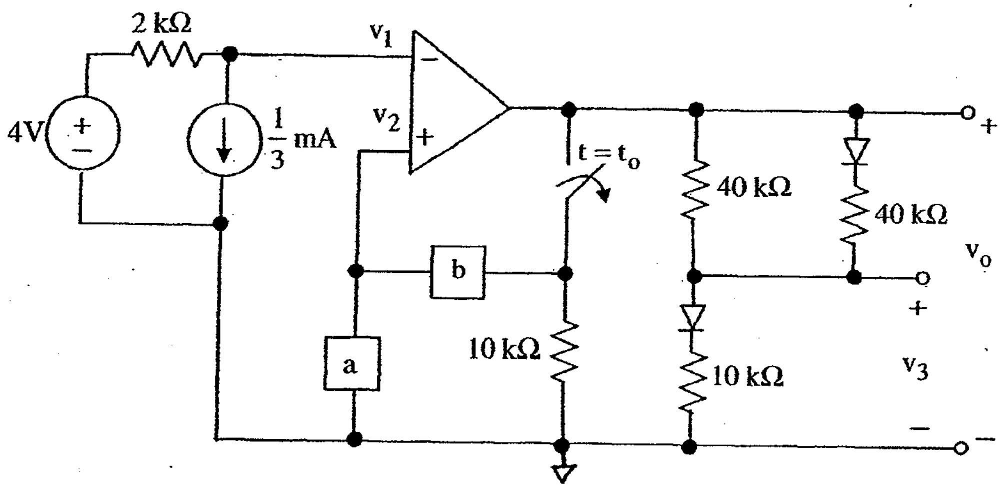
HW #10 cont.

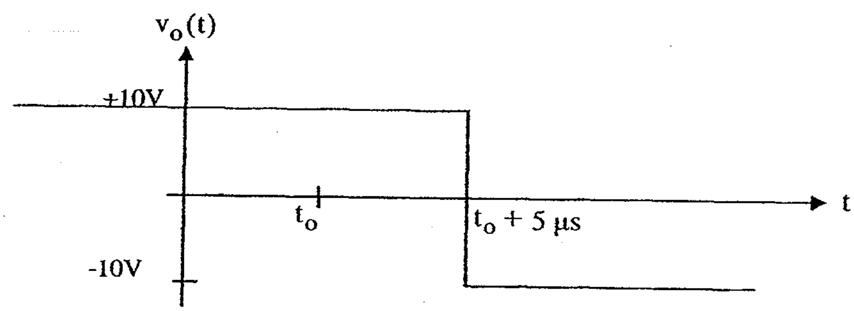
Su 2005

2.



Rail voltages = ±10 V

After being closed for a long time, the switch closes at $t = t_o$.



a. Choose either an R or L to go in box a and either an R or L to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value.

b. Sketch $v_1(t)$, showing numerical values appropriately.

c. Sketch $v_2(t)$, showing numerical values appropriately.

d. Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 5\ \mu s$, and for $t_o + 5\ \mu s < t$. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

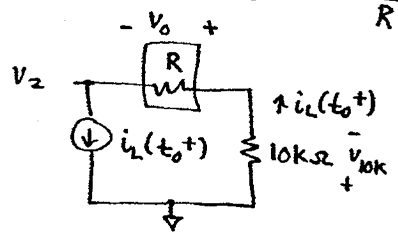
Explain your work carefully.

Sol'n: 2. a) Consider possibilities.

$a = R$ and $b = R$ — Doesn't work. V_2 would change at t_0 but never again. Delay of $5\,\mu s$ not possible.

$a = L$ and $b = L$ — Doesn't work. Before time t_0, the L's look like wires and would short V_0 to ref. But $V_0 = \pm V_{Rail} = \pm 10V$. Thus, we would have an invalid circuit.

$a = L$ and $b = R$ — Before t_0, the L looks like a wire, and $V_2 = 0V$. $i_L(t_0^-) = \frac{V_0}{R} = i_L(t_0^+)$.



Circuit at t_0^+.

At t_0^+, the current in R will be $i_L(t_0^+) = i_L(t_0^-) = \frac{V_0}{R}$ or the same as at t_0^-. Thus, the v-drop for R at t_0^+ will be V_0. Current $i_L(t_0^+)$ flowing in the 10kΩ will cause a voltage drop in series with the drop for R, resulting in a very negative voltage at V_2. This would cause V_0 to go low at $t = t_0^+$ rather than after a delay of $5\,\mu s$.

∴ This case doesn't work.

HW #10 Cont.

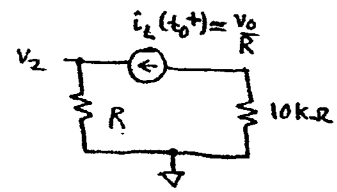
Su 05

ECE 1000

Sol'n: 2.a) cont. $\boxed{a = R \text{ and } b = L}$

$i_L(t_0^-) = \frac{V_0}{R}$ as in prev case.

At t_0^+ with $i_L(t_0^+) = i_L(t_0^-) = \frac{V_0}{R}$:



we have $V_2(t_0^+) = \underbrace{\frac{V_0}{R}}_{i_L(t_0^+)} \cdot R = V_0.$

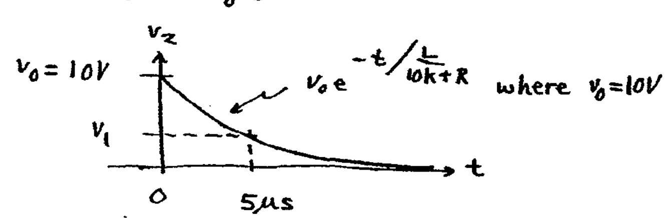
So V_2 doesn't change immediately.
This will work!
For $t \to \infty$ we will have $V_2 = 0V$
since there is no power source.

Using the general formula for $V_2(t)$, we have:

$$V_2(t) = \underbrace{V_2(t \to \infty)}_{=\,0V} + [\underbrace{V_2(t_0^+)}_{=\,V_0} - \underbrace{V_2(t \to \infty)}_{=\,0V}]\, e^{-t/\frac{L}{R_{Th}}}$$

where $R_{Th} = 10k\Omega + R$

$$V_2(t) = V_0\, e^{-t/\frac{L}{10k+R}}$$



The output, V_0, will drop at $t_0 + 5\mu s$ if $V_2(t_0 + 5\mu s) = V_1$.

