b) We know $v_x = (3\,mA - i_1)\,10k\Omega$

$$" = \frac{3}{2}\,mA \cdot 10k\Omega$$

$$v_x = 15V$$

The current for the dependent src is i_2.

$$i_2 = 1\,mA$$

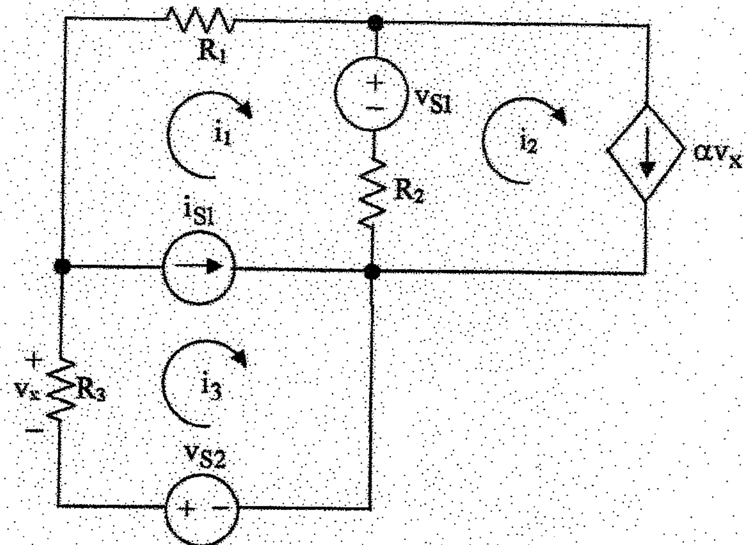
Thus, power for the dependent src is

$$p = v \cdot i = 2v_x\, i_2 = 2(15V) \cdot 1mA$$

or $p = 30\ mW$.

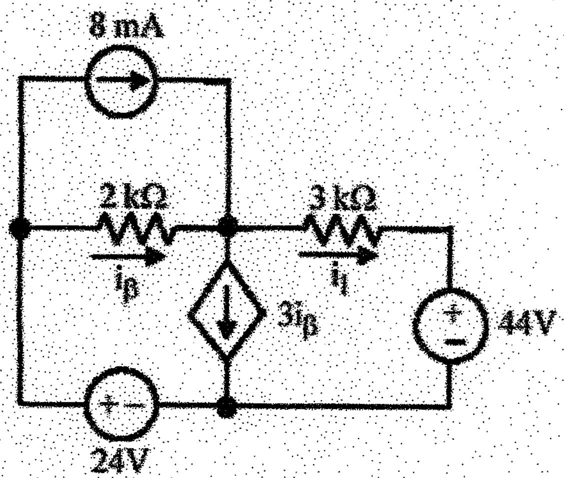
Mesh Current Example

For the circuit shown, write three independent equations for the three mesh currents i_1, i_2, and i_3. The quantity v_x must not appear in the equations.



(77)

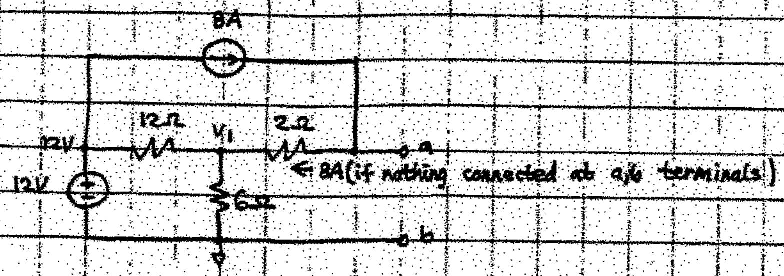
Mesh Current Example



a. Use the mesh-current method to find i_1.

b. Find the power dissipated by the dependent current source.

ex:



Find the Thevenin equivalent with respect to terminals a,b.
(In other words, create a circuit with a V-source and internal R that has same i and v at its terminals as above circuit has at a,b terminals.)

Use Node-V method to find v_1, then use $v_a = v_1 + 8A \cdot 2\Omega = v_{Th}$.
(We are finding v_{Th} by calculating voltage at a,b terminals with nothing connected to a,b terminals.)

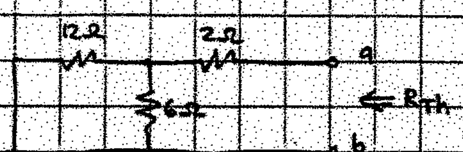
Node v_1 eqn: $\dfrac{v_1 - 12V}{12\Omega} + \dfrac{v_1}{6\Omega} - 8A = 0A$

or $\dfrac{v_1}{12\Omega \| 6\Omega} = 8A + \dfrac{12V}{12\Omega} = 9A$

or $v_1 = 9A \cdot 12\Omega \| 6\Omega = 9A \cdot 6\Omega\,(2\|1) = 54V \cdot \frac{2}{3} = 36V$

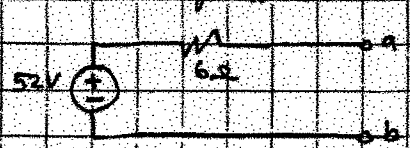
$$V_{Th} = V_a = v_1 + 8A \cdot 2\Omega = 36V + 16V = 52V$$

To find R_{Th} we turn independent sources to zero and find R looking into a,b terminals.



