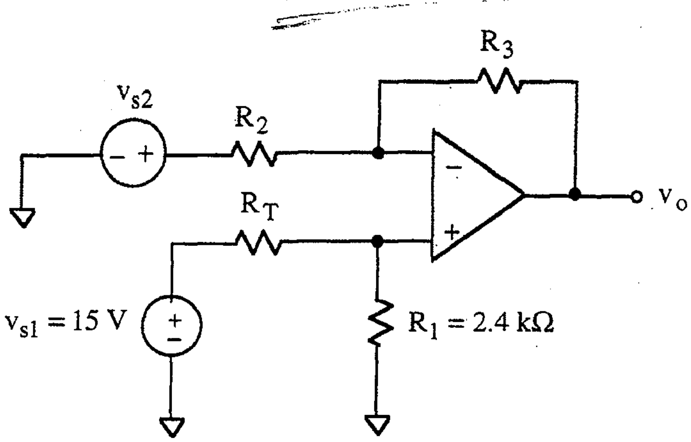
EE 1000

Final Exam soln

Spring 2001

1. (70 points)



Rail voltage = ± 15 V

Design an electronic thermometer using the circuit diagram shown above. The voltage v_o is used to indicate temperature. Use a thermister with a resistance described by

$$R_T = R_o e^{1120\left(\frac{1}{T} - \frac{1}{300}\right)}$$

where $R_o = 2490\ \Omega$ and T is temperature in °K.

Pts

10 a. ~~Calculate the numerical values of R_T (273°K) and R_T (373°K).~~

25 b. Derive a symbolic expression for v_o. The expression must contain not more than the parameters v_{s1}, v_{s2}, R_T, R_1, R_2, and R_3.
Hint: Use superposition.

25 c. Choose v_{s2}, R_2, and R_3 that will produce the following:

$v_o = 0$ V when T = 273°K
$v_o = 10$ V when T = 373°K

10 d. Using the component values you chose in (c), calculate v_o when T = 323°K.

sol'n 1.a)

$$R_T(273\,°K) = 2490\, e^{1120\left(\frac{1}{273} - \frac{1}{300}\right)} = 3.6\,k\Omega \qquad \boxed{R_T(273\,°K) = 3.6\,k\Omega}$$

$$R_T(373\,°K) = 2490\, e^{1120\left(\frac{1}{373} - \frac{1}{300}\right)} = 1.2\,k\Omega \qquad \boxed{R_T(373\,°K) = 1.2\,k\Omega}$$

b) Superposition:

V_{S1} on, V_{S2} off $\Rightarrow$ $V_p = V_{S1} \cdot \dfrac{R_1}{R_1 + R_T}$, $V_n = V_p$

$$i_f\,(\text{thru } R_2) = \frac{0 - V_n}{R_2} = -V_{S1}\,\frac{R_1}{R_1 + R_T}\,\frac{1}{R_2}$$

$$i_f\,(\text{thru } R_3) = \frac{V_n - V_{01}}{R_3} = -V_{S1}\,\frac{R_1}{R_1 + R_T}\,\frac{1}{R_3}$$

$$i_f\,(\text{thru } R_2) = i_f\,(\text{thru } R_3) \Rightarrow \frac{-V_n}{R_2} = \frac{V_n - V_o}{R_3}$$

or $$V_{01} = V_n\left(1 + \frac{R_3}{R_2}\right) = V_{S1}\,\frac{R_1}{R_1 + R_T}\left(1 + \frac{R_3}{R_2}\right)$$

V_{S1} off, V_{S2} on $\Rightarrow$ $V_p = 0V$, $V_n = V_p$

$$i_f\,(\text{thru } R_2) = \frac{V_{S2}}{R_2} \qquad i_f\,(\text{thru } R_3) = -\frac{V_{02}}{R_3}$$

$$i_f\,(\text{thru } R_2) = i_f\,(\text{thru } R_3) \Rightarrow V_{02} = -V_{S2}\,\frac{R_3}{R_2}$$

$$V_o = V_{01} + V_{02} \qquad \boxed{V_o = V_{S1}\,\frac{R_1}{R_1 + R_T}\left(1 + \frac{R_3}{R_2}\right) - V_{S2}\,\frac{R_3}{R_2}}$$

c) At $273\,°K$, $$V_{S1}\,\frac{R_1}{R_1 + R_T} = 15V\,\frac{2.4k\Omega}{2.4k\Omega + 3.6k\Omega} = 15V \cdot \frac{2}{5} = 6V = V_p$$

