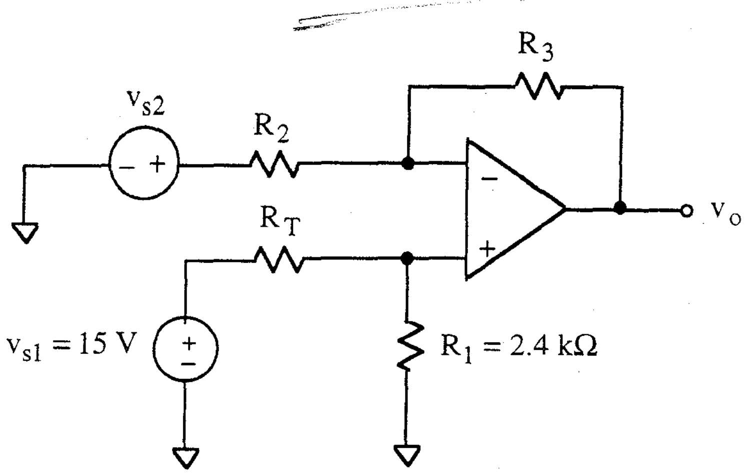
EE 1000 Final Exam sol'n Spring 2001

1. (70 points)



Rail voltage = ± 15 V

Design an electronic thermometer using the circuit diagram shown above. The voltage v_o is used to indicate temperature. Use a thermister with a resistance described by

$$R_T = R_o e^{1120\left(\frac{1}{T} - \frac{1}{300}\right)}$$

where $R_o = 2490\ \Omega$ and T is temperature in °K.

Pts

10 a. Calculate the numerical values of R_T (273°K) and R_T (373°K).

25 b. Derive a symbolic expression for v_o. The expression must contain not more than the parameters v_{s1}, v_{s2}, R_T, R_1, R_2, and R_3.
Hint: Use superposition.

25 c. Choose v_{s2}, R_2, and R_3 that will produce the following:

$v_o = 0$ V	when	T = 273°K
$v_o = 10$ V	when	T = 373°K

10 d. Using the component values you chose in (c), calculate v_o when T = 323°K.

sol'n 1.a) $R_T(273°K) = 2490\, e^{1120\left(\frac{1}{273} - \frac{1}{300}\right)} = 3.6\,k\Omega$ $\boxed{R_T(273°K) = 3.6\,k\Omega}$

$R_T(373°K) = 2490\, e^{1120\left(\frac{1}{373} - \frac{1}{300}\right)} = 1.2\,k\Omega$ $\boxed{R_T(373°K) = 1.2\,k\Omega}$

b) Superposition:

V_{S1} on, V_{S2} off $\Rightarrow$ $V_p = V_{S1} \cdot \frac{R_1}{R_1 + R_T}$, $V_n = V_p$

$$i_f\,(\text{thru } R_2) = \frac{0 - V_n}{R_2} = -V_{S1} \frac{R_1}{R_1 + R_T} \frac{1}{R_2}$$

$$i_f\,(\text{thru } R_3) = \frac{V_n - V_{01}}{R_3} = -V_{S1} \frac{R_1}{R_1 + R_T} \frac{1}{R_3}$$

$$i_f\,(\text{thru } R_2) = i_f\,(\text{thru } R_3) \Rightarrow \frac{-V_n}{R_2} = \frac{V_n - V_0}{R_3}$$

or $V_{01} = V_n\left(1 + \frac{R_3}{R_2}\right) = V_{S1} \frac{R_1}{R_1 + R_T}\left(1 + \frac{R_3}{R_2}\right)$

V_{S1} off, V_{S2} on $\Rightarrow$ $V_p = 0V$, $V_n = V_p$

$i_f\,(\text{thru } R_2) = \frac{V_{S2}}{R_2}$ $\quad$ $i_f\,(\text{thru } R_3) = -\frac{V_{02}}{R_3}$

$$i_f\,(\text{thru } R_2) = i_f\,(\text{thru } R_3) \Rightarrow V_{02} = -V_{S2} \frac{R_3}{R_2}$$

