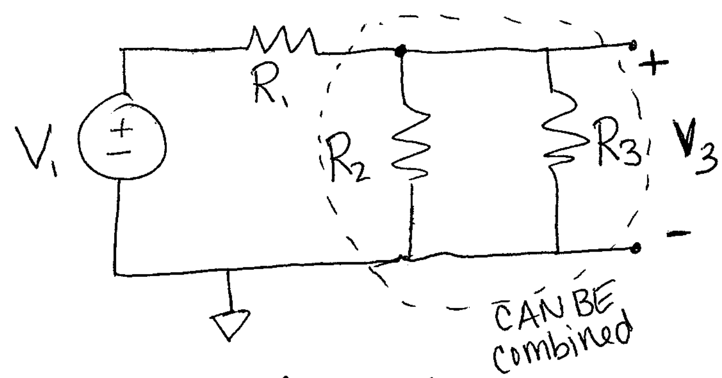
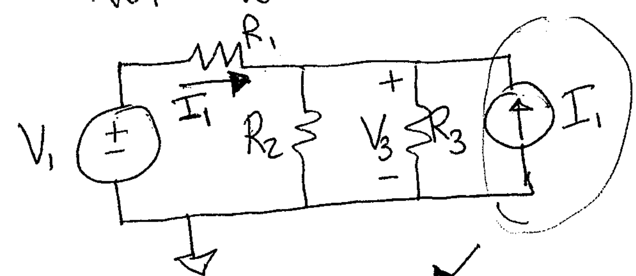
Requirements to use voltage divider:

- Current is <u>same</u> through all R's in loop.

Ex: Able to use V-divider



Not valid to use



A voltage or current source will alter the current so that I_1 does not split equally through R_2 & R_3

$$V_3 = \frac{V_1 (R_2 \parallel R_3)}{(R_2 \parallel R_3) + R_1}$$

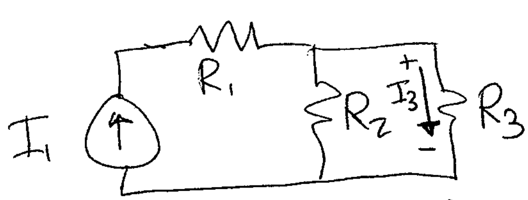
$\left[\dfrac{\text{Voltage Src} * \text{R\_desired V}}{\text{total R}}\right]$

Requirement to use current divider:

- Same current going into R's comes out.
- voltage is <u>equal</u> across all R's.

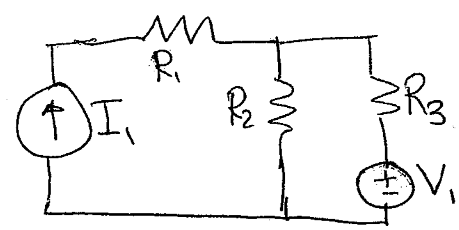
Ex:

<u>VALID</u>



$$I_3 = \frac{I_1 (R_2)}{R_2 + R_3}$$

<u>NOT VALID</u>



$\left[\dfrac{\text{I src} * \text{(opposite branch R)}}{\text{total R(in branches)}}\right]$

↑ note that R_1 does nothing in this circuit

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HOMEWORK #1 Examples

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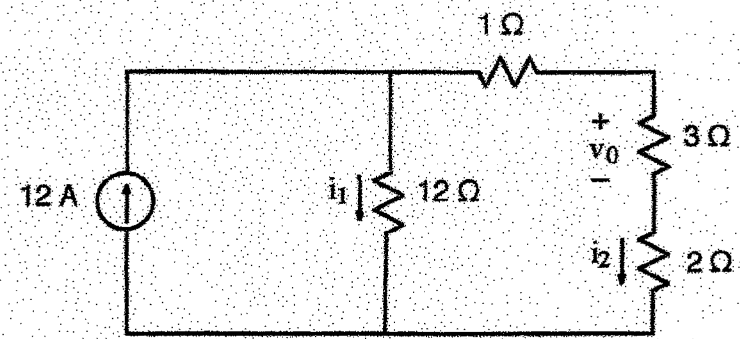
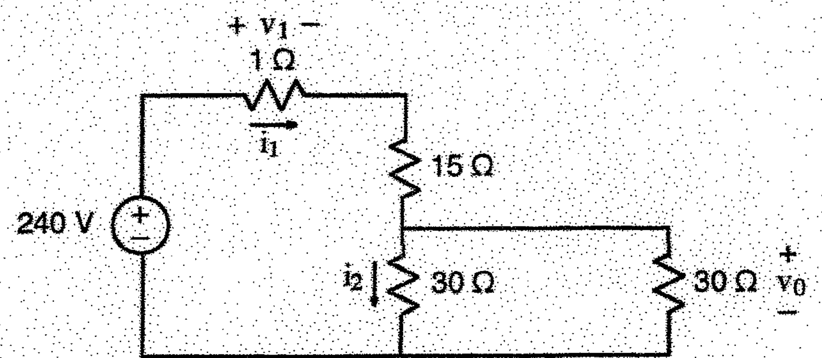
1.



a) Calculate i_1, i_2, and v_0.

