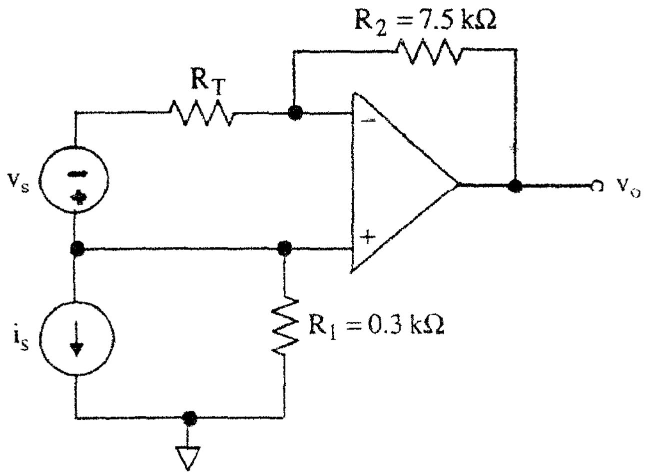
Homework #9 Solution

1. (75 points)



Rail voltage = ± 15 V

Design an electronic thermometer using the circuit diagram shown above. The voltage v_0 is used to indicate temperature. Use a thermister with a resistance described by

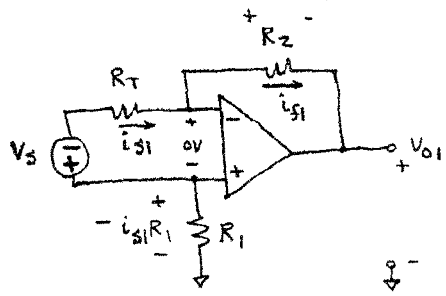
$$R_T = R_0 e^{\beta\left(\frac{1}{T} - \frac{1}{300}\right)}$$

where $R_0 = 2.625\ k\Omega$, $\beta = 1200°K$, and T is temperature in °K.

Pts		
30	a.	Derive a symbolic expression for v_0. The expression must contain not more than the parameters i_s, V_s, R_1, R_2, and R_T. **Hint: Use superposition.**
10	b.	Calculate the numerical values of R_T (273°K) and R_T (373°K).
15	c.	Determine a value for v_s such that $v_0(T = 373°K) - v_0(T = 273°K) = 1V$
10	d.	Using your answer to (c), determine a value of i_s such that $v_0 (T = 273°K) = 0\ V$.
10	e.	Using the component values you chose above, calculate v_0 when T = 323°K. Make a rough sketch of v_0 vs. T on the basis of the values when T = 273°K, 323°K, and 373°K. On the same axes, sketch the ideal linear response.

Sol'n: 1. a) Use superposition

Case I: v_s on, i_s off = open



Negative feedback ⇒ 0V across + and - terminals.

From v loop around v_s, R_T, and +- terminals,

we have $i_{s1} = \dfrac{-v_s}{R_T}$.

From v loop around R_1, +- terminals, R_2, and v_o,

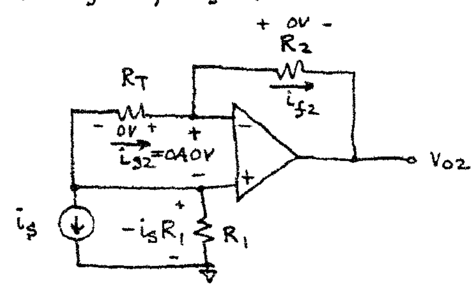
we have $-i_{s1}R_1 + 0V - i_{f1}R_2 - v_{o1} = 0V$

Now use $i_{s1} = i_{f1}$ and solve for v_{o1}:

$$- -\frac{v_s}{R_T} R_1 + 0V - -\frac{v_s}{R_T}R_2 - v_{o1} = 0V$$

$$v_{o1} = v_s \frac{R_1 + R_2}{R_T}$$

Case II: i_s on, v_s off = wire



We 0V across R_T ⇒ $i_s = 0$.

Since $i_{s2} = i_{f2} = 0A$, we have $v_{o2} = v_- = v_+$

$v_1 = -i_sR_1 \Rightarrow v_{o2} = -i_sR_1$

Sum $\boxed{v_o = v_{o1} + v_{o2} = v_s \dfrac{R_1 + R_2}{R_T} - i_sR_1}$

soln: 1. b) $R_T(273°K) = 2.625k\Omega\, e^{1200°K\left(\frac{1}{273°K} - \frac{1}{300°K}\right)}$

$\boxed{R_T(273°K) \doteq 3.9k\Omega}$

$R_T(373°K) = 2.625k\Omega\, e^{1200°K\left(\frac{1}{373°K} - \frac{1}{300°K}\right)}$

$\boxed{R_T(373°K) \doteq 1.2k\Omega}$

c) $1V = V_o(373°K) - V_o(273°K) = V_s \frac{(R_1 + R_2)}{R_T(373°K)} - \cancel{i_s R_1}$

$- \left(V_s \frac{R_1 + R_2}{R_T(273°K)} - \cancel{i_s R_1}\right)$

$= V_s (R_1 + R_2)\left(\frac{1}{R_T(373°K)} - \frac{1}{R_T(273°K)}\right)$

$= V_s \underbrace{(0.3k\Omega + 7.5k\Omega)}_{7.8k\Omega}\left(\frac{1}{1.2k\Omega} - \frac{1}{3.9k\Omega}\right)$

$V_s = \frac{1.2k\Omega \| (-3.9k\Omega)}{7.8k\Omega} V = \frac{\cancel{0.3k\Omega}}{\cancel{0.3k\Omega}} \cdot \frac{4 \| -13}{26} V = \frac{-4 \cdot \cancel{13}^{\,2}}{(4-13)\cancel{26}} V$