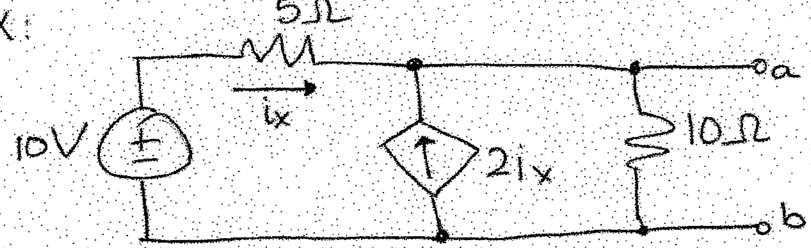
$$\begin{aligned} R_{Th} &= (6\Omega \| 12\Omega) + 2\Omega \\ &= 6\Omega \cdot (1\|2) + 2\Omega \\ &= 6\Omega \cdot \tfrac{2}{3} + 2\Omega \\ &= 6\Omega \end{aligned}$$

Thevenin equivalent:



Thévenin Equivalent

Ex:



· Find Thévenin equivalent circuit between nodes a and b.

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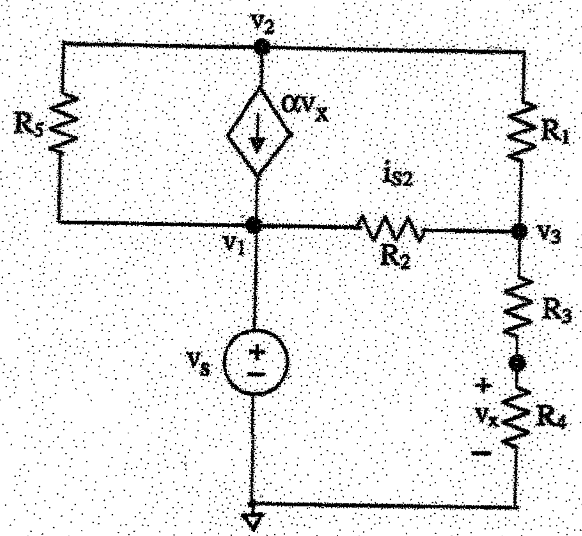
1.



For the circuit shown, write three independent equations for the node voltages v_1, v_2, and v_3. The quantity v_x must not appear in the equations.

2. Make a consistency check on your equations for problem 1 by setting resistors and sources to values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and v_1, v_2, v_3 for your consistency check, and show that your equations for problem 1(a) are satisfied for these values. (In other words, plug the values into your equations for problem 1(a).)

sol'n: 1. i) $v_1 = v_s$ (connected to ref node by v-src)

2) $$\frac{v_2 - v_1}{R_5} + \alpha \boxed{\frac{v_3 R_4}{R_3 + R_4}} + \frac{v_2 - v_3}{R_1} = 0\,A$$

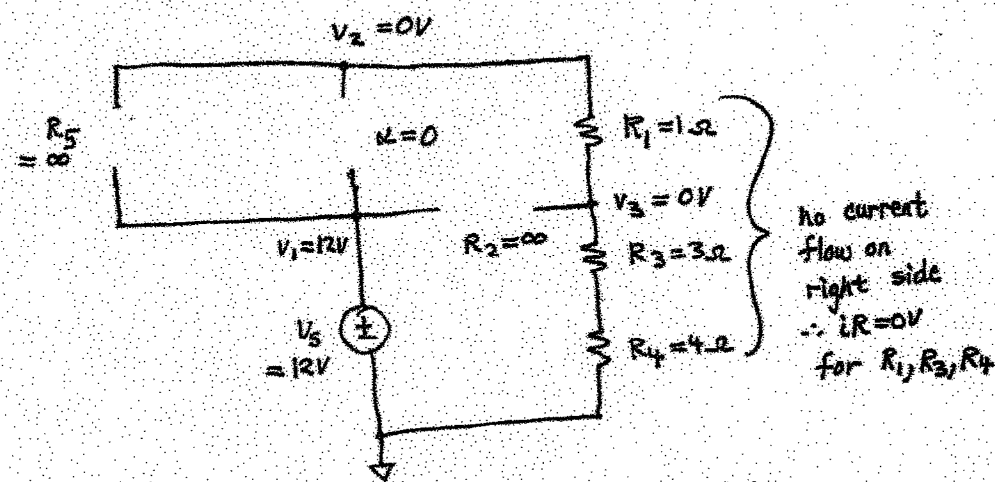
This is v_x in terms of node voltage. We are using a v-divider.

3) $$\frac{v_3 - v_2}{R_1} + \frac{v_3 - v_1}{R_2} + \frac{v_3 - 0V}{R_3 + R_4} = 0\,A$$

Note: we only have to write the eq'ns, not solve them.

2. Many consistency checks are possible.
Choose values of R's and src's such that values of v_1, v_2, v_3 are obvious from inspection.

My choices:



Plug all the component values and v's into eq'ns in prob 1 to verify that eq'ns are satisfied.

1) $12V = 12V$ ✓
$v_1 = v_s$

2) $\frac{0 - 12V}{\infty}^{0} + 0 \cdot \frac{0V \cdot 4\Omega}{3+4\Omega} + \frac{0 - 0V}{1\Omega} = 0A$ ✓

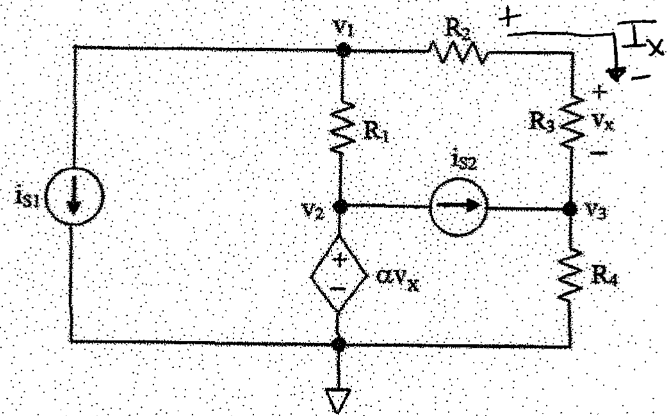
$\frac{v_2 - v_1}{R_5} + \alpha \frac{v_3 R_4}{R_3 + R_4} + \frac{v_2 - v_3}{R_1} = 0A$

3) $\frac{0 - 0V}{1\Omega} + \frac{0 - 12V}{\infty} + \frac{0V - 0V}{3+4\Omega} = 0A$ ✓

$\frac{v_3 - v_2}{R_1} + \frac{v_3 - v_1}{R_2} + \frac{v_3 - 0V}{R_3 + R_4} = 0A$

Homework #4 Example

1.



