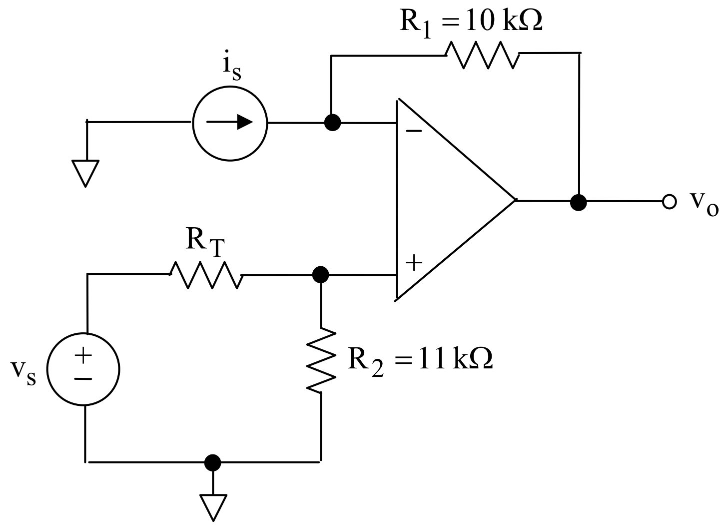
UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

ECE 1000 HOMEWORK #9 Spring 2005

1.



Rail voltage = ± 15 V

Design an electronic thermometer using the circuit diagram shown above. The voltage v_o is used to indicate temperature. Use a thermister with a resistance described by

$$R_T = R_o e^{\beta\left(\frac{1}{T}-\frac{1}{300}\right)}$$

where $R_o = 12.25\ k\Omega$, $\beta = 170^\circ K$, and T is temperature in °K. Derive a symbolic expression for v_o. The expression must contain not more than the parameters i_s, V_s, R_1, R_2, and R_T. **Hint: Use superposition**.

2. a. Calculate the numerical values of R_T (273°K) and R_T (373°K).

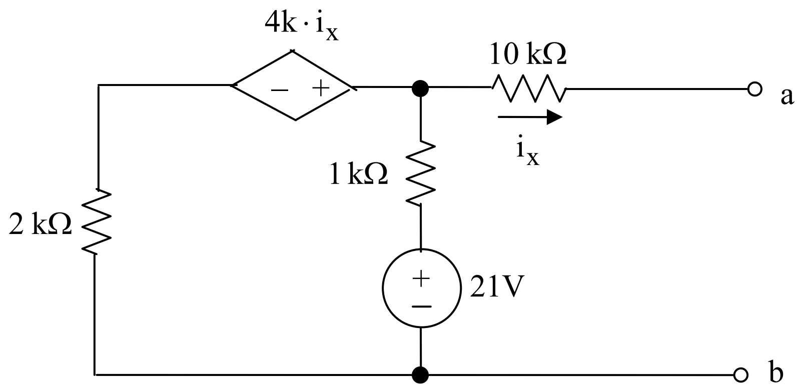
b. Determine a value for v_s such that $v_o(T = 373^\circ K) - v_o(T = 273^\circ K) = 1V$

c. Using your answer to (c), determine a value of i_s such that

$$v_o\ (T = 273°K) = 0\ V\ .$$

d. Using the component values you chose above, calculate v_o when T = 323°K. Make a rough sketch of v_o vs. T on the basis of the values when T = 273°K, 323°K, and 373°K. On the same axes, sketch the ideal linear response.

3.



a. Find the Thevenin equivalent of the above circuit relative to terminals a and b.

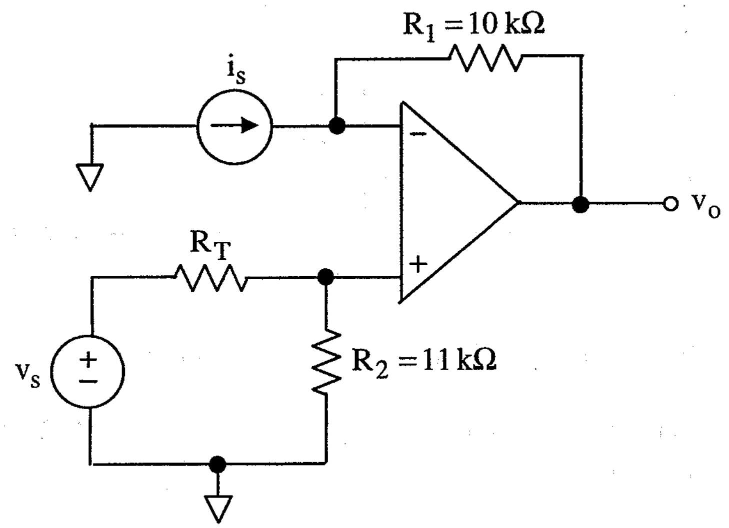
b. If we attach R_L to terminals a and b, find the value of R_L that will absorb maximum power.

c. Calculate the value of that maximum power absorbed by R_L.

UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

ECE 1000 HOMEWORK #9 Spring 2005

1.



Rail voltage = ± 15 V

Design an electronic thermometer using the circuit diagram shown above. The voltage v_o is used to indicate temperature. Use a thermister with a resistance described by

$$R_T = R_o e^{\beta\left(\frac{1}{T}-\frac{1}{300}\right)}$$

where $R_o = 12.25\,k\Omega$, $\beta = 170^\circ K$, and T is temperature in °K. Derive a symbolic expression for v_o. The expression must contain not more than the parameters i_s, V_s, R_1, R_2, and R_T. **Hint: Use superposition.**

2. a. Calculate the numerical values of R_T (273°K) and R_T (373°K).

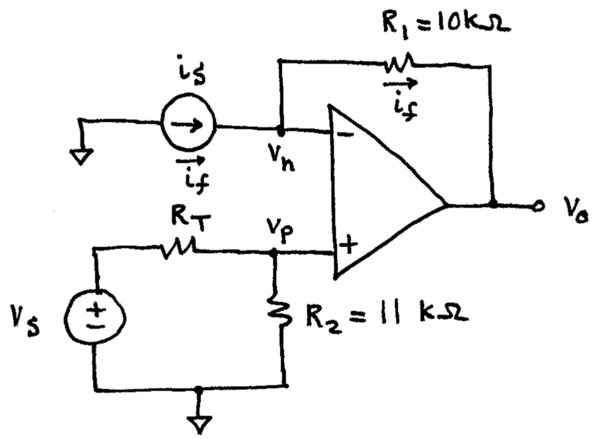
b. Determine a value for v_s such that $v_o(T = 373^\circ K) - v_o(T = 273^\circ K) = 1V$

c. Using your answer to (c), determine a value of i_s such that

$$v_o\,(T = 273^\circ K) = 0\text{ V}.$$

d. Using the component values you chose above, calculate v_o when T = 323°K. Make a rough sketch of v_o vs. T on the basis of the values when T = 273°K, 323°K, and 373°K. On the same axes, sketch the ideal linear response.

sol'n: 1.



Find v_p:

$$v_p = v_s \frac{R_2}{R_2 + R_T} \qquad \text{V-divider}$$

$v_n = v_p$

Find i_f on left: $i_f = i_s$

Find i_f on right: $i_f = \dfrac{v_n - v_o}{R_1}$

Set i_f's equal and use $v_n = v_p$

$$i_s = \frac{v_n - v_o}{R_1} \quad \text{or} \quad v_o = v_n - i_s R_1$$

$$\boxed{v_o = v_s \frac{R_2}{R_2 + R_T} - i_s R_1}$$

2. a) $R_T(273°K) = 12.25\,k\Omega \cdot e^{170°K\left(\frac{1}{273°K} - \frac{1}{300°K}\right)} = 13\,k\Omega$

$R_T(373°K) = 12.25\,k\Omega \cdot e^{170°K\left(\frac{1}{373°K} - \frac{1}{300°K}\right)} = 11\,k\Omega$

$$\boxed{\begin{array}{l} R_T(273°K) = 13k\Omega \\ R_T(373°K) = 11k\Omega \end{array}}$$

b) $1V = v_o(373°K) - v_o(273°K) = v_s \dfrac{R_2}{R_2 + R_T(373°K)} - \cancel{i_s R_1}$

$$- \left(v_s \frac{R_2}{R_2 + R_T(273°K)} - \cancel{i_s R_1}\right)$$

$$1V = v_s \cdot 11\,k\Omega \cdot \left(\frac{1}{11k\Omega + 11k\Omega} - \frac{1}{11\,k\Omega + 13k\Omega}\right)$$

$$\boxed{v_s = \frac{1V}{11k} \frac{22k\Omega \cdot 24k\Omega}{24\,k\Omega - 22k\Omega} = 24\ V}$$