$\boxed{V_s = \frac{2}{9} V}$

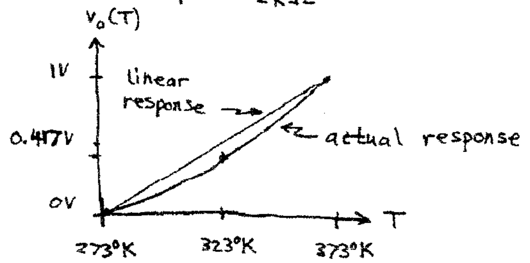
d) $0V = V_o(273°K) = V_s \frac{R_1 + R_2}{R_T(273°K)} - i_s R_1 = \frac{2}{9} V \frac{\cancel{7.8}^{\,2} k\Omega}{\cancel{3.9}k\Omega} - i_s\, 0.3k\Omega$

$i_s\, 0.3k\Omega = \frac{4}{9} V \Rightarrow i_s = \frac{4}{9} \frac{1V}{0.3k\Omega} = \frac{4}{2.7} mA$

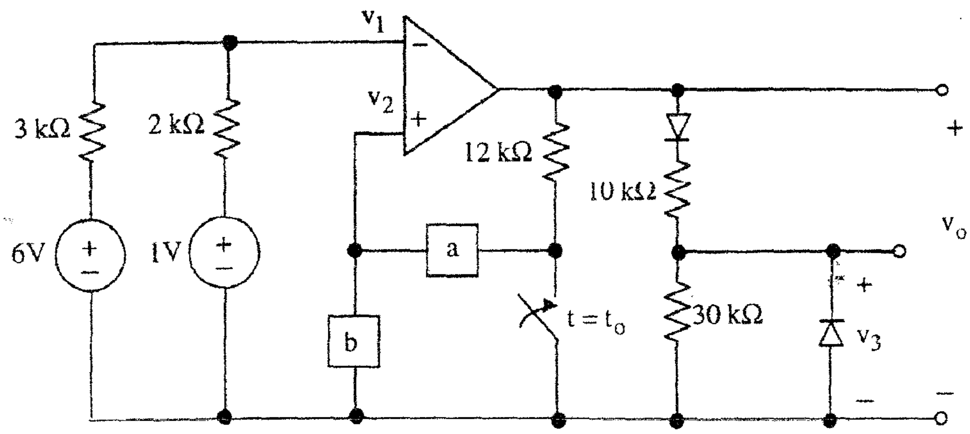
$\boxed{i_s \doteq 1.48\, mA \approx 1.5\, mA}$

e) $R_T(323°K) = 2.625k\Omega\, e^{1200°K\left(\frac{1}{323°K} - \frac{1}{300°K}\right)} = 1.97k\Omega \approx 2k\Omega$

$V_o(323°K) \approx \frac{2}{9} V \frac{7.8k\Omega}{2k\Omega} - 1.5mA\; 0.3k\Omega = 0.417V$

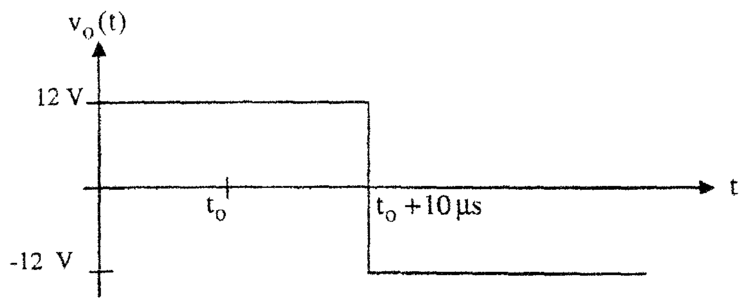


2. (65 points)



Rail voltages = ±12 V

After being open for a long time, the switch closes at $t = t_o$.



Pts		
35	a.	Choose either an R or L to go in box a and either an R or L to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value.
5	b.	Sketch $v_1(t)$, showing numerical values appropriately.
15	c.	Sketch $v_2(t)$, showing numerical values appropriately.
10	d.	Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 10\ \mu s$, and for $t_o + 10\ \mu s < t$. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

Explain your work carefully.

soln: 2.a) Ignore diode and resistor network on output (since it doesn't affect V_o).

For $t < t_o$, $V_o = +12V \Rightarrow V_2 > V_1$.

Calculate V_1 using node-v method: $\frac{V_1 - 6V}{3k\Omega} + \frac{V_1 - 1V}{2k\Omega} = 0A$

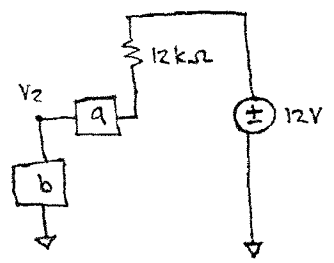
$$V_1\left(\frac{1}{3k\Omega} + \frac{1}{2k\Omega}\right) = \frac{6V}{3k\Omega} + \frac{1V}{2k\Omega}$$

mult both sides by $6k\Omega$

$$V_1\ (2+3) = 6V \cdot 2 + 1V \cdot 3 = 15V, \quad V_1 = \frac{15V}{5} = 3V$$

So we need $\underline{\underline{V_2 > 3V.}}$

Circuit model: $V_o = 12V$, switch open



If we have an L, it will be equivalent to a wire.