(a) For the circuit shown, write three independent equations for the node voltages v_1, v_2, and v_3. The quantity v_x must not appear in the equations.

(b) Make a consistency check on your equations for (a) by setting resistors and sources to values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and v_1, v_2, v_3 for your consistency check, and show that your equations for problem (a) are satisfied for these values. (In other words, plug the values into your equations for problem (a).)

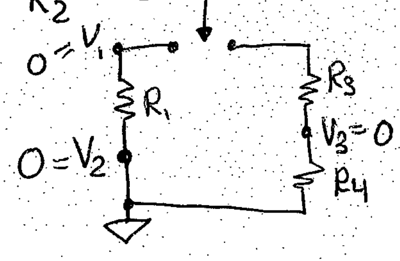
(a) node voltage: $\dfrac{(V_1-V_3)R_3}{R_3+R_2} = V_x$ (V-divider)

OR $V_x = I_x R_3 = \underbrace{\left[\dfrac{(V_1-V_3)}{R_2+R_3}\right]}_{I_x} R_3$ (same

$V_2 = \alpha V_x = \boxed{\dfrac{\alpha(V_1-V_3)R_3}{R_2+R_3} = V_2}$

$\boxed{i_{s1} + \dfrac{(V_1-V_2)}{R_1} + \dfrac{(V_1-V_3)}{R_2+R_3} = 0}$ $\boxed{\dfrac{V_3-V_1}{R_2+R_3} - i_{s2} + \dfrac{V_3}{R_4} = 0}$

(b) Let $\alpha = 0$, $i_{s1} = 0$, $i_{s2} = 0$ [Redraw Circuit]
$R_2 = \infty$



From eq: $V_2 = \dfrac{0\,(V_1-V_3)R_3}{R_2+R_3} = 0$

$0 + \dfrac{(V_1-0)}{R_1} + \cancel{\dfrac{(V_1-V_3)}{\infty}}^{\,0} = 0 \Rightarrow V_1 = 0$

$\dfrac{V_3-0}{R_2+R_3} - 0 + \dfrac{V_3}{R_4} = 0 \Rightarrow V_3 = 0$

Homework #4 Examples

Sp 05

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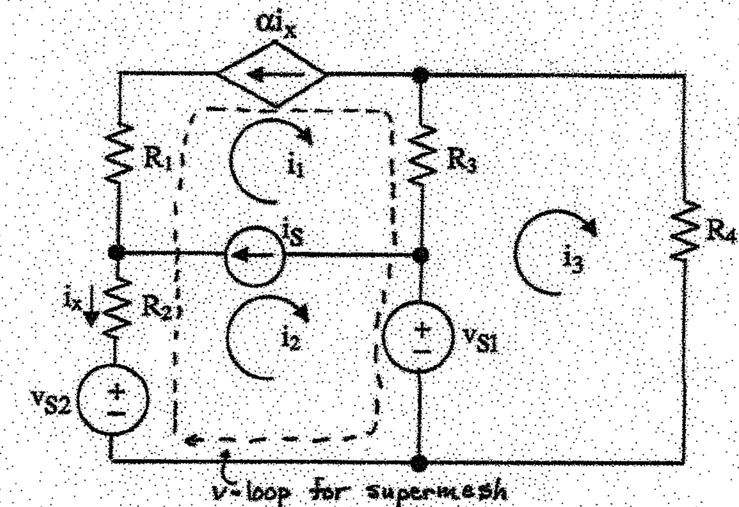
3.



For the circuit shown, write three independent equations for the three mesh currents i_1, i_2, and i_3. The quantity i_x must not appear in the equations.

sol'n: First, write i_x in terms of mesh currents.

$i_x = -i_2$ (use mesh currents only)

Now do v-loops for mesh currents.

we have current source between loops for i_1 and i_2.

∴ we have supermesh loop around outside of i_1 and i_2 loops (that avoids having to define v for i_s).

don't use i_x here!

supermesh v-loop: $+v_{S2} - i_2 R_2 - i_1 R_1 -$ stop! current src not allowed

we must not need this v-loop. Indeed, we have a current src on the outside edge, giving us $\boxed{i_1 = -\alpha\underbrace{(-i_2)}_{i_x}}$

we must also have a current eq'n for i_s source to complete super mesh:

$$\boxed{i_s = i_1 - i_2}$$

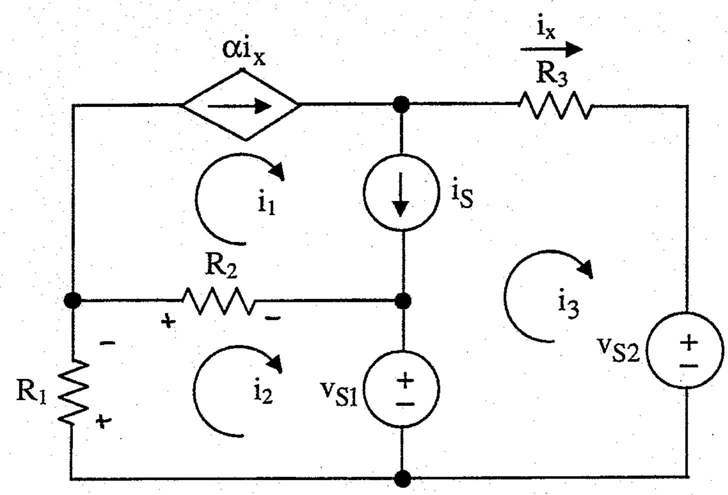
← flows against i_s so is minus

Normal v-loop for i_3:

$$\boxed{\begin{aligned} &+v_{S1} - i_3 R_3 - i_3 R_4 = 0V \\ &\qquad + i_1 R_3 \end{aligned}}$$

Question only requires 3 eq'ns we could solve, but we don't have to solve them.

