HOMEWORK #2 Solution Prob 3 (cont.)

Now that we have solved the circuit, we can find i_o from an i-sum eq'n for the node on the right.

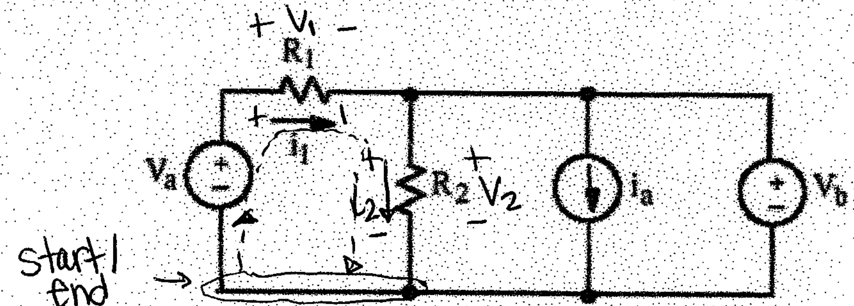
$$-i_2 - i_1 - i_q + i_o = 0\text{A}$$

or $\quad i_o = i_1 + i_2 + i_q$

or $\quad i_o = -\dfrac{v_s}{R_1} - \dfrac{v_q + v_s}{R_2} + i_q$

HW #2 Examples

3. **Derive an expression for i_1. The expression must not contain more than the circuit parameters V_a, V_b, i_a, R_1, and R_2.**



- Label all I's & V's
- Shortcut: (Use Ohms Law immediately in V-loops)

(1) $+V_a - \underbrace{i_1 R_1}_{\text{Ohms Law for } V_1} - \underbrace{i_2 R_2}_{\text{Ohms Law for } V_2} = 0$

(2) $+V_a - i_1 R_1 - V_b = 0$

(3) $+i_2 R_2 - V_b = 0 \longrightarrow i_2 = \frac{V_b}{R_2}$

- Can not do current summation because of V_b

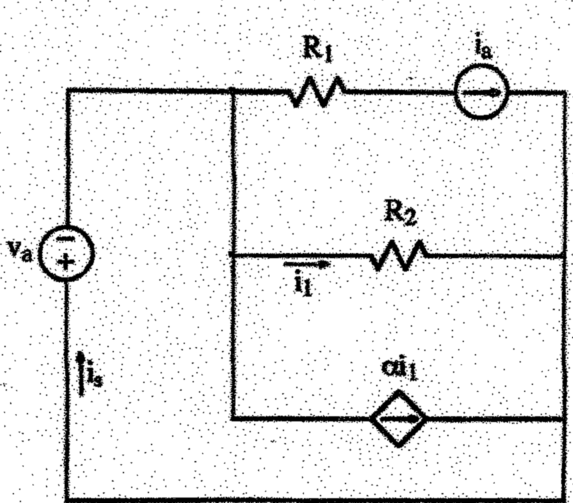
Solving (2) $\Rightarrow$ $\boxed{i_1 = \frac{V_a - V_b}{R_1}}$

OR solving (1) with (3) plugged in

$$V_a - i_1 R_1 - \frac{V_b}{R_2}(R_2) = 0$$

$$i_1 = \frac{V_a - V_b}{R_1} \text{ (same)}$$

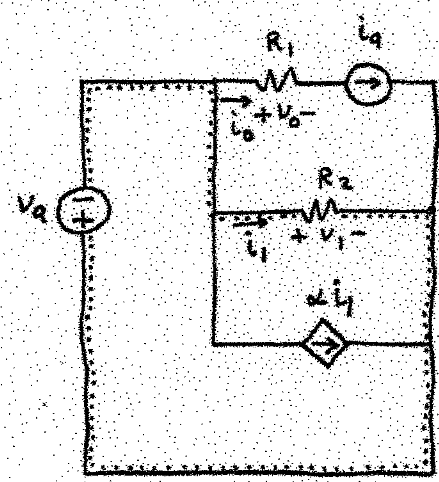
Ex:



a) **Derive an expression for i_s. The expression must not contain more than the circuit parameters α, i_a, v_a, R_1, and R_2. (Make sure to eliminate i_1 from the answer.)**

b) **Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.**

sol'n: a) Label R's



Only v-loop without current source is thru v_a and R_2, (dotted line).