$V_0 = V_{01} + V_{02}$ $\quad$ $\boxed{V_0 = V_{S1} \frac{R_1}{R_1 + R_T}\left(1 + \frac{R_3}{R_2}\right) - V_{S2} \frac{R_3}{R_2}}$

c) At 273°K, $V_{S1} \frac{R_1}{R_1 + R_T} = 15V \frac{2.4k\Omega}{2.4k\Omega + 3.6k\Omega} = 15V \cdot \frac{2}{5} = 6V = V_p$

At 373°K, $V_{S1} \frac{R_1}{R_1 + R_T} = 15V \frac{2.4k\Omega}{2.4k\Omega + 1.2k\Omega} = 15V \cdot \frac{2}{3} = 10V = V_p$

$$V_0 = \underbrace{V_p\left(1 + \frac{R_3}{R_2}\right)}_{\text{proportional to } V_p} - \underbrace{V_{S2} \frac{R_3}{R_2}}_{\text{constant}}$$

The change in V_0 vs change in V_p:

$$\Delta V_0 = \Delta V_p \left(1 + \frac{R_3}{R_2}\right)$$

$\Delta V_p = V_p(373°K) - V_p(273°K) = 10V - 6V = 4V$

$\Delta V_0 = V_0(373°K) - V_0(273°K) = \underbrace{10V - 0V}_{\text{from prob statement}} = 10V$

sol'n 1.c) cont.

$$\Delta V_o = \Delta V_P \left(1 + \frac{R_3}{R_2}\right) \quad \text{or} \quad 10V = 4V \left(1 + \frac{R_3}{R_2}\right)$$

$$\therefore \; 1 + \frac{R_3}{R_2} = 2.5 \quad \text{or} \quad \frac{R_3}{R_2} = 1.5$$

Let $\boxed{R_3 = 15\,k\Omega}$, $\boxed{R_2 = 10\,k\Omega}$.

$$V_o(273°K) = \underset{\substack{\text{from}\\\text{prob}\\\text{statement}}}{0V} = V_P\left(1 + \frac{R_3}{R_2}\right) - V_{S2}\frac{R_3}{R_2} = 6V(2.5) - V_{S2}(1.5)$$

$$\text{or} \quad V_{S2} = \frac{6(2.5)V}{1.5} \quad \text{or} \quad \boxed{V_{S2} = 10V}$$

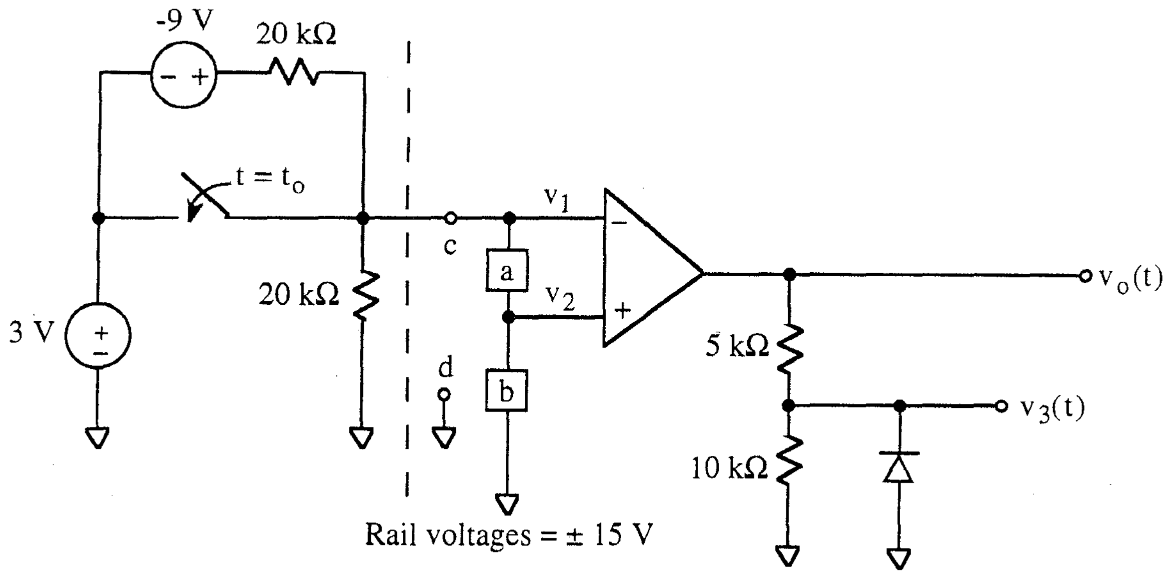
d) $$R_T(323°K) = 2490\, e^{1120\left(\frac{1}{323} - \frac{1}{300}\right)} = 1.9\,k\Omega$$