[Assume $t_0 = 0$ for convenience.]

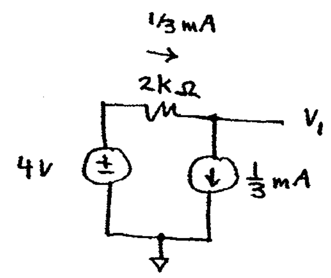
Now find V_1.

HW #10 Cont.

ECE 1000

Su 05

sol'n: 2.a) cont.



$$v_1 = 4V - \tfrac{1}{3}mA \cdot 2k\Omega$$

$$v_1 = 4V - \tfrac{2}{3}V = \tfrac{10}{3}V$$

$\therefore$ we want $v_2(5\mu s) = v_1 = \frac{10}{3}V$

or $\cancel{10}V\, e^{-5\mu s / \frac{L}{10k\Omega + R}} = \frac{\cancel{10}}{3}V$

$$-5\mu s \Big/ \frac{L}{10k\Omega + R} = \ln \tfrac{1}{3} \doteq -1.1$$

or $5\mu s = 1.1 \dfrac{L}{10k\Omega + R}$

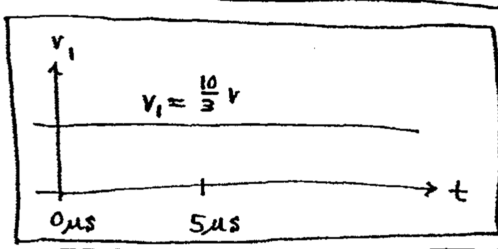
If we use R = 12k we get a convenient value for L. Many solutions for R and L will work, however.

R = 12kΩ gives $L = 5\mu s \cdot \dfrac{10k\Omega + 12k\Omega}{1.1}$

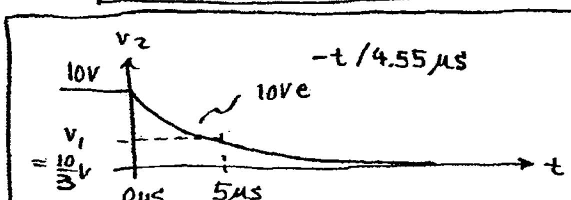
$$L = 5\mu s \cdot 20k\Omega = 100mH$$

$\boxed{R = 12k\Omega \qquad L = 100mH}$ One solution among many.

b)



c)



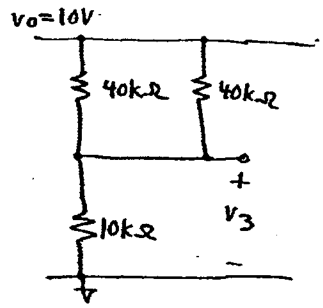
Note: $\dfrac{L}{R_{Th}} = \dfrac{100mH}{22k\Omega} = 4.55\mu s$

HW #10 Cont.

ECE 1000

Su 05

sol'n: 2.d) For $V_o = +10V$, both diodes are forward biased and look like wires.

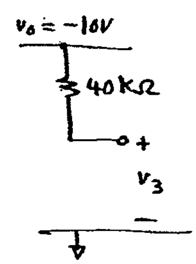


$$V_3 = 10V \cdot \frac{10k\Omega}{10k\Omega + 40k\Omega \| 40k\Omega}$$

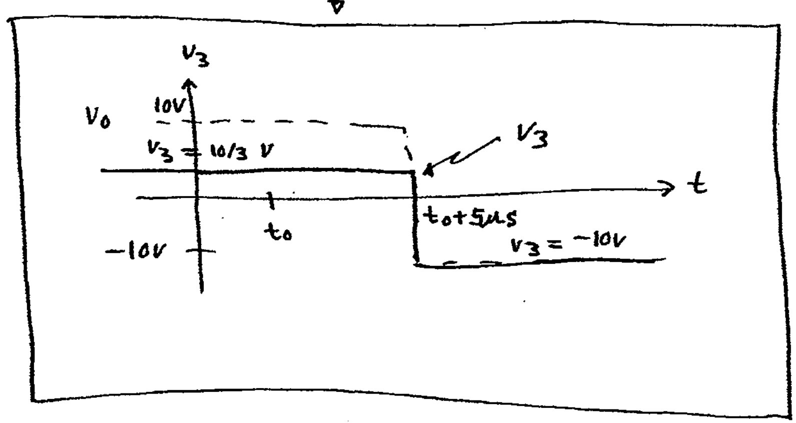
$$" = 10V \cdot \frac{10k\Omega}{10k\Omega + 20k\Omega}$$

$$V_3 = \frac{10}{3}\ V$$

For $V_o = -10V$, both diodes are reverse biased and look like open circuits.