$-v_q - v_1 = 0V$

We look for nodes where we can write i-sum eq'ns. Here, however, we really only have two nodes, and they are connected by only v-src v_q.

Thus, we have no i-sum eq'ns.

We look for components in series carrying the same current.

$i_o = i_q$

From Ohm's law:

$v_o = i_o R_1 = i_q R_1$

$v_1 = i_1 R_2$

Substituting for v_1 in our v-loop eq'n:

$-v_q - i_1 R_2 = 0V$

or $i_1 = -\frac{v_q}{R_2}$

It follows that $\alpha i_1 = -\alpha \frac{v_q}{R_2}$.

Now we write i-sum eq'n (for node consisting of wire on right side) to find i_g.

$$i_s - \alpha i_1 - i_1 - i_q = 0A$$

or $\quad i_s = \alpha i_1 + i_1 + i_q$

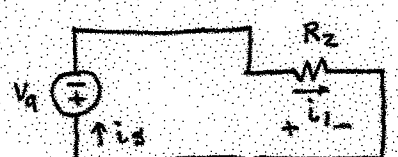
or $\quad i_s = (\alpha+1)\left(\dfrac{-v_q}{R_2}\right) + i_q$

or $\quad i_s = i_q - (\alpha+1)\dfrac{v_q}{R_2}$

b) Many consistency checks are possible. The idea is to pick component values that make the circuit so simple that we can solve it by inspection.

One example is to eliminate current sources:

Let $i_q = 0A$ and $\alpha = 0$:



We have $i_s = i_1 = -\dfrac{v_q}{R_2}$.

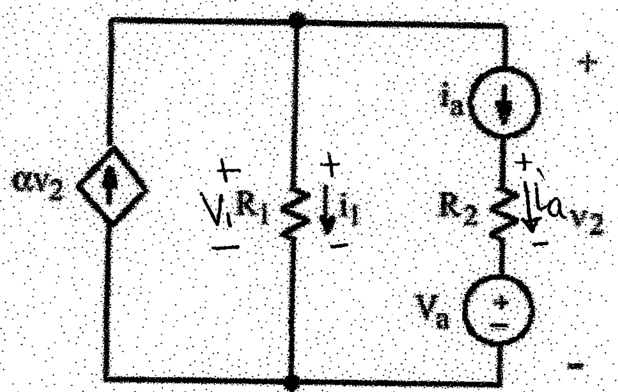
Now we verify that our eq'n from (a) agrees:

$$i_s = 0 - (0+1)\frac{v_q}{R_2} = -\frac{v_q}{R_2} \checkmark$$

Another example is to set $v_q = 0V$, $\alpha = 0$. Then $i_s = i_q$. Our eq'n gives $i_s = i_q - (0+1)\cdot\dfrac{0}{R_2} = i_q \checkmark$

HW #2 Examples

4. Derive an expression for i_1. The expression must not contain more than the circuit parameters α, V_a, i_a, R_1, and R_2.



Make at least one consistency check (other than a units check) on your expression for problem 4. Explain the consistency check clearly.