soln: 1.c) $0V = v_o(273°K) = V_s \dfrac{R_2}{R_2 + R_T(273°K)} - i_s R_1$

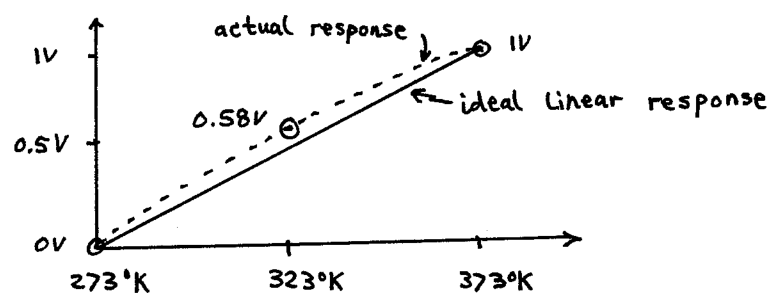
$$i_s = \frac{1}{R_1} V_s \frac{R_2}{R_2 + R_T(273°K)} = \frac{1}{10k\Omega} \cdot \cancel{24V} \cdot \frac{11k\Omega}{\cancel{11k\Omega + 13k\Omega}}$$

$$'' = \frac{11}{10k} A = 1.1\ mA$$

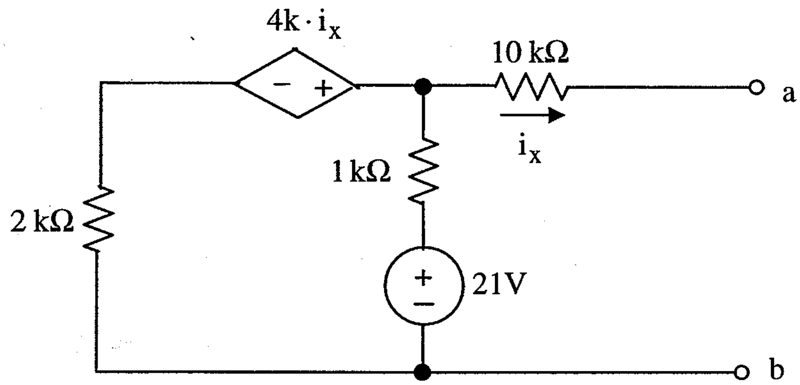
$$\boxed{i_s = 1.1\ mA}$$

d) $R_T(323°K) = 12.25k\Omega \cdot e^{170°K\left(\frac{1}{323°K} - \frac{1}{300°K}\right)} = 11.8k\Omega$

$$v_o(323°K) = 24V \frac{11k\Omega}{11k\Omega + 11.8k\Omega} - 1.1mA \cdot 10k\Omega = 0.58V$$



3.

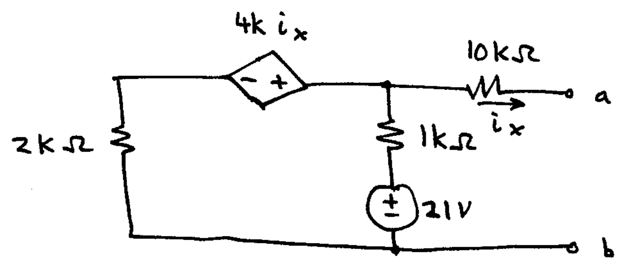


a. Find the Thevenin equivalent of the above circuit relative to terminals a and b.

b. If we attach R_L to terminals a and b, find the value of R_L that will absorb maximum power.

c. Calculate the value of that maximum power absorbed by R_L.

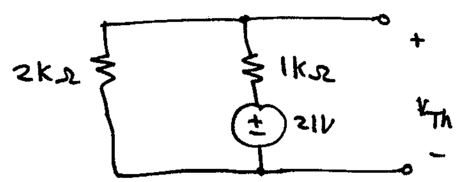
soln: 3. a)



$V_{Th} = V_{ab}$ open circuit

$i_x = 0$ for open circuit a, b. $\therefore 4k \cdot i_x = 0V$

No $i_x \Rightarrow$ no v drop across 10kΩ $\Rightarrow$ ignore 10kΩ

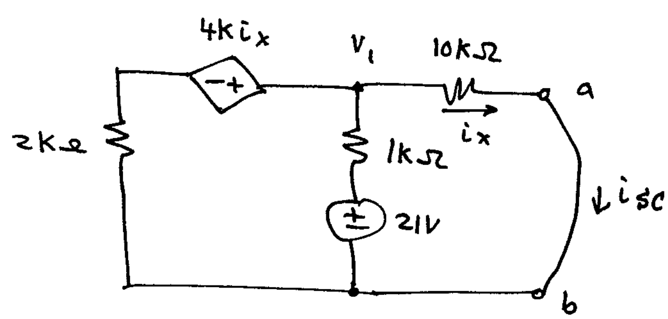


This is V-divider.