At $373\,°K$, $$V_{S1}\,\frac{R_1}{R_1 + R_T} = 15V\,\frac{2.4k\Omega}{2.4k\Omega + 1.2k\Omega} = 15V \cdot \frac{2}{3} = 10V = V_p$$

$$V_o = \underbrace{V_p\left(1 + \frac{R_3}{R_2}\right)}_{\text{proportional to } V_p} - \underbrace{V_{S2}\,\frac{R_3}{R_2}}_{\text{constant}}$$

The change in V_o vs change in V_p:

$$\Delta V_o = \Delta V_p\left(1 + \frac{R_3}{R_2}\right)$$

$$\Delta V_p = V_p(373\,°K) - V_p(273\,°K) = 10V - 6V = 4V$$

$$\Delta V_o = V_o(373\,°K) - V_o(273\,°K) = \underbrace{10V - 0V}_{\text{from prob statement}} = 10V$$

sol'n 1.c) cont. $\Delta V_o = \Delta V_p\left(1+\frac{R_3}{R_2}\right)$ or $10V = 4V\left(1+\frac{R_3}{R_2}\right)$

$\therefore\ 1+\frac{R_3}{R_2} = 2.5$ or $\frac{R_3}{R_2} = 1.5$

Let $\boxed{R_3 = 15\,k\Omega}$, $\boxed{R_2 = 10\,k\Omega}$.

$V_o(273°K) = 0V$ (from prob statement) $= V_p\left(1+\frac{R_3}{R_2}\right) - V_{s2}\frac{R_3}{R_2} = 6V(2.5) - V_{s2}(1.5)$

or $V_{s2} = \frac{6(2.5)V}{1.5}$ or $\boxed{V_{s2} = 10V}$

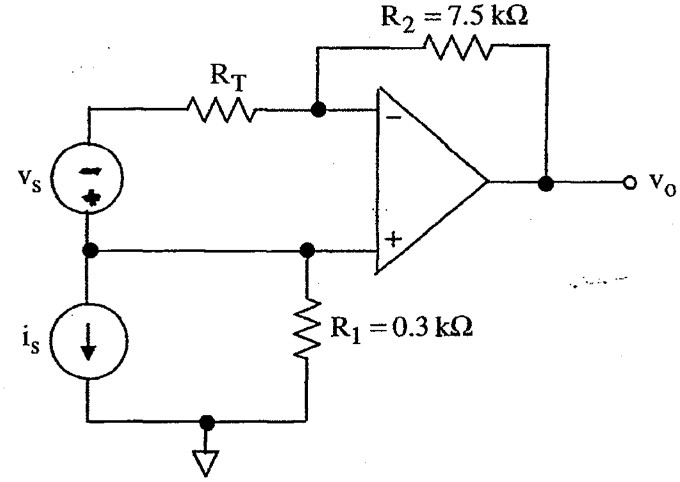
d) $R_T(323°K) = 2490\, e^{1120\left(\frac{1}{323}-\frac{1}{300}\right)} = 1.9\,k\Omega$

$V_p = 15V \cdot \frac{2.4k\Omega}{2.4k\Omega + 1.9k\Omega} = 8.37\ V$

$V_o = V_p\left(1+\frac{R_3}{R_2}\right) - V_{s2}\frac{R_3}{R_2} = 8.37(2.5) - 10(1.5)\ V$

$\boxed{V_o(323°K) = 5.9\,V}$ off by 18% from linear value of 5V.

1. (75 points)



Rail voltage = ± 15 V

Design an electronic thermometer using the circuit diagram shown above. The voltage v_O is used to indicate temperature. Use a thermister with a resistance described by

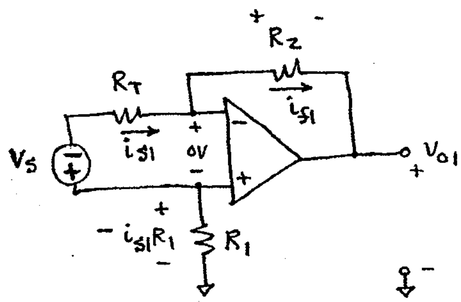
$$R_T = R_o e^{\beta\left(\frac{1}{T}-\frac{1}{300}\right)}$$

where $R_o = 2.625\ k\Omega$, $\beta = 1200°K$, and T is temperature in °K.