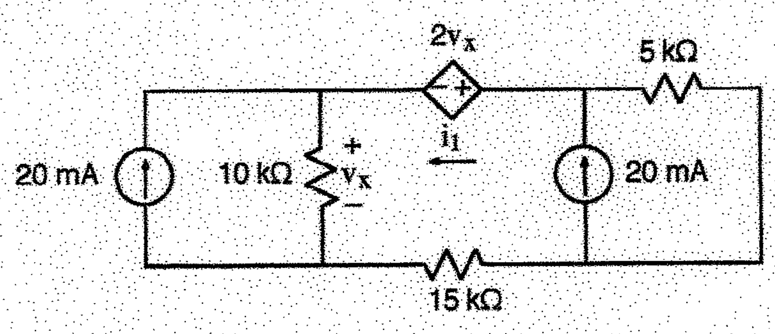
b) Find the power dissipated for every component, including the current source.

2.



Calculate i_1, i_2, and v_0.

3.



Find v_x, i_1, and the power dissipated by the dependent source.

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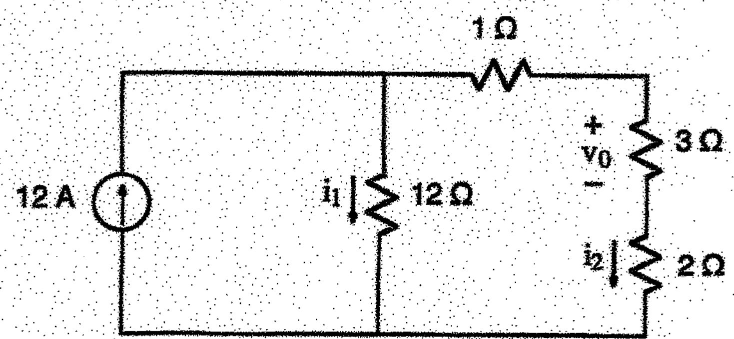
HOMEWORK #1 Prob 1. sol'n

Ex:

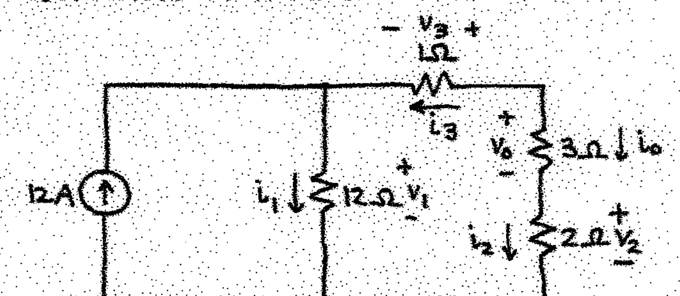


a) Calculate i_1, i_2, and v_0.

b) Find the power dissipated for every component, including the current source.

sol'n: a) We first label current and voltage for each resistor. We follow the passive sign convention: the arrow for the direction of current measurement points toward the − sign of the voltage measurement.

For the 1Ω resistor, we may define the current measurement in either direction. For the sake of illustration, we define the direction of current measurement in a way that is somewhat awkward.



Now we write eq'ns for voltage loops. We try to write v-loop eq'ns for inner loops, but we avoid loops that include a current source. (The reason we do so is to avoid defining a new variable that requires another eq'n.) In this problem, there is only one v-loop without a current source. Going around the inner loop on the right side in a clockwise direction and using the sign where we exit a component, we have

$$+v_1 + v_3 - v_o - v_2 = 0V$$

Next, we write eq'ns for current summations at nodes. We sum the currents measured flowing away from the top center node.

$$-12A + i_1 - i_3 = 0A$$

There is always one redundant node. So we only need this one eq'n.

Now we look for components in series that carry the same current.

$i_3 = -i_o$ (minus sign because currents measured in opposite directions)

$i_o = i_2$

Our last set eq'ns comes from Ohm's Law for each resistor.

$$v_1 = i_1 \cdot 12\Omega$$

$$v_o = i_o \cdot 3\Omega$$

$$v_2 = i_2 \cdot 2\Omega$$

To solve the simultaneous eq'ns, we substitute i_2 for i_o and $-i_2$ for i_3. Then we substitute for v's using the Ohm's law eq'ns.

