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Unit 2

a) Solve following eqns for V_1 and V_2 in terms of R's.

$$V_1\left(\frac{1}{R_2}+\frac{1}{R_1}+\frac{1}{R_3}\right)-V_2\frac{1}{R_3}=10$$

$$\frac{V_1-V_2}{R_3}=\frac{V_2}{R_4}$$

soln: Rewrite 2nd eqn as $\frac{V_1}{R_3}=V_2\left(\frac{1}{R_3}+\frac{1}{R_4}\right)$

Use parallel R notation $R_1\|R_2=\frac{R_1R_2}{R_1+R_2}=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$

Our two eqns become:

$$\frac{V_1}{R_1\|R_2\|R_3}-\frac{V_2}{R_3}=10$$

$$\frac{V_1}{R_3}-\frac{V_2}{R_3\|R_4}=0$$

Now use 2nd eqn to write V_2 in terms of V_1:

$$V_2=V_1\frac{R_3\|R_4}{R_3}=V_1\frac{\cancel{R_3}R_4}{(R_3+R_4)\cdot\cancel{R_3}}=V_1\frac{R_4}{R_3+R_4}$$

Now substitute this into first eqn:

$$\frac{V_1}{R_1\|R_2\|R_3}-\frac{V_1}{R_3}\frac{R_4}{R_3+R_4}=10\text{ A}$$

$$V_1=10\Big/\left(\frac{1}{R_1\|R_2\|R_3}-\frac{R_4}{R_3}\frac{1}{R_3+R_4}\right)=10\Big/\left(\frac{1}{R_1\|R_2\|R_3}-\frac{R_3\|R_4}{R_3^2}\right)$$

$$V_2=V_1\frac{R_3\|R_4}{R_3}=10\Big/\frac{R_3}{R_3\|R_4}\left(\frac{1}{R_1\|R_2\|R_3}-\frac{R_4}{R_3}\frac{1}{R_3+R_4}\right)$$

$$V_2=10\Big/\left(\frac{R_3+R_4}{R_4}\frac{1}{R_1\|R_2\|R_3}-\frac{R_3\|R_4}{R_3(R_3\|R_4)}\right)$$

$$V_2=10\Big/\left(\frac{R_3/R_4\|R_3}{R_1\|R_2\|R_3}-\frac{1}{R_3}\right)=10R_3\Big/\left(\frac{R_3^2/R_3\|R_4}{R_1\|R_2\|R_3}-1\right)$$

b) Make at least two consistency checks on answer to (a).

soln: Units: $v_1 = A \bigg/ \left(\frac{1}{\Omega} - \frac{\Omega}{\Omega} \frac{1}{\Omega} \right) = A \cdot \Omega = V$ ✓

$v_2 = A\Omega \bigg/ \left(\frac{\Omega^2/\Omega}{\Omega} - 1 \right) = A \cdot \Omega = V$ ✓

When $R_3 \to \infty$ 2nd eq'n gives $v_2 = 0$, 1st eq'n gives

$v_1 = 10A \bigg/ \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$.

In our v_1 eq'n with $R_3 \to \infty$ we get $R_1 \| R_2 \| R_3 = R_1 \| R_2$,

$\frac{R_4}{R_3} \frac{1}{R_3 + R_4} = \frac{R_4}{\infty(\infty + R_4)} = \frac{R_4}{\infty} = 0$

$\therefore v_1 = \frac{10}{1/R_1 \| R_2} = 10 \cdot R_1 \| R_2 = 10 \cdot \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$ ✓

$v_2 = 10 \bigg/ \left(\frac{\overset{\infty}{\infty + R_4}/\infty}{R_1 \| R_2} - \overset{0}{\frac{1}{\infty}} \right) = 10 \cdot \frac{R_1 \| R_2}{\infty} = 0$ ✓

Make another consistency check.

$R_4 \to \infty$ gives $v_1 = v_2$ from 2nd eq'n and $R_3 \| R_4 = R_3$.

