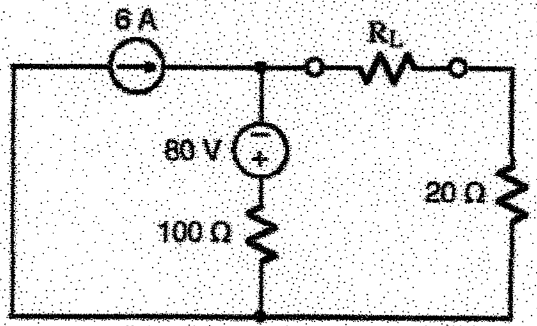
Homework #6/Review Examples

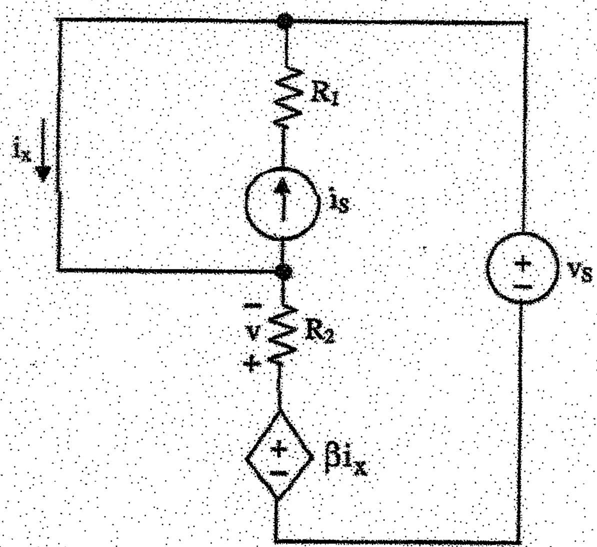
3.



a) Calculate the value of R_L that would absorb maximum power.

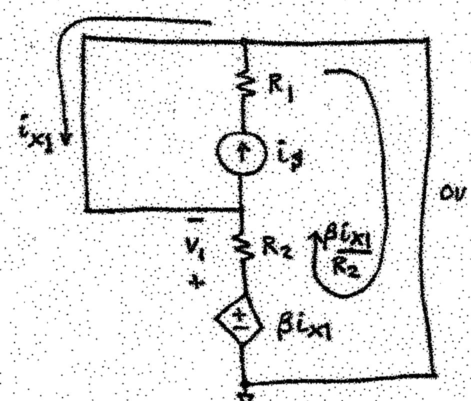
b) Calculate that value of maximum power R_L could absorb.

5.



Using superposition, derive an expression for v that contains no circuit quantities other than i_s, v_S, R_1, R_2, and β. Note: $\beta > 0$.

soln: case I: i_s on, v_s off = wire



$V_1 = \beta i_{x1}$ from outer v loop

Current summation at center node gives

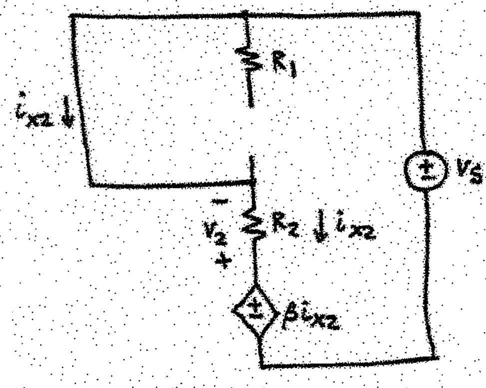
$$-i_s + i_{x1} + \frac{\beta i_{x1}}{R_2} = 0A$$

or $i_{x1}\left(1 + \frac{\beta}{R_2}\right) = i_s$

$$i_{x1} = \frac{i_s}{1 + \frac{\beta}{R_2}} = i_s \cdot \frac{R_2}{R_2 + \beta}$$

$$v_1 = \beta i_{x1} = \beta i_s \frac{R_2}{R_2 + \beta} = i_s \cdot R_2 \| \beta$$

sol'n: 5. cont. case II: i_s off, v_s on
= open



v loop around outside gives

$$\beta i_{x2} - - i_{x2} \cdot R_2 - v_s = 0V$$

$$i_{x2}(\beta + R_2) = v_s$$

$$i_{x2} = \frac{v_s}{\beta + R_2}$$

$$v_2 = -i_{x2} R_2 = -v_s \frac{R_2}{\beta + R_2} = -v_s \frac{R_2}{R_2 + \beta}$$