Pts		
30	a.	Derive a symbolic expression for v_O. The expression must contain not more than the parameters i_s, V_S, R_1, R_2, and R_T. **Hint: Use superposition.**
10	b.	Calculate the numerical values of R_T (273°K) and R_T (373°K).
15	c.	Determine a value for v_s such that $v_o(T = 373°K) - v_o(T = 273°K) = 1V$
10	d.	Using your answer to (c), determine a value of i_s such that $v_O(T = 273°K) = 0\ V$.
10	e.	Using the component values you chose above, calculate v_O when T = 323°K. Make a rough sketch of v_O vs. T on the basis of the values when T = 273°K, 323°K, and 373°K. On the same axes, sketch the ideal linear response.

Sol'n: 1. a) Use superposition

Case I: v_s on, i_s off = open



Negative feedback $\Rightarrow$ 0V across + and - terminals.

From v loop around v_s, R_T, and +- terminals,

we have $i_{s1} = \frac{-v_s}{R_T}$.

From v loop around R_1, +- terminals, R_2, and v_o,

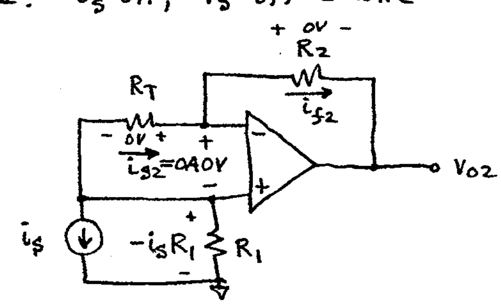
we have $-i_{s1}R_1 + 0V - i_{f1}R_2 - v_{o1} = 0V$

Now use $i_{s1} = i_{f1}$ and solve for v_{o1}:

$$- -\frac{v_s}{R_T} R_1 + 0V - -\frac{v_s}{R_T} R_2 - v_{o1} = 0V$$

$$v_{o1} = v_s \frac{R_1 + R_2}{R_T}$$

Case II: i_s on, v_s off = wire



We 0V across R_T $\Rightarrow$ $i_s = 0$.

Since $i_{s2} = i_{f2} = 0A$, we have $v_{o2} = v_- = v_+$

$v_+ = -i_s R_1 \Rightarrow v_{o2} = -i_s R_1$

Sum $\boxed{v_o = v_{o1} + v_{o2} = v_s \frac{R_1 + R_2}{R_T} - i_s R_1}$

soln: 1. b) $R_T(273^\circ K) = 2.625\,k\Omega\, e^{1200^\circ K\left(\frac{1}{273^\circ K}-\frac{1}{300^\circ K}\right)}$

$$\boxed{R_T(273^\circ K) \doteq 3.9\,k\Omega}$$

$$R_T(373^\circ K) = 2.625\,k\Omega\, e^{1200^\circ K\left(\frac{1}{373^\circ K}-\frac{1}{300^\circ K}\right)}$$

$$\boxed{R_T(373^\circ K) \doteq 1.2\,k\Omega}$$

c) $$1V = V_o(373^\circ K) - V_o(273^\circ K) = V_s\frac{(R_1+R_2)}{R_T(373^\circ K)} - \cancel{i_s R_1} - \left(V_s\frac{R_1+R_2}{R_T(273^\circ K)} - \cancel{i_s R_1}\right)$$

$$= V_s(R_1+R_2)\left(\frac{1}{R_T(373^\circ K)} - \frac{1}{R_T(273^\circ K)}\right)$$

$$= V_s\,(\underbrace{0.3k\Omega + 7.5k\Omega}_{7.8k\Omega})\left(\frac{1}{1.2k\Omega} - \frac{1}{3.9k\Omega}\right)$$

$$V_s = \frac{1.2k\Omega \,\|\, (-3.9k\Omega)\,V}{7.8\,k\Omega} = \frac{\cancel{0.3k\Omega}}{\cancel{0.3k\Omega}}\cdot\frac{4\,\|\,-13V}{26} = \frac{-4\cdot\cancel{13}}{(4-13)\,\cancel{26}_{2}}\,V$$