$$V_{Th} = 21V \cdot \frac{2k\Omega}{2k\Omega + 1k\Omega} = 14V$$

$$\boxed{V_{Th} = 14V}$$

To find R_{Th} we can use the i_{sc} method:



$$R_{Th} = \frac{V_{Th}}{i_{sc}}$$

Need to find i_{sc}. $\therefore$ Find v_1

Find v_1 by node-voltage method:

$$i_x = \frac{v_1}{10k\Omega} \quad \text{so we can eliminate } i_x$$

$$\frac{v_1 - 4k\frac{v_1}{10k\Omega}}{2k\Omega} + \frac{v_1 - 21V}{1k\Omega} + \frac{v_1}{10k\Omega} = 0A$$

mult everything by 10kΩ

$$v_1 \cdot 5 - v_1 \cdot 2 + v_1 \cdot 10 + v_1 \cdot 1 = 210\ V$$

$v_1(5 - 2 + 10 + 1) = 210V$ $\quad v_1 \cdot 14 = 210V$ $\quad v_1 = 15V$

$i_{sc} = i_x = \frac{v_1}{10k\Omega} = 1.5\ mA$ $\quad R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{14V}{1.5mA}$

$$\boxed{R_{Th} = 9.33\ k\Omega}$$

sol'n: 3.b) max pwr when $\boxed{R_L = R_{Th} = 9.33\ k\Omega}$

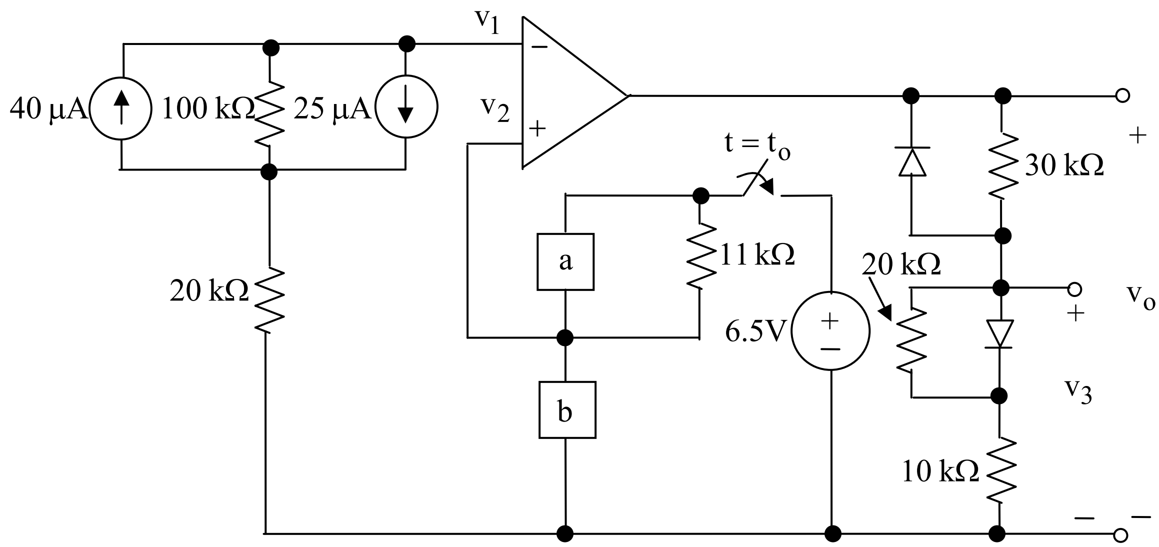
c) max pwr $P_{max} = \dfrac{V_{Th}^2}{4R_{Th}} = \dfrac{(14V)^2}{4 \cdot \frac{14}{1.5}\ k\Omega} = \dfrac{14\ (1.5)}{4}\ mW$

$$\boxed{P_{max} = \frac{21}{4}\ mW = 5.25\ mW}$$

UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

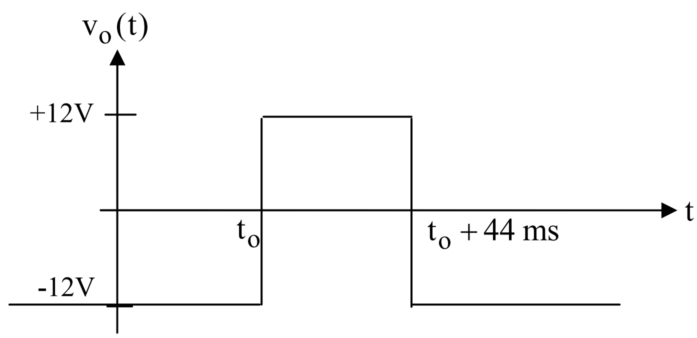
ECE 1000 HOMEWORK #10 Spring 2005

1.