- Label all I's & V's across R's with polarity
- V-loop (with ohm's Law)

  (1) $+i_1 R_1 - V_2 = 0 \Rightarrow i_1 = \frac{V_2}{R_1}$
- Current summations:

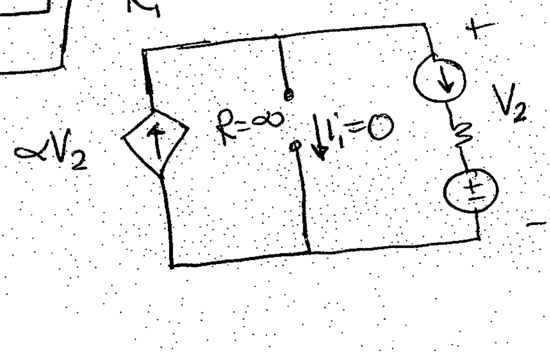
  (2) $+\alpha V_2 - i_1 - i_a = 0$
- plug (1) into (2) $\Rightarrow +\alpha V_2 - \frac{V_2}{R_1} - i_a = 0$ (1 unknown)

  $V_2 (\alpha - \frac{1}{R_1}) = i_a \Rightarrow V_2 = \frac{i_a}{(\alpha - \frac{1}{R_1})}$

  $i_1 = \frac{i_a}{(\alpha - \frac{1}{R_1})R_1} = \boxed{\frac{i_a}{\alpha R_1 - 1} = i_1}$

consistency check: • set values • redraw circuit

$R_1 = \infty$ (open)

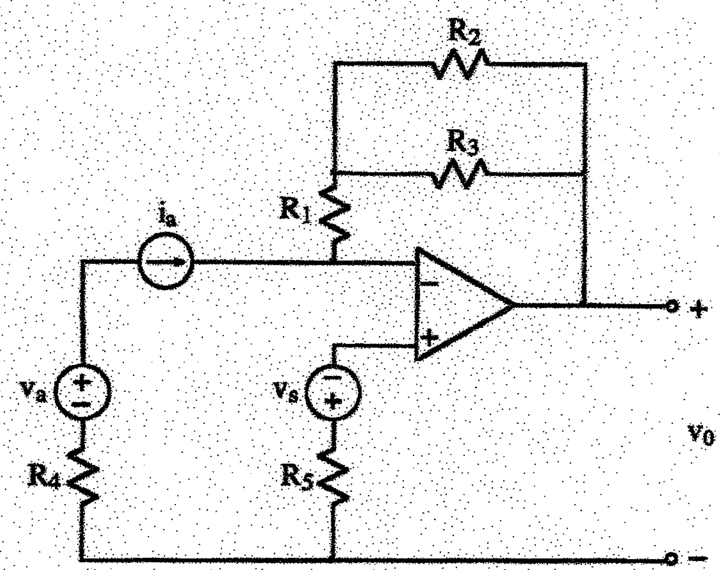


$\boxed{i_1 = 0}$

From eq: $\frac{i_a}{\infty} = \boxed{i_1 = 0}$

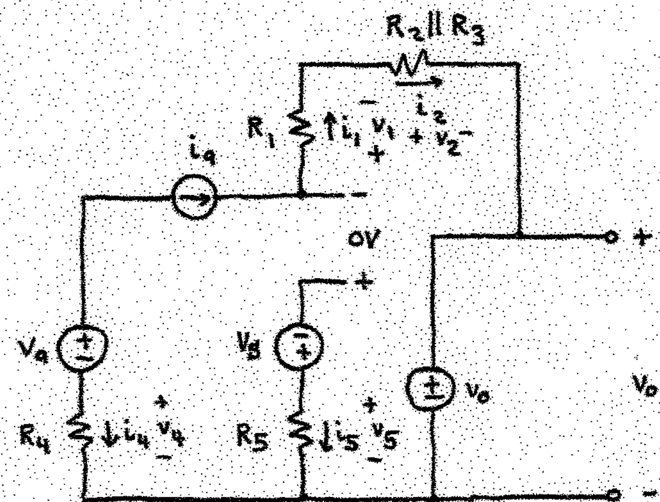
(anything of $\infty = 0$)

EX:



The op-amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for v_0 in terms of not more than v_a, v_s, i_a, R_1, R_2, R_3, R_4 and R_5.

soln: Replace op-amp with src called v_o and assume v-drop across + and − terminals is 0V. We also combine R_2 and R_3.