1. c. (20 points)



For the circuit shown, write three independent equations for the three mesh currents i_1, i_2, and i_3. The quantity i_x must not appear in the equations.

soln: αi_x dependent src in terms of i_1, i_2, i_3:

$\alpha i_x = i_1$, $i_x = i_3$ so $\boxed{\alpha i_3 = i_1}$

supermesh for i_1 and i_3 loops:

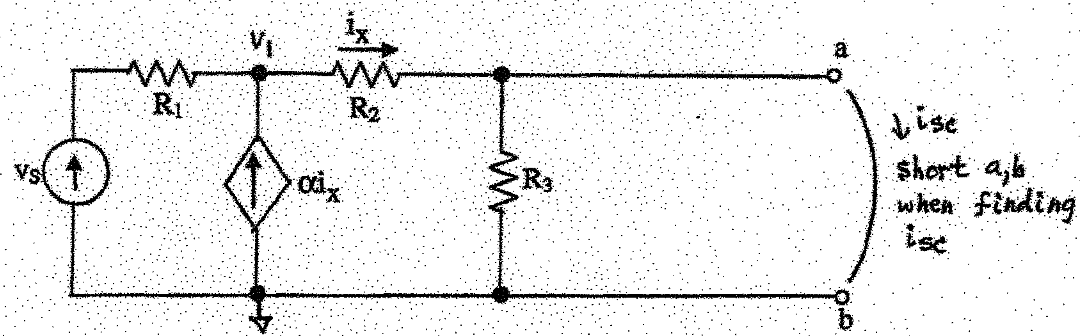
$\boxed{i_s = i_1 - i_3}$

outer loop for i_1 and i_3? No, because of αi_x src.

Mesh eqn for i_2:

$\boxed{-i_2 R_1 - i_2 R_2 - v_{s1} = 0V}$

4.



Find the Thevenin's equivalent circuit at terminals a-b. i_x must not appear in your solution. **Hint:** Use the node voltage method. **Note:** $\alpha > 0$.

soln: Assuming v_s is current src:

$V_{Thev} = V_{a,b}$ open circuit

Use node-voltage method to find v_1:

$$-v_s - \alpha \underbrace{\boxed{\frac{v_1}{R_2+R_3}}}_{= i_x} + \frac{v_1}{R_2+R_3} = 0A$$

$$\text{or} \quad v_1 \frac{1-\alpha}{R_2+R_3} = v_s$$

$$v_1 = v_s \frac{R_2+R_3}{1-\alpha}$$

$$V_{Thev} = v_1 \cdot \frac{R_3}{R_2+R_3} = v_s \frac{R_3}{1-\alpha} \quad \text{v-divider}$$

Now short a,b terminals and find i_{sc} flowing from a to b.

Note that R_3 will carry no current; it all flows thru the short. ∴ we may ignore R_3.

Use node voltage to find v_1.

$$-v_s - \alpha \underbrace{\boxed{\frac{v_1}{R_2}}}_{= i_x} + \frac{v_1}{R_2} = 0A$$

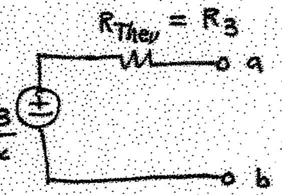
$$v_1 = v_s \frac{R_2}{1-\alpha}$$

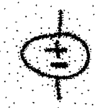
Now, $i_{sc} = i_x = \frac{v_1}{R_2} = \frac{v_s}{1-\alpha}$

$R_{Thev} = V_{Thev} / i_{sc} = R_3$

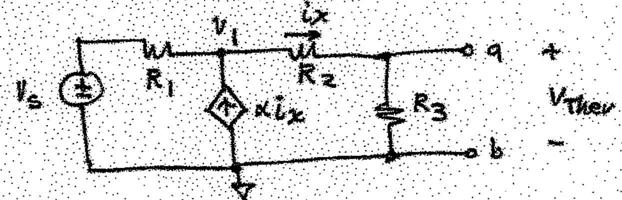
4. cont. Thevenin equiv:

$V_{Thev} = \frac{v_s R_3}{1-\alpha}$



Assuming v_s is <u>voltage src</u>  v_s:

$V_{Thev} = V_{a,b}$ open circuit



Use node-voltage method to find v_1:

$$\frac{v_1 - v_s}{R_1} - \alpha \underbrace{\boxed{\frac{v_1}{R_2+R_3}}}_{=i_x} + \frac{v_1}{R_2+R_3} = 0A$$

$$v_1\left(\frac{1}{R_1} + \frac{1-\alpha}{R_2+R_3}\right) = \frac{v_s}{R_1}$$

mult both sides by R_1 and re-arrange:

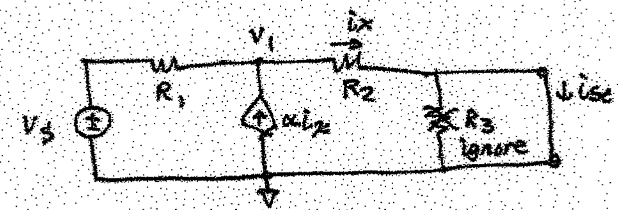
$$v_1 = \frac{v_s}{1 + \frac{(1-\alpha)R_1}{R_2+R_3}}$$

$$V_{Thev} = v_1 \cdot \frac{R_3}{R_2+R_3} = \frac{v_s R_3}{\left(1 + \frac{(1-\alpha)R_1}{R_2+R_3}\right)(R_2+R_3)}$$

$$V_{Thev} = \frac{v_s R_3}{R_2+R_3+(1-\alpha)R_1}$$

Now short a,b terminals and find i_{sc} flowing from a to b. Note that R_3 will be bypassed. All current will flow thru short.
∴ we may ignore R_3.