$$v = v_1 + v_2 = \beta i_s \frac{R_2}{R_2+\beta} - v_s \frac{R_2}{R_2+\beta} = (\beta i_s - v_s)\frac{R_2}{R_2+\beta}$$

$$\boxed{v = (\beta i_s - v_s)\frac{R_2}{R_2+\beta}}$$

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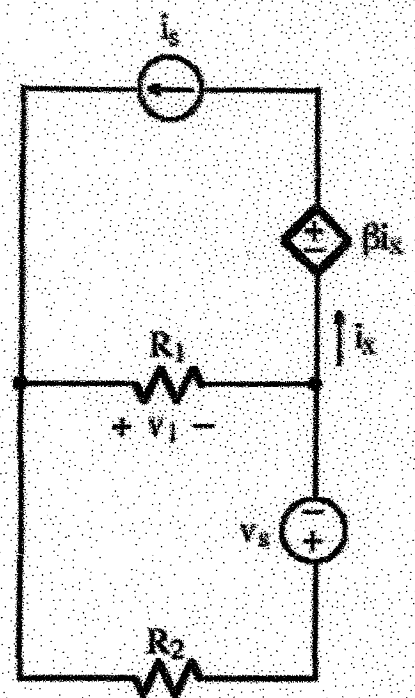
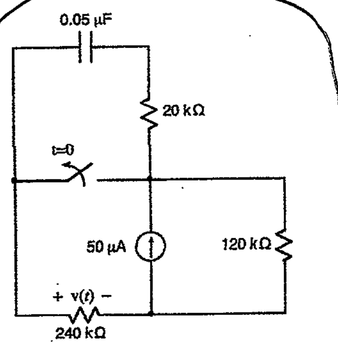
4.



Using superposition, derive an expression for v_1 that contains no circuit quantities other than i_S, v_S, R_1, R_2, and β, where $\beta > 0$.

EX:



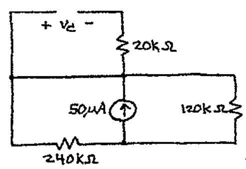
Calculate the energy stored on the capacitor at $t = 0^+$.

Sol'n: Energy $w_C = \frac{1}{2} C v_C^2(t = 0^+)$

Since capacitor voltage cannot change instantly, $v_C(0^+) = v_C(0^-)$.

At $t = 0^-$, C acts like open circuit and switch is closed.

$t = 0^-$:



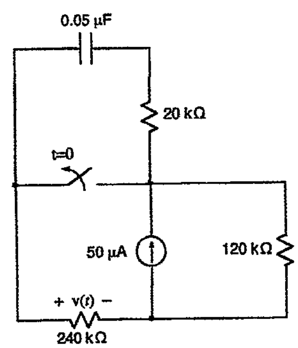
The short created by the switch creates a voltage loop on top left with 0V across C and across the 20kΩ resistor.

Thus $v_C(0^-) = 0V = v_C(0^+)$.

$\therefore$ $w_C(0^+) = 0\,J$

Note: The units for energy are Joules.

EX:



Write a numerical expression for $v(t)$ for $t > 0$.

Sol'n: We use general form of solution for RC circuits.

$$v(t>0) = v(t\to\infty) + [v(0^+) - v(t\to\infty)]\, e^{-t/R_{Th}C}$$

We find $v(0^+)$, $v(t\to\infty)$, and R_{Th}.

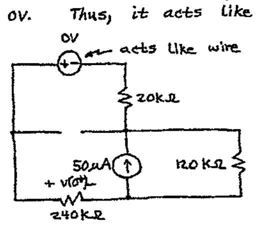
We start at $t = 0^-$ to find voltage on C at $t = 0^+$.

$t = 0^-$: C acts like open circuit. Switch is closed.

Switch creates a short circuit, and $v_C(0^-) = 0V$.

$t = 0^+$: $v_C(0^+) = v_C(0^-) = 0V$ since v_C can't change instantly.

We model C as voltage source of 0V. Thus, it acts like a wire.



This is a current-divider circuit with $20k\Omega + 240k\Omega = 260k\Omega$ on one side and $120k\Omega$ on the other side.