Our eq'ns: $v_1 = 10 \bigg/ \left(\frac{1}{R_1 \| R_2 \| R_3} - \frac{R_3}{R_3^2} \right) = 10 \bigg/ \left(\frac{1}{R_1 \| R_2 \| R_3} - \frac{1}{R_3} \right)$

$v_2 = 10 \bigg/ \left(\frac{R_3/R_3}{R_1 \| R_2 \| R_3} - \frac{1}{R_3} \right) = 10 \bigg/ \left(\frac{1}{R_1 \| R_2 \| R_3} - \frac{1}{R_3} \right)$ ✓

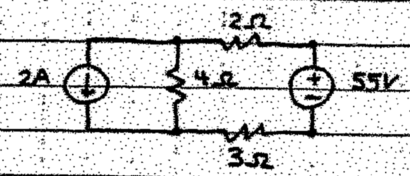
One more check. $R_1 = \infty$ $R_2 = \infty$ $R_3 = R_4$ Then $v_1 = 2v_2$.

$v_2 = \frac{v_1}{2}$, $v_1 - v_2 = 10 R_3$, $v_1 - \frac{v_1}{2} = 10 R_3$, $v_1 = 2 \cdot 10 R_3$.

Our eq'ns: $v_1 = 10 \bigg/ \left(\frac{1}{R_3} - \frac{R_3/2}{R_3^2} \right) = 10 \bigg/ \frac{1}{2R_3} = 2 \cdot 10 R_3$ ✓

$v_2 = 10 \bigg/ \left(\frac{\cancel{R_3}/(R_3/2)}{\cancel{R_3}} - \frac{1}{R_3} \right) = 10 / 1/R_3 = 10 R_3 = \frac{v_1}{2}$ ✓

ex:



Use Node-Voltage method to find how much power the 2A source extracts from circuit.

First we use terminology:

\# nodes = 4 (two or more circuit elements join)

\# essential nodes = 2 (three or more circuit elements join; they are nodes for 4Ω resistor, top & bottom)

paths: 2A → 4Ω , 4Ω → 2Ω → 55V → 3Ω , 2Ω → 55V , 2A → 2Ω → 55V are a few examples (trace of connected circuit elements without passing thru any element twice)

branch: (path that connect 2 nodes) 2A, 4Ω, 2Ω, 3Ω, 2Ω → 55V, 55V → 3Ω, 3Ω → 55V (either direction OK), 55V, 2Ω → 55V → 3Ω

essential branch: (path connecting essential node w/o passing thru essential node) 2A, 4Ω, 2Ω → 55V → 3Ω, or 3Ω → 55V → 2Ω

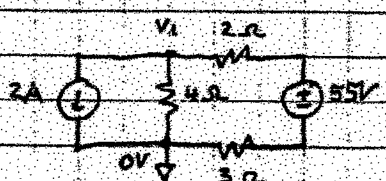
loops: (path with last node = start node) 2A → 4Ω, 2A → 2Ω → 55V → 3Ω, 4Ω → 2Ω → 55V → 3Ω

mesh: (loop not enclosing any other loop) 2A → 4Ω, 4Ω → 2Ω → 55V → 3Ω but not 2A → 2Ω → 55V → 3Ω

planar circuit: (can draw circuit w/o crossover branches) is planar

For Node-V method, we use all but 1 essential nodes after we define a ref node.

Choose node at bottom of 4Ω as ref node (i.e. 0V.)
symbol ≡ 0V



Node at top of 4Ω is the other essential node. Label it V_1

Although we call it the Node-V method, (because we get an equation that we solve for voltage), we are writing an equation for sum of currents out of node = 0.

$$2A + \frac{v_1 - 0V}{4\Omega} + \frac{v_1 - 55V}{2\Omega + 3\Omega} = 0A$$

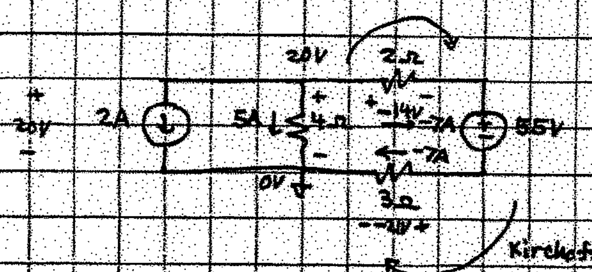
Note that current thru 2Ω is equal to total V-drop (i.e. $v_1 - 55V$) across the 2Ω and 3Ω R's.