4. cont.



use node-voltage to find v_1:

$$\frac{v_1 - v_s}{R_1} - \alpha \underbrace{\boxed{\frac{v_1}{R_2}}}_{= i_x} + \frac{v_1}{R_2} = 0A$$

$$v_1\left(\frac{1}{R_1} + \frac{1-\alpha}{R_2}\right) = \frac{v_s}{R_1}$$

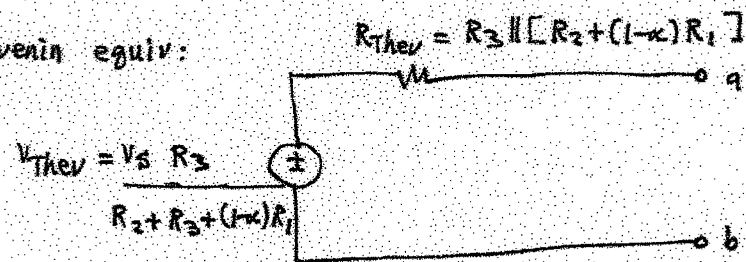
$$\text{or} \quad v_1 = \frac{v_s}{1 + \frac{(1-\alpha)R_1}{R_2}}$$

$$i_{sc} = i_x = \frac{v_1}{R_2} = \frac{v_s}{R_2 + (1-\alpha)R_1}$$

$$R_{The} = \frac{v_{Thev}}{i_{sc}} = \frac{R_3}{R_2 + R_3 + (1-\alpha)R_1} \cdot R_2 + (1-\alpha)R_1$$

$$R_{Thev} = R_3 \,\|\, [R_2 + (1-\alpha)R_1]$$

Thevenin equiv:



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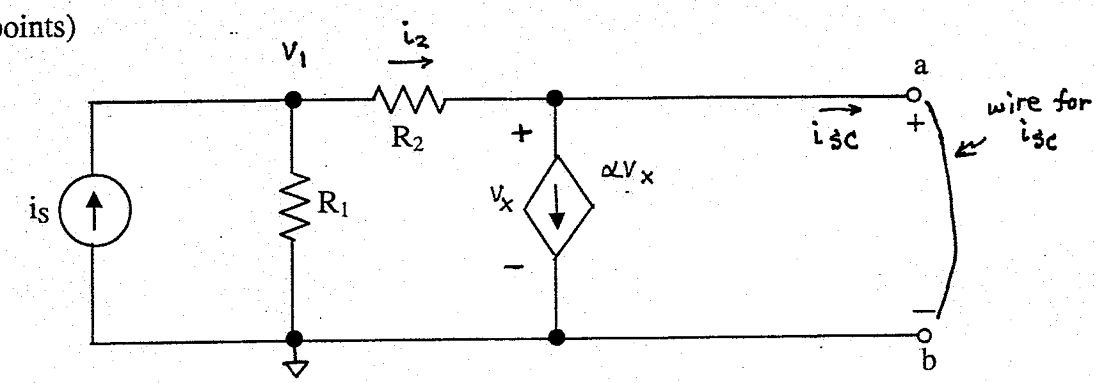
2. (25 points)



Find the Thevenin's equivalent circuit at terminals a-b. v_x must not appear in your solution. **Hint:** Use node voltage method to find v above R_1. **Note:** $\alpha > 0$.

sol'n: $V_{Th} = V_{ab}$ with no load connected to a, b.

$R_{Th} = \frac{V_{Th}}{i_{sc}}$ where i_{sc} = i out of 'a' terminal with wire from a to b

Find v_1: Use Node V method

write v_x in terms of v_1: $v_x = v_1 - \alpha v_x R_2$

$v_x = v_1/(1+\alpha R_2)$

Node v_1 eqn: $-i_s + \frac{v_1}{R_1} + \frac{v_1 - v_x}{R_2} = 0\text{A}$

$$v_1\left[\frac{\left(1 - \frac{1}{1+\alpha R_2}\right)}{R_2}\right] = \frac{\frac{\cancel{1}+\alpha \cancel{R_2} \cancel{-1}}{1+\alpha R_2}}{\cancel{R_2}} = v_1 \frac{\alpha}{1+\alpha R_2}$$

$$= v_1\left(1 - \alpha \frac{R_2}{R_1}\right)$$

$$v_1\left(\frac{1}{R_1} + \frac{\alpha}{1+\alpha R_2}\right) = i_s$$

$$v_1 = i_s \frac{1}{\frac{1}{R_1} + \frac{\alpha}{1+\alpha R_2}}$$

$$V_{Th} = v_x = \frac{v_1}{1+\alpha R_2} = i_s \frac{1}{\frac{1+\alpha R_2}{R_1} + \alpha}$$

$$\boxed{V_{Th} = \frac{i_s}{\frac{1+\alpha R_2}{R_1} + \alpha} = i_s\left[R_1 \| (R_2 + 1/\alpha)\right] \cdot \frac{1/\alpha}{R_2 + 1/\alpha}}$$

Latter formula from using $1/\alpha$ for R_{eq} of dep. src

$R_{Th} = \frac{V_{Th}}{i_{sc}}$

For i_{sc} we have $v_x = 0\text{V}$.