$$V_P = 15V \cdot \frac{2.4\,k\Omega}{2.4\,k\Omega + 1.9\,k\Omega} = 8.37\,V$$

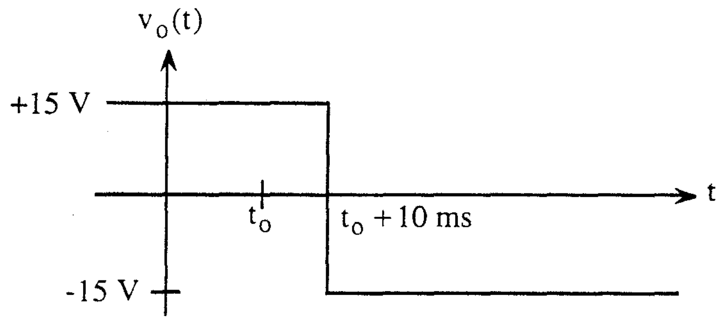
$$V_o = V_P\left(1 + \frac{R_3}{R_2}\right) - V_{S2}\frac{R_3}{R_2} = 8.37\,(2.5) - 10(1.5)\;V$$

$\boxed{V_o(323°K) = 5.9\,V}$ Off by 18% from linear value of 5V.

2. (70 points)



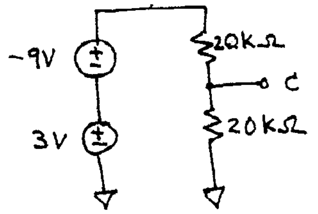
After being open for a very long time, the switch closes at $t = t_o$.



Pts		
10	a.	Find the Thevenin equivalent of the circuit left of the dashed line with respect to terminals c, d for $\mathbf{t < t_o}$.
10	b.	Find the Thevenin equivalent of the circuit left of the dashed line with respect to terminals c, d for $\mathbf{t > t_o}$.
20	c.	Choose either an R or C to go in box a and either an R or C to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value.
20	d.	Using the elements found in (b), sketch $v_2(t)$. Show numerical values appropriately.
10	e.	Sketch $v_3(t)$. Show numerical values for $t < t_o$, $t_o < t < t_o + 10$ ms, and $t > t_o +$ 10 ms. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

Explain your work carefully.

sol'n 2. a) $t < t_0$ switch is open

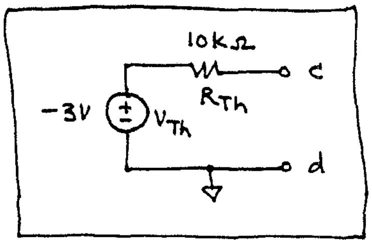


$$V_{Th} = (3V - 9V)\,\frac{20k\Omega}{20k\Omega + 20k\Omega} = -6V \cdot \frac{1}{2} = -3V$$