Consider possibilities:

case I: $a = L$ and $b = l$ $\quad L = \text{wire}$

$V_2 = 0V$ from v divider

Doesn't work.

case II: $a = R$ and $b = L = \text{wire}$

$V_2 = 0V$ from v divider

Doesn't work.

case III: $a = R_1$ and $b = R_2$

We can choose R_1 and R_2 to achieve $V_2 > 3V$, but we cannot get a delay in V_o dropping from $+12V$ to $-12V$.

case IV: $a = L$ and $b = R$

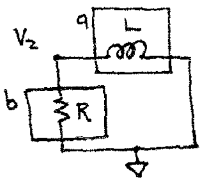
Since $L = \text{wire}$ and we can pick R, we can achieve $V_2 > 3V$. When switch moves, the L continues to carry same current initially. Thus, $V_2 > V_1$ is sustained for delay. Should work.

soln: 2.a) cont. When switch closes, we have RL circuit that determines V_2. Time constant $\tau = L/R$. Output V_o drops when V_2 drops below 3V.

As $t \to \infty$, the L in 'a' acts like a wire and the switch is closed $\Rightarrow V_2(t\to\infty) = 0V$

Without additional constraints, we may choose any V_2 between 3V and 12V. One choice is $V_2(0^-) = 6V$. Using v-divider of 12kΩ and 'b', $\boxed{b = 12k\Omega}$.

We want $V_2(t = 10\mu s) = 3V$ so V_o drops at time $t_o + 10\mu s$. (Assume $t_o = 0s$)

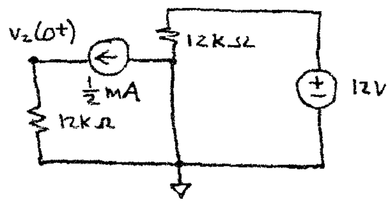


$$v_2(t>0) = v_2(t\to\infty) + [V_2(0^+) - V_2(t\to\infty)]e^{-t/\tau}$$
$$" = 0V + [V_2(0^+) - 0V]e^{-t/\tau}$$
$$" = V_2(0^+)e^{-t/\tau},$$ Now find $V_2(0^+)$.

Consider $t = 0^-$: L = wire R = 12kΩ

$$i_L(0^-) = \frac{12V}{12k\Omega + 12k\Omega} = \frac{1}{2}mA$$

$t = 0^+$: L = i src where $i_L(0^+) = i_L(0^-) = \frac{1}{2}mA$



$$V_2(0^+) = \frac{1}{2}mA \cdot 12k\Omega$$
$$V_2(0^+) = 6V$$

$$v_2(t>0) = 6V e^{-t/\tau}$$

We want $v_2(10\mu s) = 3V = 6V e^{-10\mu s/\tau}$

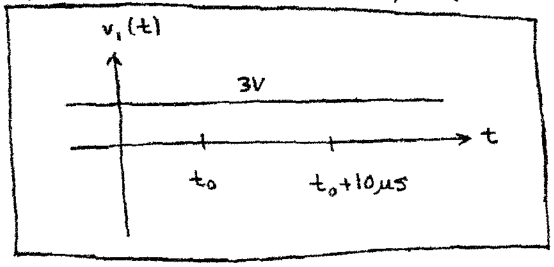
$$\frac{3V}{6V} = \frac{1}{2} = e^{-10\mu s/\tau}, \quad \ln\frac{1}{2} = -10\mu s/\tau$$

$$\tau = \frac{-10\mu s}{\ln\frac{1}{2}} = \frac{10\mu s}{\ln 2} = 14.4\mu s = \frac{L}{R} = \frac{L}{12k\Omega}$$

$$L = 14.4\mu s \cdot 12k\Omega = 173\ mH$$

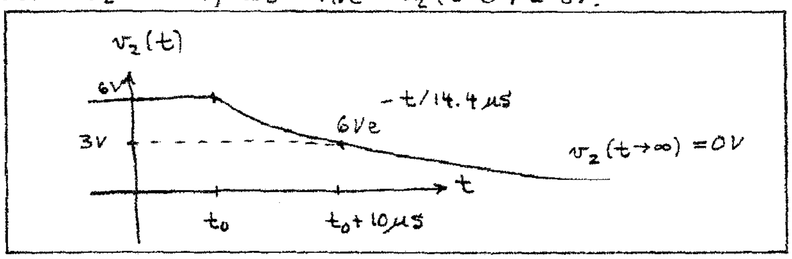
Summary: $\boxed{\begin{array}{l} a = L = 173\ mH \\ b = R = 12k\Omega \end{array}}$

soln: 2.b) As shown in soln for (a), $v_1(t) = 3V$. It never changes.

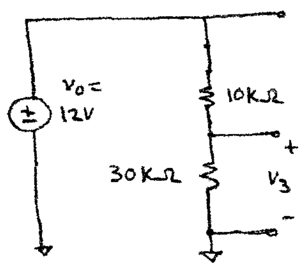


c) From soln to (a), we have $v_2(t>0) = 6V e^{-t/14.4\mu s}$

For $v_2(t<0)$, we have $v_2(t<0) = 6V$.

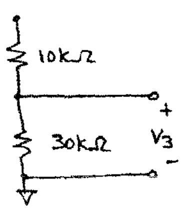


d) When $v_o > 0V$, top diode = wire, bottom diode = open.

