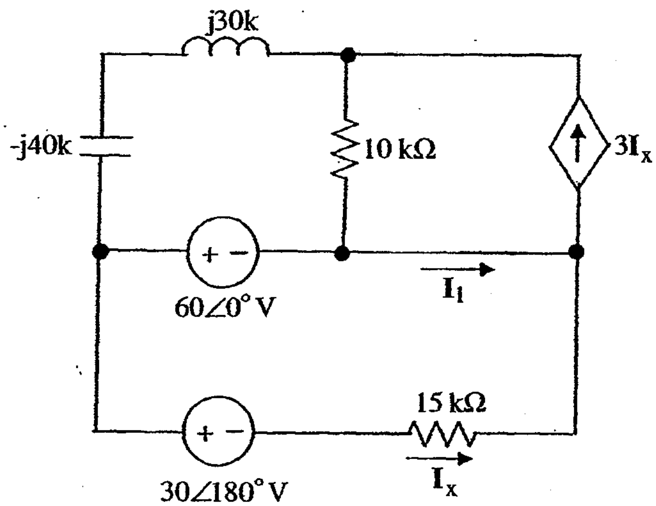
HW #10 Cont.

4.



a. A frequency-domain circuit is shown above. Write the value of phasor $\mathbf{I}_1$ in polar form.

b. Given $\omega = 53.13$ rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

HW #10 Cont.

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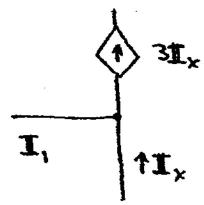
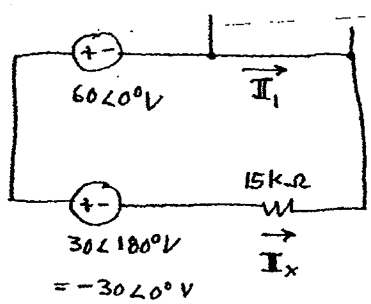
sol'n: 4. a) Sum of currents for node on right side:



We see that $\mathbf{I}_1 = 2\mathbf{I}_x$ from sum of currents out of node $= 0$.

From the bottom half of the circuit, we can compute $\mathbf{I}_x$ directly:



From V-loop we have

$$\mathbf{I}_x = \frac{60\angle 0° V - -30\angle 0° V}{15 k\Omega}$$

$$\mathbf{I}_x = \frac{90\angle 0° V}{15 k\Omega}$$

$$\mathbf{I}_x = 6\ mA\ \angle 0°$$

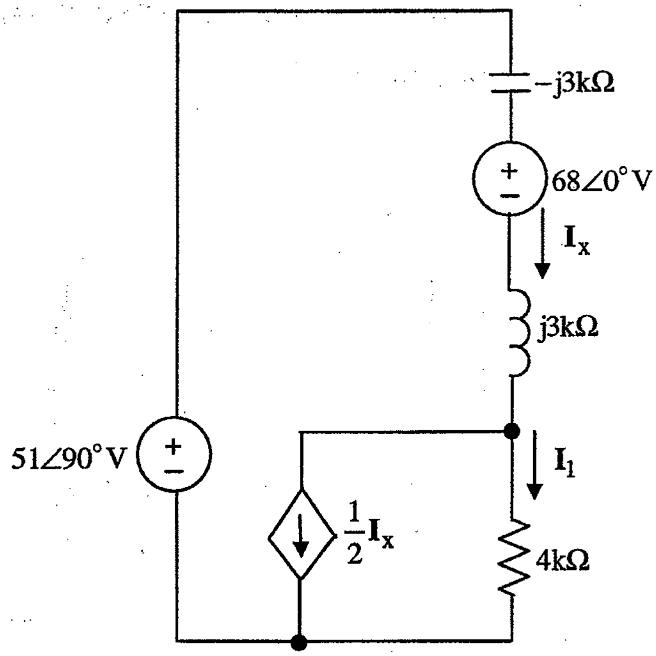
So $\mathbf{I}_1 = 2\mathbf{I}_x = 12\ mA\ \angle 0°$

$$\boxed{\mathbf{I}_1 = 12\angle 0°\ mA}$$

b) $\boxed{i_1(t) = 12\cos(53.13t)\ mA}$

or πt

3.