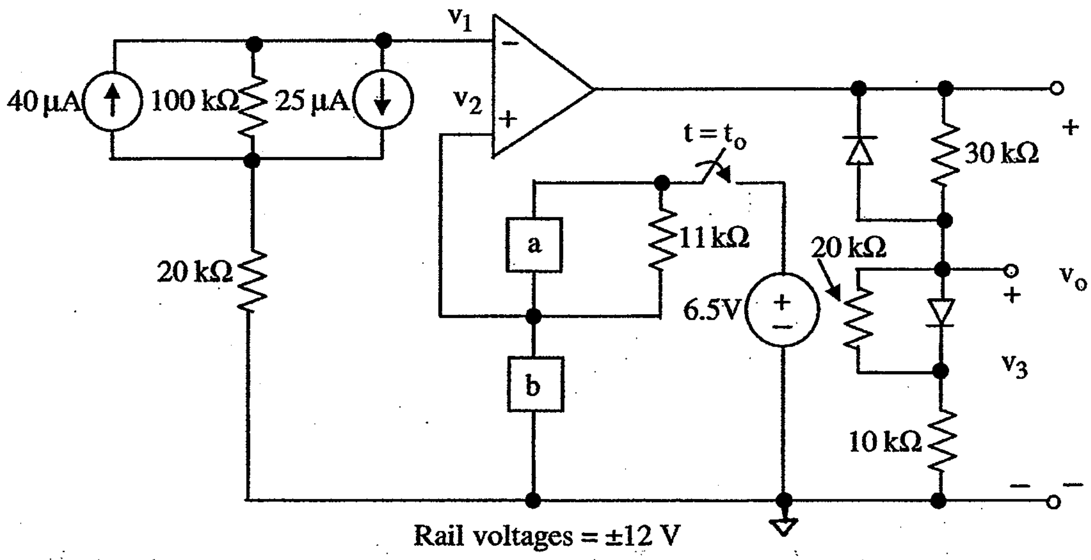
$V_3 = -10V$



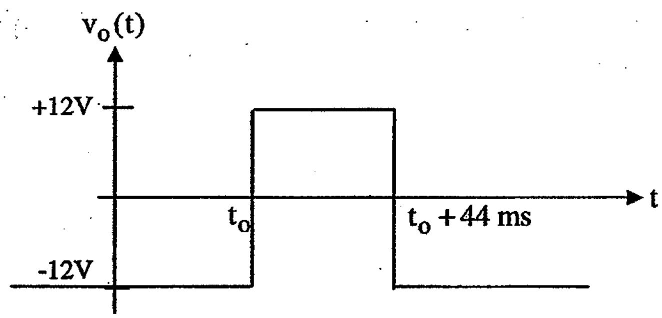
UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

ECE 1000 HOMEWORK #10 Spring 2005

1.



After being open for a long time, the switch closes at $t = t_o$.



a. Choose either an R or C to go in box a and either an R or C to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value. **Hint:** Use $v_2(t \to \infty) = 1V$

b. Sketch $v_1(t)$, showing numerical values appropriately.

2. a. Sketch $v_2(t)$, showing numerical values appropriately.

b. Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 44$ ms, and for $t_o + 44$ ms $< t$. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

Explain your work carefully.

17

sol'n: 1.a) Consider possibilities.

$a = R$ and $b = R$: v_2 would change at time $t = t_0$ but never change again. Thus, v_0 could not go high and then low again.

$a = C_1$ and $b = C_2$: Before $t = t_0$, there would be no path for C_2 in b to discharge. Thus, we would need to know $v_{C2}(t = t_0^-)$.

We would also have $v_{C1}(t = t_0^-) = 0V$ since C_1 will discharge thru the R in parallel with it.

When the switch closes, we have an invalid circuit if $v_{C2}(t_0^+) + v_{C1}(t_0^+) \neq 6.5V$. This means we must have $v_{C2}(t_0^+) = v_C(t_0^-) = 6.5V$ since $v_{C1}(t_0^+) = v_C(t_0^-) = 0V$.

But if $v_{C2}(t_0^+) = 6.5$, nothing changes when the switch closes. Thus, we could not get a waveform that goes high and then low again.

$a = R$ and $b = C$: As in the case of $a = C_1$ and $b = C_2$, we would have some value of $v_C(t_0^-) = v_C(t_0^+)$. Since $v_2 = v_C(t_0^+)$ at $t = t_0^+$, v_0 would not change at t_0. Thus, this will not work.

$\boxed{a = C \text{ and } b = R}$: The C in a will discharge thru the R in parallel with it. $\therefore v_C(t_0^-) = 0V$. Also, no current flows in R for $t < t_0$ because there is no closed circuit path in which current could flow. Thus, $v_2(t_0^-) = i \cdot R = 0V$.

sol'n: 1.a) cont.

At $t = t_o^+$, we have $v_2(t_o^+) = 6.5V - v_C(t_o^+)$ or $v_2(t_o^+) = 6.5V$ since $v_C(t_o^+) = v_C(t_o^-) = 0V$.

Thus, v_2 jumps from 0V to 6.5V at t_o.

If $v_2(t_o^+) > v_1$, then v_o would go high, (assuming $v_1 > 0V$ and $v_1 < 6.5V$).

After t_o, C will charge and v_2 will start to drop. If v_2 eventually drops below v_1, then v_o will go low again.

Thus, this will work.

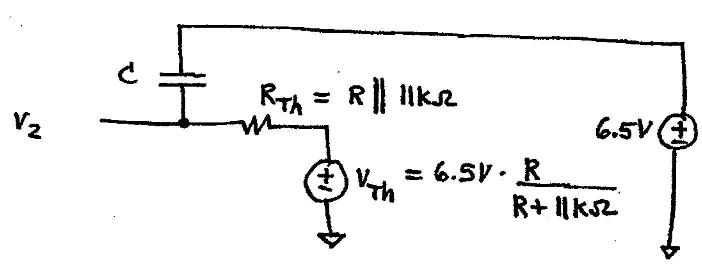
Now find values of R and C:

v_1: Since no current flows into the − input of the op amp, no current flows thru the 20kΩ resistor (lower left).

Combining the parallel current sources, we have $40\mu A - 25\mu A = 15\mu A$. This current flows thru the 100kΩ to produce voltage $15\mu A \cdot 100k\Omega = 1.5V$.

$\therefore v_1 = 2.5V$ (constant)

v_2: For $t > t_o$, we use a Thevenin equivalent circuit for the 6.5V source, the 11kΩ, and R in b.



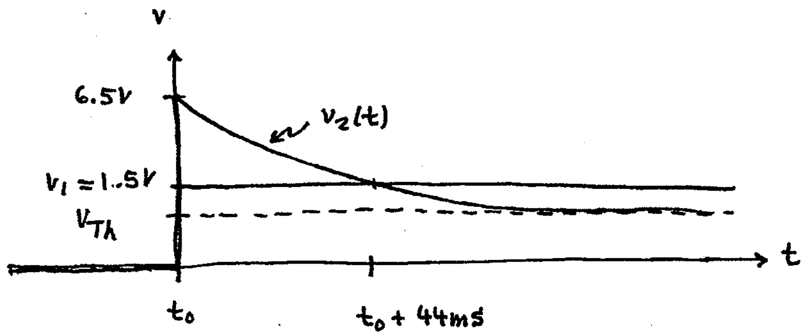
soln: 1.a) cont. We use the general RC soln for $v_2(t)$:

$$v_2(t) = v_2(t\to\infty) + [v_2(t_0^+) - v_2(t\to\infty)]\, e^{-t/R_{Th}C}$$

To find $v_2(t\to\infty)$, we observed that, when C is charged, no current flows in R_{Th}.

Thus $v_2(t\to\infty) = V_{Th} = 6.5V \dfrac{R}{R+11k\Omega} < 6.5V$.