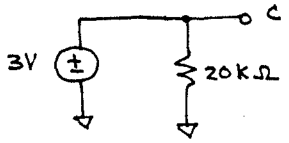
use V-divider

R_{Th} = R looking into c with V sources shorted

$= 20k\Omega \,\|\, 20k\Omega = 10k\Omega$



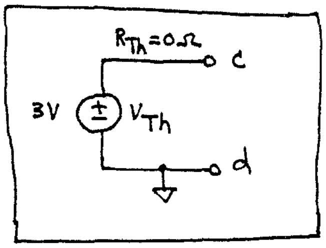
b) $t > t_0$ switch is closed (-9V source and 20kΩ bypassed)



$V_{Th} = 3V$ since c connected to 3V source

R_{Th} = R looking into c with 3V source shorted

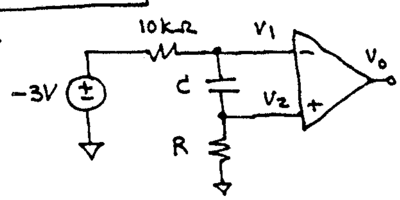
$= 0\Omega$ (20kΩ bypassed by short)



c) R in **a** and C in **b** would immediate switching: <u>wrong</u>

$\therefore$ $\boxed{\text{C in } \mathbf{a} \text{ and R in } \mathbf{b}}$ must be correct.

$t < t_0$ circuit:
$(t = t_0^-)$



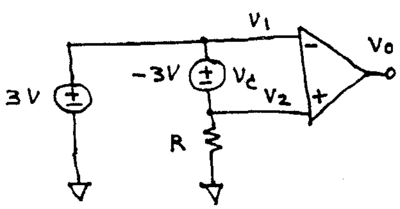
C is open circuit
no V drop across R's

$\therefore V_2 = 0V$

$V_1 = -3V$

$V_1 < V_2 \Rightarrow V_o = +15V$ (rail)

$t = t_0^+$ circuit:



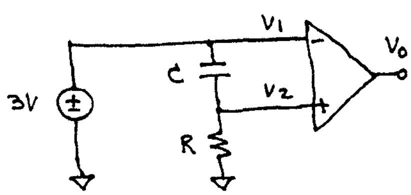
C acts like -3V source

$\therefore V_1 = 3V$

$V_2 = 3V - -3V = 6V$

$V_1 < V_2 \Rightarrow V_o = +15V$ (rail)

$t \to \infty$ circuit:



C acts like open circuit
no V drop across R

$\therefore V_1 = 3V$

$V_2 = 0V$

$V_1 > V_2 \Rightarrow V_o = -15V$ (rail) sol'n 2.c) cont. Let $t_o = 0$. Want $V_1 = V_2$ at $\Delta t \equiv 10ms$ for V_o transition.

$V_1 = 3V$ for $t > t_o = 0$

$V_2 = 3V - V_C$ for $t > t_o$.

$$V_2(t>0) = V_2(t\to\infty) + [V_2(0^+) - V_2(t\to\infty)]\, e^{-t/RC}$$

$$'' \quad = 0V + [6V - 0V]\, e^{-t/RC} = 6V\, e^{-t/RC}$$

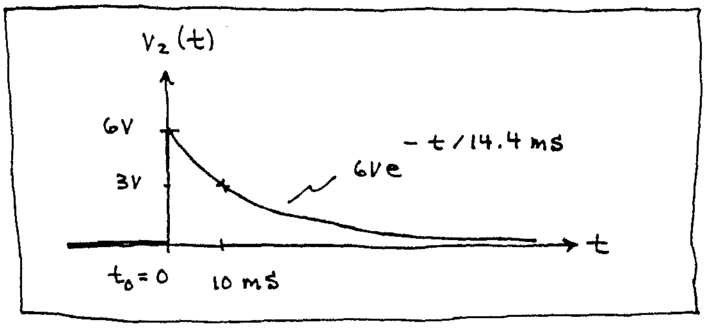
We want $V_2(\Delta t) = 3V$. $\therefore$ $6V\, e^{-\Delta t/RC} = 3V$

$$e^{-\Delta t/RC} = \frac{3V}{6V} = \frac{1}{2} \quad \text{or} \quad -\frac{\Delta t}{RC} = \ln \frac{1}{2}$$