The current thru the 240 kΩ is

$$i(0^+) = 50\mu A \cdot \frac{120k\Omega}{20k\Omega + 240k\Omega + 120k\Omega}$$

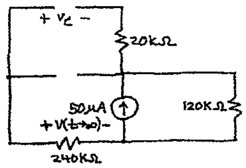
$$i(0^+) = 50\mu A \cdot \frac{120k\Omega}{380\,k\Omega}$$

$v(0^+)$ from Ohm's Law is

$$v(0^+) = 50\mu A \cdot \frac{120k\Omega}{380k\Omega} \cdot 240\,k\Omega$$

$v(0^+) = 6V \cdot \frac{12}{19}$

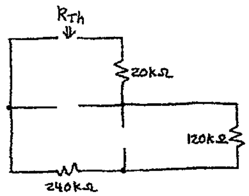
$t \to \infty$: Switch is open. C = open circuit.



No current can flow thru the 240 kΩ resistor.

$\therefore v(t \to \infty) = 0V$

R_{Th}: We look in from the terminals where C is connected, and we turn off the current source.



$R_{Th} = 20k\Omega + 120k\Omega + 240k\Omega = 380k\Omega$

The time constant is $R_{Th}C$:

$\tau = 380\ k\Omega \cdot 0.05\ \mu F = 19\ ms$

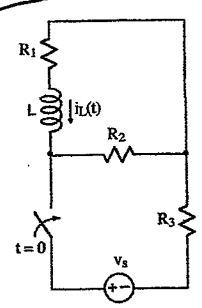
Putting results together:

$$v(t>0) = 0V + \left(6V \cdot \frac{12}{19} - 0V\right) e^{-t/19ms}$$

or

$$v(t>0) = 6V \cdot \frac{12}{19} e^{-t/19ms} \doteq 3.8 V e^{-t/19ms}$$

Ex:

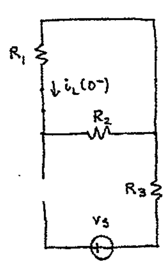


a) Write an expression for $i_L(t = 0^+)$

b) Write an expression for $i_L(t > 0)$ in terms of R_1, R_2, R_3, v_s, and L.

Sol'n: a) $i_L(0^+) = i_L(0^-)$

At $t = 0^-$, L acts like wire. Switch is open at $t = 0^-$.



There is no power source in the R_1, R_2 loop, and v_s is disconnected.
$\therefore i_L(0^-) = 0A$

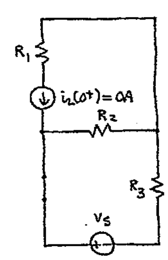
Thus, $i_L(0^+) = i_L(0^-) = 0A$

b) For $i_L(t>0)$, we use the general form of sol'n for RL problems:

$$i_L(t>0) = i_L(t\to\infty) + \left[i_L(0^+) - i_L(t\to\infty)\right] e^{-t/L/R_{Th}}$$

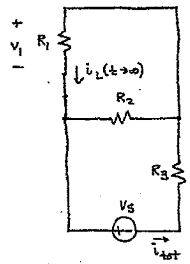
$t = 0^+$: We model L as i-src with value $i_L(0^+) = i_L(0^-) = 0A$.

Switch is closed.



Since the quantity we are looking for is $i_L(0^+)$, we do not have to solve the circuit, but this is the circuit we would use.

$t \to \infty$: L acts like wire
Switch is closed



We can calculate i_{tot} as

$$i_{tot} = \frac{-v_s}{R_1\|R_2 + R_3}$$

Then we can use a current divider to find $i_L(t\to\infty)$:

$$i_L(t\to\infty) = i_{tot}\cdot\frac{R_2}{R_1+R_2}$$

$$i_L(t\to\infty) = \frac{-v_s}{R_1\|R_2+R_3}\cdot\frac{R_2}{R_1+R_2}$$

Another way to calculate i_{tot} is to write $R_1\|R_2$ in a different way:

$$R_1\|R_2 = \frac{R_1}{1+\frac{R_1}{R_2}}$$

Then we use a voltage divider formula:

$$v_1 = -v_s\,\frac{\frac{R_1}{1+R_1/R_2}}{\frac{R_1}{1+R_1/R_2}+R_3}$$

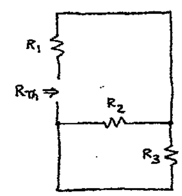
$$v_1 = -v_s\,\frac{R_1}{R_1+R_3(1+R_1/R_2)}$$

We divide v_1 by R_1 to find $i_L(t\to\infty)$:

$$i_L(t\to\infty) = -\frac{v_s}{R_1+R_3(1+R_1/R_2)}$$

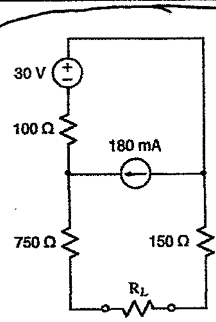
This answer is equivalent to our previous answer