Rail voltages = ±12 V

After being open for a long time, the switch closes at $t = t_o$.



a. Choose either an R or C to go in box a and either an R or C to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value. **Hint:** Use $v_2(t \to \infty) = 1V$

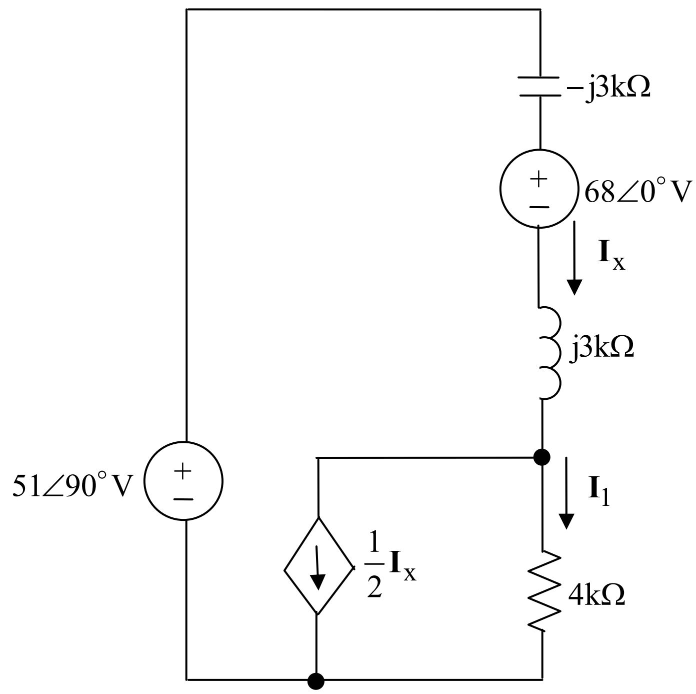
b. Sketch $v_1(t)$, showing numerical values appropriately.

2. a. Sketch $v_2(t)$, showing numerical values appropriately.

b. Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 44$ ms, and for $t_o + 44$ ms $< t$. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

Explain your work carefully.

3.



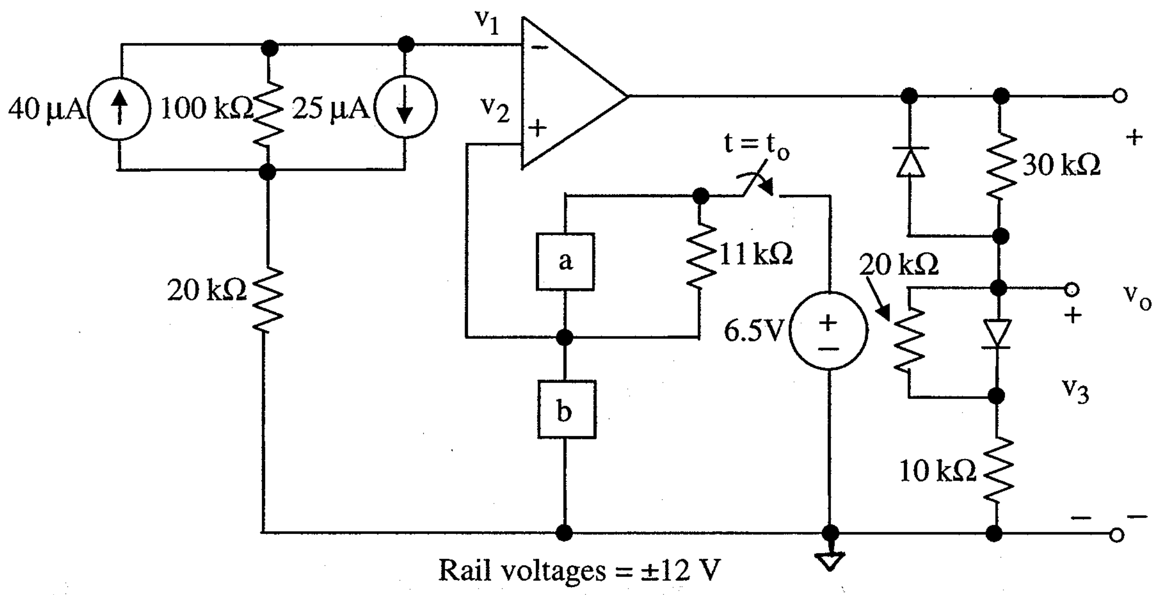
a. A frequency-domain circuit is shown above. Write the value of phasor $\mathbf{I}_1$ in polar form.

b. Given $\omega = \pi$ rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

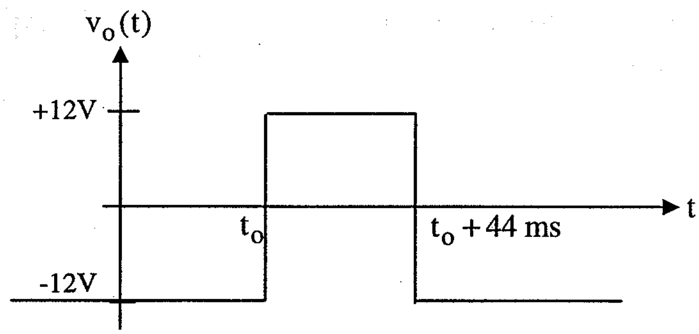
ECE 1000 HOMEWORK #10 Spring 2005

1.