Write v-loops passing thru 0V across + and − terminals:

$+v_4 + v_q + \,?$ <u>Don't use</u> left-side v-loop because of current src.

$+v_5 - v_5 - 0V - v_1 - v_2 - v_o = 0V$

Write current sums at nodes.
The only true node is on the bottom.

$-i_4 - i_5 - i_2 = 0A$

Look for components in series carrying the same current:

$i_4 = -i_q$

$i_1 = i_q$

$i_2 = i_1 = i_q$

$i_5 = 0A$ (since it is in series with an open circuit)

We see that i_q flows all the way around the outer loop.

We need only substitute for v's in v-loop using i_q and Ohm's law for each resistor:

$V_1 = i_1 R_1 = i_q R_1$

HOMEWORK #2 Solution Prob 5 (cont.)

$$v_2 = i_2 \cdot R_2 \| R_3 = i_q \cdot R_2 \| R_3$$

$$v_5 = i_5 R_5 = 0 \cdot R_5 = 0V$$

Our v-loop becomes:

$$0V - v_s - 0V - i_q R_1 - i_q \cdot R_2 \| R_3 - v_o = 0V$$

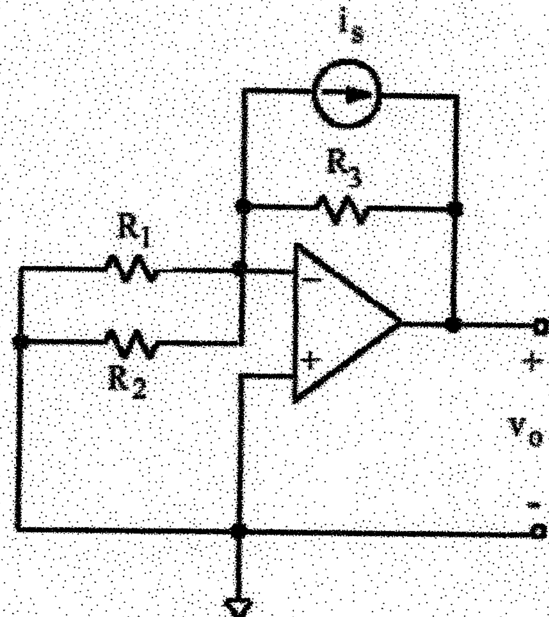
Solving v_o gives the expression we seek:

$$v_o = -v_s - i_q \left(R_1 + R_2 \| R_3\right)$$

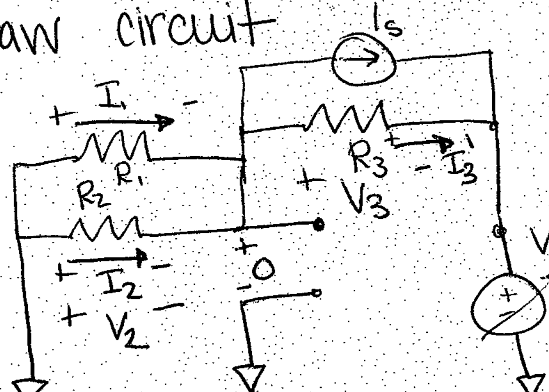
HW #2 Examples

5.

The op amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for v_o in terms of not more than i_s, R_1, R_2, and R_3.



·Redraw circuit



·Vloops:

$-V_2 - 0 = 0$

$V_2 = 0$

Ohms Laws $\begin{cases} V_2 = I_2(R_2) \Rightarrow I_2 = 0 \\ V_2 = I_1(R_1) \Rightarrow I_1 = 0 \end{cases}$

(1) $+0 - I_3 R_3 - V_o = 0$

Current summations:

$+\underset{0}{I_1} + \underset{0}{I_2} - i_s - I_3 = 0$

(no summation on right of i_s because of V_o branch)

(2) $-i_s = I_3$

plug (2) into (1)

$\boxed{+i_s R_3 = V_o}$

Review Solution

1. a. (5 points)

Calculate v_1.