From earlier, $v_2(t_0^+) = 6.5V$.



Any $V_{Th} < v_1$ will suffice. For convenience,

let $V_{Th} = 1V = 6.5V \cdot \dfrac{R}{R+11k\Omega} \Rightarrow \boxed{R = 2k\Omega}$

Assume $t_0 = 0$. For v_0 to switch at $t = 44ms$,

we have $v_2(44ms) = v_1 = 1.5V$

or $1V + [6.5V - 1V]\, e^{-44ms/R_{Th}C} = 0.5V$

or $5.5V\, e^{-44ms/R_{Th}C} = 0.5V$ where

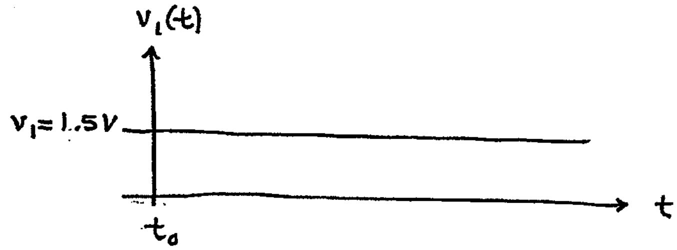
$R_{Th} = R \| 11k\Omega = 2k \| 11k\Omega = \dfrac{22}{13} k\Omega$.

Take ln of both sides of exponential RC eqn:

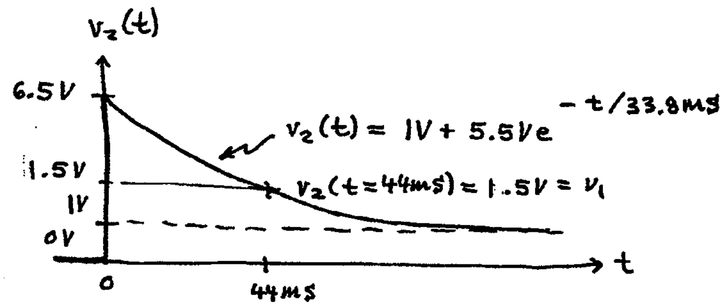
$$-\frac{\overset{2}{\cancel{44}}ms}{\frac{\cancel{22}}{13}k\Omega \cdot C} = \ln \frac{0.5V}{5.5V} \doteq -2.39$$

$$C = \frac{2}{\frac{2.39}{13}}\mu F = 10.9\mu F \qquad \boxed{C = 11\mu F}$$

sol'n: 1.b) $v_1(t) = 1.5V$ constant



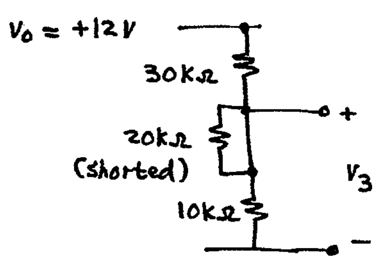
2.a)



Note: $R_{Th}\,C = \frac{22}{13}\,k\Omega \cdot 20\mu F \doteq 33.8\,ms$

Note: Assume $t_0 = 0$.

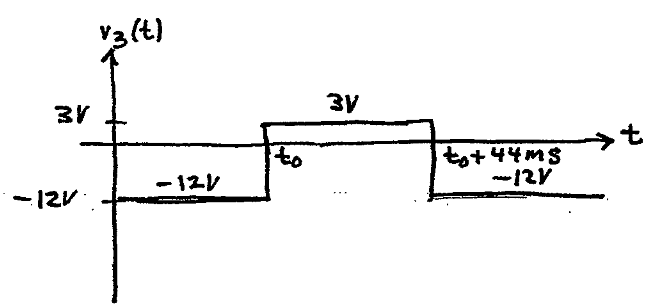
b) When $v_0 = +12V$, top diode is reverse biased = open. The bottom diode is forward biased = wire.



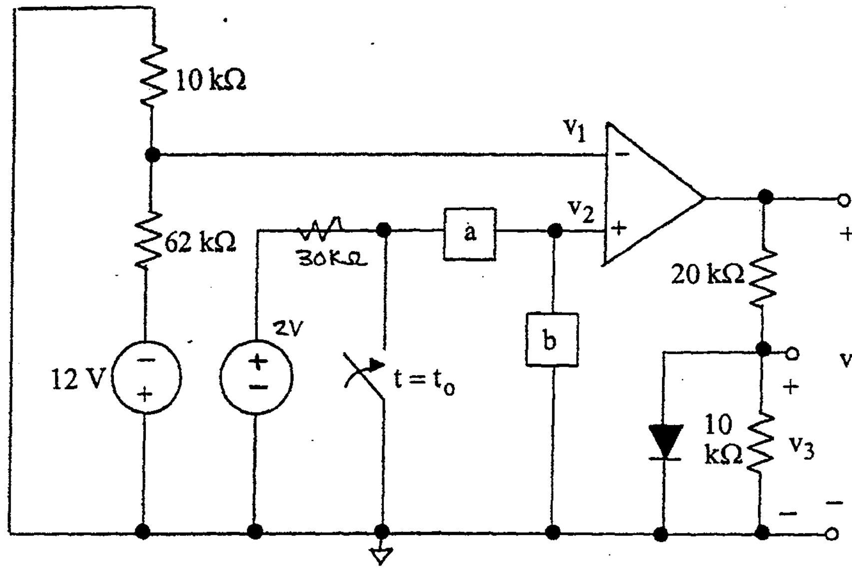
$$v_3 = v_0 \cdot \frac{10k\Omega}{10k\Omega + 30k\Omega} = 12V \cdot \frac{1}{4} = 3V$$

when $v_0 = -12V$, top diode is forward biased = wire.

Thus, v_0 is shorted to $-12V$.

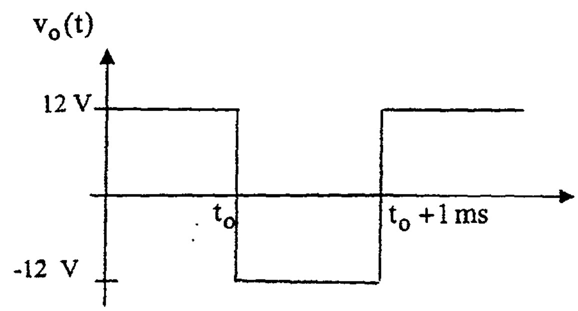


2. (65 points)



Rail voltages = ±12 V

After being open for a long time, the switch closes at $t = t_o$.