Rail voltages = ±12 V

After being open for a long time, the switch closes at $t = t_o$.



a. Choose either an R or C to go in box a and either an R or C to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value. **Hint:** Use $v_2(t \to \infty) = 1\text{V}$

b. Sketch $v_1(t)$, showing numerical values appropriately.

2. a. Sketch $v_2(t)$, showing numerical values appropriately.

b. Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 44$ ms, and for $t_o + 44$ ms $< t$. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

Explain your work carefully.

sol'n: 1.a) Consider possibilities.

$a=R$ and $b=R$: V_2 would change at time $t=t_0$ but never change again. Thus, V_0 could not go high and then low again.

$a=C_1$ and $b=C_2$: Before $t=t_0$, there would be no path for C_2 in b to discharge. Thus, we would need to know $V_{C2}(t=t_0^-)$.

We would also have $V_{C1}(t=t_0^-)=0V$ since C_1 will discharge thru the R in parallel with it.

When the switch closes, we have an invalid circuit if $V_{C2}(t_0^+)+V_{C1}(t_0^+) \neq 6.5V$. This means we must have $V_{C2}(t_0^+)=V_C(t_0^-)=6.5V$ since $V_{C1}(t_0^+)=V_C(t_0^-)=0V$.

But if $V_{C2}(t_0^+)=6.5$, nothing changes when the switch closes. Thus, we could not get a waveform that goes high and then low again.

$a=R$ and $b=C$: As in the case of $a=C_1$ and $b=C_2$, we would have some value of $V_C(t_0^-) = V_C(t_0^+)$. Since $V_2=V_C(t_0^+)$ at $t=t_0^+$, V_0 would not change at t_0. Thus, this will not work.

$\boxed{a=C \text{ and } b=R}$: The C in a will discharge thru the R in parallel with it. $\therefore V_C(t_0^-)=0V$. Also, no current flows in R for $t<t_0$ because there is no closed circuit path in which current could flow. Thus, $V_2(t_0^-)=i\cdot R=0V$.

soln: 1.a) cont.

At $t = t_o^+$, we have $v_2(t_o^+) = 6.5V - v_C(t_o^+)$ or $v_2(t_o^+) = 6.5V$ since $v_C(t_o^+) = v_C(t_o^-) = 0V$.

Thus, v_2 jumps from 0V to 6.5V at t_o.

If $v_2(t_o^+) > v_1$, then v_o would go high, (assuming $v_1 > 0V$ and $v_1 < 6.5V$).

After t_o, C will charge and v_2 will start to drop. If v_2 eventually drops below v_1, then v_o will go low again.

Thus, this will work.

Now find values of R and C:

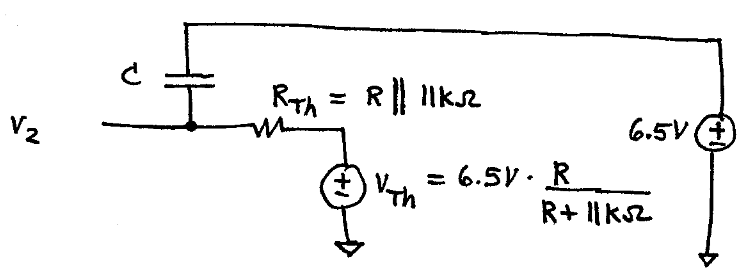
v_1: Since no current flows into the − input of the op amp, no current flows thru the 20kΩ resistor (lower left).

Combining the parallel current sources, we have $40\mu A - 25\mu A = 15\mu A$. This current flows thru the 100kΩ to produce voltage

$15\mu A \cdot 100k\Omega = 1.5V$.

$\therefore v_1 = 2.5V$ (constant)

v_2: For $t > t_o$, we use a Thevenin equivalent circuit for the 6.5V source, the 11kΩ, and R in b.