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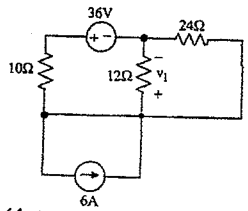
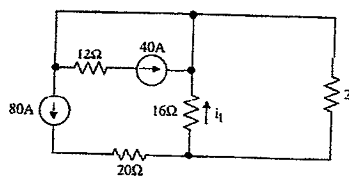


soln: The 6A source across the wire may be ignored. Its current flows through the wire but produces no V-drop. Without the 6A src we have a V-divider:

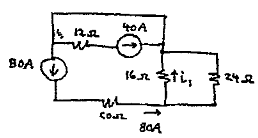
b. (5 points) $v_1 = 36V \cdot 12\Omega \| 24\Omega \,/\, 12\Omega\|24\Omega + 10\Omega = 36V \cdot 8\Omega / 18\Omega = 16V$

$\boxed{v_1 = 16V}$

Calculate i_1.



soln: If we redraw the circuit, we see a current divider:



$i_1 = 80A \cdot \frac{24\Omega}{16+24\Omega} = 80A \cdot \frac{24\Omega}{40\Omega}$

$\boxed{i_1 = 48A}$

2. (30 points)

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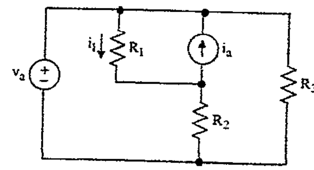
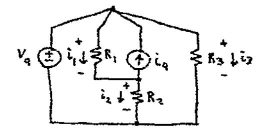
Derive an expression for i_1. The expression must not contain more than the circuit parameters v_a, i_a, R_1, R_2, and R_3.



soln: Redraw with top as one node:



Current sum at top or bottom node? No, because we would have to define a current for source V_a.

Current at center node: $i_a - i_1 + i_2 = 0A$

V-loop around left inner loop: $V_a - i_1R_1 - i_2R_2 = 0V$

No V-loop for other inner loops because we would have to define V-drop for i_a.

Next larger loop is R_1, R_2, R_3: $i_2R_2 + i_1R_1 - i_3R_3 = 0V$

Now we have 3 eqns in 3 unknowns, and we want to find i_1. We observe, however, that the first two eqns have only two unknowns. So we don't actually need the 3rd eqn. Use 1st eqn to find $i_2 = i_1 - i_a$.

Substitute into 2nd eqn: $v_a - i_1R_1 - (i_1 - i_a)R_2 = 0V$

or $i_1(-R_1 - R_2) = -v_a - i_aR_2$ or $\boxed{i_1 = \frac{v_a + i_aR_2}{R_1 + R_2}}$

3. (30 points)

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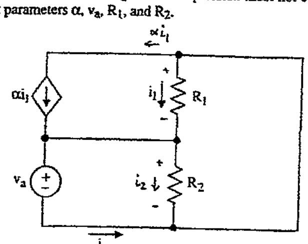
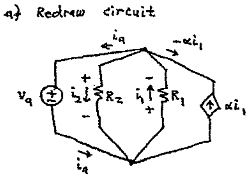
a. Derive an expression for i_a. The expression must not contain more than the circuit parameters α, v_a, R_1, and R_2.



b. Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

soln: a) Redraw circuit



No current sums at nodes because of V_a.

V-loop on left: $V_a - i_2R_2 = 0V \Rightarrow i_2 = \frac{V_a}{R_2}$

V-loop in middle: $i_2R_2 + i_1R_1 = 0V \Rightarrow i_1 = -\frac{V_a}{R_1}$

($i_2 = \frac{V_a}{R_2}$)

we could also just observe that V_a is across R_1 and R_2.