Pts

35 a. Choose either an R or C to go in box a and either an R or C to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value.

5 b. Sketch $v_1(t)$, showing numerical values appropriately.

15 c. Sketch $v_2(t)$, showing numerical values appropriately.

10 d. Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 1$ ms, and for $t < t_o + 1$ ms. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

22

soln: 2. a) At $t = t_0^-$: $v_1 = -12V \dfrac{10k\Omega}{10k\Omega + 62k\Omega}$ V-divider

$v_1 = -\frac{5}{3}V$ Actually, $v_1 = -\frac{5}{3}V$ for all t

$v_0 = +12V \Rightarrow v_2 > v_1 = -\frac{5}{3}V$

Consider $a = R$, $b = R$: v_0 would never switch because v_2 would be $0V$ after switch closed, and $v_2 > v_1$ for all time

Will <u>not</u> work.

Consider $a = C$, $b = C$: C's will charge to $V_{tot} = 2V$.

Part of $2V$ across a, part across b.

When switch closes, the C's instantly charge to $V_{tot} = 2V$.

Then their voltages remain fixed.

$\therefore v_0$ would not switch after 1 ms.

Will <u>not</u> work.

Consider $a = R$, $b = C$: C will charge to $2V$. $v_2 = 2V$

When switch closes, v_2 will start at v_2 and charge toward $0V$.

$v_2 > v_1$ for all time.

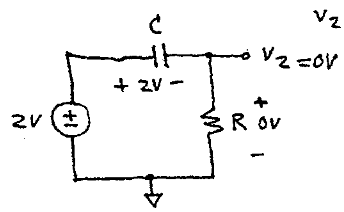
v_0 never switches.

Will <u>not</u> work.

Consider $\boxed{a = C,\ b = R}$: C will charge to $2V$ at t_0^-

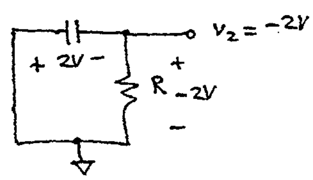
$v_2 > v_1$ so $v_0 = +12V$ at t_0^- ✓

t_0^-:



When switch closes, v_c will stay at $2V$ for $t = t_0^+$:

t_0^+:



Thus, v_2 drops to $-2V$.

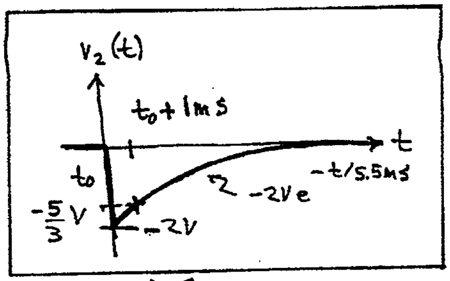
Then $v_2 < v_1$ and v_0 drops to $-12V$. ✓

$t > t_0$: C charges to $0V$ and v_2 climbs toward $0V$.

We want $v_2 = v_1 = -\frac{5}{3}V$ at $t_0 + 1ms$. ✓

<u>Works</u>

sol'n: 2.a) (cont.)



sol'n: 2.c) ↗

Let $t_0 = 0$

$v_2(t) = -2V\, e^{-t/RC}$

$v_2(1ms) = -2V e^{-1ms/RC} = -\frac{5}{3}V$

$e^{-1ms/RC} = \frac{5}{6}$

$-1ms/RC = \ln \frac{5}{6}$

$RC = \frac{-1ms}{\ln \frac{5}{6}} = 5.48ms$

$RC \approx 5.5\ ms$

Let $\boxed{C = 1\mu F, \quad R = 5.5k\Omega \text{ or } 5.6k\Omega \text{ (standard value)}}$

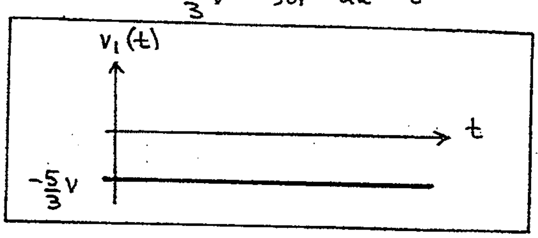
Any $RC = 5.5\ ms$ acceptable if $1pF < C < 1F$ and $1\Omega < R < 1G\Omega$.

Note: If R is $\frac{1}{8}W$ then we would really want $\max i_R^2 R < \frac{1}{8}W$.

$\max i_R^2 = \left(\frac{2V}{R}\right)^2 \Rightarrow \max i_R^2 R = \frac{4V^2}{R} < \frac{1}{8}W$

$\therefore R > \frac{4V^2}{1/8\ W} = 32\Omega$ required.

2.b) $v_1 = -\frac{5}{3}V$ for all t

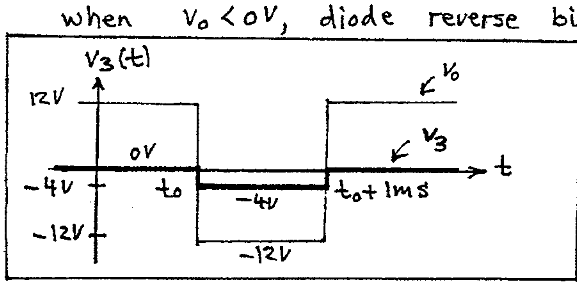


2.c) See plot of $v_2(t)$ in sol'n to 2.a), above.