or $$v_1\left(\frac{1}{4\Omega} + \frac{1}{5\Omega}\right) = -2A + \frac{55V}{5\Omega}$$

or $$\frac{v_1}{4\Omega \| 5\Omega} = 11A - 2A = 9A$$

or $$v_1 = 9A \cdot 4\Omega \| 5\Omega = 9A \cdot \frac{4\Omega \cdot 5\Omega}{4\Omega + 5\Omega} = 20\ V$$

check: $$\frac{v_1}{4\Omega} = \frac{20V}{4\Omega} = 5A \qquad \frac{v_1 - 55V}{2\Omega + 3\Omega} = -\frac{35V}{5\Omega} = -7A$$

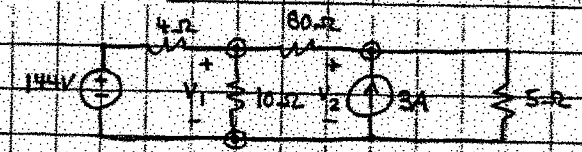


$$-55V - -21V + 20V - -14V$$
$$= -55V + (21 + 20 + 14)\ V$$
$$= 0V \checkmark$$

check: current out of top node for 4Ω is $2A + 5A - 7A = 0$ ✓

Calculate power for 2A source: $p = i \cdot v = 2A \cdot 2V = 40W$
$p > 0 \Rightarrow$ power <u>absorbed</u>.

ex:

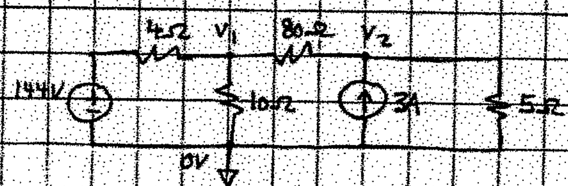


Use Node-V method to find v_1 & v_2.

sol'n: nodes are marked by dots. Essential nodes are marked by ⊙. Note that node under 3A source is considered to be part of essential node under 10Ω Resistor, (because they are connected by a wire).

3 essential nodes ⇒ 3 − 1 = 2 equations needed.

Put ref V of 0V, (i.e. ⏚), at bottom so top nodes are v_1 and v_2.



Node-V eqn's give sum of currents out of node = 0.

node 1: (v_1) $\quad \dfrac{v_1 - 144\text{V}}{4\,\Omega} + \dfrac{v_1 - 0\text{V}}{10\,\Omega} + \dfrac{v_1 - v_2}{80\,\Omega} = 0\text{A}$

node 2 (v_2) $\quad \dfrac{v_2 - v_1}{80\,\Omega} + -3\text{A} + \dfrac{v_2 - 0\text{V}}{5\,\Omega} = 0\text{A}$

or node 1: $\quad v_1\left(\dfrac{1}{4\,\Omega} + \dfrac{1}{10\,\Omega} + \dfrac{1}{80\,\Omega}\right) - \dfrac{v_2}{80\,\Omega} = \dfrac{144\text{V}}{4\,\Omega}$

node 2: $\quad -v_1 \dfrac{1}{80\,\Omega} + v_2\left(\dfrac{1}{80\,\Omega} + \dfrac{1}{5\,\Omega}\right) = 3\text{A}$

or node 1: $\quad \dfrac{v_1}{4\,\Omega \| 10\,\Omega \| 80\,\Omega} - \dfrac{v_2}{80\,\Omega} = \dfrac{144\text{V}}{4\,\Omega}$

node 2: $\quad -\dfrac{v_1}{80\,\Omega} + \dfrac{v_2}{80\,\Omega \| 5\,\Omega} = 3\text{A}$

$$4\,\Omega \parallel 10\,\Omega \parallel 80\,\Omega = \left(4\,\Omega \parallel 10\,\Omega\right) \parallel 80\,\Omega$$

$$= \frac{40}{14}\,\Omega \parallel 80\,\Omega$$

$$= 40\,\Omega \cdot \frac{1}{14} \parallel 2$$

$$= 40\,\Omega \cdot \frac{2/14}{2 + 1/14}$$

$$= 40\,\Omega \cdot \frac{2}{29} \qquad \text{(mult top \& bottom by 14)}$$

$$= \frac{80}{29}\,\Omega$$

$$80\,\Omega \parallel 5\,\Omega = 5\,\Omega \cdot 16 \parallel 1 = 5\,\Omega \cdot \frac{16}{17} = \frac{80}{17}\,\Omega$$