$$\boxed{V_s = \frac{2}{9}\,V}$$

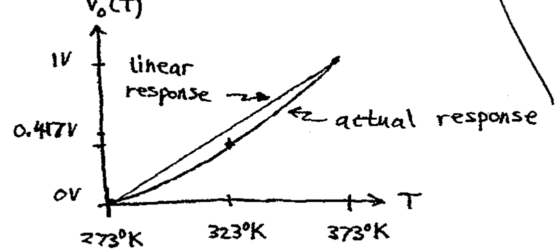
d) $$0V = V_o(273^\circ K) = V_s\frac{R_1+R_2}{R_T(273^\circ K)} - i_s R_1 = \frac{2}{9}V\,\frac{\cancel{7.8k\Omega}^{\,2}}{\cancel{3.9k\Omega}} - i_s\,0.3k\Omega$$

$$i_s\,0.3k\Omega = \frac{4}{9}V \Rightarrow i_s = \frac{4}{9}\,\frac{1V}{0.3k\Omega} = \frac{4}{2.7}\,mA$$

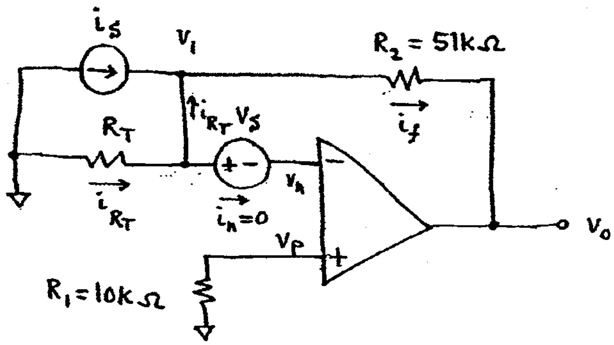
$$\boxed{i_s \doteq 1.48\,mA \approx 1.5\,mA}$$

e) $$R_T(323^\circ K) = 2.625\,k\Omega\, e^{1200^\circ K\left(\frac{1}{323^\circ K}-\frac{1}{300^\circ K}\right)} = 1.97\,k\Omega \approx 2k\Omega$$

$$V_o(323^\circ K) \approx \frac{2}{9}V\,\frac{7.8k\Omega}{2k\Omega} - 1.5\,mA\;0.3k\Omega = 0.417\,V$$



sol'n: 1.a)



- Find v_p: $v_p = 0V$ since no current in R_1 so no v-drop for R_1

$v_n = v_p = 0V$

- Find i_f on left side (total current into v_1 node from i_s and R_T)

$v_1 = v_n + v_s = v_s$, and no current flows thru v_s since no current flows into op-amp.

$\therefore$ i_f on left side $= i_s + i_{R_T} = i_s + \frac{0 - v_1}{R_T} = i_s + \frac{-v_s}{R_T}$

- Find i_f on right side

i_f on right side $= \frac{v_1 - v_o}{R_2} = \frac{v_s - v_o}{R_2}$

- Set i_f on left side $= i_f$ on right side

$$i_s - \frac{v_s}{R_T} = \frac{v_s - v_o}{R_2} \quad \text{or} \quad \boxed{v_o = v_s\left(1 + \frac{R_2}{R_T}\right) - i_s R_2}$$

verify: Superposition $v_s = 0 \Rightarrow i_{R_T} = 0 \Rightarrow v_o = 0V - i_s R_2$ ✓

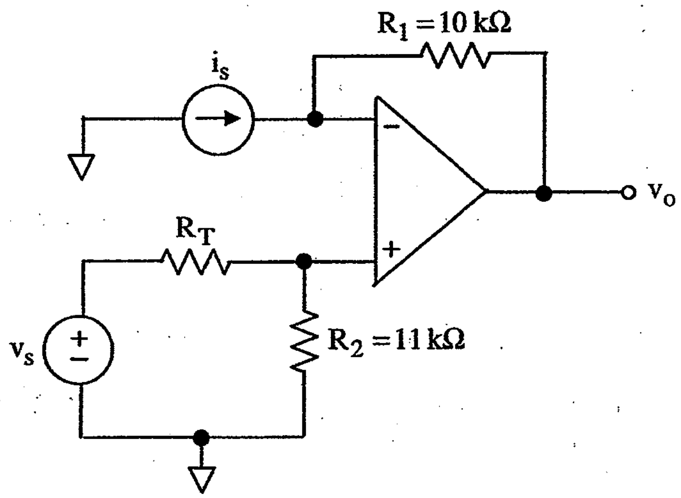
$i_s = 0$ $\quad i_{R_T} = \frac{-v_s}{R_T} = i_f = \frac{v_s - v_o}{R_2}$

Same as having $v_p = v_s$ $\quad v_o = v_s\left(1 + \frac{R_2}{R_T}\right)$ ✓

UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

ECE 1000 HOMEWORK #9 Spring 2005

1.