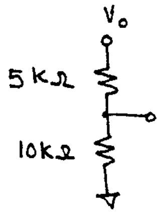
$$RC = \frac{-\Delta t}{\ln \frac{1}{2}} = \frac{\Delta t}{\ln 2} = \frac{10ms}{\ln 2} = 14.4\ ms$$

Use $\boxed{C = 1\mu F,\quad R = 14.4\,k\Omega}$ (15kΩ is closest standard value)

d) $V_2(t<0) = 0V$ $\quad V_2(t>0) = 6V e^{-t/14.4ms}$ $\quad V_2(10ms) = 3V$

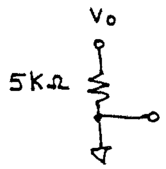


e) When $V_o = +15V$, the diode is reverse biased ⇒ open circuit (i.e. disappears)

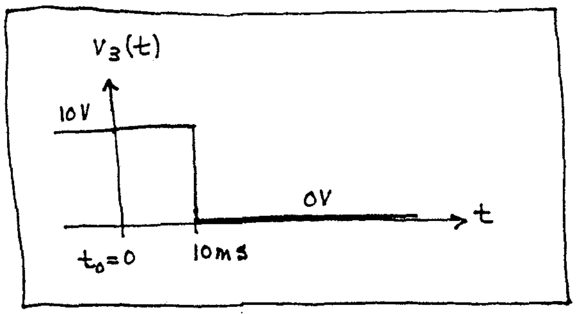


$$V_3 = V_o \cdot \frac{10k\Omega}{10k\Omega + 5k\Omega} = 15V \cdot \frac{2}{3} = 10V$$

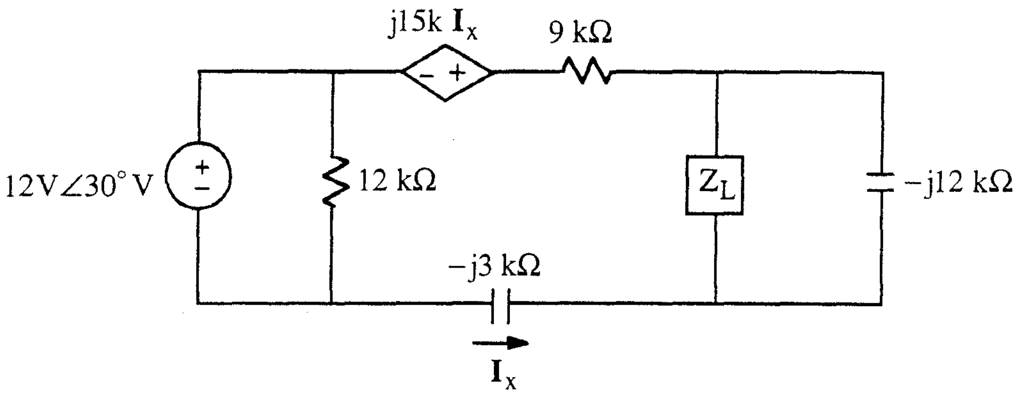
When $V_o = -15V$, the diode is forward biased ⇒ short



$V_3 = 0V$



3. (30 points)