Thus, we have: node 1 $\quad v_1 \dfrac{29}{80\,\Omega} - \dfrac{v_2}{80\,\Omega} = 36\text{ A}$

node 2 $\quad \dfrac{-v_1}{80\,\Omega} + v_2 \dfrac{17}{80\,\Omega} = 3\text{ A}$

or node 1 $\quad 29v_1 - v_2 = 2880\text{ V}$

node 2 $\quad -v_1 + 17v_2 = 240\text{ V}$

From node 1: $-v_2 = 2880\text{V} - 29v_1$; substitute this into Node 2:

node 2: $\quad -v_1 - 17(2880\text{V} - 29v_1) = 240\text{V}$

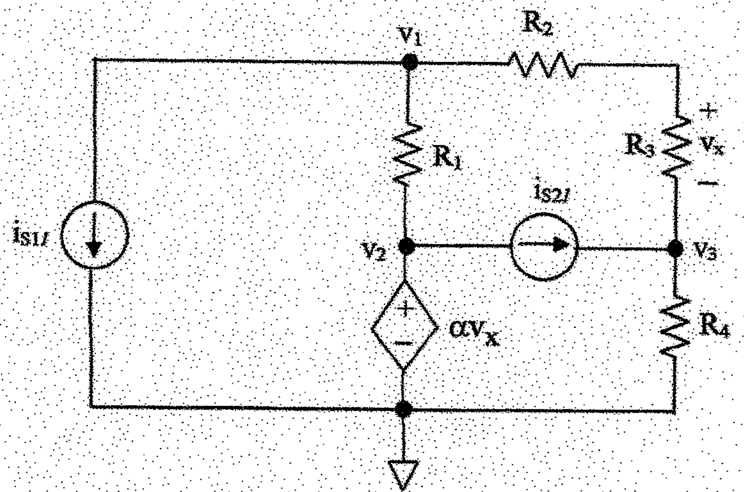
$$492v_1 = 240\text{V} + 48960$$

$$v_1 = 100\text{ V}$$

$$v_2 = \frac{240\text{V} + v_1}{17} = \frac{340\text{V}}{17} = 20\text{V}$$

Node Voltage Example

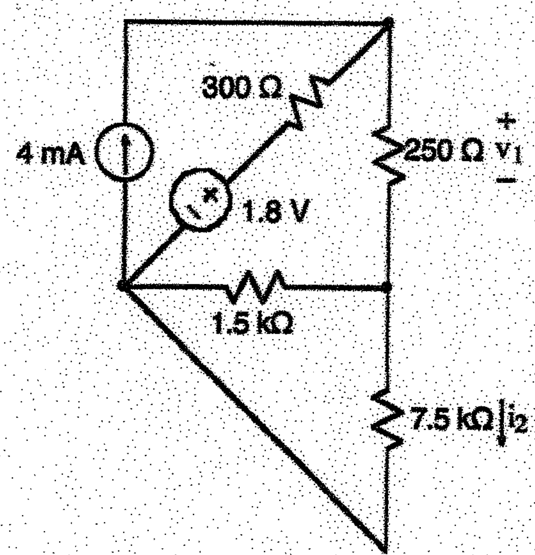
1. a.



For the circuit shown, write three independent equations for the node voltages v_1, v_2, and v_3. The quantity v_x must not appear in the equations.

b. Make a consistency check on your equations for problem 1 by setting resistors and sources to values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and v_1, v_2, v_3 for your consistency check, and show that your equations for problem 1(a) are satisfied for these values. (In other words, plug the values into your equations for problem 1(a).)

Ex:



a) Use the node-voltage method to calculate v_1 and i_2.

b) Calculate the power in the 300 Ω resistor.