Rail voltage = ± 15 V

Design an electronic thermometer using the circuit diagram shown above. The voltage v_o is used to indicate temperature. Use a thermister with a resistance described by

$$R_T = R_o e^{\beta\left(\frac{1}{T}-\frac{1}{300}\right)}$$

where $R_o = 12.25\,k\Omega$, $\beta = 170°K$, and T is temperature in °K. Derive a symbolic expression for v_o. The expression must contain not more than the parameters i_s, V_s, R_1, R_2, and R_T. **Hint: Use superposition.**

2. a. ~~Calculate the numerical values of R_T (273°K) and R_T (373°K).~~

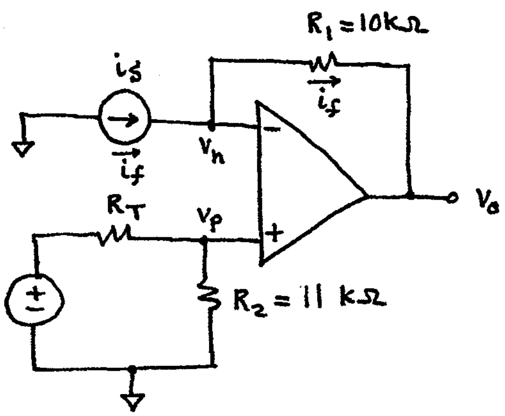
b. ~~Determine a value for v_s such that $v_o(T = 373°K) - v_o(T = 273°K) = 1V$~~

c. ~~Using your answer to (c), determine a value of i_s such that~~

v_o ~~(T = 273°K) = 0~~ V.

d. ~~Using the component values~~ you chose above, calculate v_o when T = 323°K. Make a rough sketch ~~of v_o vs. T on the basis of the~~ values when T = 273~~°K, 323°K, and 373°K. On the same axes, sketch the ideal~~ linear response.

soln: 1.



Find v_p:

$$v_p = v_s \frac{R_2}{R_2 + R_T} \qquad \text{V-divider}$$

$v_n = v_p$

Find i_f on left: $i_f = i_s$

Find i_f on right: $i_f = \dfrac{v_n - v_o}{R_1}$

Set i_f's equal and use $v_n = v_p$

$$i_s = \frac{v_n - v_o}{R_1} \quad \text{or} \quad v_o = v_n - i_s R_1$$

$$\boxed{v_o = v_s \frac{R_2}{R_2 + R_T} - i_s R_1}$$

2. a) $R_T(273°K) = 12.25k\Omega \cdot e^{170°K\left(\frac{1}{273°K} - \frac{1}{300°K}\right)} = 13k\Omega$

$R_T(373°K) = 12.25k\Omega \cdot e^{170°K\left(\frac{1}{373°K} - \frac{1}{300°K}\right)} = 11k\Omega$

$$\boxed{\begin{array}{l} R_T(273°K) = 13k\Omega \\ R_T(373°K) = 11k\Omega \end{array}}$$

b) $1V = v_o(373°K) - v_o(273°K) = v_s \dfrac{R_2}{R_2 + R_T(373°K)} - \cancel{i_s R_1}$

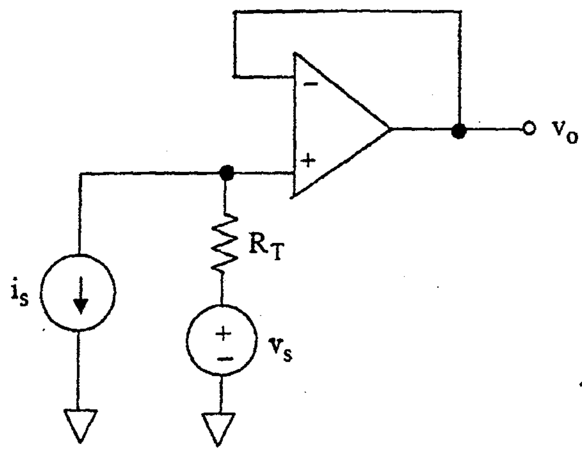
$- \left(v_s \dfrac{R_2}{R_2 + R_T(273°K)} - \cancel{i_s R_1}\right)$