2.d) when $v_0 > 0V$, diode forward biased = wire. $\therefore V_3 = 0V$

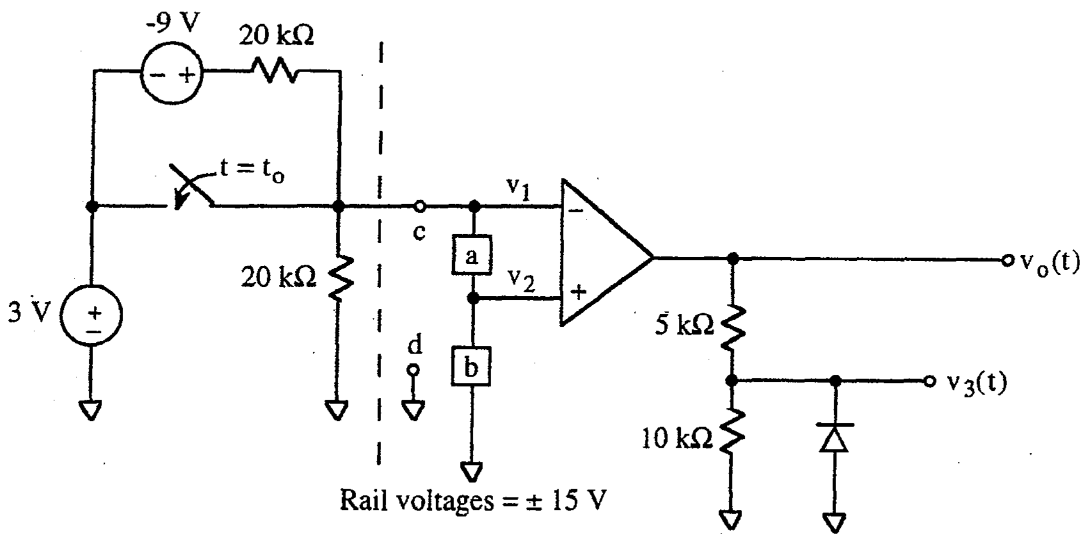
when $v_0 < 0V$, diode reverse biased = open. $\therefore V_3 = V_0 \frac{10k\Omega}{10k\Omega + 20k\Omega}$

$V_3 = -12V \cdot \frac{1}{3} = -4V$

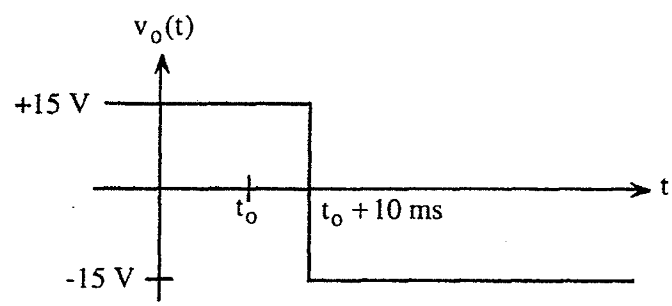


24

2. (70 points)



After being open for a very long time, the switch closes at $t = t_o$.

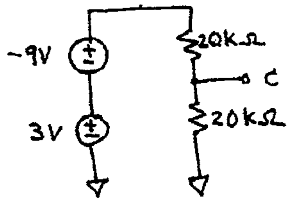


Pts		
10	a.	Find the Thevenin equivalent of the circuit left of the dashed line with respect to terminals c, d for $\mathbf{t < t_o}$.
10	b.	Find the Thevenin equivalent of the circuit left of the dashed line with respect to terminals c, d for $\mathbf{t > t_o}$.
20	c.	Choose either an R or C to go in box a and either an R or C to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value.
20	d.	Using the elements found in (b), sketch $v_2(t)$. Show numerical values appropriately.
10	e.	Sketch $v_3(t)$. Show numerical values for $t < t_o$, $t_o < t < t_o + 10$ ms, and $t > t_o + 10$ ms. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

Explain your work carefully.

25

sol'n 2. a) $t < t_o$ switch is open

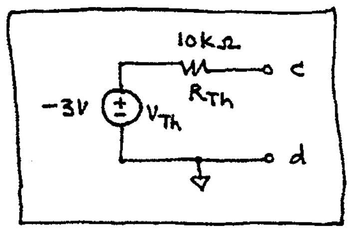


$$V_{Th} = (3V - 9V)\,\frac{20k\Omega}{20k\Omega + 20k\Omega} = -6V \cdot \frac{1}{2} = -3V$$

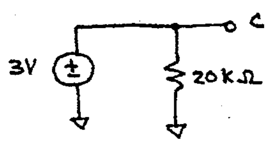
use V-divider

R_{Th} = R looking into c with V sources shorted

$= 20k\Omega \parallel 20k\Omega = 10\,k\Omega$



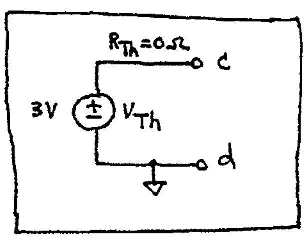
b) $t > t_o$ switch is closed (-9V source and 20kΩ bypassed)



$V_{Th} = 3V$ since c connected to 3V source

R_{Th} = R looking into c with 3V source shorted

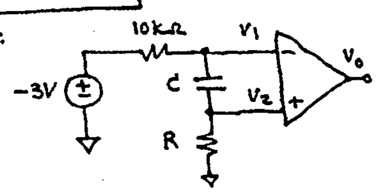
$= 0\Omega$ (20kΩ bypassed by short)



c) R in a and C in b would immediate switching: <u>wrong</u>

∴ $\boxed{\text{C in a and R in b}}$ must be correct.

$t < t_o$ circuit:
$(t = t_o^-)$



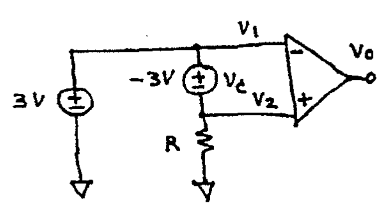
C is open circuit
no V drop across R's

∴ $V_2 = 0V$

$V_1 = -3V$

$V_1 < V_2 \Rightarrow V_o = +15V$ (rail)

$t = t_o^+$ circuit:



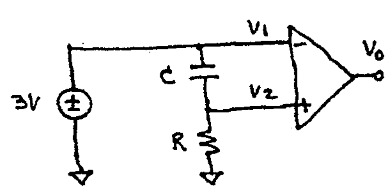
C acts like -3V source

∴ $V_1 = 3V$

$V_2 = 3V - -3V = 6V$

$V_1 < V_2 \Rightarrow V_o = +15V$ (rail)

$t \to \infty$ circuit:



C acts like open circuit
no V drop across R

∴ $V_1 = 3V$

$V_2 = 0V$

$V_1 > V_2 \Rightarrow V_o = -15V$ (rail) sol'n 2.c) cont. Let $t_0 = 0$. Want $V_1 = V_2$ at $\Delta t \equiv 10\,ms$ for V_o transition.