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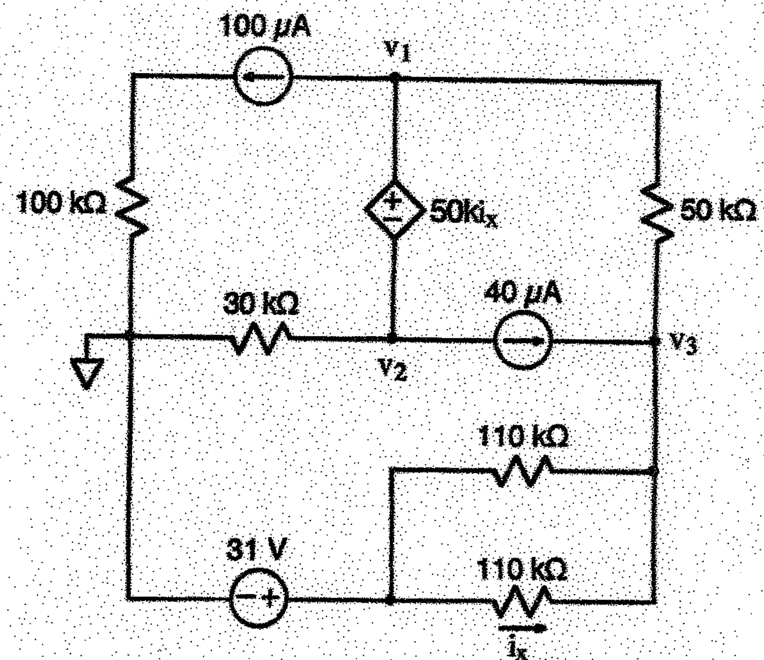
HOMEWORK #3 Solution Prob 3

Ex:



Use the node-voltage method to find v_1, v_2, and v_3.

sol'n: We first write the variable for the dependent source, i_x, in terms of node V's.

$$i_x = \frac{31V - v_3}{110\,k\Omega}$$

We have a super node for v_1 and v_2 since these nodes are connected only by a V src. Thus, we sum all the currents flowing out of a bubble drawn around v_1, v_2, and the V-src between them.

v_1, v_2 node: $100\mu A + \dfrac{v_1 - v_3}{50\,k\Omega} + \dfrac{v_2}{30\,k\Omega} + 40\mu A = 0A$

We also write a voltage eq'n for v_1 & v_2:

$$v_1 = v_2 + 50\text{k}\Omega\, i_x = v_2 + 50\text{k}\Omega \left(\frac{31\text{V} - v_3}{110\text{k}\Omega}\right)$$

(Remember to use only node V's in eq'ns.)

For the v_3 node, we only have a current sum.

$$v_3 \text{ node: } \frac{v_3 - v_1}{50\text{k}\Omega} + -40\mu\text{A} + \frac{v_3 - 31\text{V}}{\underbrace{110\text{k}\Omega \| 110\text{k}\Omega}_{55\text{k}\Omega}} = 0\text{A}$$

Now we solve the 3 eq'ns. We put terms multiplying v_1, v_2, and v_3 on the left side and constant terms on the right side.

$$v_1 \frac{1}{50\text{k}\Omega} + v_2 \frac{1}{30\text{k}\Omega} + v_3\left(\frac{-1}{50\text{k}\Omega}\right) = -100\mu\text{A} - 40\mu\text{A}$$

$$v_1 - v_2 + v_3 \frac{50\text{k}\Omega}{110\text{k}\Omega} = 31\text{V} \cdot \frac{50\text{k}\Omega}{110\text{k}\Omega}$$

$$v_1\left(\frac{-1}{50\text{k}\Omega}\right) + v_3\left(\frac{1}{50\text{k}\Omega} + \frac{1}{55\text{k}\Omega}\right) = 40\mu\text{A} + \frac{31\text{V}}{55\text{k}\Omega}$$

We multiply both sides of the 1st eq'n by 150 kΩ to clear the denominators.

$$v_1 \cdot 3 + v_2 \cdot 5 + v_3(-3) = -140\mu\text{A} \cdot 150\text{k}\Omega = -21\text{V}$$

We multiply both sides of the 2nd eq'n by 11Ω to clear the denominators.