$1V = v_s \cdot 11k\Omega \cdot \left(\dfrac{1}{11k\Omega + 11k\Omega} - \dfrac{1}{11k\Omega + 13k\Omega}\right)$

$$\boxed{v_s = \frac{1V}{11k} \frac{22k\Omega \cdot 24k\Omega}{24k\Omega - 22k\Omega} = 24\ V}$$

ECE 1000 Final Exam sol'n Sp 03

1. (75 points)



Rail voltage = ± 15 V

Design an electronic thermometer using the circuit diagram shown above. The voltage v_O is used to indicate temperature. Use a thermister with a resistance described by

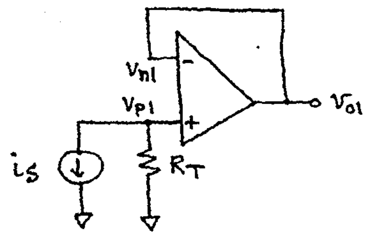
$$R_T = R_o e^{\beta\left(\frac{1}{T}-\frac{1}{300}\right)}$$

where $R_o = 2.8\ \text{k}\Omega$, $\beta = 1300^\circ\text{K}$, and T is temperature in °K.

Pts		
30	a.	Derive a symbolic expression for v_O. The expression must contain not more than the parameters i_S, V_S, and R_T. **Hint: Use superposition.**
10	b.	Calculate the numerical values of R_T (273°K) and R_T (373°K).
15	c.	Determine a value for i_S such that $v_O(T = 373^\circ\text{K}) - v_O(T = 273^\circ\text{K}) = 1\text{V}$
10	d.	Using your answer to (c), find the numerical value of $\mathbf{v}_S$ such that $v_O\ (T = 273°K) = 0\ V$.
10	e.	Using the component values you chose above, calculate v_O when T = 323°K. Make a rough sketch of v_O vs. T on the basis of the values when T = 273°K, 323°K, and 373°K. On the same axes, sketch the ideal linear response.

10

sol'n: 1.a) Superposition case I: i_s on, v_s off = wire

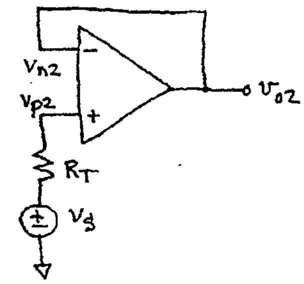


$v_{p1} = -i_s R_T$

$v_{n1} = v_{p1} = -i_s R_T$

$v_{o1} = v_{n1} = -i_s R_T$

case II: i_s off = open, v_s on



$v_{p2} = v_s$ (no current in R_T so no V-drop)

$v_{n2} = v_{p2} = v_s$

$v_{o2} = v_{n2} = v_s$

$$v_o = v_{o1} + v_{o2}$$

$$\boxed{v_o = -i_s R_T + v_s}$$

b) $R_T(273\,°K) = 2.8\,k\Omega\, e^{1300\,°K\left(\frac{1}{273\,°K} - \frac{1}{300\,°K}\right)}$

$\boxed{R_T(273\,°K) = 4.3\,k\Omega}$

$R_T(373\,°K) = 2.8\,k\Omega\, e^{1300\,°K\left(\frac{1}{373\,°K} - \frac{1}{300\,°K}\right)}$

$\boxed{R_T(373\,°K) = 1.2\,k\Omega}$

c) Change in v_o: $\Delta v_o = -i_s \,\Delta R_T$ $\Delta \equiv$ "change"
(for $\Delta T = 100\,°K$) $= 1V$ (desired) (v_s constant does not change with T)

$\Delta R_T = 1.2k\Omega - 4.3k\Omega$

$\Delta R_T = -3.1\,k\Omega$

$\therefore\ i_s = \dfrac{-\Delta v_o}{\Delta R_T} = \dfrac{-1V}{-3.1k\Omega} = 0.323\ mA$

$\boxed{i_s = 0.323\,mA}$

d) $v_o(273\,°K) = 0V = \underbrace{-0.323mA}_{=\,-i_s} \cdot \underbrace{4.3k\Omega}_{=\,R_T} + v_s$

$\therefore\ v_s = 0.323\,mA \cdot 4.3\,k\Omega$

$\boxed{v_s = 1.39\,V \text{ or } 1.4V}$