Our v-loop eq'n becomes

$$i_1 \cdot 12\Omega - i_2 \cdot 1\Omega - i_2 \cdot 3\Omega - i_2 2\Omega = 0V.$$

Our i-sum eq'n becomes

$$-12A + i_1 + i_2 = 0A.$$

Solving the second eq'n, we have, for i_2,

$$i_2 = 12A - i_1.$$

Substituting for i_2 in the v-loop eq'n:

$$i_1 \cdot 12\Omega - (12A - i_1)(\underbrace{1\Omega + 3\Omega + 2\Omega}_{6\Omega}) = 0V$$

or $i_1 (12\Omega + 6\Omega) = 12A \cdot 6\Omega$

or $i_1 = 12A \cdot 6\Omega / 18\Omega = 4A$

Using an earlier eqn:

$$i_2 = 12A - i_1 = 12A - 4A = 8A$$

From earlier eqn:

$$v_0 = i_0 \cdot 3\Omega = i_2 \cdot 3\Omega = 8A \cdot 3\Omega = 24V$$

b) Power dissipated: $p = i \cdot v = i^2 R$ for R's

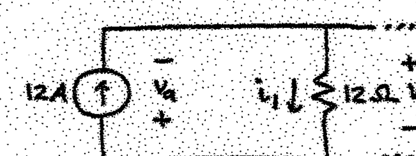
12Ω: $p = i_1^2 \cdot 12\Omega = 4A^2 \cdot 12\Omega = 192W$

1Ω: $p = i_3^2 \cdot 1\Omega = (-8A)^2 \cdot 1\Omega = 64W$

3Ω: $p = i_0^2 \cdot 3\Omega = (8A)^2 \cdot 3\Omega = 192W$

2Ω: $p = i_2^2 \cdot 2\Omega = (8A)^2 \cdot 2\Omega = 128W$

For the current source, we find the voltage drop from a v-loop on the left side.



We have $-v_a - v_1 = -v_a - i_1 \cdot 12\Omega = 0V$

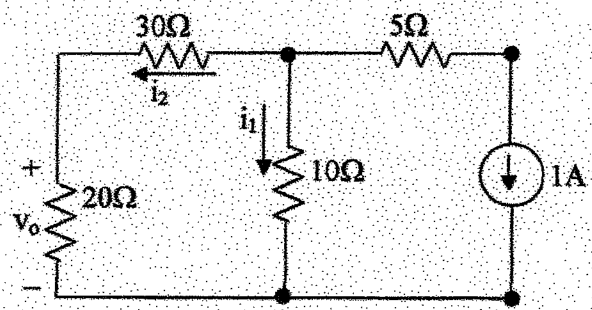
$$v_a = -i_1 \cdot 12\Omega = -4A \cdot 12\Omega = -48V$$

Power for 12A src: $p = 12A \cdot v_a = 12A(-48V)$

$$p = -576W$$

Homework #1 Examples

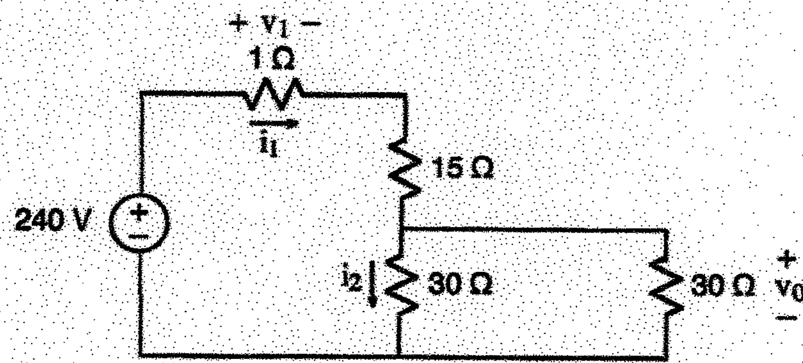
1.



(a) Calculate i_1, i_2, and v_o.
(b) Find the power dissipated for every component including the current source.

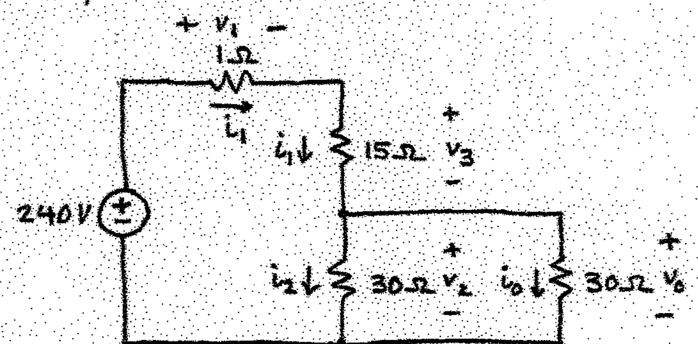
HOMEWORK #1 Solution Problem 2

Ex:



Calculate i_1, i_2, and v_0.

sol'n: First, we label R's.