$$v_1(11) + v_2(-11) + v_3(5) = 31\text{V}(5) = 155\text{ V}$$

We multiply the 3rd eq'n by $1100\,k\Omega = 1.1\,M\Omega$

$$v_1(-22) + v_3(22+20) = 40\,\mu A \cdot 1.1\,M\Omega + 31V(20)$$

or $v_1(-22) + v_3(42) = 44V + 620V = 664V$

Now we start eliminating variables. From the 1st eqn, we have

$$v_2 = \frac{-21V - 3v_1 + 3v_3}{5}$$

Our 2nd and 3rd eq'ns with this v_2 become

$$11v_1 - 11\left(\frac{-21V - 3v_1 + 3v_3}{5}\right) + 5v_3 = 155V$$

and

$$-22v_1 + 42v_3 = 664V$$

Solving the last eq'n for v_3 gives

$$v_3 = \frac{664V + 22v_1}{42}$$

Substituting into the 1st eq'n (and collecting terms multiplying v_3) yields the following eq'n:

$$\left(11 + \frac{33}{5}\right)v_1 + \left(-\frac{33}{5} + 5\right)\left(\frac{664V + 22v_1}{42}\right) = 155V - \frac{21(11)V}{5}$$

or, after multiplying both sides by 5,

$$88v_1 - 8\left(\frac{664V + 22v_1}{42}\right) = 775V - 231V = 544V.$$

Dividing both sides by 4 and moving constant term:

$$22v_1 - \frac{2(22)}{42}v_1 = 136V + \frac{2(664)}{42}V.$$

Multiplying both sides by 42 gives

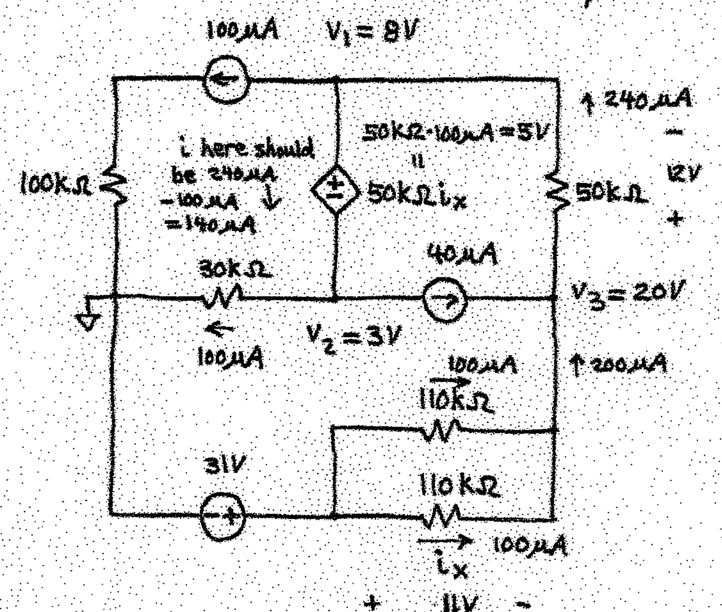
$$[42(22) - 44]v_1 = 136V(42) + 2(664)V$$

or $$v_1 = \frac{5712 + 1328}{924 - 44}V = \frac{7040}{880}V = 8V$$

Then $$v_3 = \frac{664V + 22(8V)}{42} = 20V$$

and $$v_2 = \frac{-21V - 3(8V) + 3(20V)}{5} = 3V.$$

Consistency check: Calculate currents from these voltages and verify that currents sum to zero at nodes... They do!



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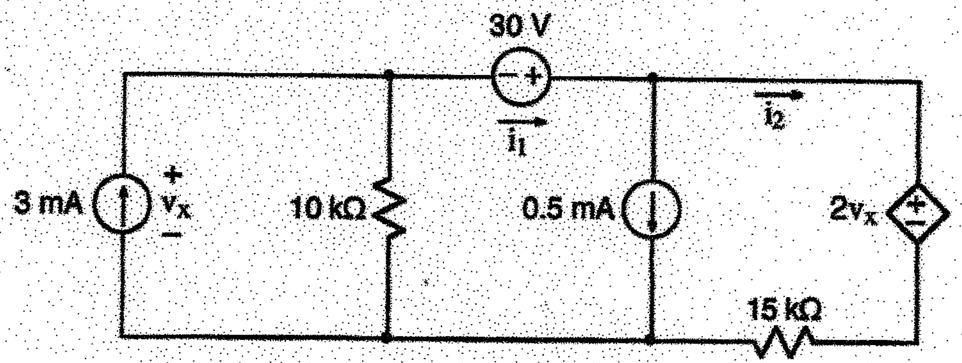
HOMEWORK #3 Solution Prob 4

Ex:



a) Use the mesh-current method to find i_1 and i_2.