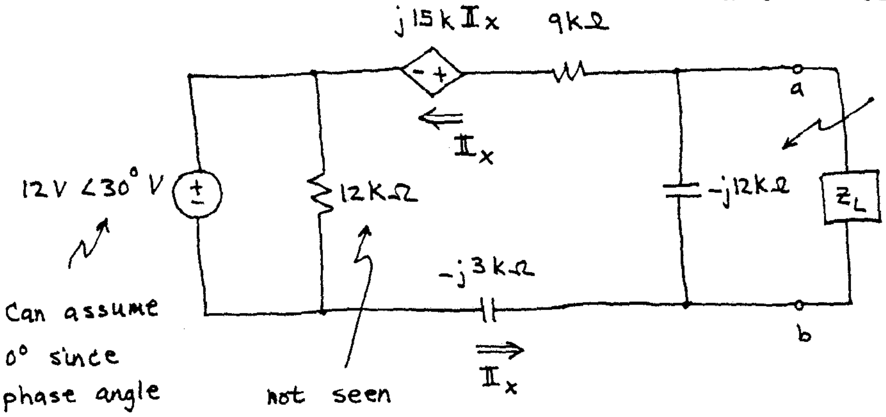
Pts

15 a. Choose the value of Z_L that will absorb maximum average power.

15 b. Calculate the value of that maximum average power absorbed by Z_L.

sol'n 3. a) $Z_L = Z_{Th}^*$ for max power xfer

Since I_x flows thru it, this looks like inductor with $z = j15k\Omega$.

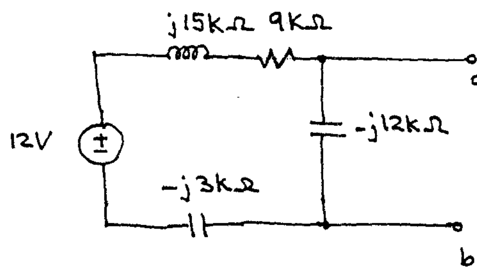


We can put C on left side of Z_L since they are in parallel

Can assume 0° since phase angle doesn't affect ave pwr

not seen in Thevenin equivalent since it is across V source

circuit for determining Thevenin equivalent:



$\Leftarrow Z_{Th}$ = z seen looking into a, b with 12V source shorted

$$\therefore Z_{Th} = -j12k\Omega \,\|\, (-j3k\Omega + j15k\Omega + 9k\Omega)$$

$$'' = -j12k\Omega \,\|\, (9k\Omega + j12k\Omega)$$

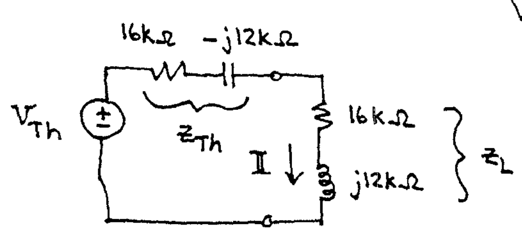
$$'' = \frac{-j12k\Omega\,(9k\Omega + j12k\Omega)}{-j\cancel{12k\Omega} + 9k\Omega + j\cancel{12k\Omega}}$$

$$'' = -j\,\frac{12k\Omega}{9k\Omega}\,(9k\Omega + j12k\Omega)$$

$$'' = -j12k\Omega + 16k\Omega$$

$$\boxed{Z_L = Z_{Th}^* = 16k\Omega + j12k\Omega}$$

b)



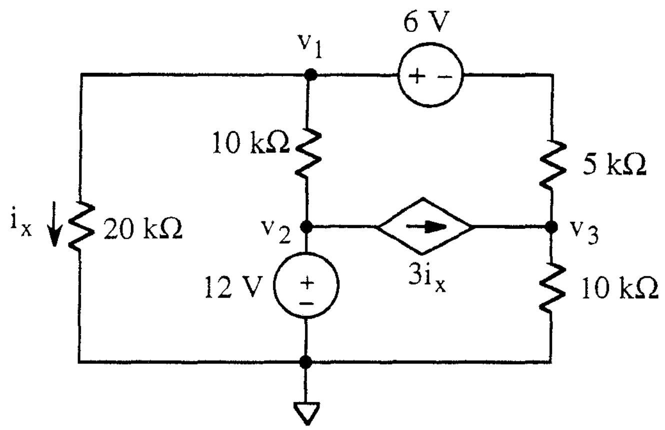
$$P \equiv (\text{ave pwr in } Z_L) = \frac{|I|^2}{2} R \quad \text{where } R = 16k\Omega \text{ in } Z_L$$