$V_1 = 3V$ for $t > t_0 = 0$

$V_2 = 3V - V_c$ for $t > t_0$.

$$V_2(t>0) = V_2(t\to\infty) + [V_2(0^+) - V_2(t\to\infty)]\,e^{-t/RC}$$

$$" \quad = 0V + [6V - 0V]\,e^{-t/RC} = 6V\,e^{-t/RC}$$

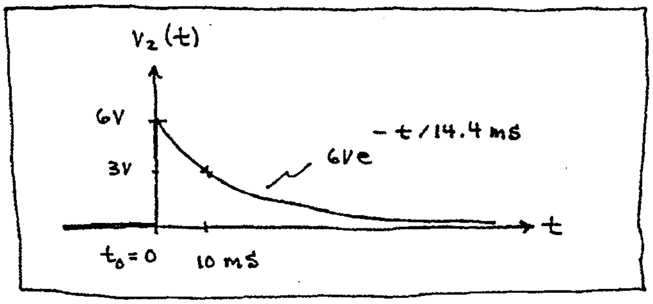
We want $V_2(\Delta t) = 3V$. $\therefore \quad 6V\,e^{-\Delta t/RC} = 3V$

$$e^{-\Delta t/RC} = \frac{3V}{6V} = \frac{1}{2} \quad \text{or} \quad -\frac{\Delta t}{RC} = \ln\frac{1}{2}$$

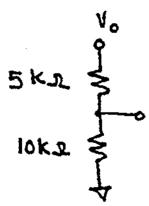
$$RC = \frac{-\Delta t}{\ln\frac{1}{2}} = \frac{\Delta t}{\ln 2} = \frac{10\,ms}{\ln 2} = 14.4\ ms$$

Use $\boxed{C = 1\,\mu F, \quad R = 14.4\,k\Omega}$ (15kΩ is closest standard value)

d) $V_2(t<0) = 0V$ $\quad V_2(t>0) = 6V e^{-t/14.4\,ms}$ $\quad V_2(10\,ms) = 3V$

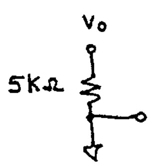


e) When $V_o = +15V$, the diode is reverse biased ⇒ open circuit (i.e. disappears)

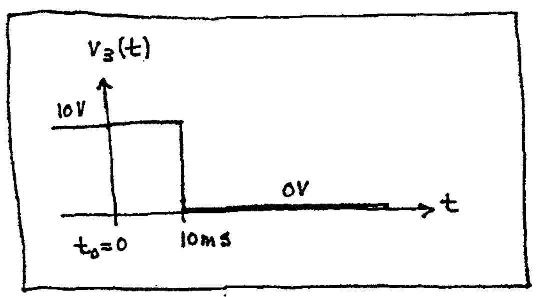


$$V_3 = V_o \cdot \frac{10k\Omega}{10k\Omega + 5k\Omega} = 15V \cdot \frac{2}{3} = 10V$$

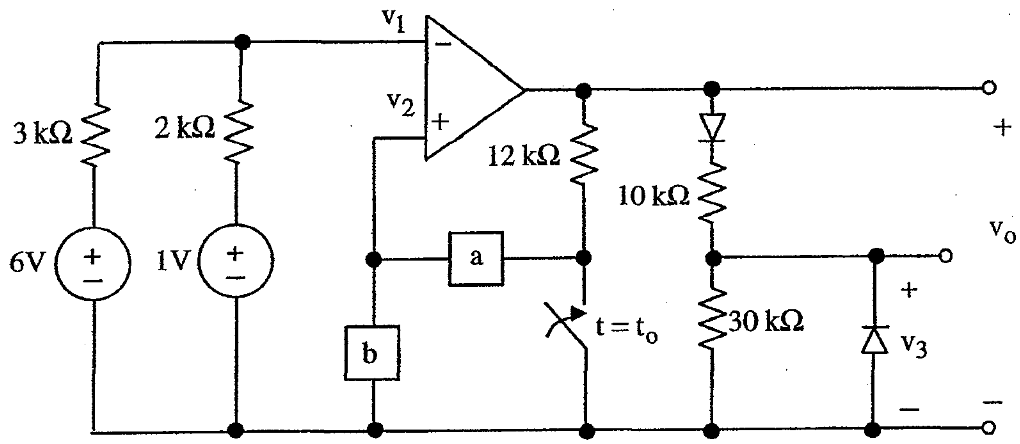
When $V_o = -15V$, the diode is forward biased ⇒ short



$V_3 = 0V$

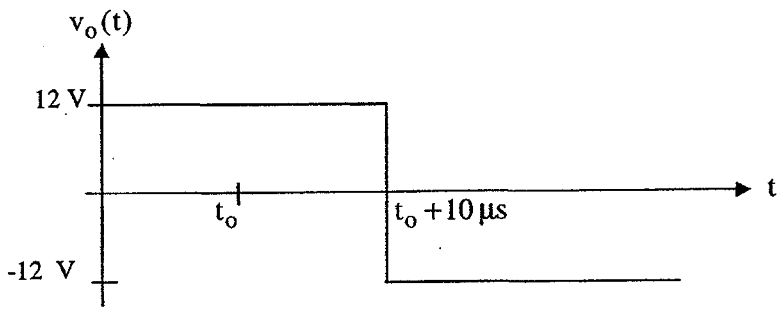


2. (65 points)



Rail voltages = ±12 V

After being open for a long time, the switch closes at $t = t_o$.



Pts

35 a. Choose either an R or L to go in box a and either an R or L to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value.

5 b. Sketch $v_1(t)$, showing numerical values appropriately.

15 c. Sketch $v_2(t)$, showing numerical values appropriately.

10 d. Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 10\ \mu s$, and for $t_o + 10\ \mu s < t$. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

Explain your work carefully.

28

soln: 2.a) Ignore diode and resistor network on output (since it doesn't affect v_o).

For $t < t_o$, $v_o = +12V \Rightarrow v_2 > v_1$.