Then $v_1 = i_s \cdot R_1 \| R_2$ (no dep src)

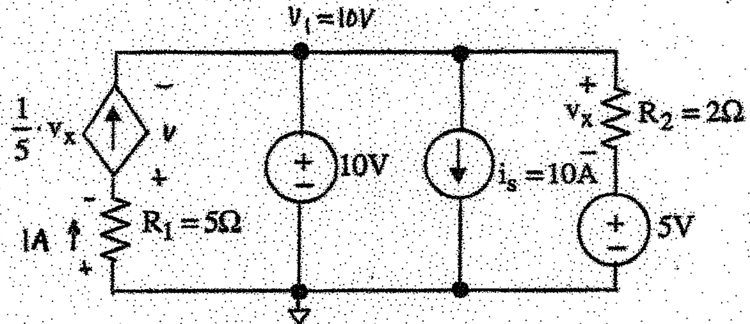
and $i_{sc} = \frac{i_s R_1}{R_1 + R_2}$

$$R_{Th} = \frac{\cancel{i_s}}{\frac{1+\alpha R_2}{R_1} + \alpha} \cdot \frac{R_1 + R_2}{\cancel{i_s} R_1}$$

$$\boxed{R_{Th} = \frac{R_1 + R_2}{1 + \alpha(R_1 + R_2)} = R_1 \| \left(R_2 + \frac{1}{\alpha}\right)}$$

Latter formula from using $1/\alpha$ for dep src and looking in from a, b with i_s src off.

5.



Calculate the power consumed by the dependent current source, (labeled $\frac{1}{5}v_x$). **Note:** If a source supplies power, the power it consumes is negative.

soln: V on top rail = 10V from 10V src (if ref on bottom).

Then $V_x = 5V = 10V - 5V$

$\therefore \frac{1}{5\Omega} V_x = \frac{1}{5\Omega} 5V = 1A$ for dependent src

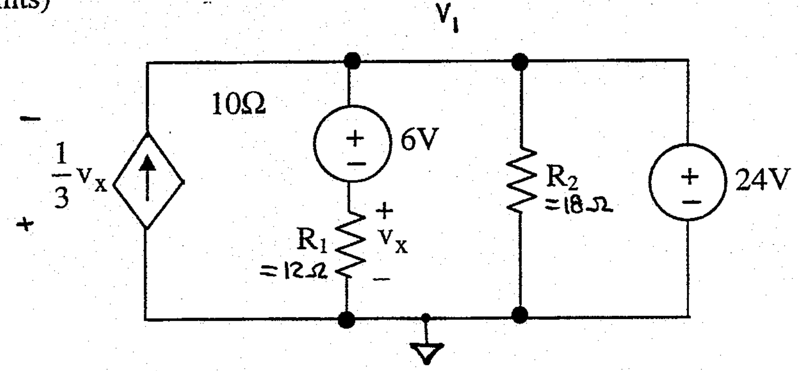
Using v-loop on left, $-1A \cdot R_1 - v - 10V = 0V$

or $v = -10V - 1A \cdot \underset{5\Omega}{\underset{\parallel}{R_1}} = -15V$

pwr $\equiv p = \frac{1}{5} V_x \cdot v = 1A\,(-15V) = -15$ W

$p = -15$ W

3. (25 points)



Calculate the power consumed by the dependent current source, (labeled $\frac{1}{3}v_x$). **Note:** If a source supplies power, the power it consumes is negative.

soln: Find v_1. Then v across dep src is $-v_1$.

Write v_x in terms of v_1: $v_x = v_1 - 6V$

Node V for v_1 node: $-\frac{1}{3}(v_1 - 6v) + \frac{v_1 - 6v}{R_1} + \frac{v_1}{R_2} + $?!

We don't need eq'n! $v_1 = 24V$ from src on right.

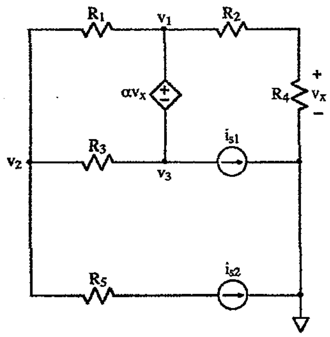
$$\therefore \quad p = \frac{1}{3}v_x \cdot (-v_1) = \frac{1}{3}(v_1 - 6v)(-v_1)$$

$$= \frac{1}{3}(24V - 6V)(-24V)$$

$$= \frac{1}{3}(18V)(-24V)$$

$$\boxed{p = -144\ W}$$

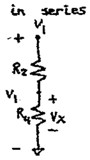
Ex:



For the circuit shown, write three independent equations for the node-voltages, v_1, v_2, and v_3. The quantity v_x must not appear in the equations.

soln: We first write v_x in terms of node-v's.

We use a v-divider since we have v_1 across R_2 in series with R_4:



$$v_x = v_1 \cdot \frac{R_4}{R_2 + R_4}$$

We have a v-src connecting v_1 to v_3. So v_1, v_3 form a supernode.

We write a current summation eq'n for v_1, v_3. We find sum of i's flowing out of bubble containing v_1, v_3, and the dependent v-src.

(1) v_1, v_3 node: $\dfrac{v_1 - v_2}{R_1} + \dfrac{v_1 - 0V}{R_2 + R_4} + \dfrac{v_3 - v_2}{R_3} + i_{s1} = 0A$

We also write a voltage eq'n for v_1 and v_3. Note that we substitute for v_x to obtain an eq'n containing only node voltages.

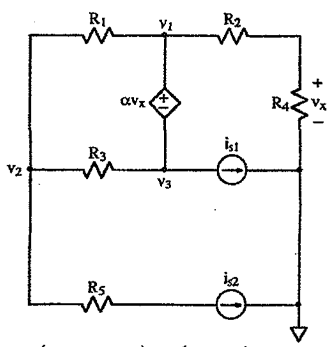
(2) $$v_1 = v_3 + \alpha \underbrace{\left(v_1 \frac{R_4}{R_2 + R_4} \right)}_{v_x}$$

For v_2, we just sum currents out of node.