Second, we write v-loop eq'ns. We write eq'ns for both inner loops.

$$+240V - v_1 - v_3 - v_2 = 0V$$

$$+v_2 - v_o = 0V$$

Third, we write current-sum eq'ns for all but one node. For the node between R's on the right side we have

$$-i_1 + i_2 + i_o = 0A$$

Fourth, we equate currents for components in series. We have already done this, however, by using i_1 for both the 1Ω and 15Ω R's.

Fifth, we write Ohm's law eq'ns for every R:

$$v_1 = i_1 \cdot 1\Omega$$

$$v_3 = i_1 \cdot 15\Omega$$

$$v_2 = i_2 \cdot 30\Omega$$

$$v_o = i_o \cdot 30\Omega$$

Now we use the Ohm's law eq'ns to substitute for v's:

$$+240V - i_1 \cdot 1\Omega - i_1 \cdot 15\Omega - i_2 \cdot 30\Omega = 0V$$

$$+ i_2 \cdot 30\Omega - i_o \cdot 30\Omega = 0V$$

Our current sum eq'n is unchanged:

$$-i_1 + i_2 + i_o = 0A$$

From the 2nd of the above 3 eq'ns we have

$$i_o = i_2$$

Using this in the 3rd eq'n gives

$$i_1 = 2i_2$$

Using this in the 1st eq'n gives

$$+240V - 2i_2(1\Omega + 15\Omega) - i_2 \cdot 30\Omega = 0V$$

or $$i_2(2 \cdot 16\Omega + 30\Omega) = 240V$$

or $$i_2 = \frac{240V}{62\Omega} = \frac{120}{31} A \approx 3.87A$$

$$i_1 = 2i_2 = \frac{240}{31} A \approx 7.74A$$

$$v_o = i_o \cdot 30\Omega = i_2 \cdot 30\Omega = \frac{120}{31}(30) V$$

or $$v_o = \frac{3600}{31} V \approx 116 V$$

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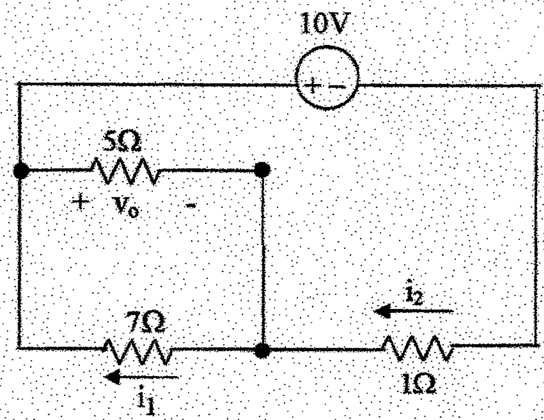
2.



Calculate i_1, i_2, and v_o.

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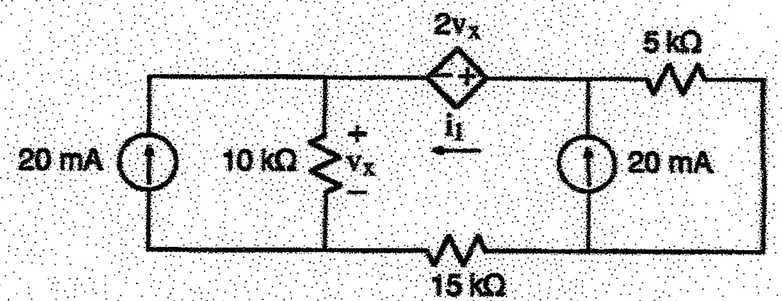
HOMEWORK #1 Solution Prob 3.

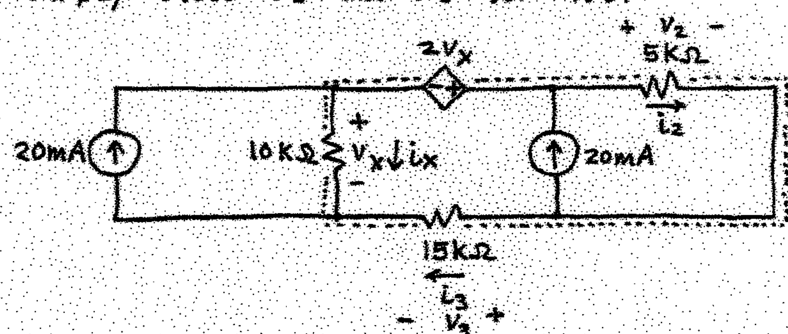
Ex:



Find v_x, i_1, and the power dissipated by the dependent source.

sol'n: We find i_1, the current for the dependent v-src <u>after</u> we solve the circuit.