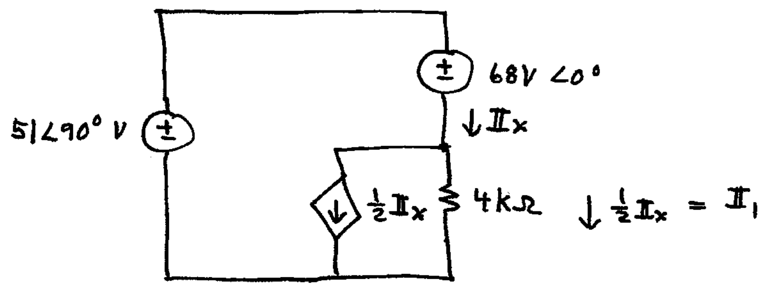


a. A frequency-domain circuit is shown above. Write the value of phasor $\mathbf{I}_1$ in polar form.

b. Given $\omega = \pi$ rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

43

sol'n: 3.a) The $-j3k\Omega$ and $j3k\Omega$ sum to zero and act like a wire. Thus, they do not affect $\mathbb{I}_x$. So we have:



Clearly, $\frac{1}{2}\mathbb{I}_x$ flows thru the $4k\Omega$ (for sum of currents at node above $4k\Omega = 0$).

But the current thru $4k\Omega$ is $\dfrac{51\angle 90° \text{ V} - 68\angle 0° \text{ V}}{4k\Omega}$

or $\frac{1}{2}\mathbb{I}_x = \mathbb{I}_1 = \dfrac{17 \cdot 3\angle 90° - 17 \cdot 4\angle 0° \text{ V}}{4k\Omega}$

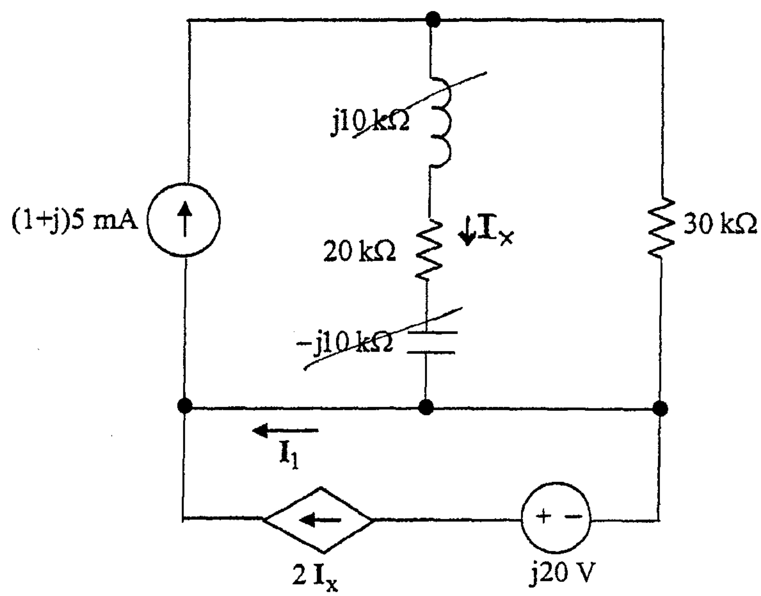
$$\mathbb{I}_1 = 17\,\frac{j3 - 4}{4k\Omega} = \frac{17}{4}(-4 + j3) \text{ mA} = \frac{17}{4} \cdot 5\text{V}\angle 143° \text{ mA}$$

$$\boxed{\mathbb{I}_1 = -17 + j\,12.75 \text{ mA} = 21.25\angle 143° \text{ mA}}$$

b)

$$\boxed{\begin{aligned} i_1(t) &= 17\cos(\pi t + 180°) - 12.75\sin(\pi t) \text{ mA} \\ '' &= 21.25\cos(\pi t + 143°) \text{ mA} \end{aligned}}$$

4. (25 points)



Pts

20 pts a. A frequency-domain circuit is shown above. Write the value of $\mathbf{I}_1$ in polar form.

5 pts b. Given ω = 100 k rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

sol'n: 4. a) $j10k\Omega - j10k\Omega = 0\Omega$ so L and C cancel.

