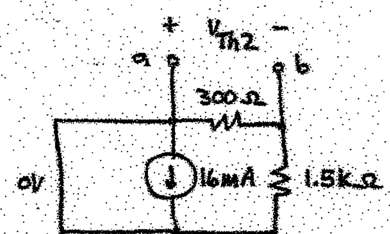


We have v-divider.

$$V_{Th1} = 2V \cdot \frac{300\,\Omega}{300\,\Omega + 1.5\,k\Omega}$$

$$V_{Th1} = \frac{2V}{6}$$

case II: 2V src off, 16 mA src on
= wire



We have i-divider.
All current flows thru wire.

$\therefore V_{Th2} = 0V$

$$V_{Th} = V_{Th1} + V_{Th2} = \frac{2V}{6} + 0V = \frac{2V}{6} = \frac{1}{3}V$$

$$\text{max pwr} = \frac{V_{Th}^2}{4R_{Th}} = \frac{\left(\frac{1}{3}V\right)^2}{4 \cdot 250\,\Omega} = \frac{1}{9}\,mW$$

$$\boxed{\text{max pwr} = \frac{1}{9}\,mW}$$