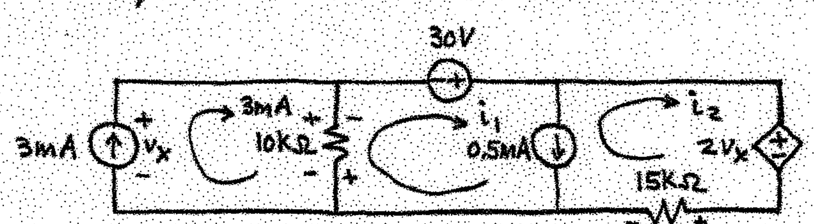
b) Find the power dissipated by the dependent source.

sol'n: a) We follow a step-by-step procedure:

1) We define mesh currents. If, however, we have any current sources on outside edges of the circuit, the mesh currents for those loops will be the same as the current source.

In this circuit, we have a current source on the left edge. Thus, the mesh current for the left loop is 3mA.

Since i_1 and i_2, as defined, are on the outside edge of the circuit, we may use them as our mesh currents.



2) We define the voltage from the dependent src, v_x, in terms of mesh currents. Here, we observe that v_x is across the 10kΩ resistor, too. For the 10kΩ resistor, we have

$$v_x = 3\,mA \cdot 10k\Omega - i_1 \cdot 10k\Omega$$

3) We look for loops with a current source in between, meaning we have a super mesh. This is the case for the i_1, i_2 loops. For the i_1, i_2 supermesh, we take a v-loop around the outside edge of the i_1 and i_2 loops, (bypassing the 0.5 mA src).

$$i_1, i_2 \text{ v-loop: } -i_1 \cdot 10k\Omega + 30V - 2\overbrace{(3mA - i_1)10k\Omega}^{v_x} + 3mA \cdot 10k\Omega$$

$$- i_2 \cdot 15k\Omega = 0V$$

Add a current eq'n for the 0.5 mA src between the loops:

$$i_1 - i_2 = 0.5\,mA = \frac{1}{2}\,mA$$

Note: we have $-i_2$ for current measured opposite the arrow in the current src.

4) We solve our eq'ns for i_1 and i_2.

We group i_1 and i_2 terms on the left and move constant to the right side.

$$i_1 \left(\underbrace{-10\,k\Omega + 2 \cdot 10\,k\Omega}_{=10\,k\Omega} \right) + i_2(-15\,k\Omega) = -60V + 60V$$

$$i_1 \qquad\qquad - i_2 = \tfrac{1}{2}\,mA$$

Solving the 2^{nd} eq'n for i_1, we have

$$i_1 = i_2 + \tfrac{1}{2}\,mA$$

Substituting into 1^{st} eq'n, we have

$$\left(i_2 + \tfrac{1}{2}\,mA\right) 10\,k\Omega + i_2(-15\,k\Omega) = 30V$$

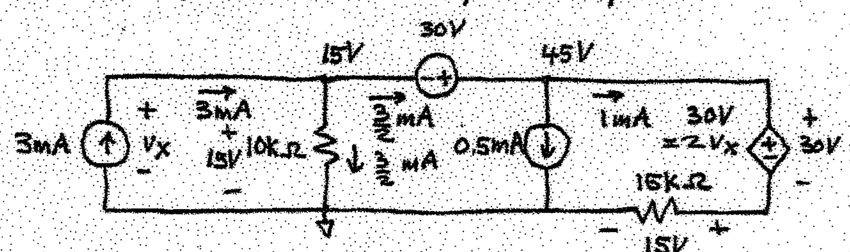
or $\quad i_2\,(10\,k\Omega - 15\,k\Omega) = 0V - \tfrac{1}{2}\,mA \cdot 10\,k\Omega$

or $\quad -i_2\,(5\,k\Omega) = -5\,V$

or $\quad i_2 = 1\,mA$

Then $\quad i_1 = 1\,mA + \tfrac{1}{2}\,mA = \tfrac{3}{2}\,mA.$

Consistency check: calculate v-drops for i_1, i_2 and verify v-loops.



$$v_x = \tfrac{3}{2}\,mA \cdot 10\,k\Omega = 15\,V$$

All v-loops sum to 0V, and all current sums at nodes = 0A. ✓