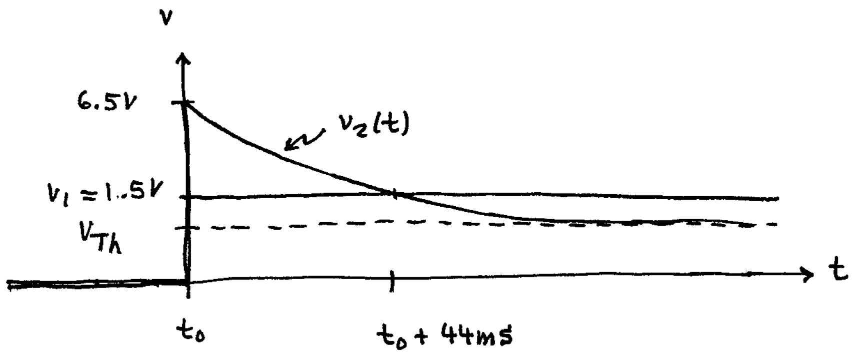
Sol'n: 1.a) cont. We use the general RC sol'n for $v_2(t)$:

$$v_2(t) = v_2(t\to\infty) + [v_2(t_0^+) - v_2(t\to\infty)]\, e^{-t/R_{Th}C}$$

To find $v_2(t\to\infty)$, we observed that, when C is charged, no current flows in R_{Th}.

Thus $v_2(t\to\infty) = V_{Th} = 6.5V \dfrac{R}{R + 11k\Omega} < 6.5V$.

From earlier, $v_2(t_0^+) = 6.5V$.



Any $V_{Th} < v_1$ will suffice. For convenience,

let $V_{Th} = 1V = 6.5V \cdot \dfrac{R}{R + 11k\Omega} \Rightarrow \boxed{R = 2k\Omega}$

Assume $t_0 = 0$. For v_0 to switch at $t = 44ms$,

we have $v_2(44ms) = v_1 = 1.5V$

or $1V + [6.5V - 1V]\, e^{-44ms/R_{Th}C} = 0.5V$

or $5.5V\, e^{-44ms/R_{Th}C} = 0.5V$ where

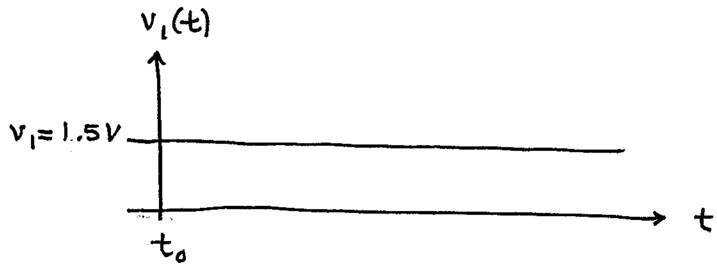
$$R_{Th} = R \| 11k\Omega = 2k \| 11k\Omega = \frac{22}{13} k\Omega.$$

Take $\ln$ of both sides of exponential RC eq'n:

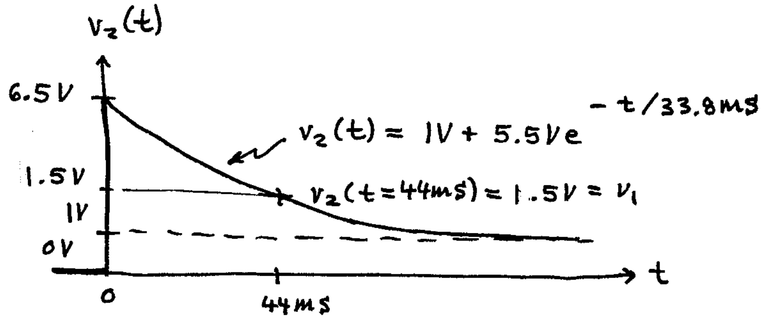
$$-\frac{\overset{2}{44ms}}{\frac{\cancel{22}}{13} k\Omega \cdot C} = \ln \frac{0.5V}{5.5V} \doteq -2.39$$

$$C = \frac{2}{\frac{2.39}{13}} \mu F = 10.9 \mu F \qquad \boxed{C = 11 uF}$$

sol'n: 1.b) $v_1(t) = 1.5V$ constant



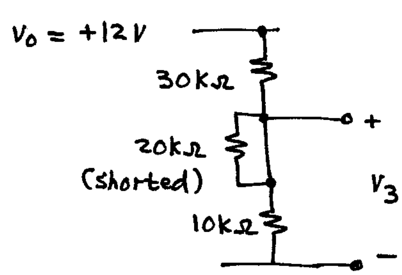
2.a)



Note: $R_{Th} C = \frac{22}{13} k\Omega \cdot 20\mu F \doteq 33.8 ms$

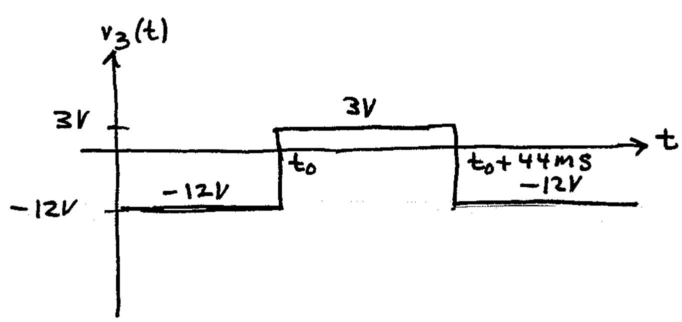
Note: Assume $t_0 = 0$.

b) When $v_0 = +12V$, top diode is reverse biased = open. The bottom diode is forward biased = wire.