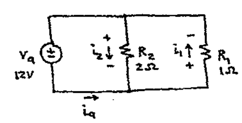
Now that we have found i_1 and i_2, we use a current at top node to find i_a:

$i_a + i_2 - i_1 - \alpha i_1 = 0A$ or $i_a + \frac{V_a}{R_2} + \frac{V_a}{R_1} + \alpha\frac{V_a}{R_1} =$

or $\boxed{i_a = -V_a\left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{\alpha}{R_1}\right)}$ or $i_a = -\frac{V_a}{R_1 \| \frac{R_1}{\alpha} \| R_2}$

soln: 3.b) Many possible answers.

Example: Suppose $\alpha = 0$. Choose other simple values:



we see that i_a is current thru $R_1 \| R_2$ with $R_1 \| R_2$ across $-V_a$.

$R_1 \| R_2 = 1\|2\ \Omega = \frac{1\cdot 2}{1+2} = \frac{2}{3}\Omega$

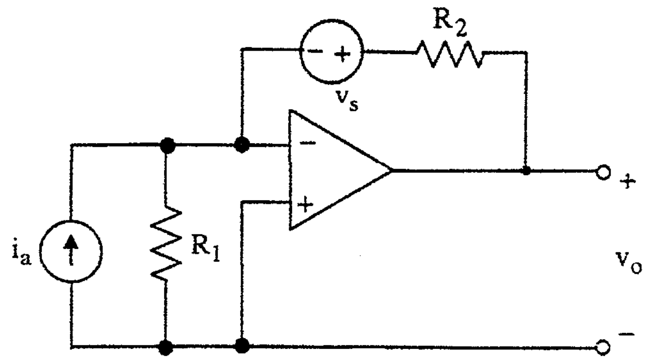
$\therefore i_a = -v_a / R_1\|R_2 = -\frac{12V}{\frac{2}{3}\Omega} = -12V \cdot \frac{3}{2\Omega} = -18A$

Use formula from (a) with these component values: $i_a = -12V\left(\frac{1}{2\Omega} + \frac{1}{1\Omega} + \frac{0}{1\Omega}\right) = -12V \cdot \frac{3}{2}$

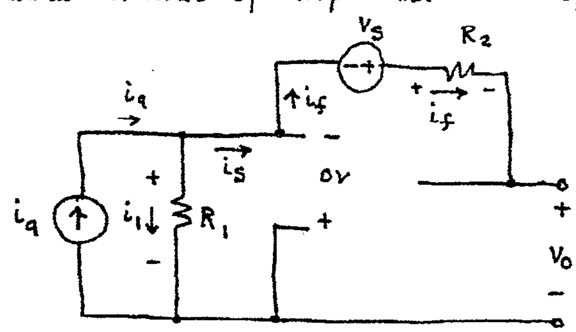
$i_a = -18V$ ✓ agrees with obvious soln for this simple case

4. (30 points)

The op-amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for v_o in terms of not more than v_s, i_a, R_1, and R_2.



sol'n: Redraw without op-amp and 0V drop across + and − inputs:



V-loop on left thru R_1 and 0V drop:

$$i_1 R_1 + 0V = 0V \quad \text{or} \quad i_1 = 0$$

Current sum at node above R_1:

$$-i_a + \underset{0}{\underset{\parallel}{i_1}} + i_s = 0A \quad \text{or} \quad i_s = i_a$$

V-loop on right thru 0V drop, v_s, R_2, and v_o:

$$-0V + v_s - i_f R_2 - v_o = 0V \quad \text{or} \quad i_f = \frac{v_s - v_o}{R_2}$$

Now use $i_s = i_f$.

$$i_a = \frac{v_s - v_o}{R_2}$$

Thus $v_s - v_o = i_a R_2$

or $\boxed{v_o = v_s - i_a R_2}$