$$I = \frac{V_{Th}}{16k\Omega - j\cancel{12k\Omega} + 16k\Omega + j\cancel{12k\Omega}} = \frac{V_{Th}}{32k\Omega}$$

$$V_{Th} = 12V \cdot \frac{-j12k\Omega}{-j\cancel{12k\Omega} + -j\cancel{3k\Omega} + j\cancel{15k\Omega} + 9k\Omega} = 12V\left(\frac{-j12k\Omega}{9k\Omega}\right) = -j16V$$

$$\therefore P = \frac{\left|\frac{-j16V}{32k\Omega}\right|^2}{2} \cdot 16k\Omega = \frac{|-j|^2 V^2}{|2k\Omega|^2} \cdot \frac{16k\Omega}{2} = \frac{1V^2 \cdot 8}{4k\Omega} \quad \text{or} \quad \boxed{P = 2\text{ mW}}$$

4. (30 points)



<u>Pts</u>

15 a. Write equations for node voltages v_1, v_2, and v_3 in the form:

$$
\begin{aligned}
g_{11}v_1 + g_{12}v_2 + g_{13}v_3 &= i_1 \\
g_{21}v_1 + g_{22}v_2 + g_{23}v_3 &= i_2 \\
g_{31}v_1 + g_{32}v_2 + g_{33}v_3 &= i_3
\end{aligned}
$$

List the numerical values of g_{ij}'s and i's.

15 b. Show exactly what you would type into MATLAB™ to:

i. Create a vertical array (called "ivec") containing your values for i_1, i_2, and i_3,

ii. Using ivec, create an array (called "ivec2") containing values for i_1^2, i_2^2, and i_3^2,

iii. Using ivec2, create a variable (called "sum_sq_i") equal to $i_1^2 + i_2^2 + i_3^2$.

sol'n 4a) $i_x = \frac{v_1}{20k\Omega}$ $\frac{v_1}{20k\Omega} + \frac{v_1 - v_2}{10k\Omega} + \frac{(v_1 - 6V) - v_3}{5k\Omega} = 0A$

$v_2 = 12V$ (connected to ref by V source)

$\frac{v_3}{10k\Omega} - 3\frac{v_1}{20k\Omega} + \frac{v_3 - (v_1 - 6V)}{5k\Omega} = 0A$

$$\boxed{\begin{array}{llll} g_{11} = \frac{1}{20k\Omega} + \frac{1}{10k\Omega} + \frac{1}{5k\Omega} = \frac{7}{20k\Omega} & g_{12} = -\frac{1}{10k\Omega} & g_{13} = -\frac{1}{5k\Omega} & i_1 = \frac{6V}{5k\Omega} \\ g_{21} = 0 & g_{22} = \frac{1}{\Omega} & g_{23} = 0 & i_2 = 12A \\ g_{31} = -\frac{1}{20k\Omega} - \frac{1}{5k\Omega} = \frac{-5}{20k\Omega} \quad g_{32} = 0 & g_{33} = \frac{1}{10k\Omega} + \frac{1}{5k\Omega} = \frac{3}{10k\Omega} & & i_3 = -\frac{6V}{5k\Omega} \end{array}}$$

b) i. `ivec = [6/5000; 12; -6/5000]`

ii. `ivec2 = ivec .* ivec`

iii. `sum_sq_i = sum(ivec2)`

wj
EL EN 1000
Final Exam
May 2, 2001

Name ______________________________

SCORE:

Problem 1	__________	of a possible 70 points
Problem 2	__________	of a possible 70 points
Problem 3	__________	of a possible 30 points
Problem 4	__________	of a possible 30 points
Total	__________	of a possible 200 points