First, label i's and v's for R's:



Second, write v-loop eq'ns for loops not containg current src's. There is only one such loop, indicated by the dotted line:

$$+v_x + 2v_x - v_2 - v_3 = 0V$$

Third, write current-sum eq'ns for nodes (unless nodes are connected only by v src's). We don't use the nodes on top since they are connected by only the $2v_x$ source.

For the node on the bottom, left of center, we have

$$+20mA - i_x - i_3 = 0A \qquad (1)$$

For the node on the bottom, right of center, we have

$$i_3 + 20mA - i_2 = 0A \qquad (2)$$

Fourth, we look components in series carrying the same current. Here, we lack any such components.

Fifth, we write Ohm's Law eq'ns for all the R's:

$$v_x = i_x \cdot 10k\Omega$$

$$v_2 = i_2 \cdot 5k\Omega$$

$$v_3 = i_3 \cdot 15k\Omega$$

Now substitute Ohm's Law for v's in v-loop eq'n:

$$i_x \cdot 10k\Omega + 2\, i_x \cdot 10k\Omega - i_2 \cdot 5k\Omega - i_3 \cdot 15k\Omega = 0V \qquad (3)$$

Solve one the three eq'ns (1-3) for a current:

$$i_3 = i_2 - 20mA$$

Substitute this in eq'ns (1) and (2):

$$20\,mA - i_x - (i_2 - 20\,mA) = 0A$$

$$i_x(10k\Omega + 20k\Omega) - i_2(5k\Omega) - (i_2 - 20mA)15k\Omega = 0V$$

Solving the first of these eq'ns for i_2 gives

$$i_2 = 40mA - i_x$$

Using this in the second of the two eq'ns gives:

$$i_x(30k\Omega) - (40mA - i_x)(5k\Omega + 15k\Omega) = -20mA \cdot 15k\Omega$$

or $i_x(30k\Omega + 20k\Omega) = 40mA(20k\Omega) - 20mA \cdot 15k\Omega$

or $i_x(50k\Omega) = 500V$

or $i_x = \dfrac{500V}{50k\Omega} = 10\,mA$

Now we can find i_1 from a current sum at the node on top to the left of center:

$$-20mA + i_x - i_1 = 0mA$$

or $i_1 = -20mA + i_x = -20mA + 10mA = -10mA$

The power dissipated by the dependent source is

$$p = i_1 \cdot 2v_x = -10mA \cdot 2 \cdot \overbrace{10\cancel{m}A \cdot 10\cancel{k}\Omega}^{v_x \text{ from Ohm's Law}}$$

or $p = -2\,W$

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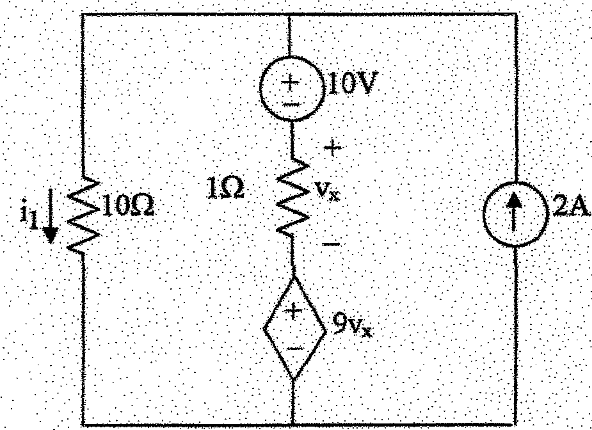
3.



Find v_x, i_1, and the power dissipated by the dependent source.