Current divider for 20kΩ and 30kΩ.

$$\therefore \ \mathbf{I}_x = (1+j)\,5\,mA \cdot \frac{30k\Omega}{20k\Omega + 30k\Omega} = (1+j)\,3\,mA$$

Find $\mathbf{I}_1$ from sum of currents at node on left side:

$$(1+j)\,5\,mA - 2\underbrace{(1+j)3mA}_{\mathbf{I}_x} - \mathbf{I}_1 = 0$$

$$\mathbf{I}_1 = (1+j)\,5mA - 2(1+j)\,3mA = (1+j)(5-6)\,mA$$

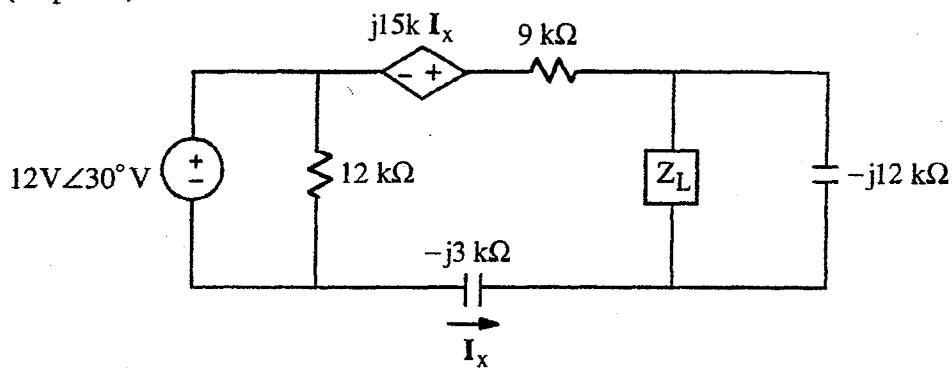
$$\mathbf{I}_1 = (1+j)(-1)\ mA = -(1+j1)\ mA$$

$$\boxed{\mathbf{I}_1 = -\sqrt{2}\ \angle 45°\ mA} = \boxed{\sqrt{2}\ \angle -135°\ mA \text{ or } 225°}$$

b) $\boxed{i_1(t) = \sqrt{2}\cdot \cos(100kt - 135°)\ mA}$

45

3. (30 points)



<u>Pts</u>

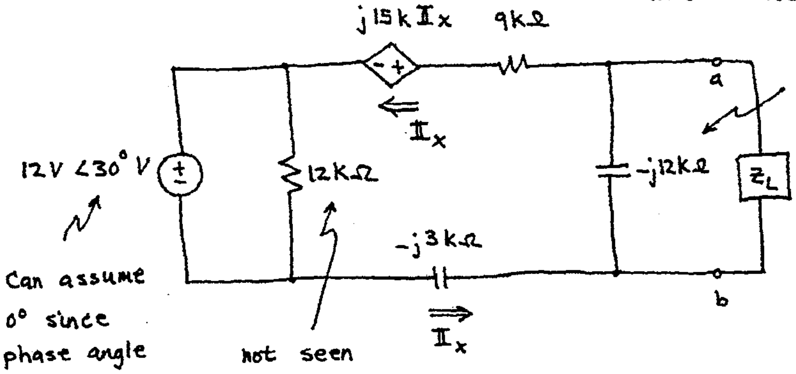
15 a. Choose the value of Z_L that will absorb maximum average power.

15 b. Calculate the value of that maximum average power absorbed by Z_L.

46

soln 3. a) $Z_L = Z_{Th}^*$ for max power xfer

Since I_x flows thru it, this looks like inductor with $z = j15K\Omega$.

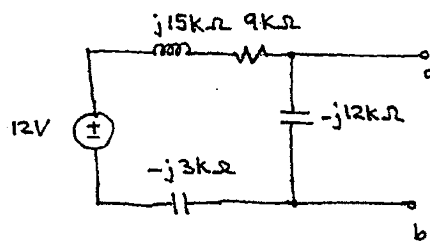


We can put C on left side of Z_L since they are in parallel

Can assume 0° since phase angle doesn't affect ave pwr

not seen in Thevenin equivalent since it is across V source

circuit for determining Thevenin equivalent:



$\Leftarrow Z_{Th} = z$ seen looking into a, b with 12V source shorted

$\therefore Z_{Th} = -j12k\Omega \,\|\, (-j3k\Omega + j15k\Omega + 9k\Omega)$

$'' = -j12k\Omega \,\|\, (9k\Omega + j12k\Omega)$

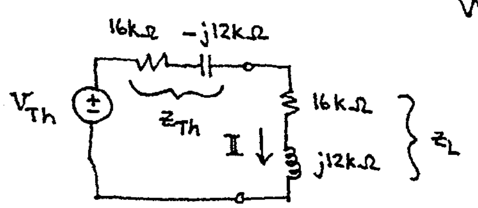
$'' = \dfrac{-j12k\Omega\,(9k\Omega + j12k\Omega)}{-j12\cancel{k\Omega} + 9k\Omega + j\cancel{12k\Omega}}$