Calculate v_1 using node-v method: $\frac{v_1 - 6V}{3k\Omega} + \frac{v_1 - 1V}{2k\Omega} = 0A$

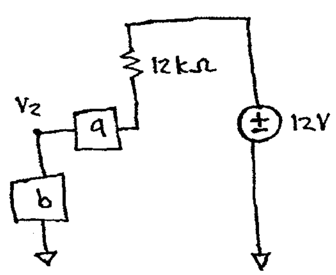
$$v_1\left(\frac{1}{3k\Omega} + \frac{1}{2k\Omega}\right) = \frac{6V}{3k\Omega} + \frac{1V}{2k\Omega}$$

mult both sides by $6k\Omega$

$$v_1\ (2+3) = 6V \cdot 2 + 1V \cdot 3 = 15V, \quad v_1 = \frac{15V}{5} = 3V$$

So we need $\underline{\underline{v_2 > 3V.}}$

Circuit model: $v_o = 12V$, switch open



If we have an L, it will be equivalent to a wire.

Consider possibilities:

case I: $a = L$ and $b = L$ $\quad L = \text{wire}$

$v_2 = 0V$ from v divider

Doesn't work.

case II: $a = R$ and $b = L = \text{wire}$

$v_2 = 0V$ from v divider

Doesn't work.

case III: $a = R_1$ and $b = R_2$

We can choose R_1 and R_2 to achieve $v_2 > 3V$, but we cannot get a delay in v_o dropping from $+12V$ to $-12V$.

case IV: $a = L$ and $b = R$

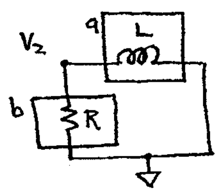
Since $L = \text{wire}$ and we can pick R, we can achieve $v_2 > 3V$. When switch moves, the L continues to carry same current initially. Thus, $v_2 > v_1$ is sustained for delay. Should work.

soln: 2.a) cont. When switch closes, we have RL circuit that determines v_2. Time constant $\tau = L/R$. Output v_o drops when v_2 drops below 3V.

As $t \to \infty$, the L in 'a' acts like a wire and the switch is closed $\Rightarrow v_2(t\to\infty) = 0V$

Without additional constraints, we may choose any v_2 between 3V and 12V. One choice is $v_2(0^-) = 6V$. Using v-divider of 12kΩ and 'b', $\boxed{b = 12k\Omega}$.

We want $v_2(t = 10\mu s) = 3V$ so v_o drops at time $t_o + 10\mu s$. (Assume $t_o = 0s$)

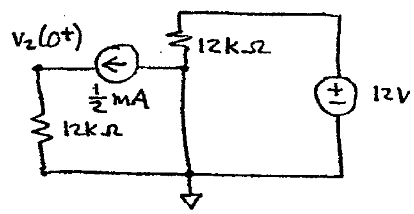


$$v_2(t>0) = v_2(t\to\infty) + [v_2(0^+) - v_2(t\to\infty)]e^{-t/\tau}$$
$$" \quad = 0V + [v_2(0^+) - 0V]e^{-t/\tau}$$
$$" \quad = v_2(0^+)e^{-t/\tau},$$ Now find $v_2(0^+)$.

Consider $t = 0^-$: L = wire R = 12kΩ

$$i_L(0^-) = \frac{12V}{12k\Omega + 12k\Omega} = \frac{1}{2}mA$$

$t = 0^+$: L = i src where $i_L(0^+) = i_L(0^-) = \frac{1}{2}mA$



$$v_2(0^+) = \frac{1}{2}mA \cdot 12k\Omega$$
$$v_2(0^+) = 6V$$

$$v_2(t>0) = 6V e^{-t/\tau}$$

We want $v_2(10\mu s) = 3V = 6V e^{-10\mu s/\tau}$

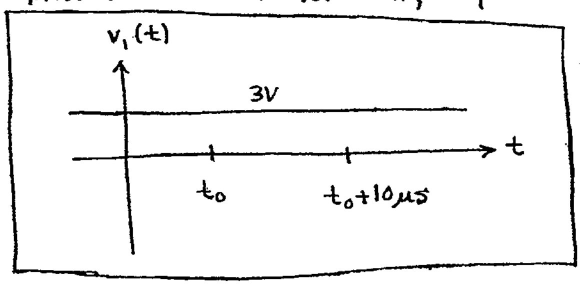
$$\frac{3V}{6V} = \frac{1}{2} = e^{-10\mu s/\tau}, \qquad \ln\frac{1}{2} = -10\mu s/\tau$$

$$\tau = \frac{-10\mu s}{\ln\frac{1}{2}} = \frac{10\mu s}{\ln 2} = 14.4\,\mu s = \frac{L}{R} = \frac{L}{12k\Omega}$$

$$L = 14.4\mu s \cdot 12k\Omega = 173\text{ mH}$$

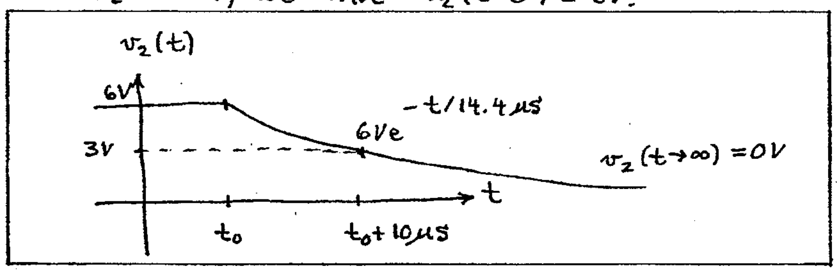
Summary: $\boxed{\begin{array}{l} a = L = 173\text{ mH} \\ b = R = 12k\Omega \end{array}}$

soln: 2.b) As shown in sol'n for (a), $v_1(t) = 3V$. It never changes.

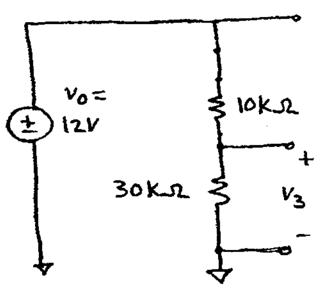


c) From sol'n to (a), we have $v_2(t>0) = 6V e^{-t/14.4\mu s}$.

For $v_2(t<0)$, we have $v_2(t<0) = 6V$.

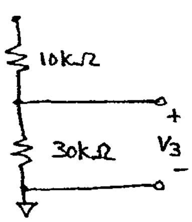


d) When $v_0 > 0V$, top diode = wire, bottom diode = open.



$$V_3 = 12V \cdot \frac{30k\Omega}{30k\Omega + 10k\Omega} = 9V$$

When $V_0 < 0V$, top diode = open, bottom diode doesn't matter since no current



$V_3 = 0V$ since no current flowing