Op-Amp Procedure:
(Linear Operation)

1. a. Replace Op-Amp with V_o source
   b. 0V across +, - inputs
   c. $i=0$ into +, - inputs

2. Follow procedure to solve for I & V (pg. 7a) by using the V-loop that includes the 0V drop across +, - inputs
   - 2 different loops can be obtained
   - make sure to use V_o in 1 loop

Op Amp Example

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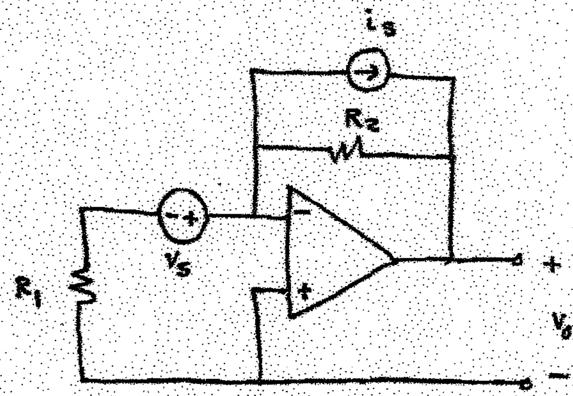
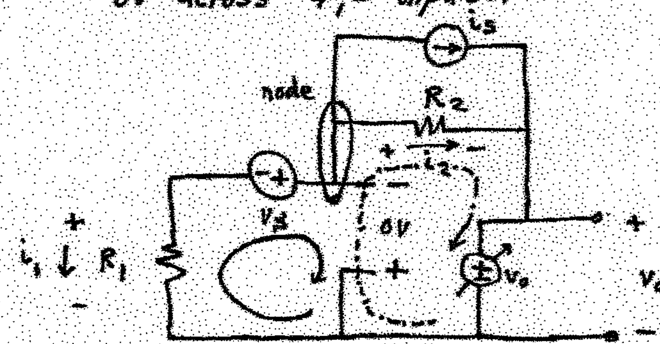


Derive expression for v_o in terms of not more than v_s, i_s, R_1, and R_2.

sol'n: Replace op-amp with v_o source and assume 0V across +,− inputs.



Use v-loop on left that includes 0V drop across +,− inputs.

$$+i_1 R_1 + v_s + 0V = 0V$$

$$\text{or} \quad i_1 = -\frac{v_s}{R_1}$$

Use v-loop on right that includes 0V drop across +,− inputs.

$$-0V - i_2 R_2 - v_o = 0V$$

$$\text{or} \quad v_o = -i_2 R_2$$

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OpAmp sol'n: cont.

Use current sum at node next to − input.

$$i_1 + i_2 + i_s = 0\,A$$

or $i_2 = -(i_1 + i_s) = -\left(\frac{-v_s}{R_1} + i_s\right) = \frac{v_s}{R_1} - i_s$

Use this in 2nd v-loop eq'n.

$$v_o = -i_2 R_2 = -\left(\frac{v_s}{R_1} - i_s\right) R_2$$

$$\boxed{v_o = \left(i_s - \frac{v_s}{R_1}\right) R_2}$$

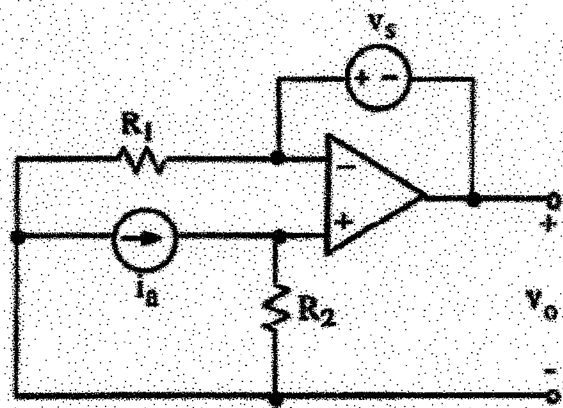
Consistency check:

If $R_2 = 0$ then 2nd v-loop implies $v_o = 0V$. $v_o = \left(i_s - \frac{v_s}{R_1}\right) \cdot 0 = 0V$ ✓

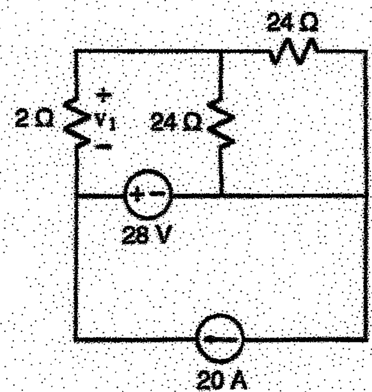
If $i_s = 0A$ and $v_s = 0V$, we should get $v_o = 0V$. $v_o = \left(0 - \frac{0}{R_1}\right) R_2 = 0V$ ✓

Op Amp Examples

5. The op-amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for v_o in terms of not more than v_s, i_a, R_1, and R_2.