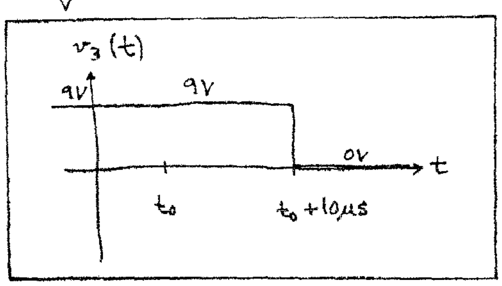


$$V_3 = 12V \cdot \frac{30k\Omega}{30k\Omega + 10k\Omega} = 9V$$

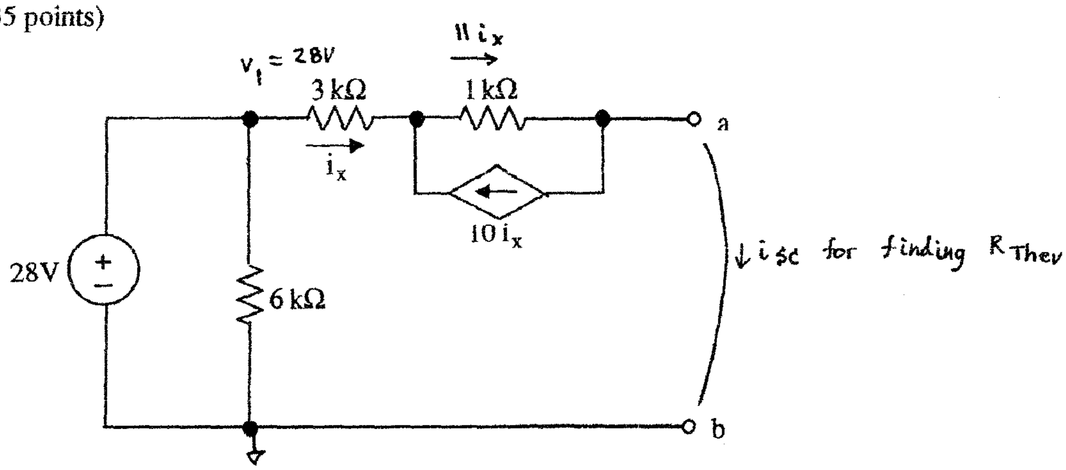
When $v_o < 0V$, top diode = open, bottom diode doesn't matter since no current



$V_3 = 0V$ since no current flowing



3. (35 points)



<u>Pts</u>

25 a. Find the Thevenin equivalent of the above circuit relative to terminals a and b.

5 b. If we attach R_L to terminals a and b, find the value of R_L that will absorb maximum power.

5 c. Calculate the value of that maximum power absorbed by R_L.

soln: a) The $6k\Omega$ resistor is across the 28V source, so it may be ignored.

For V_{Thev} we use $V_{a,b}$ with no load. Since no current flows out of the 'a' terminal, $i_x = 0$.

$\therefore$ $10 i_x = 0A$ and v drop across $3k\Omega$ and $1k\Omega$ is zero.

$\therefore$ $V_{a,b} = 28V$ from v src $\therefore$ $\boxed{V_{Thev} = 28V}$

Now find i_{sc} flowing in wire connected from a to b.

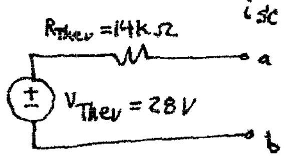
From current sum at node on left end of $1k\Omega$, we have current $11 i_x$ flowing in $1k\Omega$ resistor.

Using v drops for $3k\Omega$ and $1k\Omega$, we must have

$i_x \cdot 3k\Omega + 11 i_x \cdot 1k\Omega = 28V$ or $14k\Omega \cdot i_x = 28V$

or $i_x = 2\ mA$. Since $i_{sc} = i_x$ and $R_{Th} = \frac{V_{Th}}{i_{sc}}$,

$$\boxed{R_{Th} = \frac{28V}{2mA} = 14k\Omega}$$

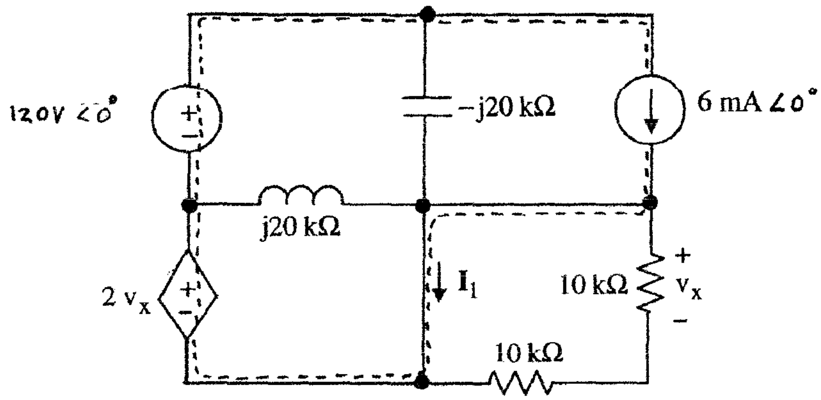


sol'n: 3. b) Max pwr when $\boxed{R_L = R_{Thev} = 14\,k\Omega}$

3. c) Max pwr $= \dfrac{V_{Thev}^2}{4R_{Thev}} = \dfrac{(28V)^2}{4 \cdot 14\,k\Omega} = 7(2)W$

$$\boxed{\text{Max pwr} = 14\ W}$$

4. (25 points)



Pts		
20	a.	A frequency-domain circuit is shown above. Write the value of phasor $\mathbf{I}_1$ in polar form.
5	b.	Given $\omega = \pi$ rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

Sol'n: a) Since the two 10kΩ resistors are shorted by wires.

∴ There is no v drop across the 10kΩ resistors,

and $v_x = 0V$.

Thus, the $2v_x$ dependent source = 0V = wire

Superposition Case I: 6mA on, 120V off = wire.