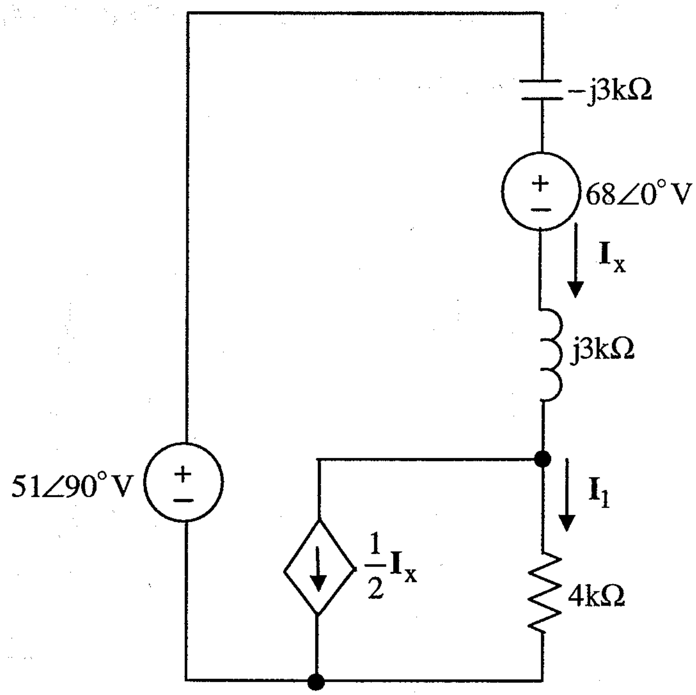


$$v_3 = v_0 \cdot \frac{10k\Omega}{10k\Omega + 30k\Omega} = 12V \cdot \frac{1}{4} = 3V$$

when $v_0 = -12V$, top diode is forward biased = wire. Thus, v_0 is shorted to $-12V$.



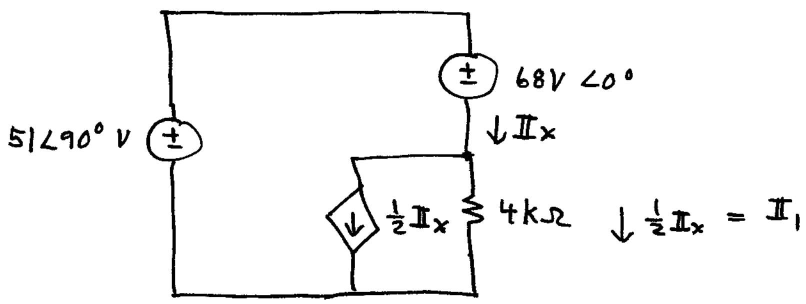
3.



a. A frequency-domain circuit is shown above. Write the value of phasor $\mathbf{I}_1$ in polar form.

b. Given $\omega = \pi$ rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

sol'n: 3.a) The $-j3k\Omega$ and $j3k\Omega$ sum to zero and act like a wire. Thus, they do not affect $\mathbb{I}_x$. So we have:



Clearly, $\frac{1}{2}\mathbb{I}_x$ flows thru the $4k\Omega$ (for sum of currents at node above $4k\Omega = 0$).

But the current thru $4k\Omega$ is $\dfrac{51\angle 90°\,V - 68\angle 0°\,V}{4k\Omega}$

or $\frac{1}{2}\mathbb{I}_x = \mathbb{I}_1 = \dfrac{17\cdot 3\angle 90° - 17\cdot 4\angle 0°\,V}{4k\Omega}$

$$\mathbb{I}_1 = 17\,\frac{j3-4}{4k\Omega} = \frac{17}{4}(-4+j3) = \frac{17}{4}\cdot 5V\angle 143°$$

mA mA

$$\boxed{\mathbb{I}_1 = -17 + j\,12.75\,mA = 21.25\angle 143°\,mA}$$

b)

$$\boxed{\begin{aligned} i_1(t) &= 17\cos(\pi t + 180°) - 12.75\sin(\pi t)\ mA \\ " &= 21.25\cos(\pi t + 143°)\ mA \end{aligned}}$$