Ex:



Calculate v_1.

soln: We can use a voltage divider consisting of the 28V src, the 2Ω Resistor, and the two 24Ω resistors in parallel:

$$v_1 = -28V \cdot \frac{2\Omega}{2\Omega + 24\Omega \| 24\Omega}$$

$$= -28V \cdot \frac{2\Omega}{2\Omega + 24\Omega \cdot 1\|1} \qquad 1\|1 = \frac{1\cdot 1}{1+1} = \frac{1}{2}$$

$$= -28V \cdot \frac{2\Omega}{2\Omega + 12\Omega}$$

$$v_1 = -4V$$

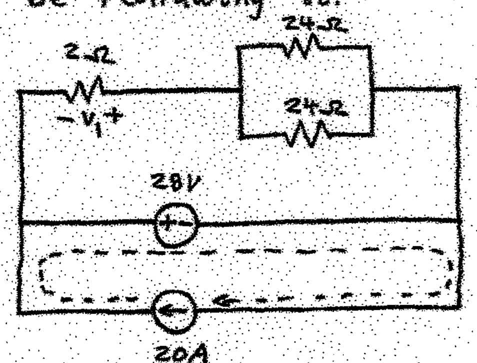
Note: We have a minus sign whenever the + sign of the resistor voltage measurement is on the side away from the + sign of the v src.

Note: We have a voltage divider when the following conditions are met:

i) The voltage across two or more R's in series is known.

ii) The current thru the R's in series is the same.

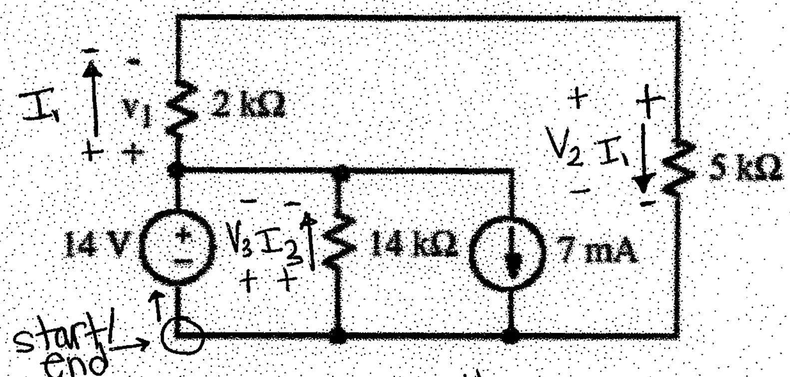
Note: We can verify that we have a v-divider in this circuit be redrawing it.



The 20A affects the current in the 28V source, but we still have 28V across the R's. The 20A just circulates in the bottom half of the circuit.

HW #2 Examples

1. Calculate v_1.



- Label all currents and voltages
- All voltage loops

$+14 + V_3 = 0 \Rightarrow V_3 = -14$

(1) $+14 - V_1 - V_2 = 0$

$-V_3 - V_1 - V_2 = 0 \Rightarrow +14 - V_1 - V_2 = 0$ (same)

- All current summations: none → Vsource in branch for all nodes

- Ohm's laws:

(2) $V_1 = I_1(2k)$

(3) $V_2 = I_1(5k)$ (one eq., 1 unknown)

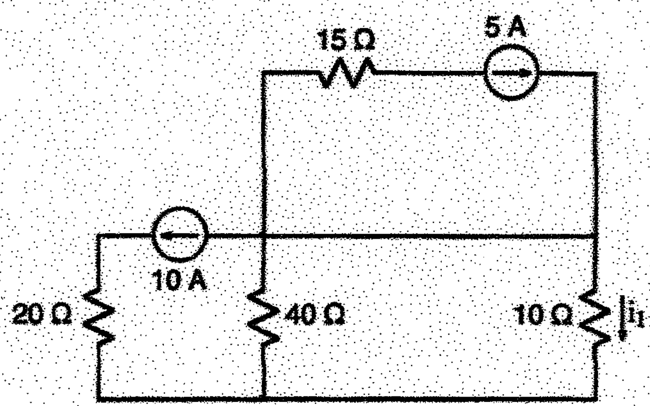
put (2), (3) into (1) $\Rightarrow +14 - I_1(2k) - I_1(5k) = 0$

$I_1(7k) = 14$

$I_1 = \underline{\underline{2A}}$

$V_1 = I_1(2k) = 2A(2k) = \boxed{4V}$

Ex:



Calculate i_1.

sol'n: We have a current divider consisting of the 10A src and the 40Ω and 10Ω R's in parallel.