It follows that all of the 6mA from the independent current source flows in the wires (shown as dashed lines above).

$\therefore\ \mathbf{I}_{11} = 6\text{ mA}\angle 0°$

Case II: 120V on, 6mA off = open circuit

we observe that the −j20kΩ is directly across the 120V source, given the wires shown as dashed lines.

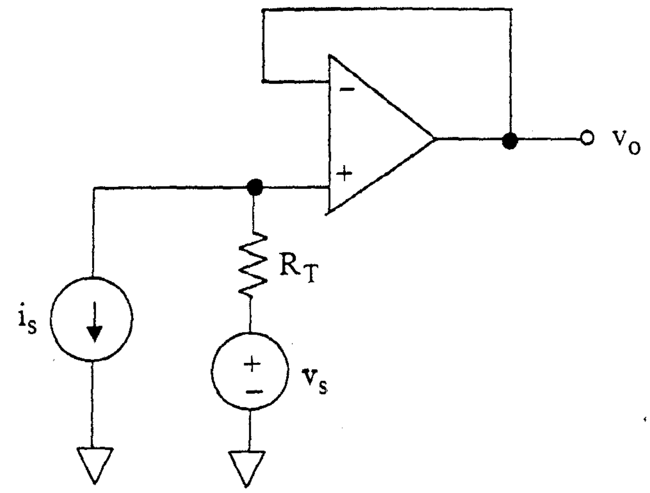
$$\therefore\ \mathbf{I}_{12} = \frac{120V\angle 0°}{-j\,20k\Omega} = j\,6mA = 6mA\angle 90°$$

Thus, $\mathbf{I}_1 = \mathbf{I}_{11} + \mathbf{I}_{12} = 6\text{ mA}\cdot(1+j)$

or $\boxed{\mathbf{I}_1 = \sqrt{2}\cdot 6\text{ mA}\angle 45°}$

b) $\boxed{i_1(t) = \sqrt{2}\cdot 6\text{ mA}\cos(\pi t + 45°)}$

1. (75 points)



Rail voltage = ± 15 V

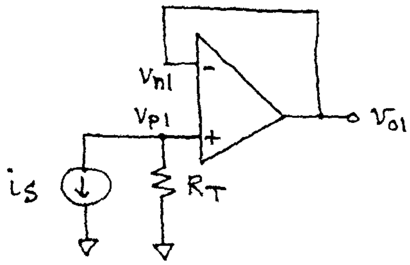
Design an electronic thermometer using the circuit diagram shown above. The voltage v_o is used to indicate temperature. Use a thermister with a resistance described by

$$R_T = R_o e^{\beta\left(\frac{1}{T}-\frac{1}{300}\right)}$$

where $R_o = 2.8\ \text{k}\Omega$, $\beta = 1300^\circ\text{K}$, and T is temperature in °K.

Pts		
30	a.	Derive a symbolic expression for v_o. The expression must contain not more than the parameters i_s, V_s, and R_T. **Hint: Use superposition.**
10	b.	Calculate the numerical values of R_T (273°K) and R_T (373°K).
15	c.	Determine a value for i_s such that $v_o(T = 373^\circ\text{K}) - v_o(T = 273^\circ\text{K}) = 1\text{V}$
10	d.	Using your answer to (c), find the numerical value of v_s such that $v_o\ (T = 273°\text{K}) = 0\ \text{V}$.
10	e.	Using the component values you chose above, calculate v_o when T = 323°K. Make a rough sketch of v_o vs. T on the basis of the values when T = 273°K, 323°K, and 373°K. On the same axes, sketch the ideal linear response.

sol'n: 1.a) Superposition case I: i_s on, v_s off = wire

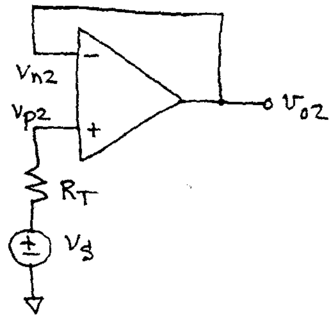


$v_{p1} = -i_s R_T$

$v_{n1} = v_{p1} = -i_s R_T$

$v_{o1} = v_{n1} = -i_s R_T$

case II: i_s off = open, v_s on



$v_{p2} = v_s$ (no current in R_T so no V-drop)

$v_{n2} = v_{p2} = v_s$

$v_{o2} = v_{n2} = v_s$

$$v_o = v_{o1} + v_{o2}$$

$$\boxed{v_o = -i_s R_T + v_s}$$

b) $R_T(273\,°K) = 2.8\,k\Omega\, e^{1300\,°K\left(\frac{1}{273\,°K} - \frac{1}{300\,°K}\right)}$

$\boxed{R_T(273\,°K) = 4.3\,k\Omega}$

$R_T(373\,°K) = 2.8\,k\Omega\, e^{1300\,°K\left(\frac{1}{373\,°K} - \frac{1}{300\,°K}\right)}$

$\boxed{R_T(373\,°K) = 1.2\,k\Omega}$

c) Change in v_o: $\Delta v_o = -i_s\,\Delta R_T$ $\Delta \equiv$ "change"
(for $\Delta T = 100\,°K$) $= 1V$ (desired) (v_s constant does not change with T)

$\Delta R_T = 1.2\,k\Omega - 4.3\,k\Omega$

$\Delta R_T = -3.1\,k\Omega$

$\therefore\ i_s = \frac{-\Delta v_o}{\Delta R_T} = \frac{-1V}{-3.1\,k\Omega} = 0.323\,mA$

$$\boxed{i_s = 0.323\,mA}$$

d) $v_o(273\,°K) = 0V = \underbrace{-0.323\,mA}_{-i_s} \cdot \underbrace{4.3\,k\Omega}_{R_T} + v_s$