(3) v_2 node: $\dfrac{v_2 - v_1}{R_1} + \dfrac{v_2 - v_3}{R_3} + i_{s2} = 0A$

We now have our 3 eq'ns for v_1, v_2, and v_3.

Ex:



$$v_1\left(\frac{1}{R_1} + \frac{1}{R_2 + R_4}\right) - v_2\left(\frac{1}{R_3} + \frac{1}{R_1}\right) + v_3 \frac{1}{R_3} + i_{s1} = 0 \text{ A}$$

$$v_1 - v_3 = \alpha v_1\left(\frac{R_4}{R_2 + R_4}\right)$$

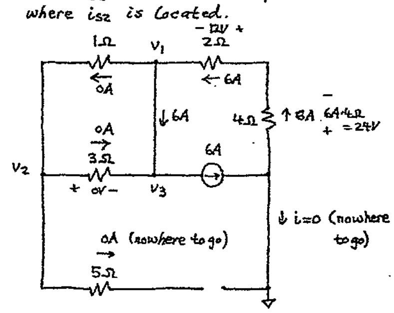
$$-v_1\left(\frac{1}{R_1}\right) + v_2\left(\frac{1}{R_1} + \frac{1}{R_3}\right) - v_3 \frac{1}{R_3} + i_{s2} = 0 \text{ A}$$

Make a consistency check on the above node-voltage equations for the circuit by setting resistors and sources to values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and v_1, v_2, v_3 for your consistency check, and show that the node-voltage equations are satisfied for these values. (In other words, plug the values into the node-voltage equations and show that they yield values of zero Amps.)

soln: Many consistency checks are possible.

One possible is to set $i_{s2} = 0A$ and $\alpha = 0$. Let $i_s = 6A$ and set $R_1 = 1\Omega$, $R_2 = 2\Omega$, ..., $R_5 = 5\Omega$.

With $\alpha = 0$, our v-src becomes a wire. With $i_{s2} = 0A$ we have an open circuit where i_{s2} is located.



All the current will flow thru the wire between v_1 and v_3, (path of least resistance).

The 6A flows thru $R_2 + R_4$, giving v-drop of $6A \cdot (2\Omega + 4\Omega) = 36V$ from reference to v_1.

Thus, $v_1 = -36V$. $v_3 = v_1 = -36V$ (wire connects v_1 to v_3)

Also, $v_2 = v_3 = -36V$ (0V across R_3; no current)

Now we plug our component values and v_1, v_2, v_3 values into the node-voltage eq'ns to see if the eq'ns are valid, (i.e. that left side of eq'n = right side of eq'n).

$$-36V\left(\frac{1}{1\Omega}+\frac{1}{2\Omega+4\Omega}\right)-(-36V)\left(\frac{1}{3\Omega}+\frac{1}{1\Omega}\right)+(-36V)\frac{1}{3\Omega}$$

$$+\ 6A$$

$$= \cancel{-36A} - \cancel{6A} + \cancel{12A} + \cancel{36A} - \cancel{12A} + \cancel{6A}$$

$= 0A$ eq'n satisfied ✓

Now we check 2nd eq'n.

$$-\cancel{36V}-(-\cancel{36V}) \stackrel{?}{=} 0\cdot\underbrace{(-36V)\frac{4\Omega}{2\Omega+4\Omega}}_{\to 0}$$

$$0 = 0 \checkmark$$

Now we check 3rd eq'n.

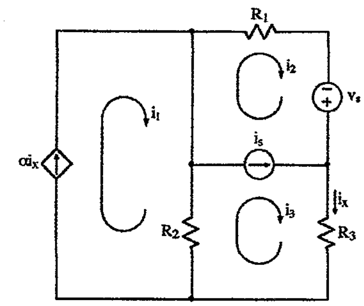
$$-(-36V)\left(\frac{1}{1\Omega}\right)+(-36V)\left(\frac{1}{1\Omega}+\frac{1}{3\Omega}\right)-(-36V)\frac{1}{3\Omega}+0A$$

$$= \cancel{36A} - \cancel{36A} - \cancel{12A} + \cancel{12A}$$

$= 0A$ ✓

This check worked. In practice we would perform others.

Ex:



For the circuit shown, write three independent equations for the three mesh currents, i_1, i_2, and i_3. The quantity i_x must not appear in the equations.

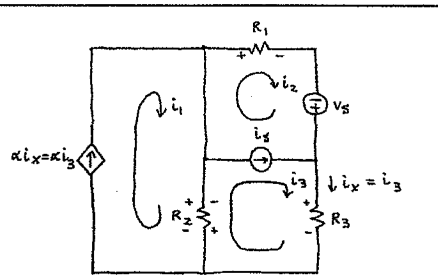
sol'n: We first write i_x in terms of mesh currents.

Since i_x is a current on the outside edge of the circuit, (flowing thru R_3), it is equal to the mesh (also called "loop") current.

$$i_x = i_3$$

Next we look for super meshes where a current src is between two loops. We have a supermesh for i_2, i_3 with i_s in between.

We draw the circuit model before writing our eq'ns.



i_2, i_3 loop: $-i_3R_2 + i_1R_2 - i_2R_1 - i_3R_3 = 0V$
(supermesh uses loop around right half of circuit)

We add a current eq'n for i_s src between loops.

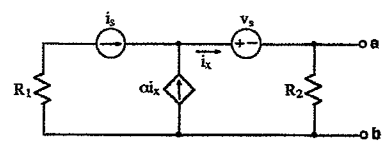
$i_s = i_3 - i_2$ (i_2 has − sign because it is measured in direction opposite to direction of i_s)

Finally, for the i_1 loop we encounter a curious situation. Since we have a current source on the outside edge of the circuit, we must have that i_1 = current for src.

Thus, $i_1 = \alpha i_3$. This is the eq'n for i_1.