R_{Th}: We turn off the v_s source and look in from the terminals where L is connected. Switch is closed.



$$R_{Th} = R_1 + R_2\|R_3$$

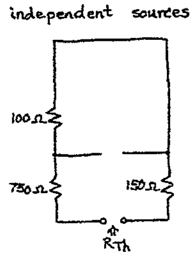
Ex:



a) Calculate the value of R_L that would absorb maximum power.
b) Calculate that value of maximum power R_L could absorb.

Sol'n: a) $R_L = R_{Th}$ for max power transfer

We find R_{Th} by looking into the terminals where R_L is connected (but without R_L) with the two independent sources turned off.



$$R_{Th} = 750\,\Omega + 100\,\Omega + 150\,\Omega$$

$$R_{Th} = 1\,k\Omega$$

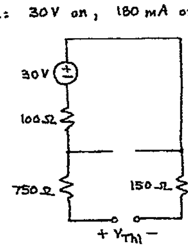
$$\therefore \boxed{R_L = 1\,k\Omega}$$

b) $\text{max pwr} = \dfrac{\left(\frac{V_{Th}}{2}\right)^2}{R_{Th}}$

We find V_{Th} as the open circuit voltage across the terminals where R_L is connected.

We find V_{Th} by using superposition.

case I: 30 V on, 180 mA off

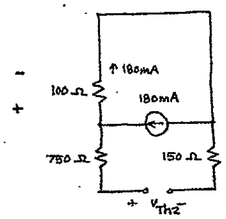


Since no current flows, there is no v-drop across the R's.

$\therefore V_{Th1} = -30\,V$ (− v src)

Note: Since we will square V_{Th}, the polarity we choose for measuring V_{Th} doesn't matter.

case II: 30V off, 180 mA on



No current flows in the 750 Ω and 150 Ω. Thus, there is no v-drop across these R's.

The v-drop across the 100 Ω is equal to v_{Th2}.

$$v_{Th2} = 180\,mA \cdot 100\,\Omega = 18V$$

We sum results to find v_{Th}.

$$v_{Th} = v_{Th1} + v_{Th2}$$

$$v_{Th} = -30V + 18V = -12V$$

$$\text{max pwr} = \frac{\left(\frac{V_{Th}}{2}\right)^2}{R_{Th}} = \frac{6^2}{1k} = 36\,mW$$

$$\text{or } i_{12} = i_s \frac{\dfrac{R_2/\alpha}{R_2 + 1/\alpha}}{\dfrac{R_2/\alpha}{R_2 + 1/\alpha} + R_1}$$

$$\text{or } i_{12} = i_s \frac{R_2/\alpha}{R_2/\alpha + R_1(R_2 + 1/\alpha)}$$

$$\text{or } i_{12} = i_s \frac{R_2}{R_2 + R_1(\alpha R_2 + 1)}$$

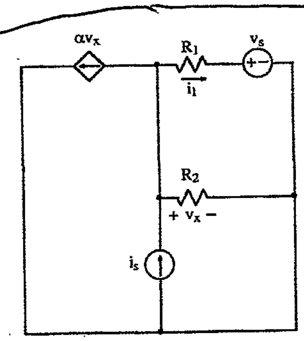
We sum i_{11} and i_{12} to get i_1:

$$i_1 = i_{11} + i_{12}$$

$$\text{or } i_1 = -\frac{v_s}{R_1 + R_2 \| \frac{1}{\alpha}} + i_s \frac{R_2}{R_2 + R_1(\alpha R_2 + 1)}$$

Note: When a current source is off it becomes an open circuit.
When a voltage source is off it becomes a wire.

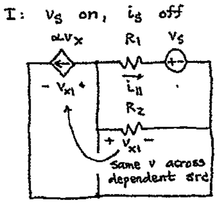
Ex:



Using superposition, derive an expression for i_1 that contains no circuit quantities other than i_s, v_S, R_1, R_2, and α, where $\alpha > 0$.

Sol'n: We turn on one source at a time. (Never turn off dependent source.)

case I: v_s on, i_s off



The dependent source is equivalent to $R_{eq} = \frac{v}{i} = \frac{v_{x1}}{\alpha v_{x1}} = \frac{1}{\alpha}$.