$\therefore\ v_s = 0.323\,mA \cdot 4.3\,k\Omega$

$$\boxed{v_s = 1.39\,V \text{ or } 1.4\,V}$$

sol'n: 1. e)

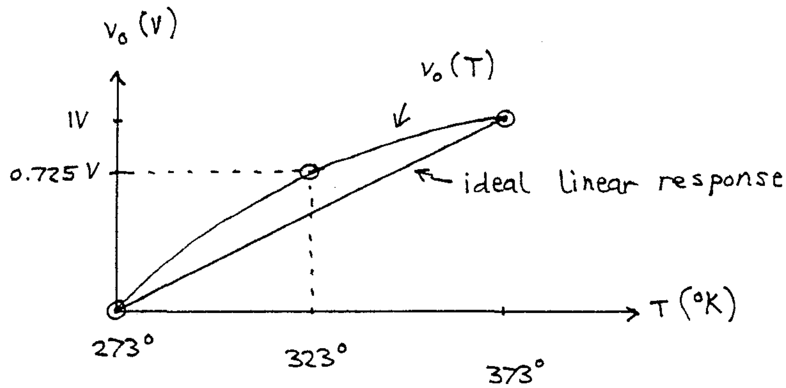
$$v_o(323^\circ K) = -i_s R_T(323^\circ K) + v_s$$

$$R_T(323^\circ K) = 2.8\,k\Omega\, e^{1300^\circ K\left(\frac{1}{323^\circ K} - \frac{1}{300^\circ K}\right)}$$

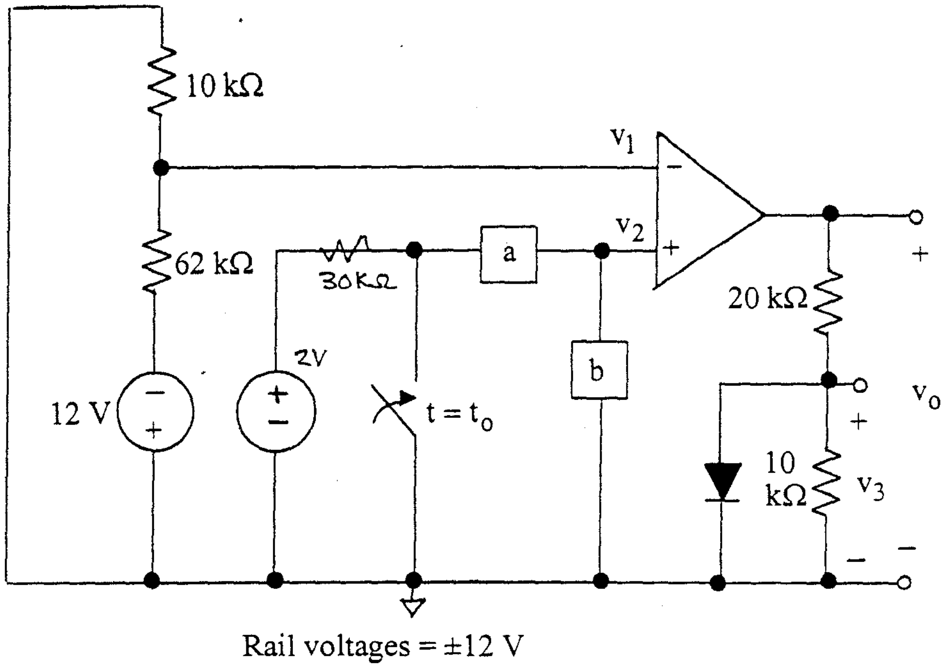
$$= 2.06\,k\Omega$$

$$v_o(323^\circ K) = -0.323\,mA \;\; 2.06\,k\Omega + 1.39\,V$$

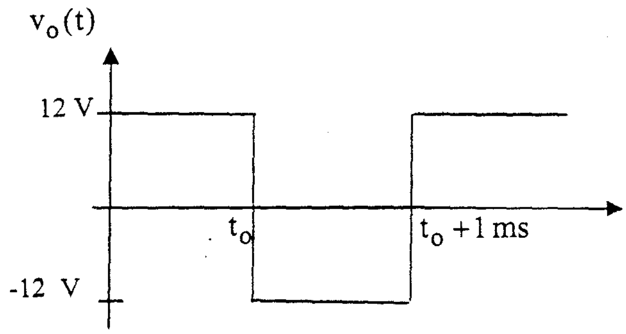
$$\boxed{v_o(323^\circ K) = 0.725\,V}$$



2. (65 points)



After being open for a long time, the switch closes at $t = t_o$.



Pts		
35	a.	Choose either an R or C to go in box a and either an R or C to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value.
5	b.	Sketch $v_1(t)$, showing numerical values appropriately.
15	c.	Sketch $v_2(t)$, showing numerical values appropriately.
10	d.	Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 1$ ms, and for $t < t_o + 1$ ms. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

sol'n: 2. a) At $t = t_o^-$: $v_1 = -12V \dfrac{10k\Omega}{10k\Omega + 62k\Omega}$ V-divider

$v_1 = -\frac{5}{3}V$ Actually, $v_1 = -\frac{5}{3}V$ for all t

$v_o = +12V \Rightarrow v_2 > v_1 = -\frac{5}{3}V$

Consider $a = R$, $b = R$: v_o would never switch because v_2 would be 0V after switch closed, and $v_2 > v_1$ for all time

Will not work.