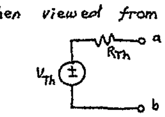
We now have 3 eq'ns in i_1, i_2, i_3 which we could solve to find i_1, i_2, and i_3.

Ex:



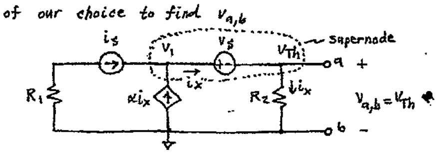
Find the Thevenin's equivalent circuit at terminals a-b. i_x must not appear in your solution. Note: $\alpha \neq 1$.

sol'n: We must find v_{Th} and R_{Th} for the Thevenin equivalent circuit that has the same behavior as the above circuit when viewed from a,b terminals.



$v_{Th} = v_{a,b}$ for circuit with nothing connected across a,b.

We can use node-v method or another method of our choice to find $v_{a,b}$



We first define i_x in terms of node voltages. Here, i_x flows thru R_2. Thus $i_x = \frac{v_{Th}}{R_2}$.

We have a supernode for v_1 and v_{Th}.

So we sum currents out of a bubble around v_1, v_{Th}, and v_s.

v_1, v_{Th} node: $-i_s - \alpha \dfrac{v_{Th}}{R_2} + \dfrac{v_{Th}}{R_2} = 0A$

We could continue on to write a voltage eq'n for v_1 and v_{Th}: $v_1 = v_{Th} + v_s$

But our first eq'n has only v_{Th} in it; we can solve the first eq'n for v_{Th} and stop there.

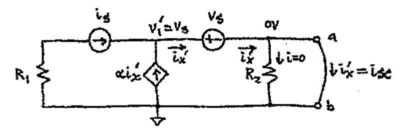
Rearranging gives $v_{Th}\left(\dfrac{1}{R_2} - \dfrac{\alpha}{R_2}\right) = i_s$

or $v_{Th} = \dfrac{i_s R_2}{1-\alpha}$

To find R_{Th}, we can use the method of shorting out a and b and measuring the current in the wire. This is i_{sc} for $i_{\text{short circuit}}$. If we look at a Thevenin equivalent circuit with a wire from a to b, we have current $i_{sc} = \dfrac{v_{Th}}{R_{Th}}$.

Thus, $R_{Th} = \dfrac{v_{Th}}{i_{sc}}$.

We redraw our circuit with a wire from a to b.



This is a different circuit than before. We have 0V at a, (instead of v_{Th}), and no current flows in R_2 since it is bypassed by a wire. Also, $i_{sc} = i_x'$.

We also have $v_1 = 0V + v_s = v_s$. Circuit is solved. Or is it? We still need to find $i_{sc} = i_x'$

Using a current summation at v_1, we have

$$-i_s - \alpha i_x' + i_x' = 0A$$

or $\quad i_x'(1-\alpha) = i_s$

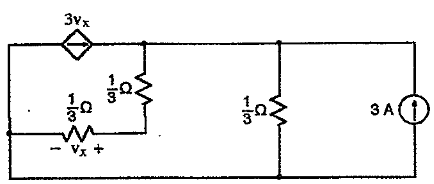
or $\quad i_x' = \dfrac{i_s}{1-\alpha} = i_{sc}$

Using $R_{Th} = \dfrac{v_{Th}}{i_{sc}}$ gives $R_{Th} = \dfrac{\frac{i_s R_2}{1-\alpha}}{\frac{i_s}{1-\alpha}}$

or $\quad R_{Th} = R_2$ (Nothing else plays a role in R_{Th})

Consistency check: Set $\alpha = 0 \Rightarrow$ dependent src = open. Then R_1, v_s in series with current src i_s irrelevant. We have Norton equiv, i_s and R_2: $v_{Th} = i_s R_2$, $R_{Th} = R_2$ ✓

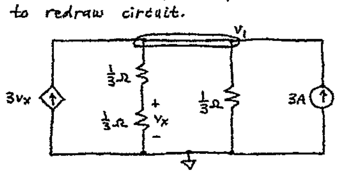
EX:



Calculate the power dissipated in the dependent current source, (labeled $3v_X$).

Sol'n: Any method of solution is allowed, (provided it is a valid approach).

We'll use node-voltage method with reference on bottom and v_1 on top. It also helps to redraw circuit.



We write v_x in terms of v_1 by using a voltage divider:

$$v_x = v_1 \cdot \frac{1/3\,\Omega}{\frac{1}{3}\Omega + \frac{1}{3}\Omega} = \frac{v_1}{2}$$

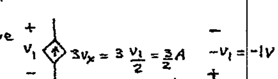
Now we write the current summation eq'n for node v_1.

$$-3\underbrace{\left(\frac{v_1}{2}\right)}_{v_x} + \frac{v_1}{\frac{1}{3}\Omega + \frac{1}{3}\Omega} + \frac{v_1}{\frac{1}{3}\Omega} - 3A = 0A$$

or $\quad v_1\left(-\dfrac{3}{2\Omega} + \dfrac{3}{2\Omega} + \dfrac{3}{\Omega}\right) = 3A$

or $\quad v_1 = 1A \cdot \Omega = 1V$

So we have

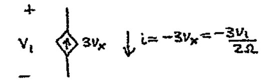


Power is $p = iv$ where i, v follow passive sign convention.

$$p = \frac{3}{2}A \cdot (-1V) = -\frac{3}{2}W$$

Note: In this problem we can replace the dependent src with a resistor, (even before we know the value of v_1).

We have voltage v_1 across dependent src and current $-3v_x = -\dfrac{3v_1}{2}$



Then $R_{eq} = \dfrac{v_1}{-\frac{3}{2\Omega}v_1} = -\dfrac{2}{3}\Omega$. $\quad -\frac{2}{3}\Omega$

Use this instead of dependent src.