To verify that we have a current divider, we observe that the circuit satisfies the following conditions:

i) The 10A is the total current flowing into one end of the 40Ω and 10Ω resistors, and

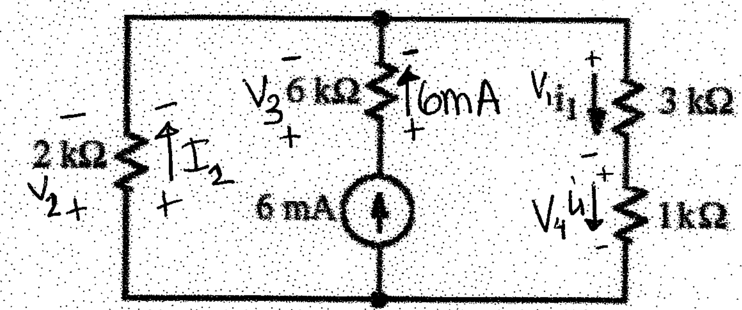
ii) The opposite ends of the 40Ω and 10Ω R's are connected so that the v-drop across the 10Ω and 40Ω resistors is the same.

Using the current-divider formula (with a minus sign because i_1 is measured in a direction opposite to the 10A src:

$$i_1 = -10A \cdot \frac{40\Omega}{40\Omega + 10\Omega} = -8A$$

HW #2 Examples

2. Calculate i_1.



- Label all I's & V's
- All V-loops: $-V_2 - V_1 - V_4 = 0$
- All I sums: (top node) $\Rightarrow$ $+I_2 + 6m - i_1 = 0$

  (bottom node) $\Rightarrow$ $-I_2 - 6m + i_1 = 0$ (same)

- Ohm's Laws:

  $V_2 = I_2(2K)$

  $V_3 = 6m(6K)$

  $V_1 = i_1(3K)$

  $V_4 = i_1(1K)$

- plug Ohm's Laws into V-loop $\Rightarrow$

  (1) $-I_2(2K) - i_1(3K) - i_1(1K) = 0$

  (2) $I_2 + 6m - i_1 = 0$

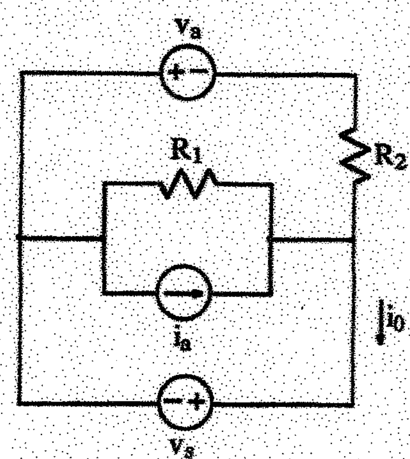
  Solve (2): $I_2 = -6m + i_1$

  plug into (1): $+6m(2K) - i_1(2K) - i_1(4K) = 0$

  $i_1(6K) = 12 \Rightarrow i_1 = 12/6K = 2mA$

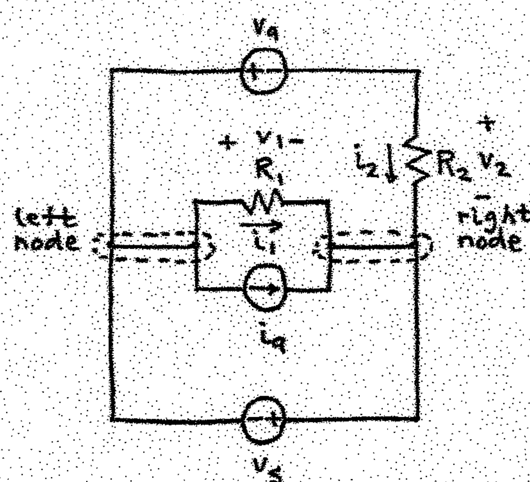
  $\boxed{i_1 = 2mA}$

EX:



Derive an expression for i_0. The expression must not contain more than the circuit parameters v_a, v_s, i_a, R_1, and R_2.

sol'n: Label R's first.



v-loops: (v_a and R_1, R_2; v_s and R_1)

$$-v_a - v_2 + v_1 = 0V$$

$$-v_1 - v_s = 0V$$

We observe that we can solve these 2 eq'ns in 2 unknowns without proceeding further.

$v_1 = -v_s$ from 2nd eq'n

$v_2 = -v_a + v_1 = -v_a - v_s$ from 1st eq'n

Note: If we try to write current-sum eq'ns, we find that the left node and right node and connected by only v-src v_s. Thus, we should not write i-sum eq'ns. (And we don't need them!)

Note: We also have no components in series that carry the same current, (except v-src v_a and R_2).

We now use Ohm's law:

$v_1 = i_1 R_1$

$v_2 = i_2 R_2$

Using v-eq'ns:

$$i_1 = \frac{v_1}{R_1} = \frac{-v_s}{R_1}$$

$$i_2 = \frac{v_2}{R_2} = \frac{-v_a - v_s}{R_2}$$