Consider $a = C$, $b = C$: C's will charge to $V_{tot} = 2V$.

Part of 2V across a, part across b.

when switch closes, the C's instantly charge to $V_{tot} = 2V$.

Then their voltages remain fixed.

$\therefore v_o$ would not switch after 1 ms.

Will not work.

Consider $a = R$, $b = C$: C will charge to 2V. $v_2 = 2V$

When switch closes, v_2 will start at v_2 and charge toward 0V.

$v_2 > v_1$ for all time.

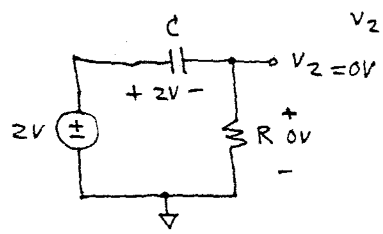
v_o never switches.

Will not work.

Consider $\boxed{a = C, \ b = R}$: C will charge to 2V at t_o^-

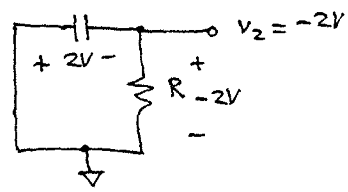
$v_2 > v_1$ so $v_o = +12V$ at t_o^- ✓

t_o^- :



when switch closes, v_c will stay at 2V for $t = t_o^+$:

t_o^+ :



Thus, v_2 drops to $-2V$.

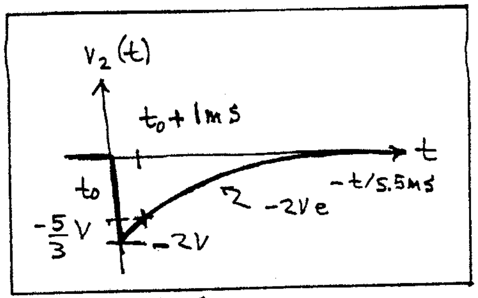
Then $v_2 < v_1$ and v_o drops to $-12V$. ✓

$t > t_o$: C charges to 0V and v_2 climbs toward 0V.

We want $v_2 = v_1 = -\frac{5}{3}V$ at $t_o + 1ms$. ✓

Works

soln: 2.a) (cont.)



soln: 2.c) ↗

Let $t_o = 0$

$v_2(t) = -2V\, e^{-t/RC}$

$v_2(1ms) = -2V e^{-1ms/RC} = -\frac{5}{3}V$

$e^{-1ms/RC} = \frac{5}{6}$

$-1ms/RC = \ln \frac{5}{6}$

$RC = \frac{-1ms}{\ln \frac{5}{6}} = 5.48\,ms$

$RC \approx 5.5\,ms$

Let $\boxed{C = 1\mu F, \quad R = 5.5k\Omega \text{ or } 5.6k\Omega \text{ (standard value)}}$

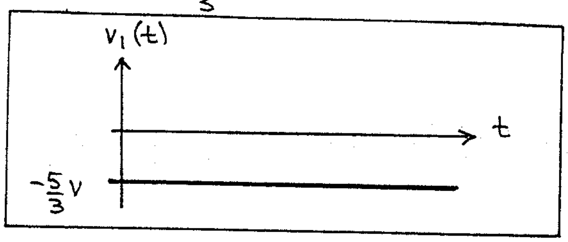
Any $RC = 5.5\,ms$ acceptable if $1pF < C < 1F$ and $1\Omega < R < 1G\Omega$.

Note: If R is $\frac{1}{8}W$ then we would really want $\max i_R^2 R < \frac{1}{8}W$.

$\max i_R^2 = \left(\frac{2V}{R}\right)^2 \Rightarrow \max i_R^2 R = \frac{4V^2}{R} < \frac{1}{8}W$.

$\therefore R > \frac{4V^2}{1/8\,W} = 32\Omega$ required.

2.b) $v_1 = -\frac{5}{3}V$ for all t

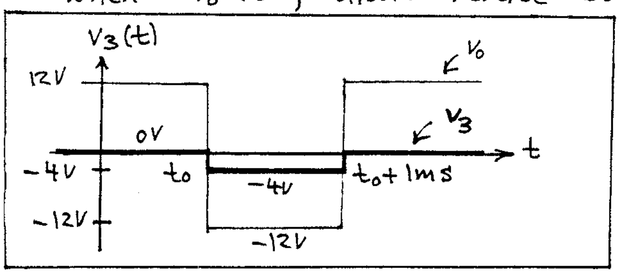


2.c) See plot of $v_2(t)$ in soln to 2.a), above.

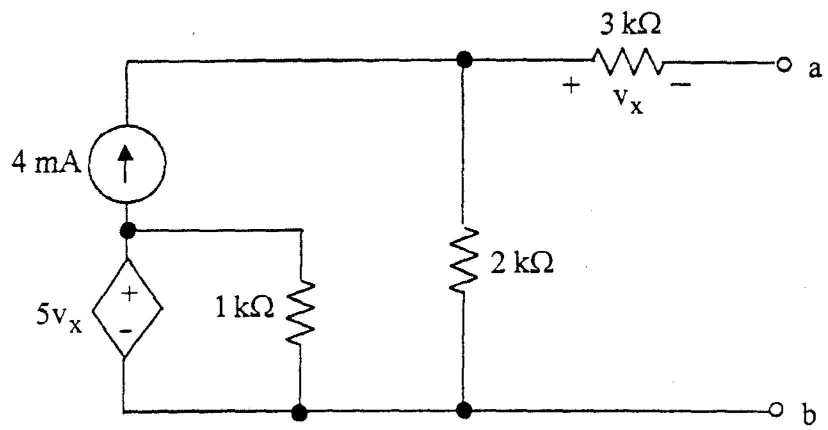
2.d) when $v_o > 0V$, diode forward biased = wire. $\therefore v_3 = 0V$

when $v_o < 0V$, diode reverse biased = open. $\therefore v_3 = v_o \frac{10k\Omega}{10k\Omega + 20k\Omega}$

$v_3 = -12V \cdot \frac{1}{3} = -4V$



3. (30 points)



Pts		
20	a.	Find the Thevenin equivalent of the above circuit relative to terminals a and b.
5	b.	If we attach R_L to terminals a and b, find the value of R_L that will absorb maximum power.
5	c.	Calculate the value of that maximum power absorbed by R_L.