$'' = -j\,\dfrac{12k\Omega}{9k\Omega}\,(9k\Omega + j12k\Omega)$

$'' = -j12k\Omega + 16k\Omega$

$$\boxed{Z_L = Z_{Th}^* = 16k\Omega + j12k\Omega}$$

b)



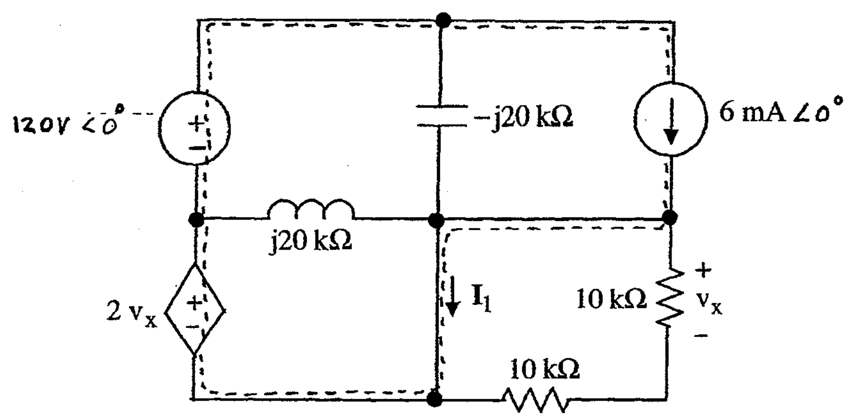
$P \equiv (\text{ave pwr in } Z_L) = \dfrac{|I|^2}{2} R$ where $R = 16k\Omega$ in Z_L

$$I = \frac{V_{Th}}{16k\Omega - j\cancel{12k\Omega} + 16k\Omega + j\cancel{12k\Omega}} = \frac{V_{Th}}{32k\Omega}$$

$$V_{Th} = 12V \cdot \frac{-j12k\Omega}{-\cancel{j12k\Omega} + -j\cancel{3k\Omega} + \cancel{j15k\Omega} + 9k\Omega} = 12V\left(\frac{-j\,12k\Omega}{9k\Omega}\right) = -j\,16\,V$$

$$\therefore P = \frac{\left|\dfrac{-j16V}{32k\Omega}\right|^2 \cdot 16k\Omega}{2} = \frac{|-j|^2\,V^2}{|2k\Omega|^2} \cdot \frac{16k\Omega}{2} = \frac{1\,V^2 \cdot 8}{4\,k\Omega} \text{ or } \boxed{P = 2\text{ mW}}$$

4. (25 points)



Pts

20 a. A frequency-domain circuit is shown above. Write the value of phasor $\mathbf{I}_1$ in polar form.

5 b. Given $\omega = \pi$ rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of $\mathbf{I}_1$.

Sol'n: a) Since the two 10kΩ resistors are shorted by wires.

∴ There is no v drop across the 10kΩ resistors,

and $v_x = 0V$.

Thus, the $2v_x$ dependent source $= 0V =$ wire

Superposition Case I: 6mA on, 120V off = wire.

It follows that all of the 6 mA from the independent current source flows in the wires (shown as dashed lines above).

$\therefore\ \mathbf{I}_{11} = 6\text{ mA}\angle 0°$

Case II: 120V on, 6mA off = open circuit

we observe that the $-j20k\Omega$ is directly across the 120V source, given the wires shown as dashed lines.

$$\therefore\ \mathbf{I}_{12} = \frac{120V\angle 0°}{-j\,20k\Omega} = j\,6mA = 6mA\angle 90°$$

Thus, $\mathbf{I}_1 = \mathbf{I}_{11} + \mathbf{I}_{12} = 6\text{ mA}\cdot(1+j)$

or $\boxed{\mathbf{I}_1 = \sqrt{2}\cdot 6\text{ mA}\angle 45°}$

b) $\boxed{i_1(t) = \sqrt{2}\cdot 6\text{ mA}\cos(\pi t + 45°)}$