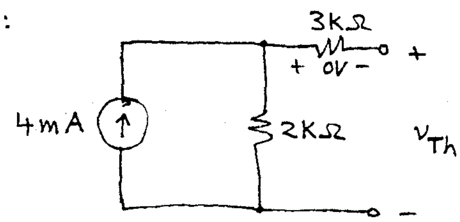
sol'n: a) $v_{Th} = v_{a,b}$ open circuit

open circuit ⇒ $v_x = 0$ since no current in 3kΩ.

∴ $5v_x$ src = 0V = wire

1kΩ across v_x shorted so can be ignored.

So we have:



$V_{Th} = 4mA \cdot 2k\Omega = 8V$

$\boxed{v_{Th} = 8V}$

$R_{Th} = \dfrac{v_{Th}}{i_{sc}}$ If we short a,b we have current divider. $i_{sc} = 4mA \cdot \dfrac{2k\Omega}{2k\Omega + 3k\Omega} = \dfrac{8}{5}$ mA

$R_{Th} = \dfrac{8V}{8/5\ mA} = 5k\Omega$ $\boxed{R_{Th} = 5k\Omega}$

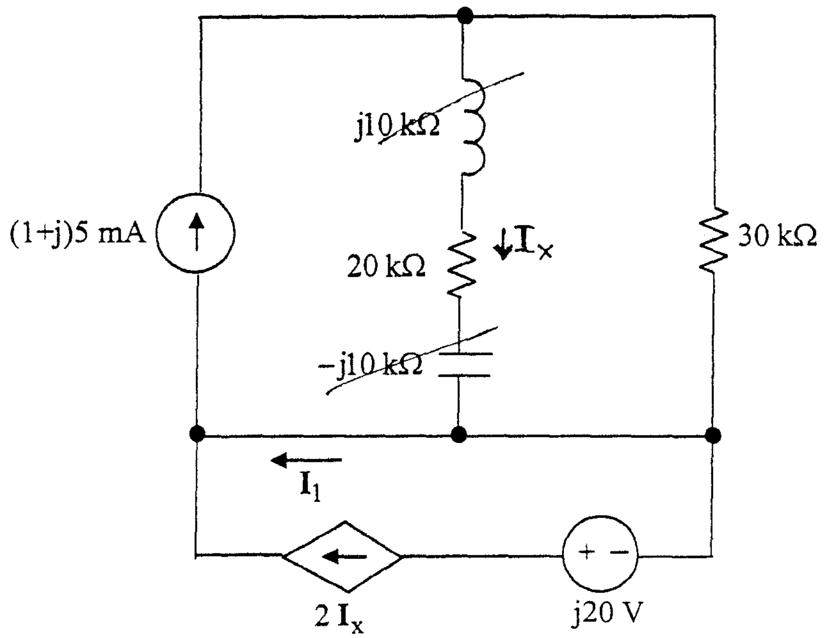
Note: Easier sol'n is to say $5v_x$ and 1kΩ don't matter because they are in series with current source. Then R_{Th} = 2kΩ + 3kΩ seen from a,b with 4mA off.

sol'n: 3.b) max pwr when $\boxed{R_L = R_{Th} = 5\,k\Omega}$

3.c) max pwr $= \dfrac{V_{Th}^2}{4 R_{Th}} = \dfrac{(8V)^2}{4 \cdot 5k\Omega} = \dfrac{64}{20}\, mW = 3.2\, mW$

$$\boxed{\text{max pwr} = 3.2\, mW}$$

4. (25 points)



Pts

20 pts a. A frequency-domain circuit is shown above. Write the value of $\mathbf{I}_1$ in polar form.

5 pts b. Given ω = 100 k rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

sol'n: 4. a) $j10\,k\Omega - j10\,k\Omega = 0\,\Omega$ so L and C cancel.

Current divider for 20kΩ and 30kΩ.

$$\therefore\ \mathbb{I}_x = (1+j)\,5\,mA \cdot \frac{30k\Omega}{20k\Omega + 30k\Omega} = (1+j)\,3\,mA$$

Find $\mathbb{I}_1$ from sum of currents at node on left side:

$$(1+j)\,5\,mA - 2\underbrace{(1+j)3\,mA}_{\mathbb{I}_x} - \mathbb{I}_1 = 0$$

$$\mathbb{I}_1 = (1+j)\,5\,mA - 2\,(1+j)\,3\,mA = (1+j)(5-6)\,mA$$

$$\mathbb{I}_1 = (1+j)(-1)\,mA = -(1+j1)\,mA$$

$$\boxed{\mathbb{I}_1 = -\sqrt{2}\ \angle 45^\circ\ mA} \boxed{= \sqrt{2}\ \angle -135^\circ\ mA \text{ or } 225^\circ}$$

b) $\boxed{i_1(t) = \sqrt{2}\ \cos(100\,kt - 135^\circ)\ mA}$